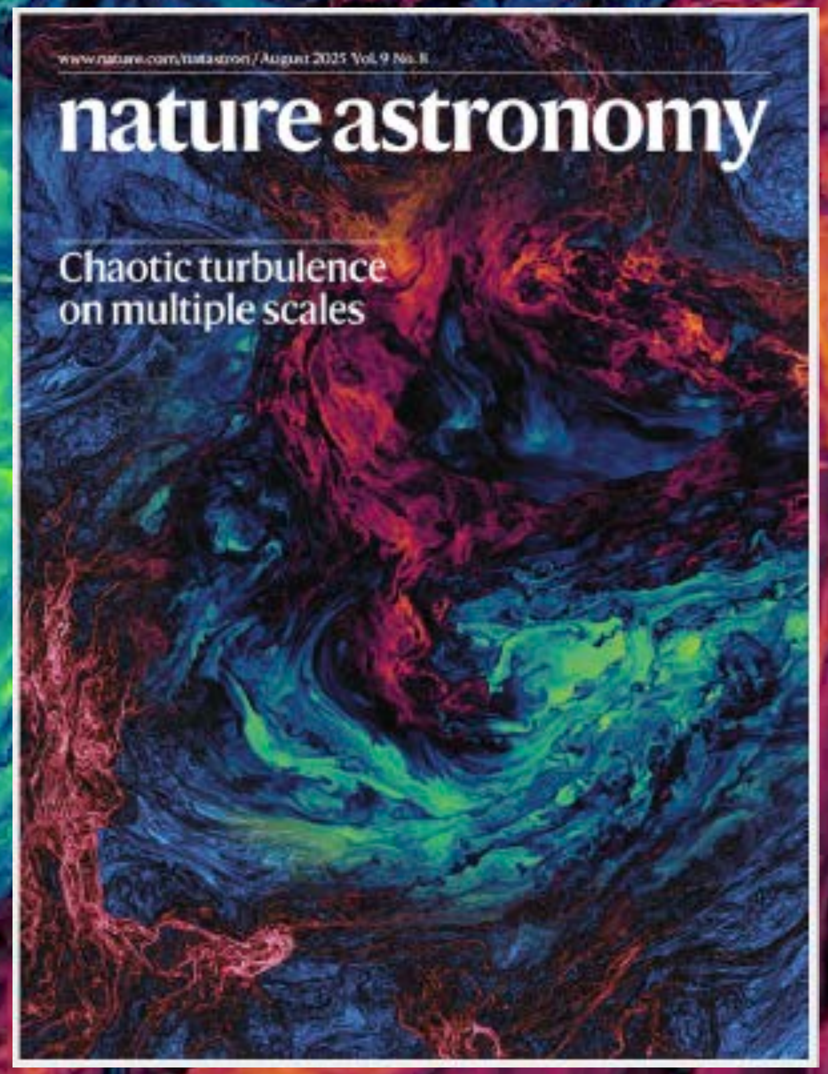

Beattie, *Federrath, Klessen, Cielo, Bhattacharjee*, 2025 (*Nature Astronomy*). The spectrum of magnetized turbulence in the interstellar medium


Izzy Connor
(undergrad. UCSC)

So long Kolmogorov: a supernovae-driven turbulence phenomenology

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MIST 2025, Cargese
James Beattie
Research Associate / Fellow
Princeton / Canadian Institute for Theoretical Astrophysics

In collaboration: Isabelle Connor (UCSC), Anne Noer Kolborg (UCSC), Enrico Ramiez Ruiz (UCSC), Amitava Bhattacharjee (Princeton), Christoph Federrath (ANU),

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Beattie + (2025; ApJ) So long Kolmogorov: the forward and backwards cascades in a supernovae-driven, multiphase ISM
Connor, **Beattie** + (2025, submitted ApJ) Cascading from the winds to the disk: universality of supernovae-driven turbulence in different galactic ISMs
Beattie (2025; submitted ApJL) Supernovae drive large-scale incompressible turbulence from small-scale instabilities

Fact: the galaxy that we reside in is in a state of highly-compressible fluid turbulence

$$E_{\rm SNe} \approx 10^{51}\,\rm erg$$

$$\gamma_{\rm SNe} \approx 0.01 - 0.03\,\rm yr^{-1}$$ Diehl + (2016)

$$\dot{E}_{\rm SNe} \approx E_{\rm SNe}\gamma_{\rm SNe} \approx 10^{41}\rm erg\,s^{-1}$$

$$E_{\rm turb} \approx 10^{54}\,\rm erg$$

Ferriere + (2020

$$t_{\rm turb} \approx \ell_0/\langle u^2 \rangle^{1/2} \approx 10^7\,\rm yr$$

$$\dot{E}_{\rm turb} \approx E_{\rm turb}/t_{\rm turb} \approx 10^{39}\rm erg\,s^{-1}$$

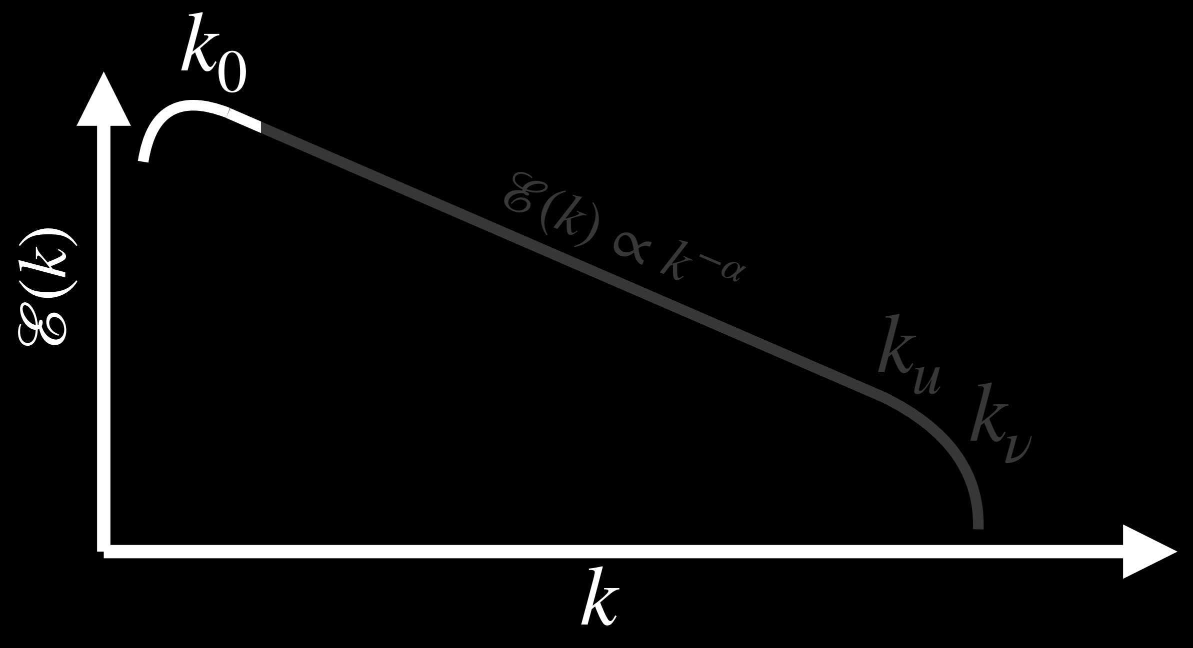
$$\dot{E}_{\rm turb}/\dot{E}_{\rm SNe} \approx 10^{-2}$$

CC-supernovae alone provide enough energy flux to fuel a continuously driven cascade

Connor, **Beattie** + (2025, submitted ApJ) Cascading from the winds to the disk: universality of supernovae-driven turbulence in different galactic ISMs

Simulation from: Goldbaum, N. J., Krumholz, M. R., & Forbes, J. C. 2016

Turbulence
Cascade



WIM: $\mathrm{Re} \sim 10^7$ $\lambda_{\mathrm{mfp}} \sim 5\ \mathrm{R}_\oplus$

WNM: $\mathrm{Re} \sim 10^7$ $\lambda_{\mathrm{mfp}} \sim 1.3\ \mathrm{AU}$

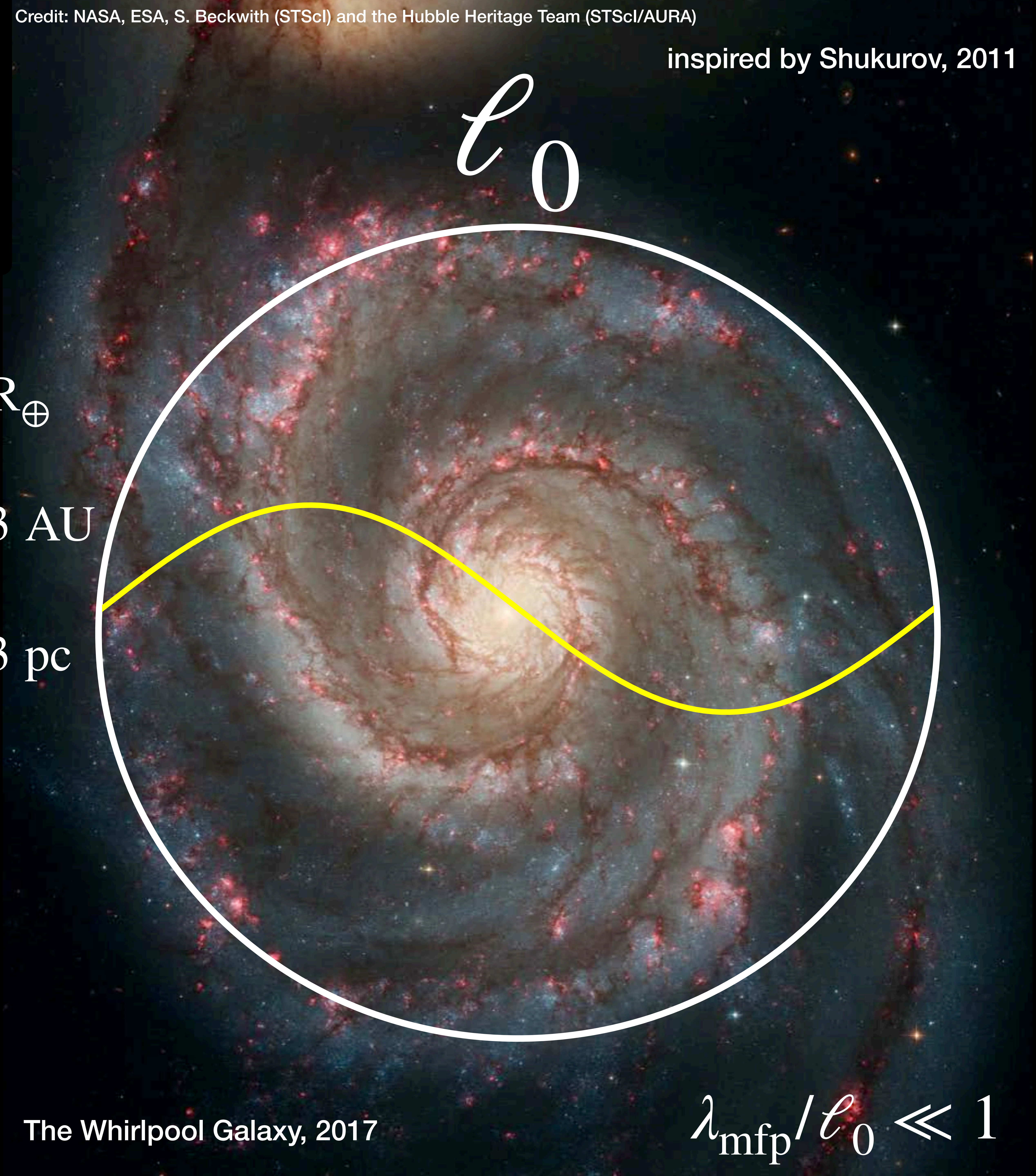
CNM: $\mathrm{Re} \sim 10^{10}$ $\lambda_{\mathrm{mfp}} \sim 0.3\ \mathrm{pc}$

Ferrière, 2020; Plasma Physics and Controlled Fusion

The quadratic nonlinear term

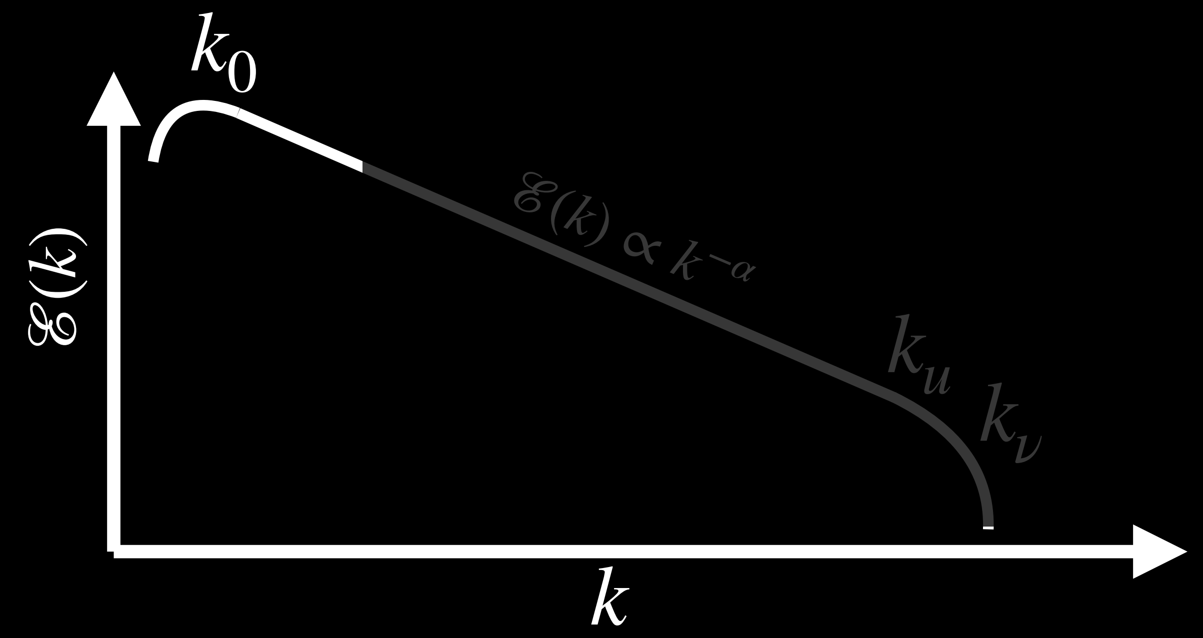
$$|\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})|$$

dominates on large scales, ℓ



The Whirlpool Galaxy, 2017

Turbulence
Cascade



The quadratic nonlinear term

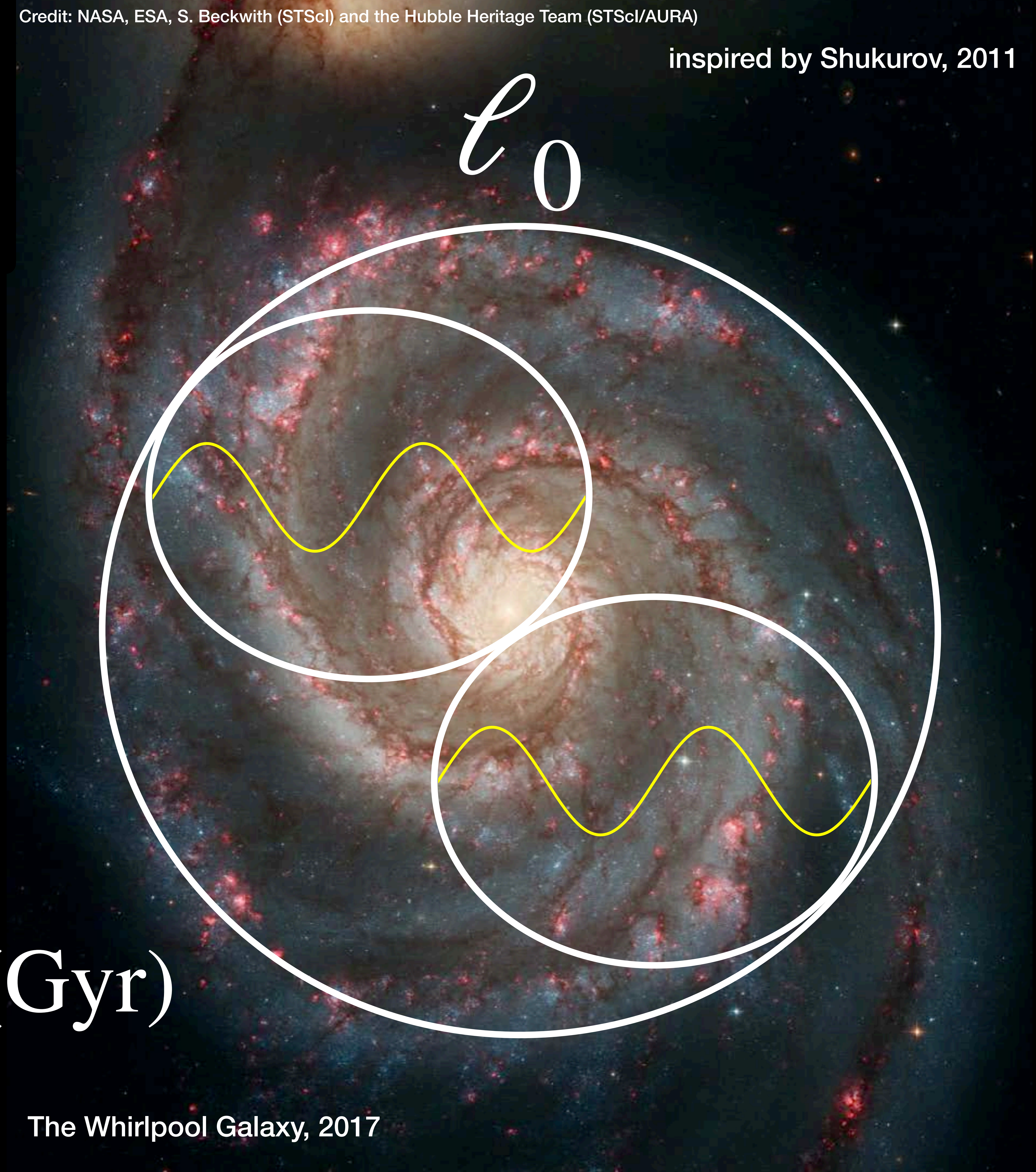
$$|\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})|$$

dominates on large scales, ℓ

Creates new modes on

$$t_{\mathrm{nl}} \sim [\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})]^{-1} \sim \mathcal{O}(\mathrm{Gyr})$$

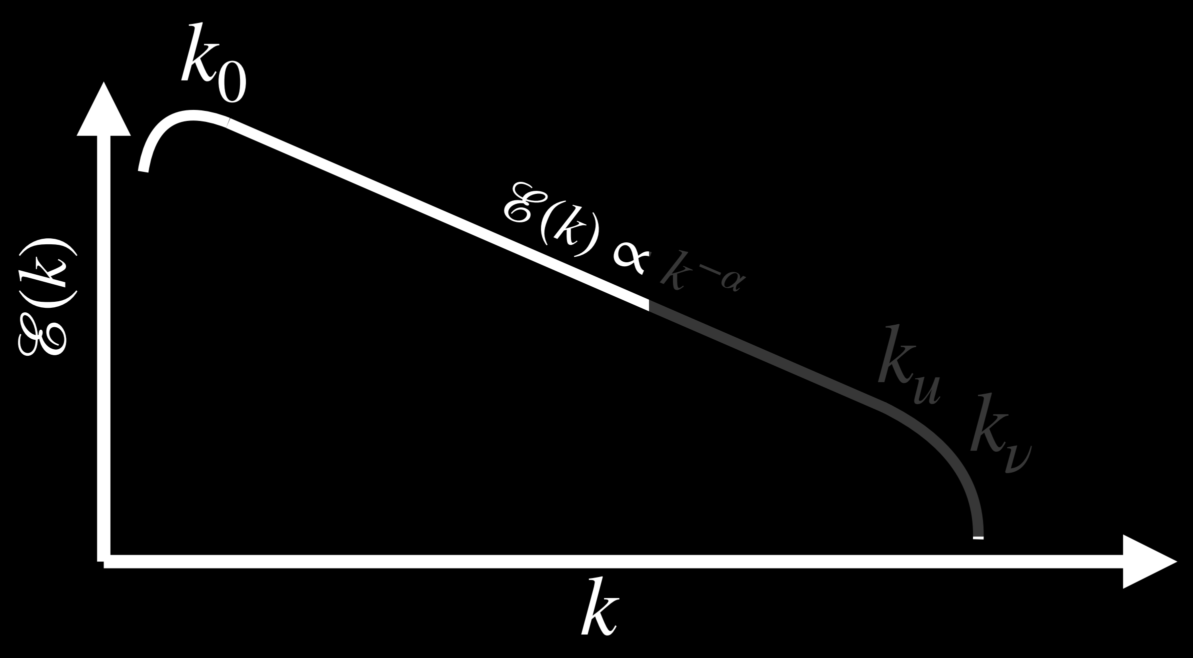
$$\dot{\varepsilon} \sim u_0^3/\ell_0$$



The Whirlpool Galaxy, 2017

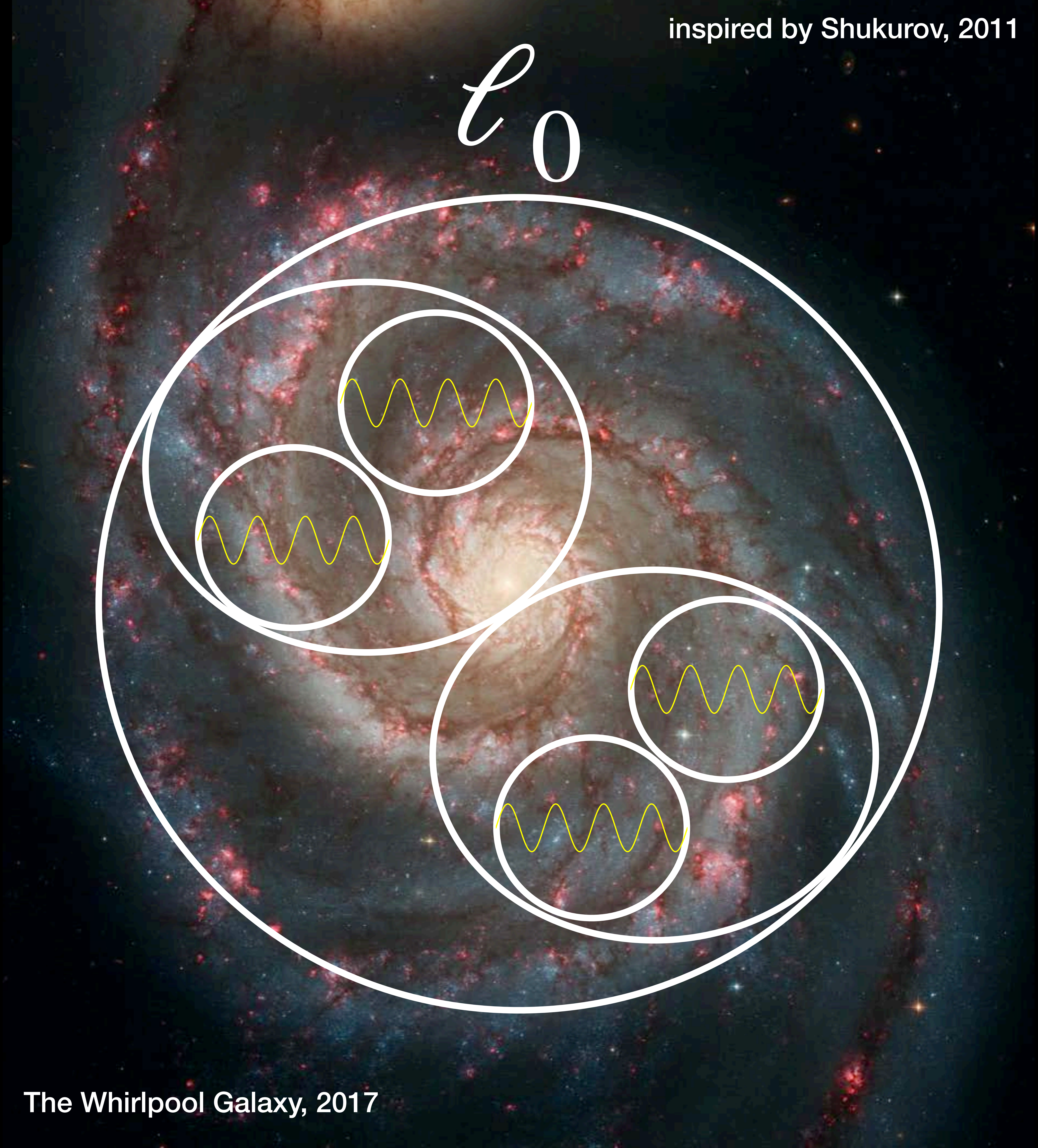
Turbulence

Cascade



inside of the cascade

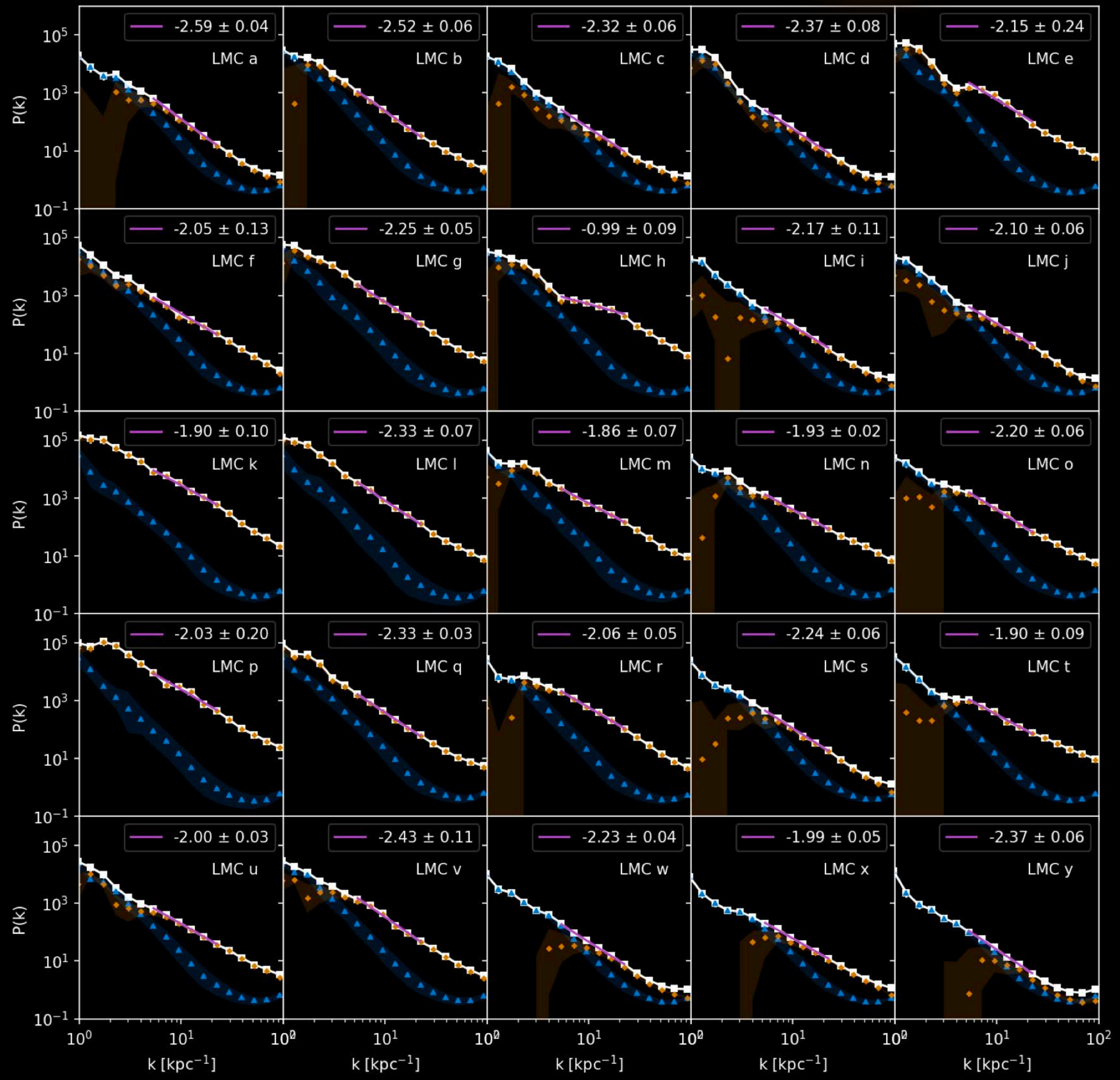
Credit: NASA, ESA, S. Beckwith (STScI) and the Hubble Heritage Team (STScI/AURA)

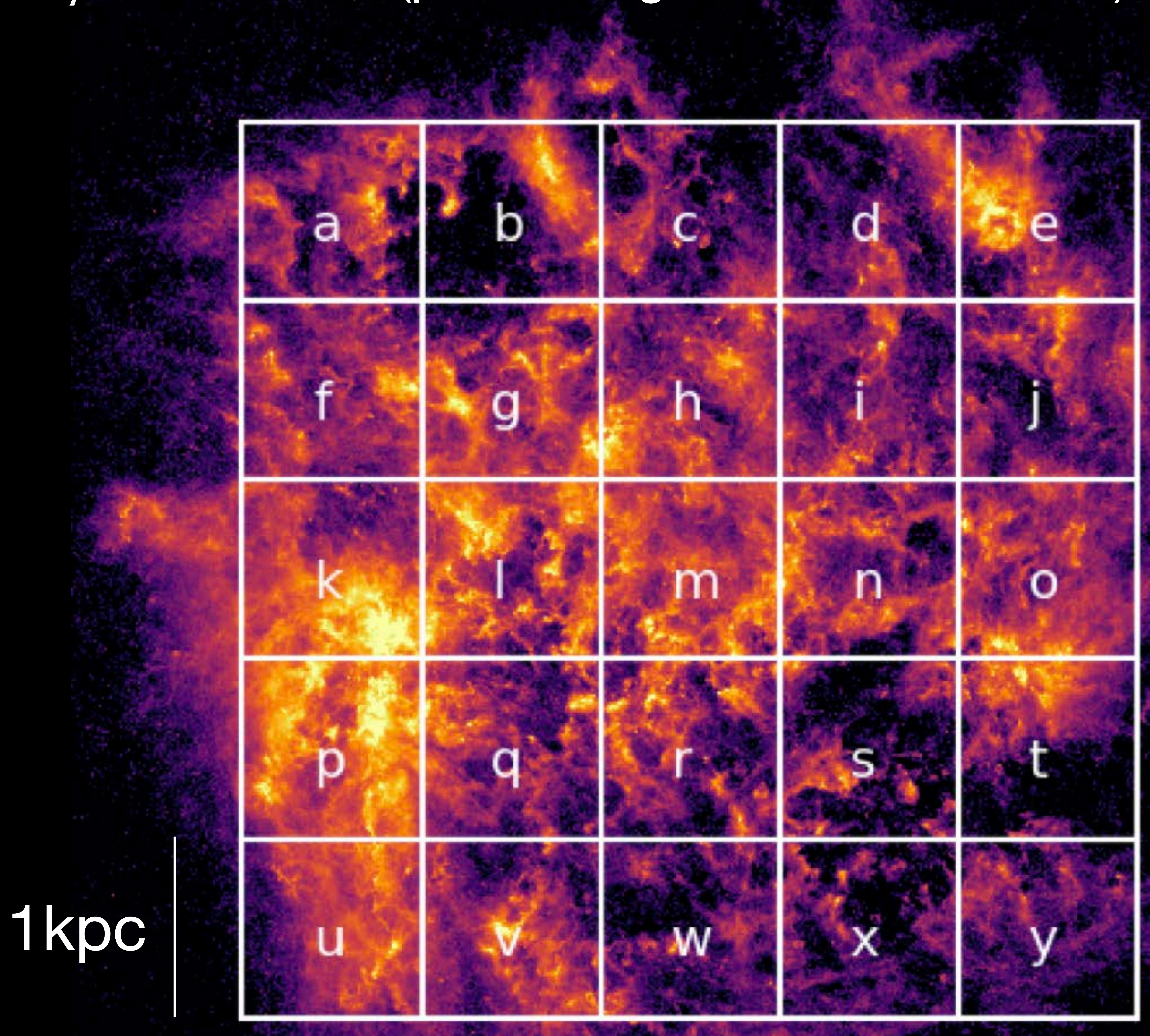


The Whirlpool Galaxy, 2017

Turbulence

LMC: 500μm, *Herschel* (processing Gordon et al. 2014)

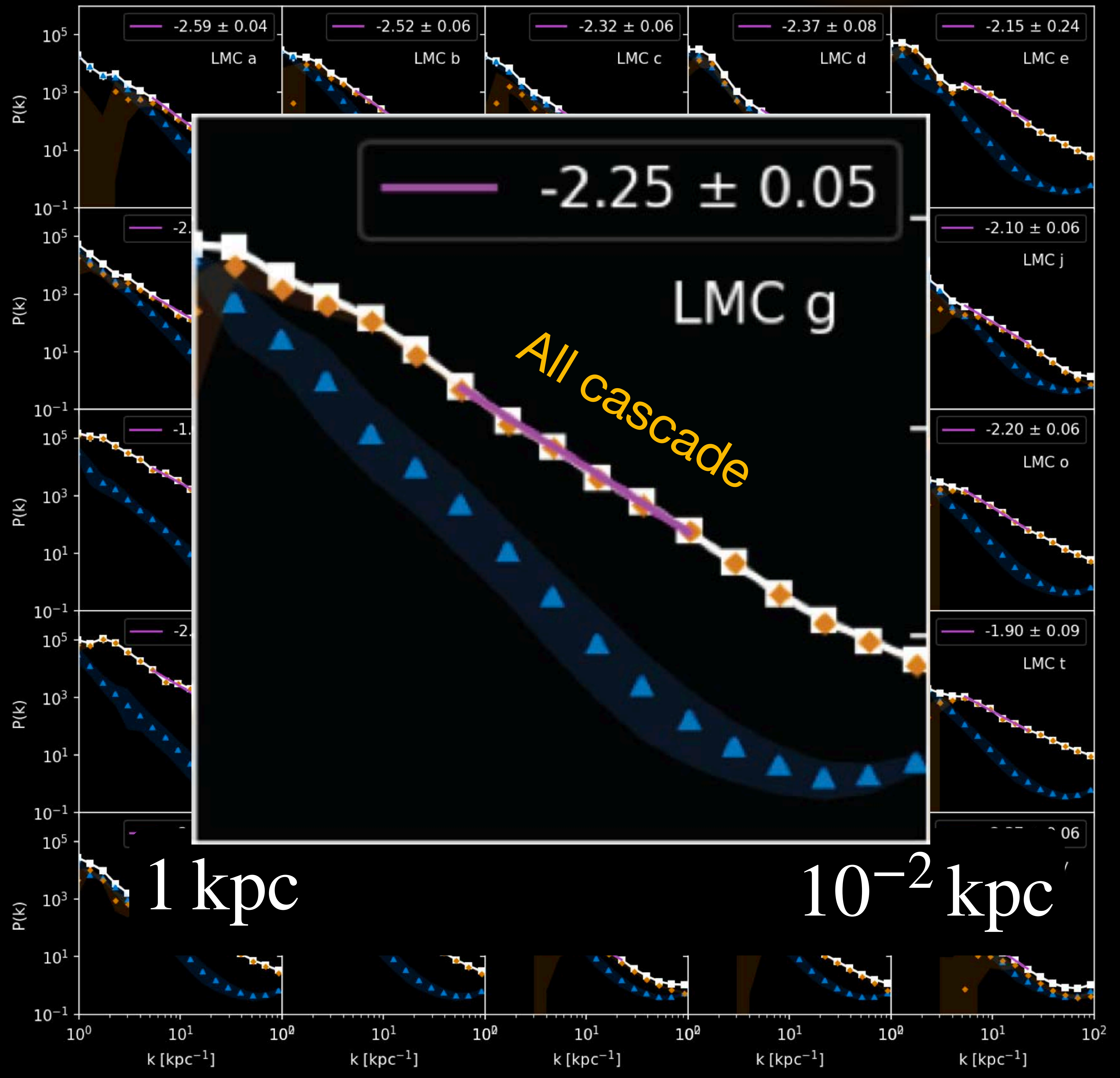


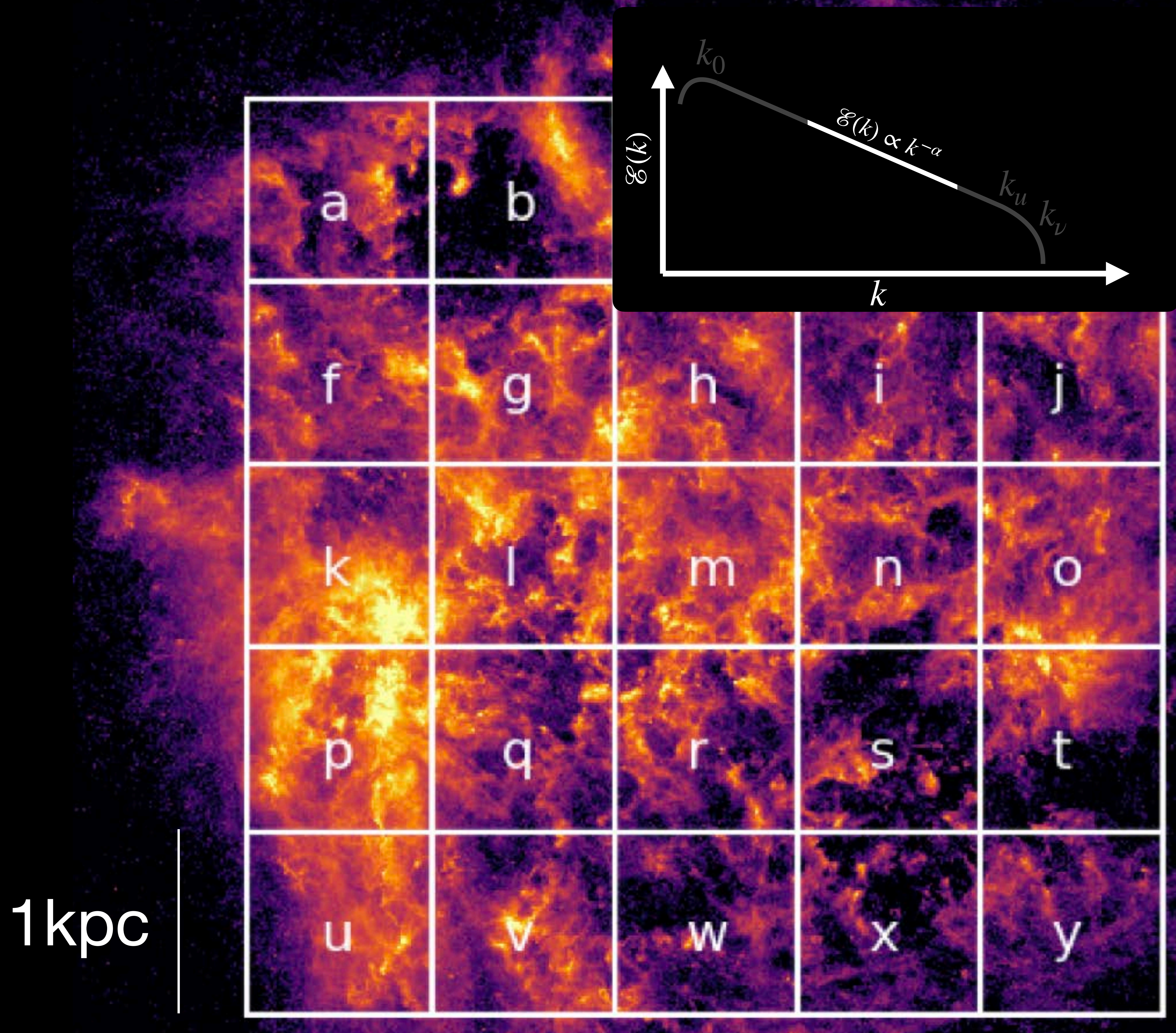


Colman, et al., 2022, MNRAS

Turbulence

LMC: 500μm, *Herschel* (processing Gordon et al. 2014)

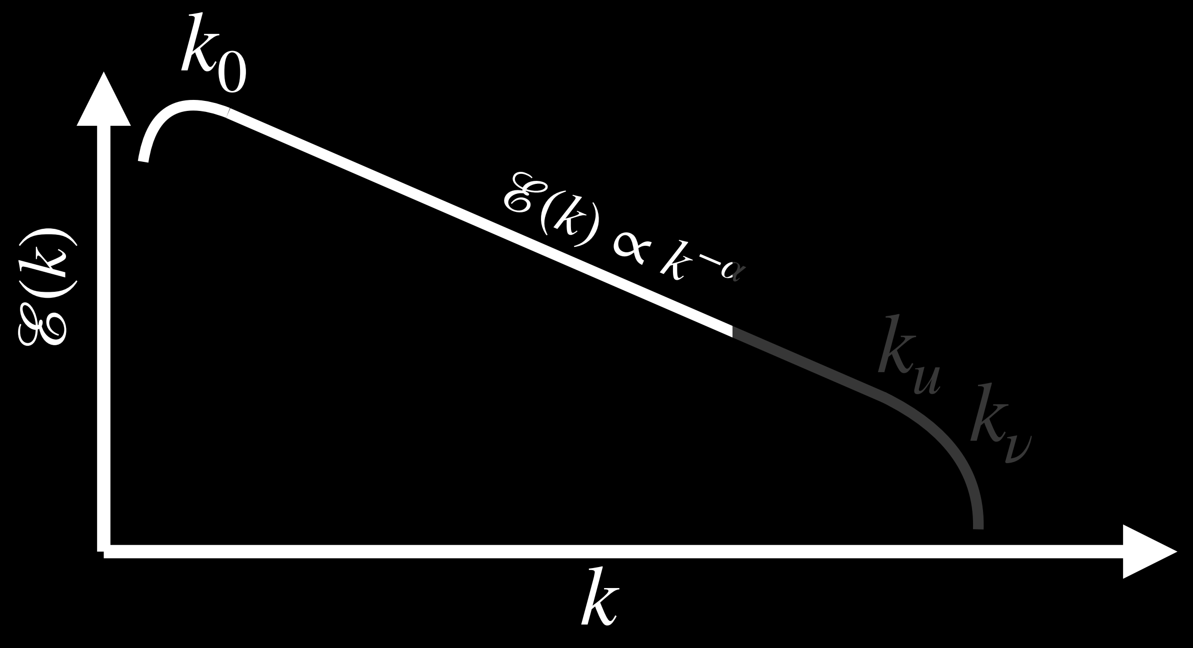




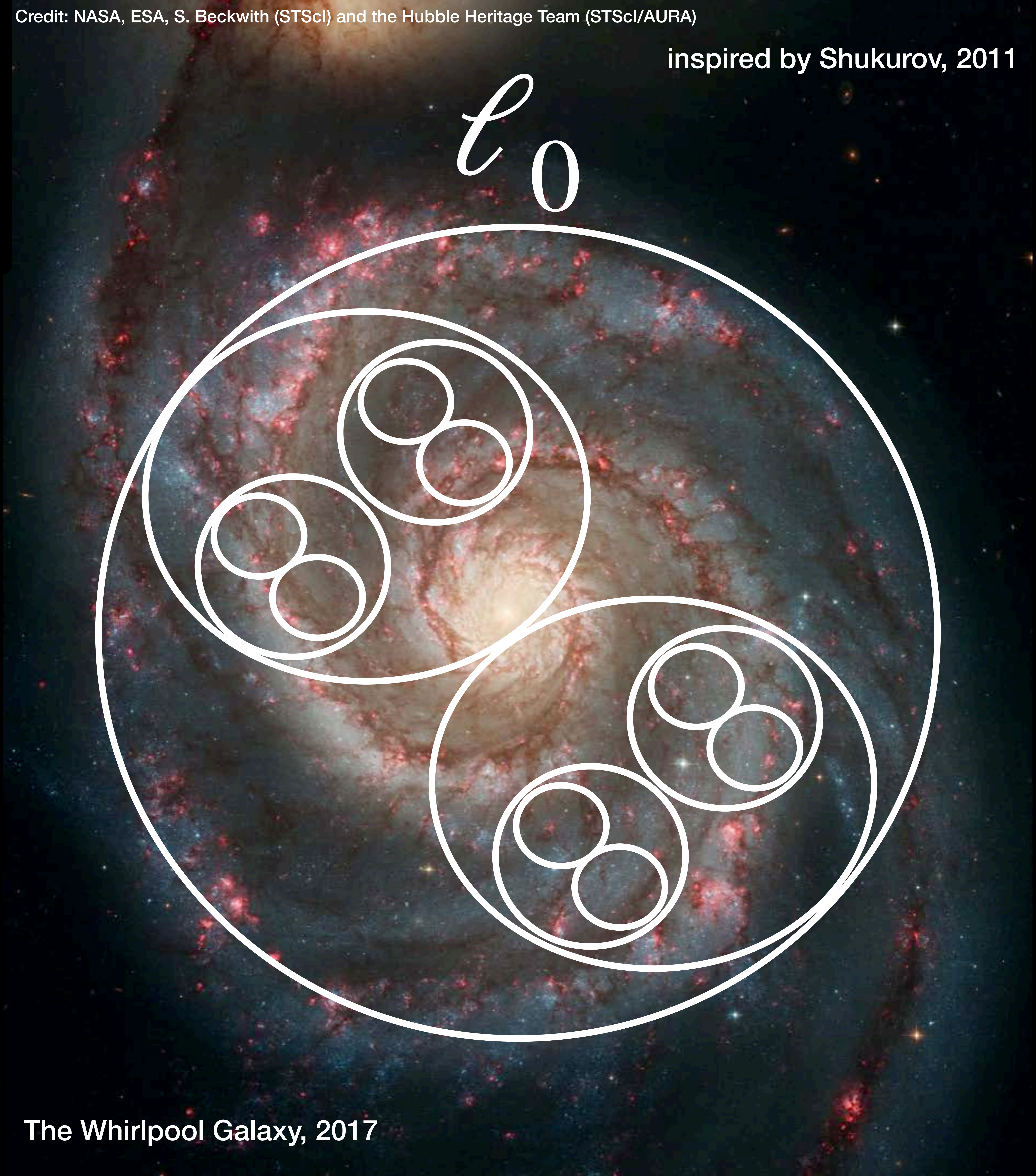
Colman, et al., 2022, MNRAS

Turbulence

Cascade



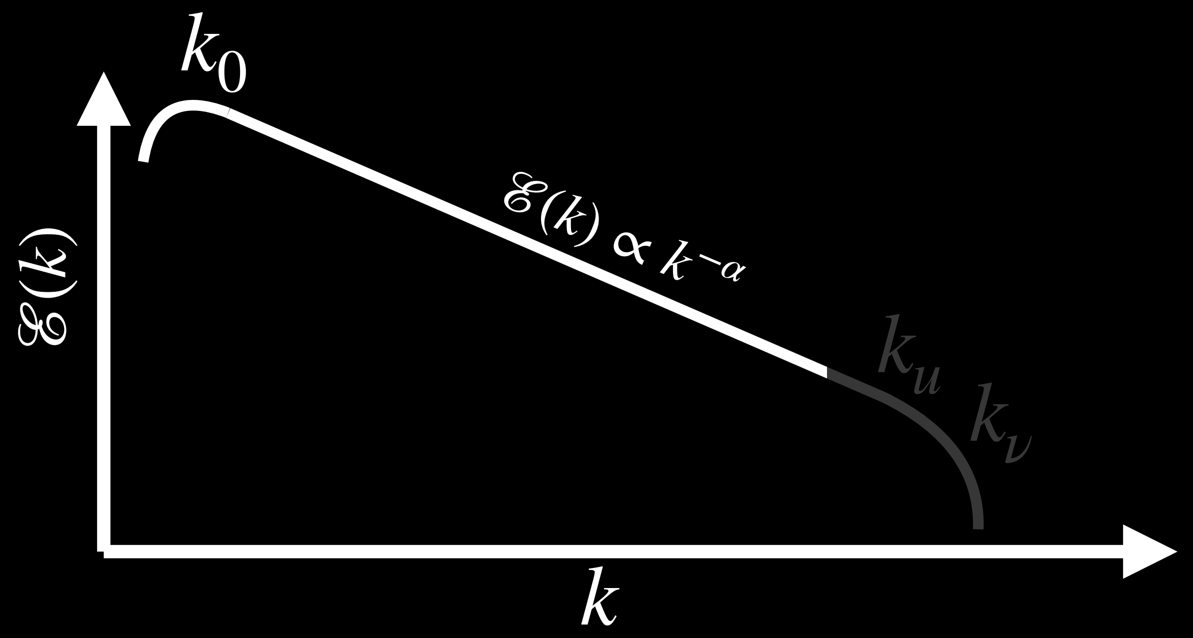
deeper into
the cascade



The Whirlpool Galaxy, 2017

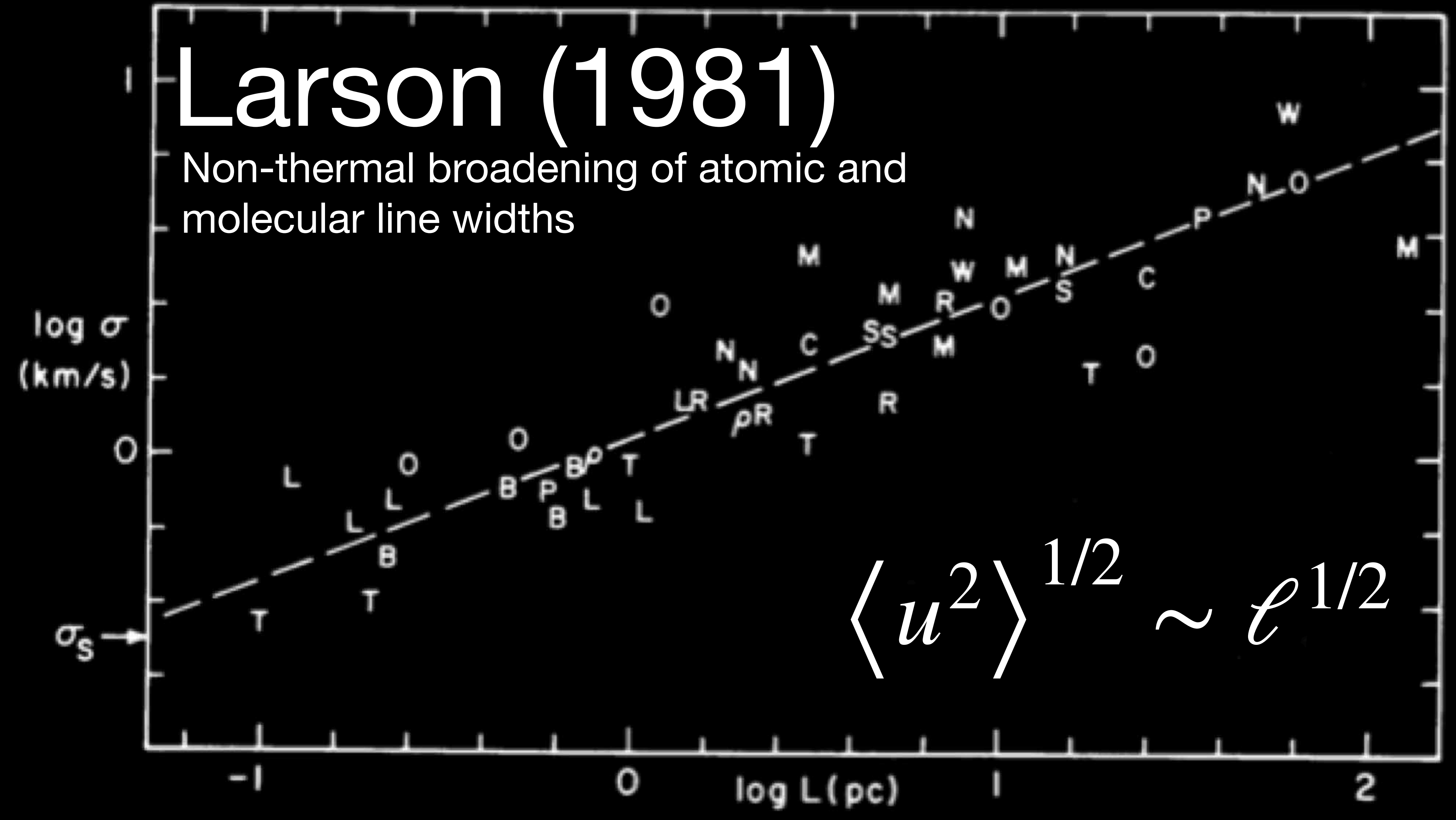
Turbulence

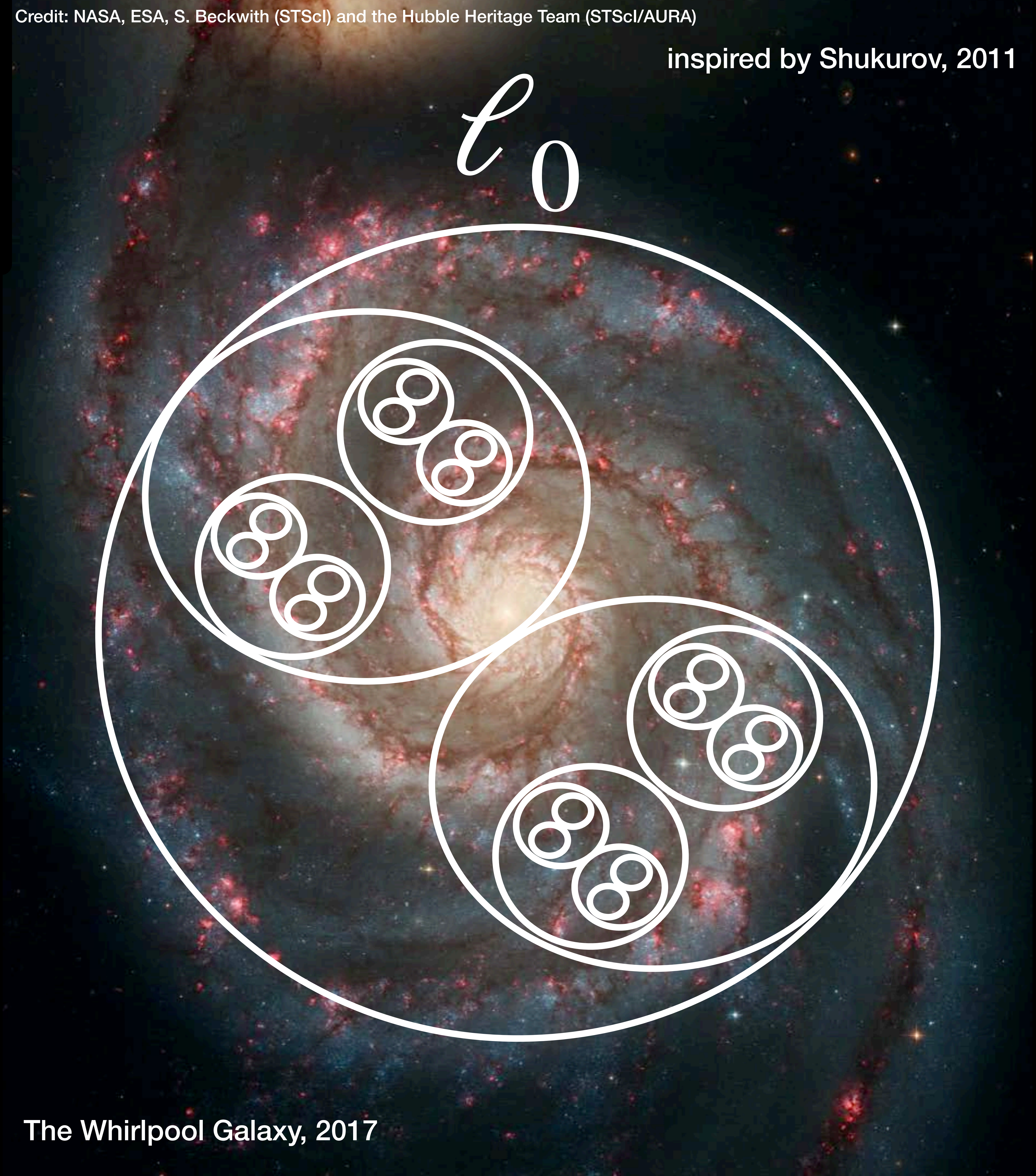
Cascade



Reynolds number is shrinking

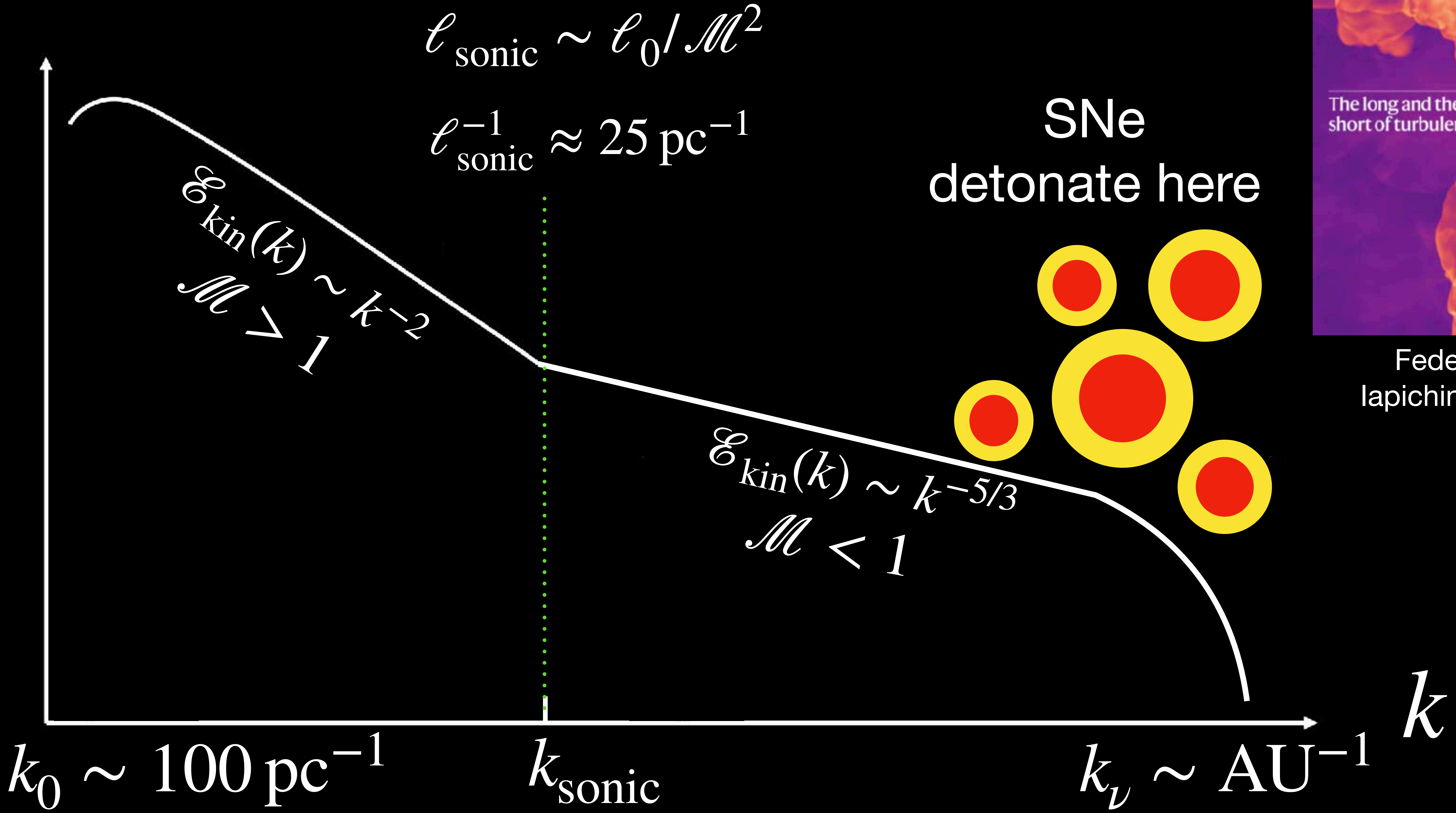
$$\mathrm{Re} \sim \langle u^2 \rangle^{1/2} \ell / \nu \sim \ell^{3/2} / \nu$$





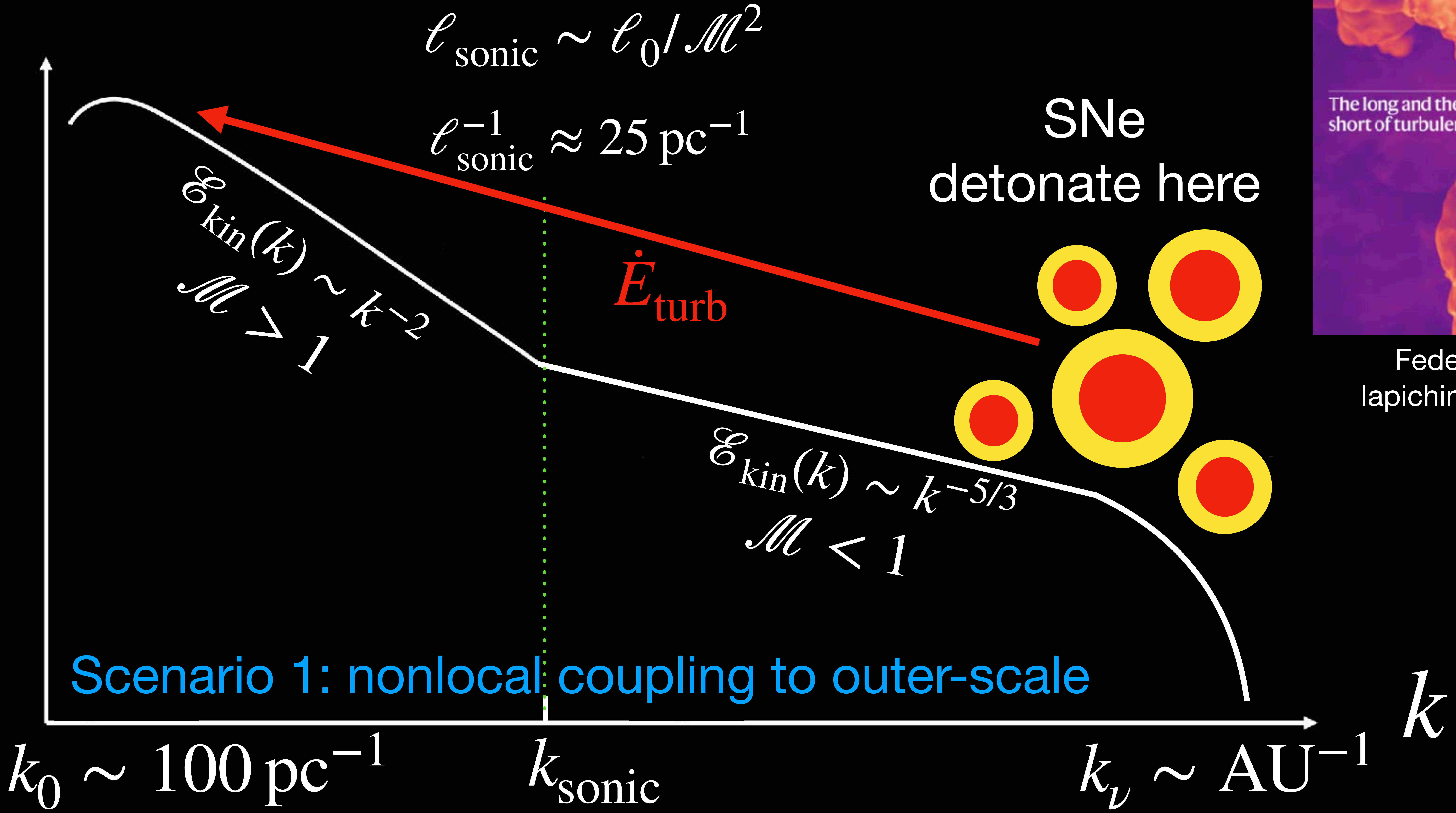

100μm+70μm
orange blue
W3/W4/W5 complex
Copyright: ESA/Herschel/NASA/JPL-Caltech, CC BY-SA 3.0 IGO; Acknowledgement: R. Hurt (JPL-Caltech)

The naive ($\mathscr{M} \approx 2$) warm ionized medium spectrum (the volume-filling spectrum)



Federrath, Klessen, Iapichino, **Beattie** (2021)

The naive ($\mathscr{M} \approx 2$) warm ionized medium spectrum (the volume-filling spectrum)



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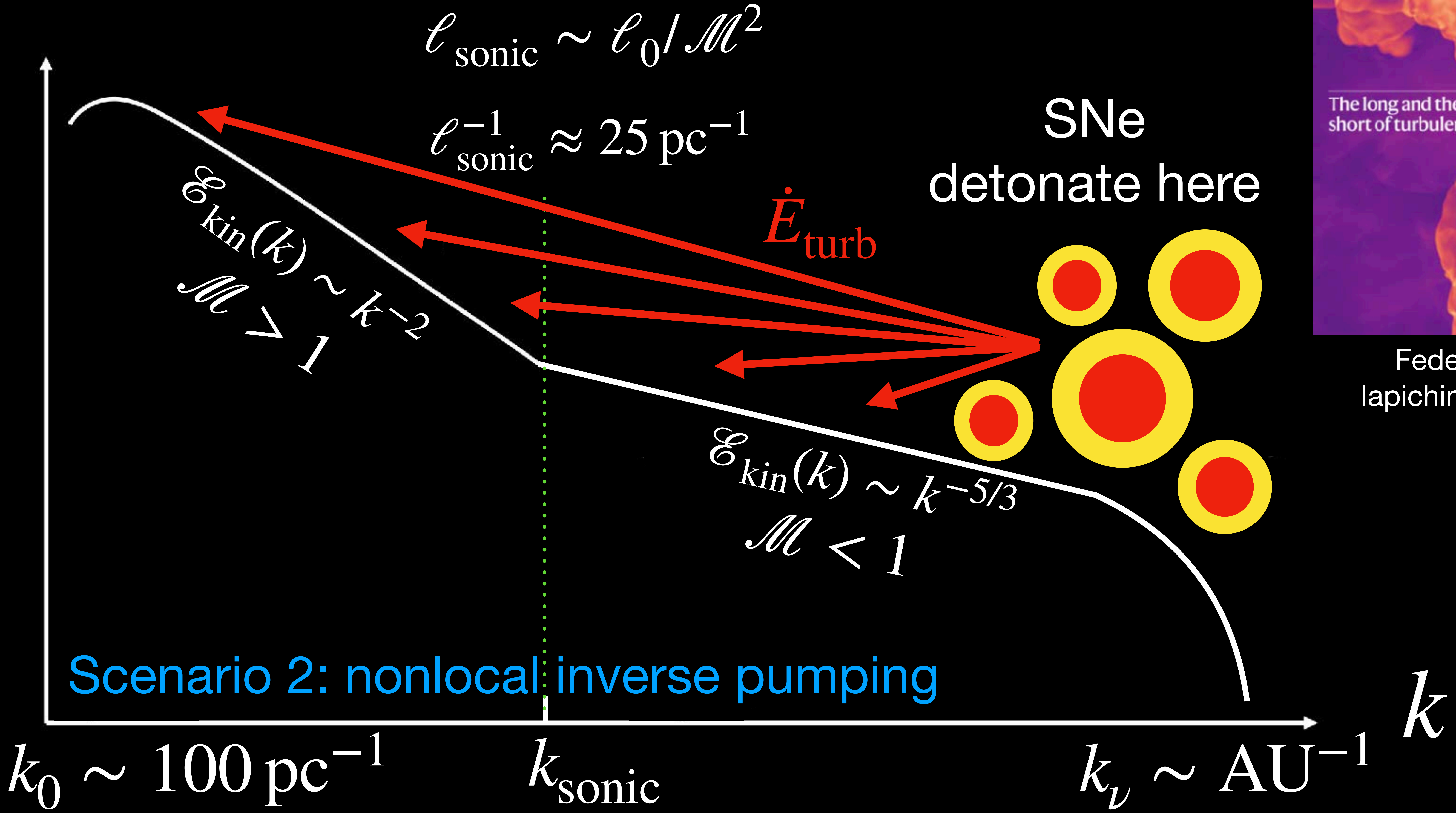

Federrath, Klessen, Iapichino, **Beattie** (2021)

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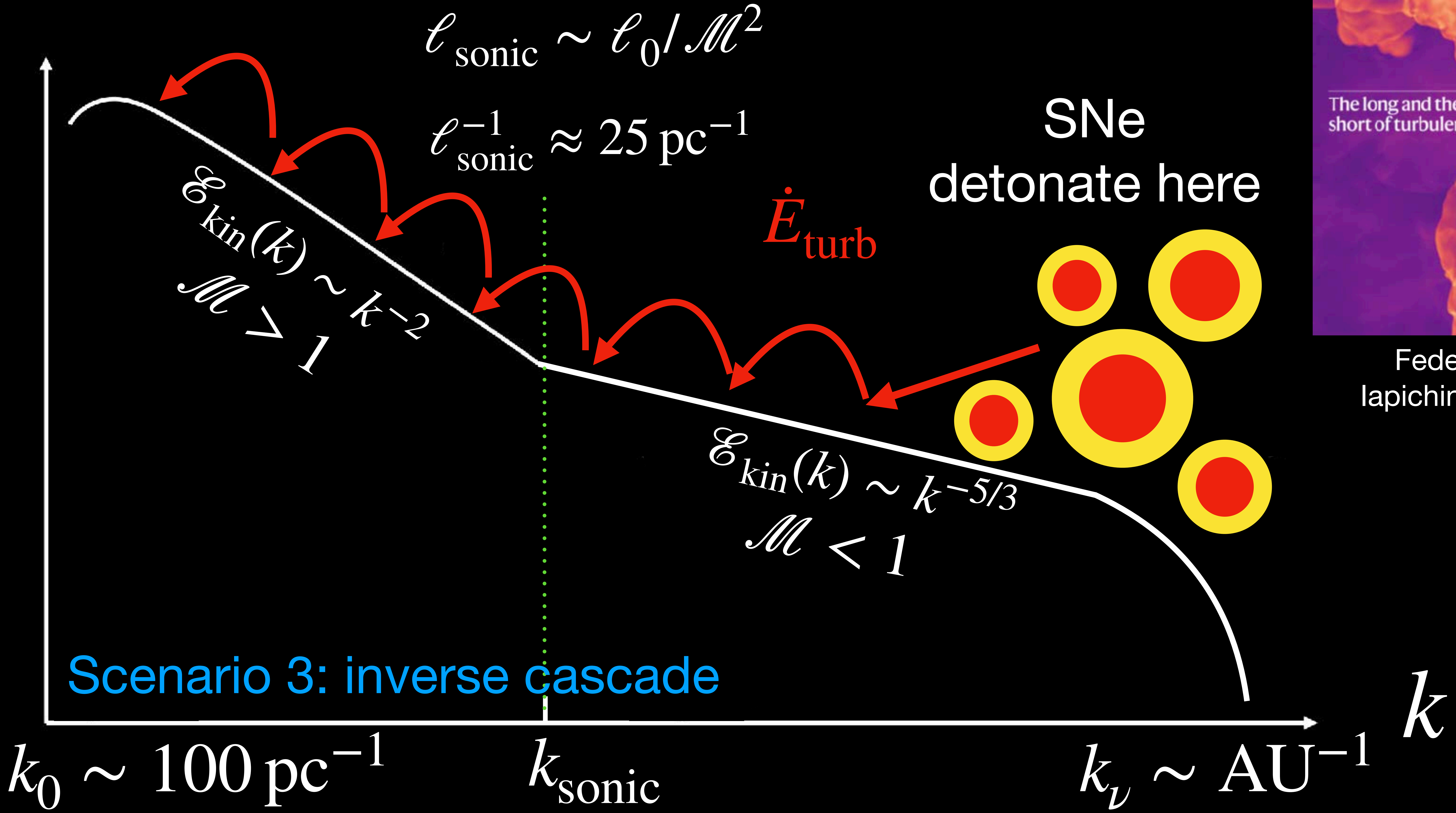
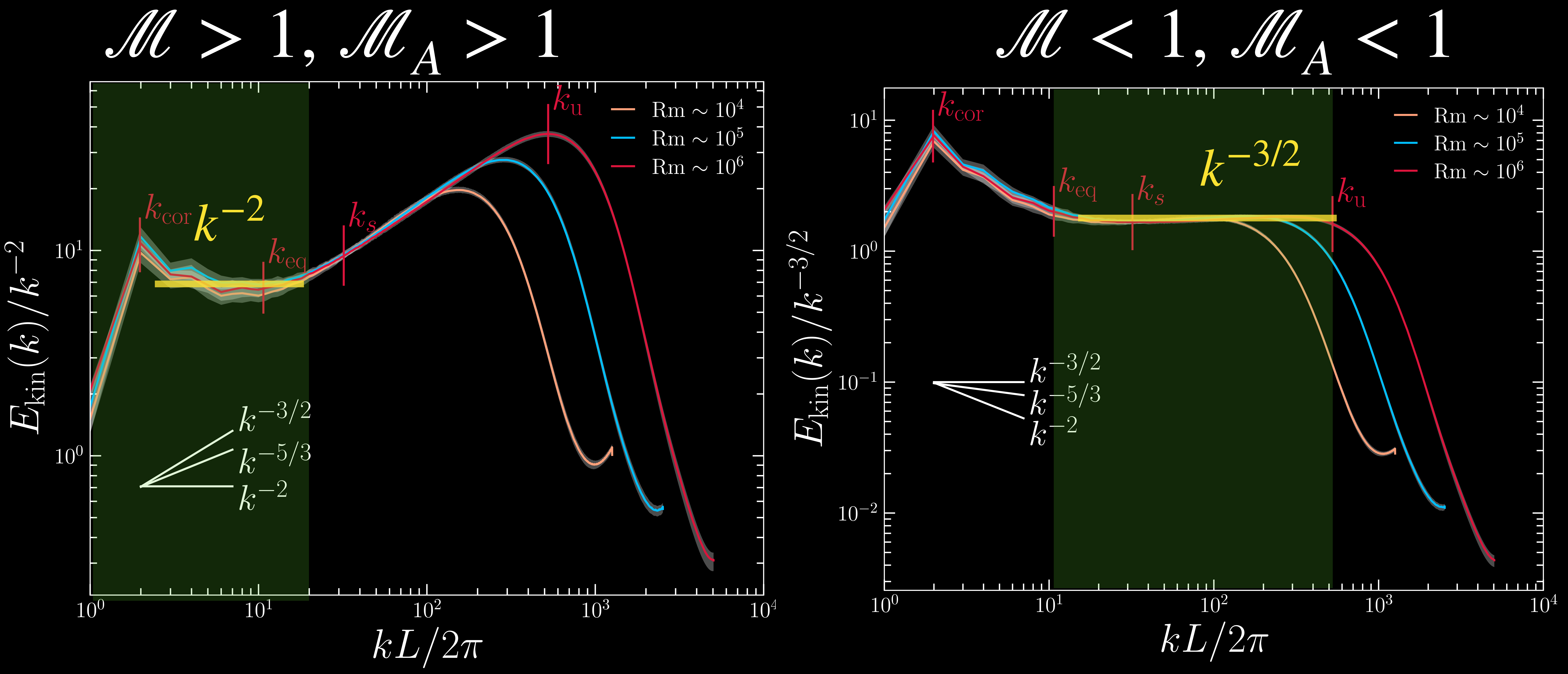


Federrath, Klessen, Iapichino, **Beattie** (2021)

Supersonic MHD turbulence admits to two power-laws

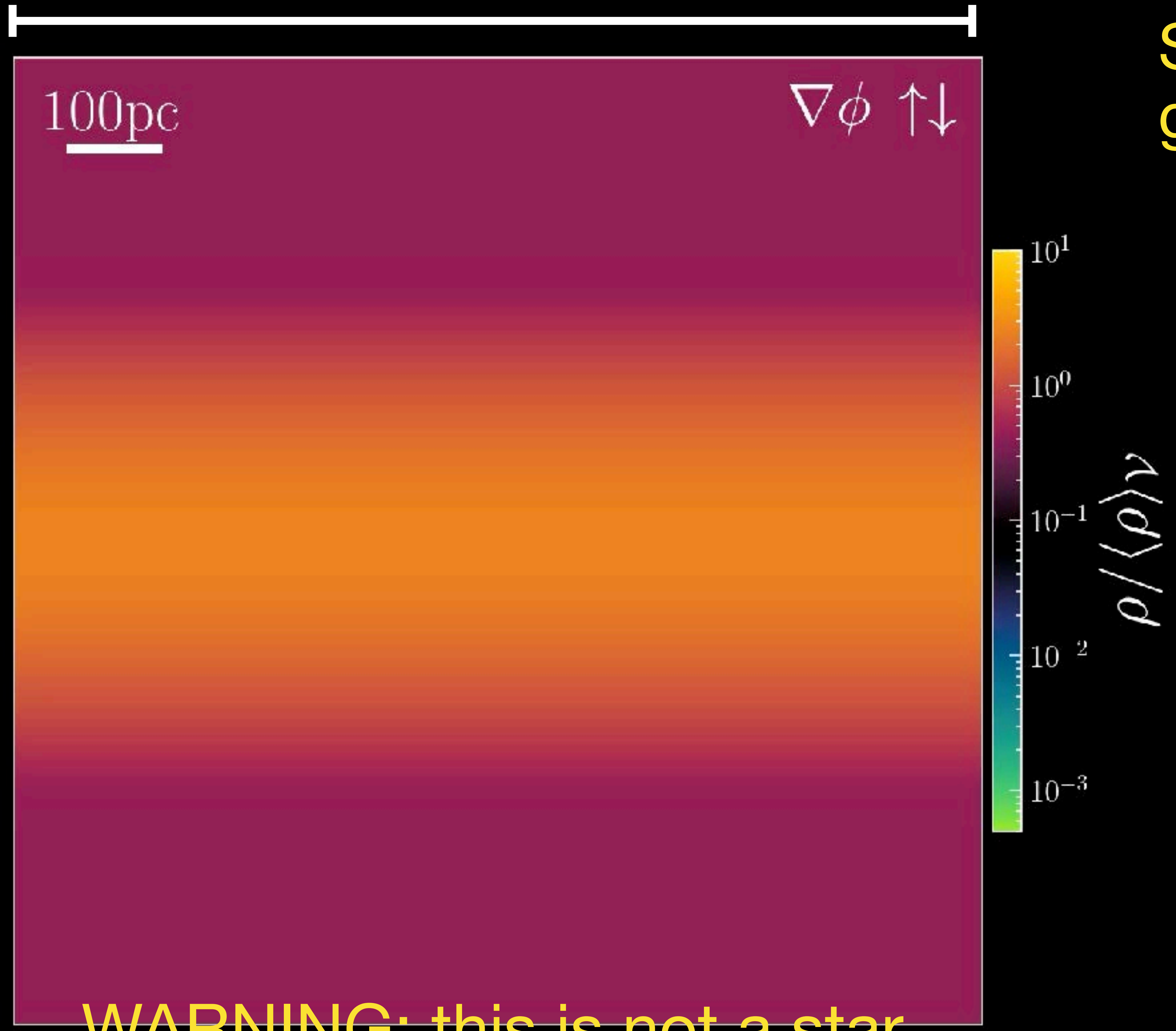


Beattie, *Federrath, Klessen, Cielo, Bhattacharjee,* 2025 (*Nature Astronomy*). The spectrum of magnetized turbulence in the interstellar medium

The simplest SNe-driven simulations possible

Martizzi+2016
RAMSES (Teyssier 2002)

$L = 1$ kpc



WARNING: this is not a star formation simulation

Supernova driven gravito-hydro dynamical model

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = \dot{n}_{\rm SNe} M_{\rm ej}, \tag{1}$$

$$\frac{\partial \rho \boldsymbol{u}}{\partial t} + \nabla \cdot (\rho \boldsymbol{u} \otimes \boldsymbol{u} + P\mathbb{I}) = -\rho \nabla \phi + \dot{n}_{\rm SNe} \mathbf{p}_{\rm SNe}(Z, n_{\rm H}), \tag{2}$$

$$\frac{\partial \rho e}{\partial t} + \nabla \cdot [\rho (e + P) \boldsymbol{u}] = -n_{\rm H}^2 \Lambda - \tag{3}$$

$$\rho \boldsymbol{u} \cdot \nabla \phi + \dot{n}_{\rm SNe} \left[E_{\rm th,SNe}(Z, n_{\rm H}) + \frac{\mathbf{p}_{\rm SNe}(Z, n_{\rm H})^2}{2(M_{\rm ej} + M_{\rm swept})} \right], \tag{4}$$

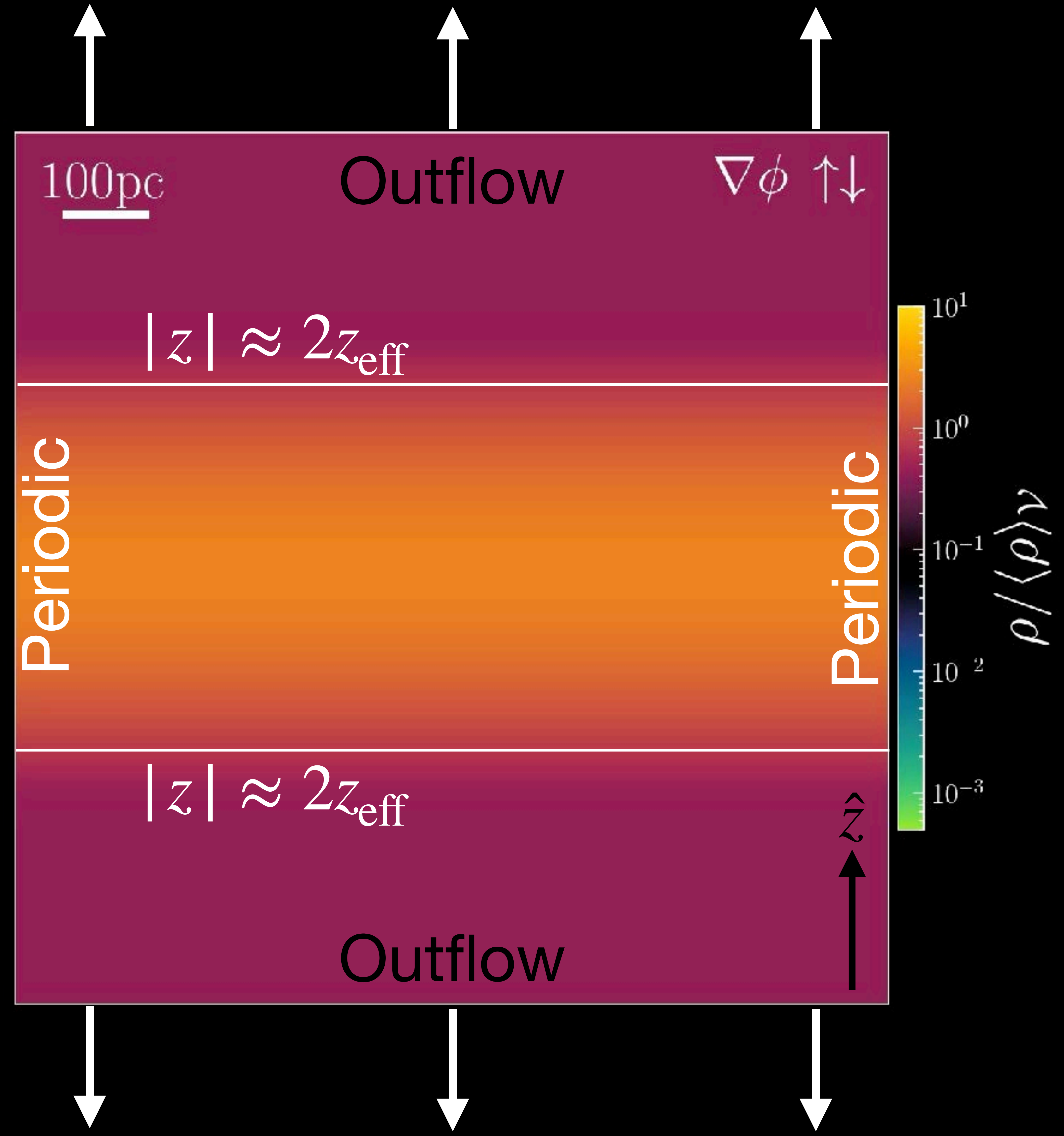
$$e = \epsilon + \frac{u^2}{2}, \quad P = \frac{2}{3} \rho e, \tag{5}$$

$$N_{\rm grid}^3 = 1024^3 \implies \Delta x \sim 1 \text{ pc}$$

$$\text{numerical diss.} \implies \Delta x \sim 10 \text{ pc}$$

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

The simplest SNe-driven simulations possible



Static gravitational potential

$$\phi(z) = 2\pi G \Sigma_* \underbrace{\left(\sqrt{z^2 - z_0^2} - z_0\right)}_{\text{stratified disk}} + \underbrace{\frac{2\pi G \rho_{\rm halo}}{3} z^2}_{\text{spherical halo}}$$

Supernova driving prescription

(following 1D evolution models for momentum and energy deposition in Martizzi + 2016)

$$\dot{n}_{\rm SNe} = \frac{\dot{\Sigma}_*}{2z_{\rm eff} 100 M_\odot}$$

1 SNe per $100 M_\odot$ of SF

$\dot{\Sigma}_* \propto \Sigma^{1.4}$

KS relation

$$p(z) = \begin{cases} \frac{1}{4z_{\rm eff}}, & |z| \leq 2z_{\rm eff} \\ 0, & |z| > 2z_{\rm eff} \end{cases}$$

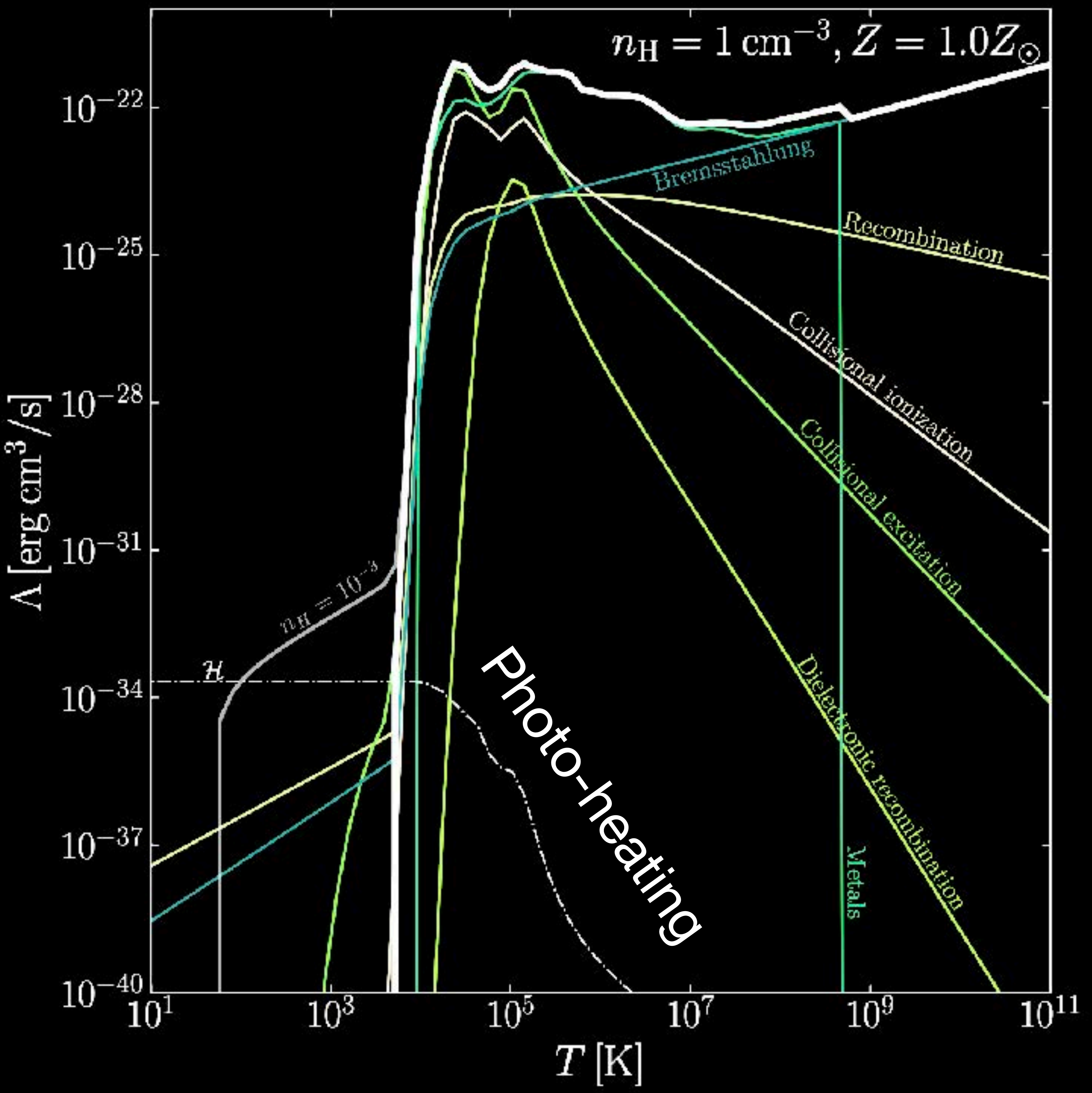
$$p(n\,|\,\lambda) = \lambda^k e^{-n}/n!$$
$$\lambda(x, y, z) = \dot{n}_{\rm SNe} \mathcal{V}_{\rm cell} \Delta t$$

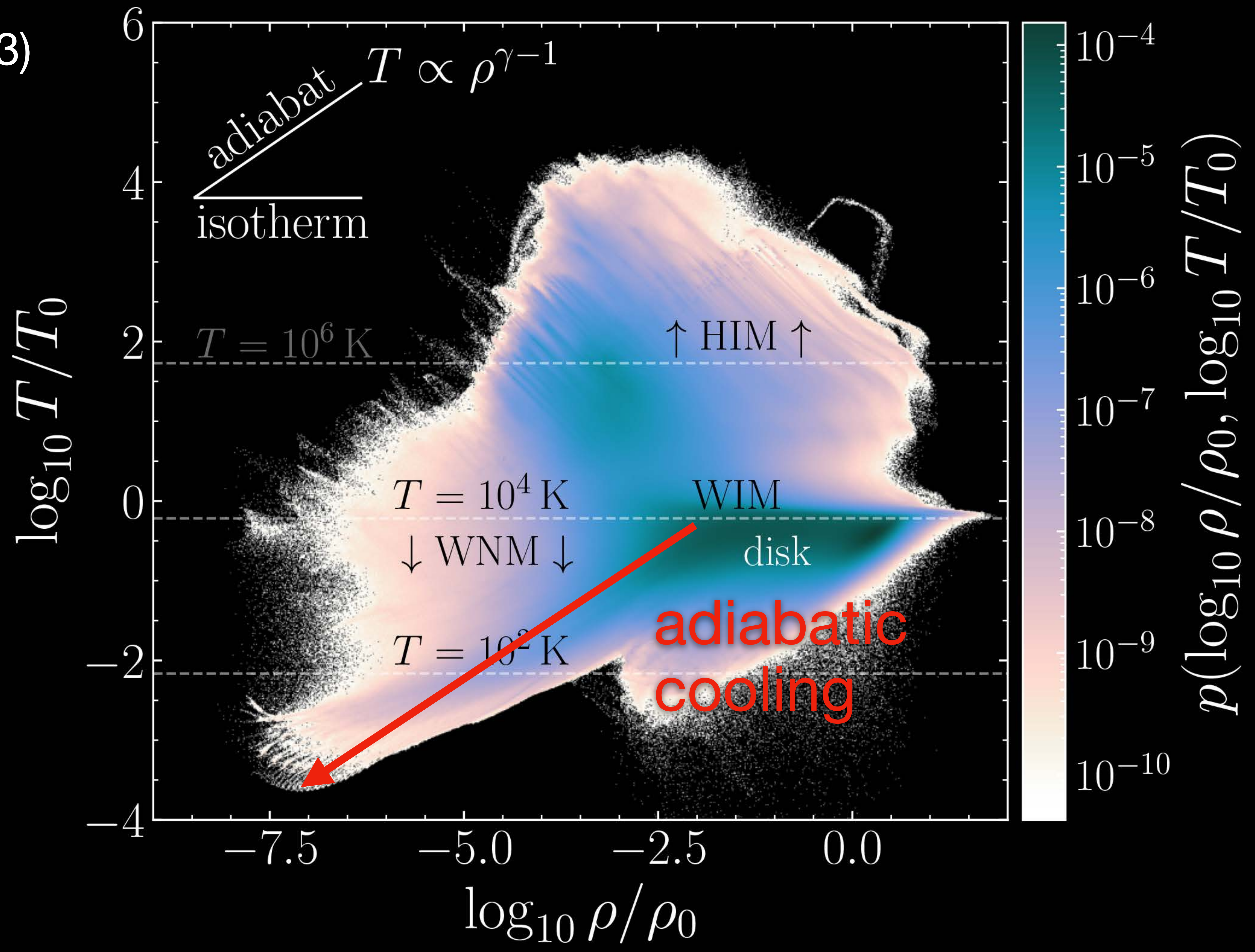
Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

The simplest SNe-driven simulations possible

Cooling function

Theuns+(1998)
Sutherland & Dopita(1993)





HI, HII, HeI, HeII, HeIII and free electrons

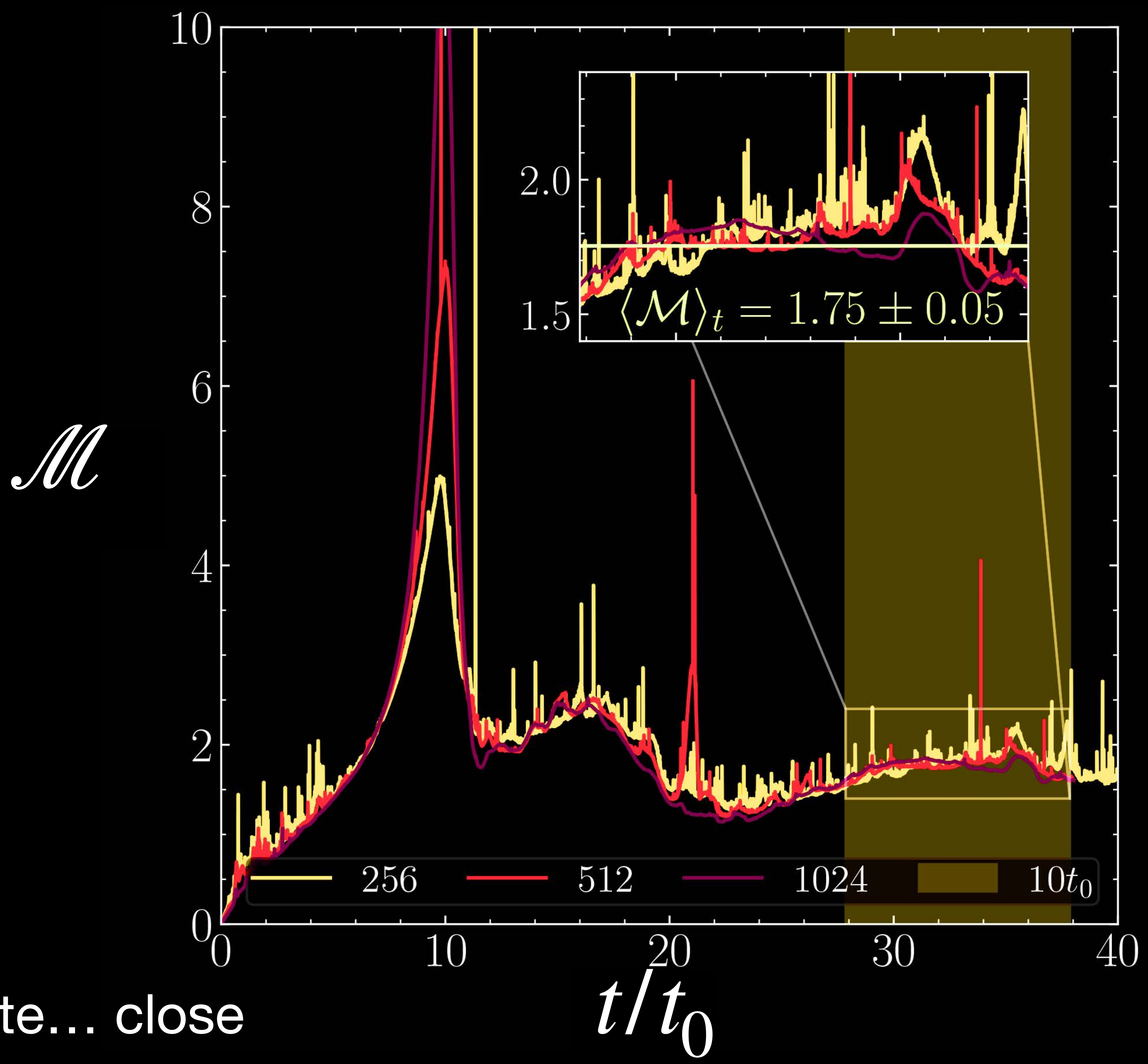
$$\rho_0 = 2.1 \times 10^{-24}\ \mathrm{g\ cm^{-3}}$$
$$P_0 = 2.2 \times 10^{-12}\ \mathrm{Ba}$$

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Stationarity and Mach number

$$\mathcal{M} = \left\langle \left(\frac{u}{c_s} \right)^2 \right\rangle^{1/2}$$

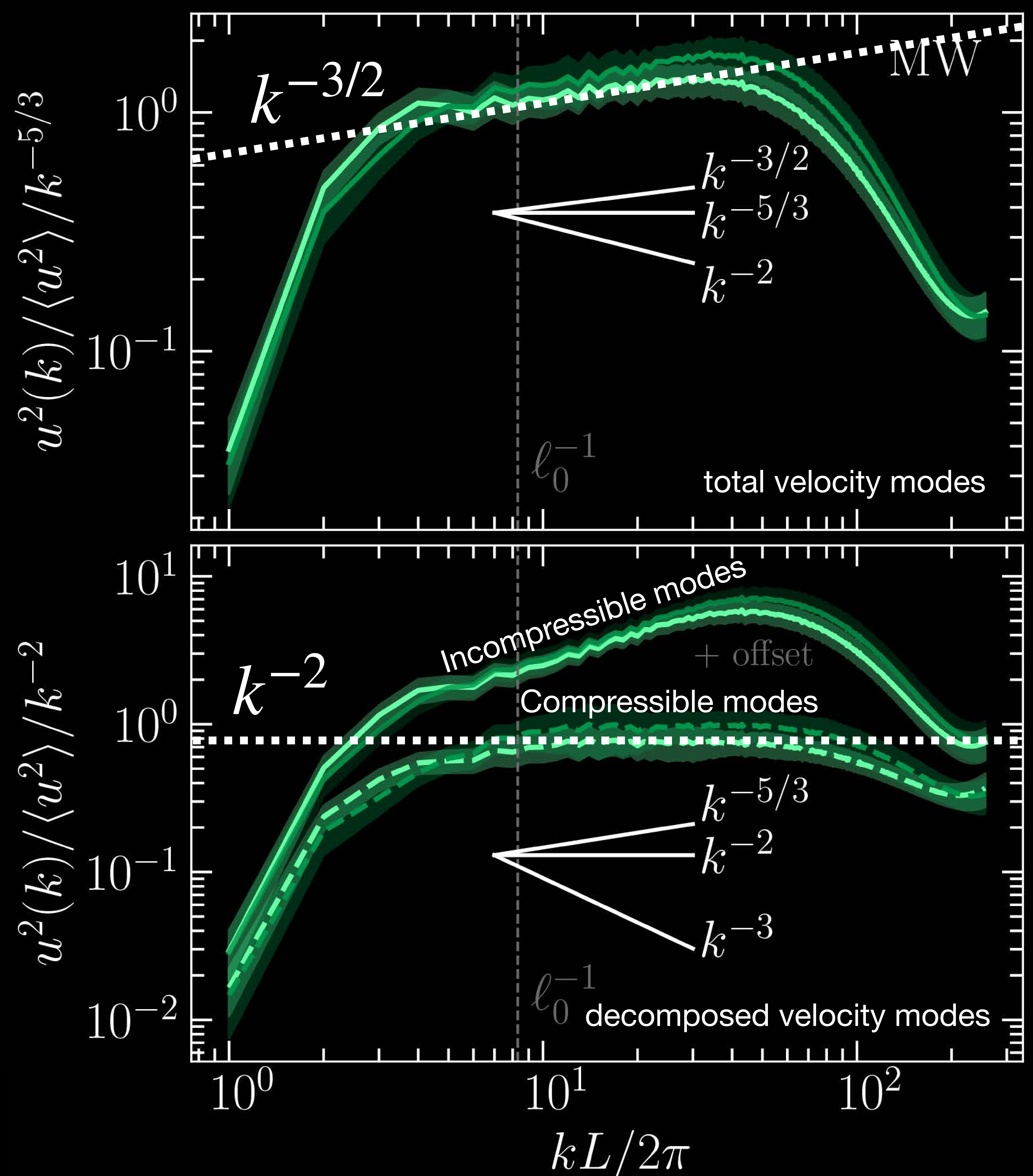
$$t_0 \sim \frac{\ell_0}{\langle u^2 \rangle^{1/2}} \sim 80 \text{ Myr}$$

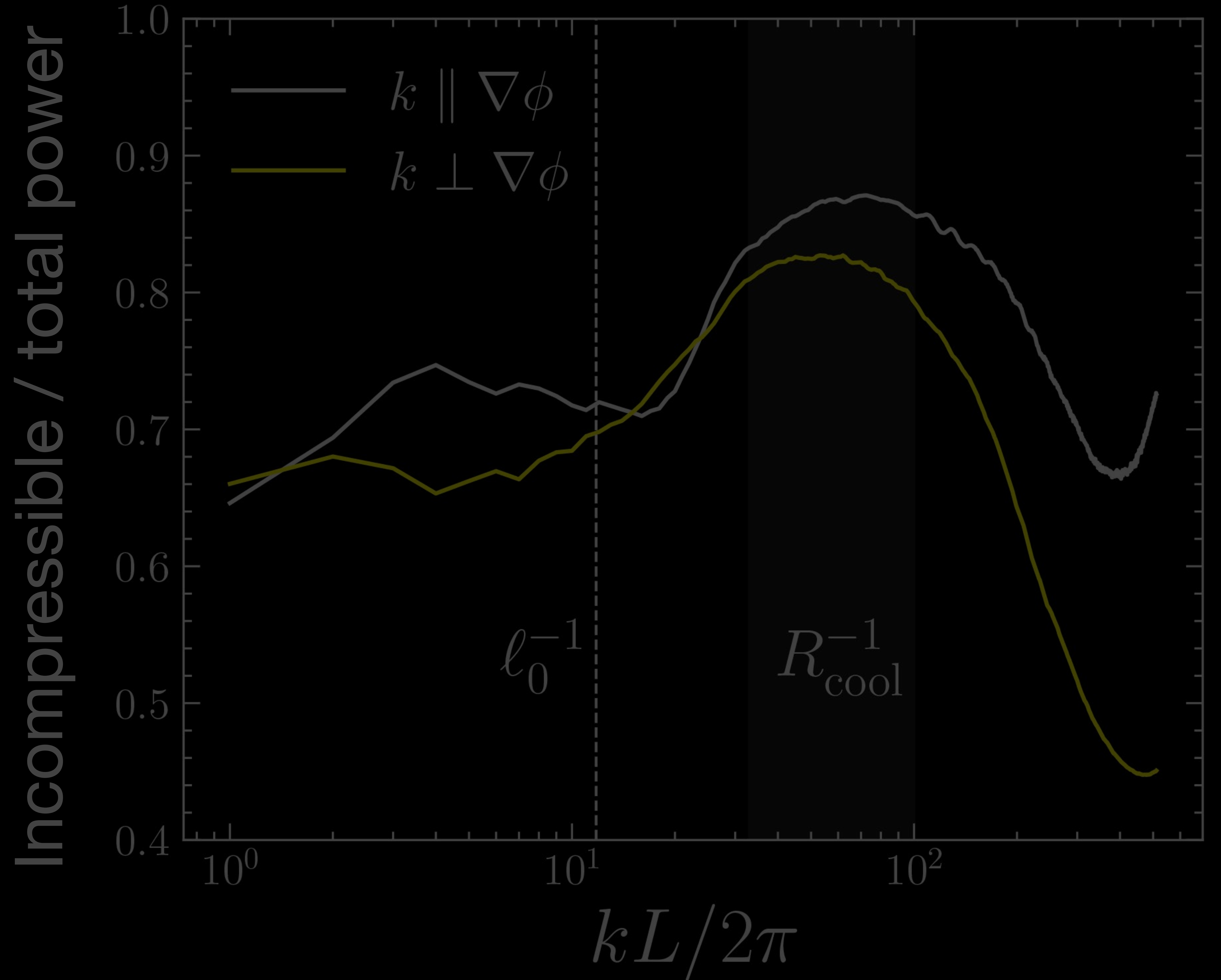


Reaches a close to stationary state… close

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Basic spectral properties

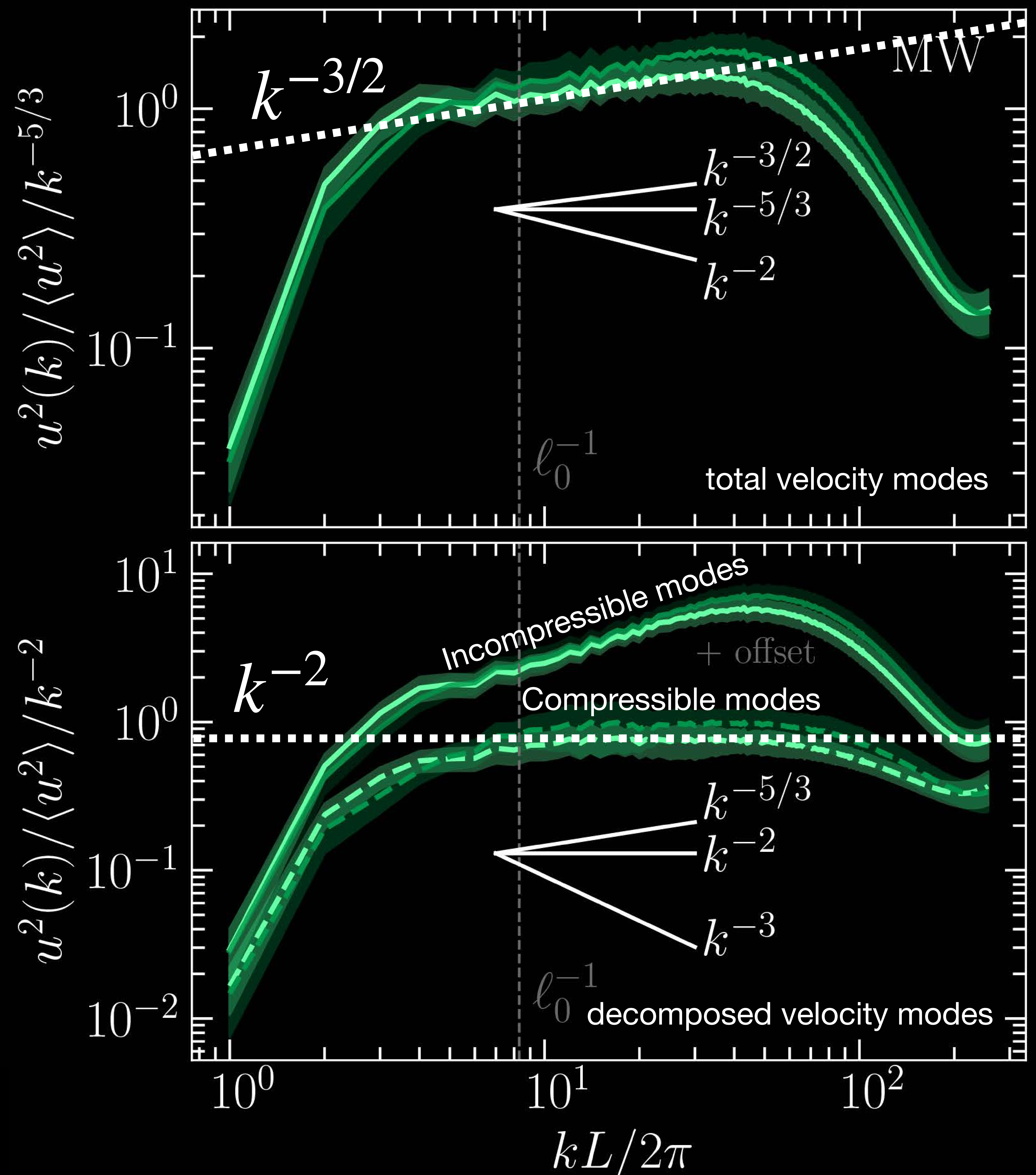


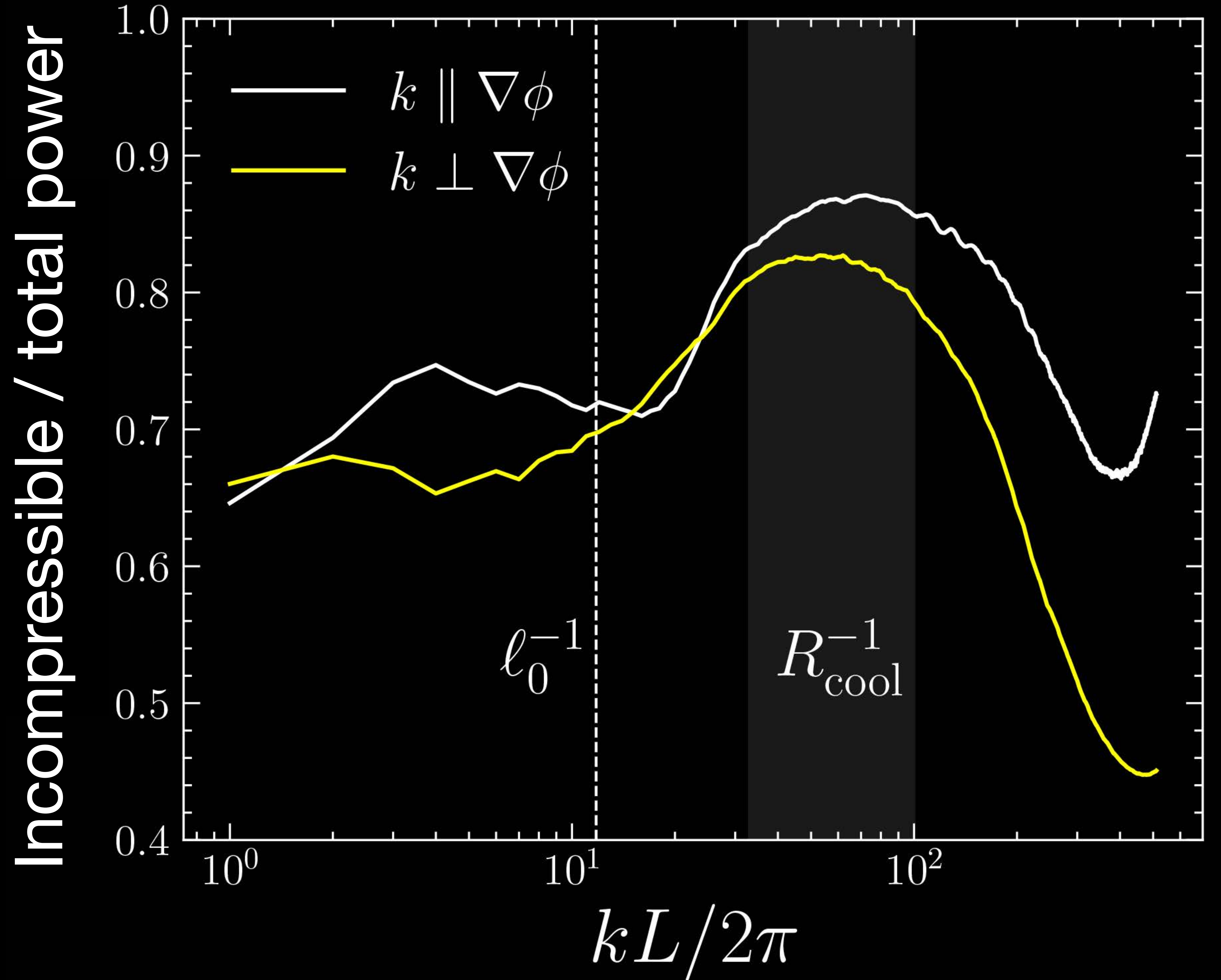


$$R_{\rm cool} \approx 14\,{\rm pc} \left(\frac{\rm E_{SN}}{10^{51}\,{\rm erg}}\right)\left(\frac{0.29}{\rm n_0\,cm^{-3}}\right)^{-0.43}$$

Cioffi + 1998
Blondin + 1998

Basic spectral properties





$$R_{\rm cool} \approx 14\,\mathrm{pc}\left(\frac{\mathrm{E_{SN}}}{10^{51}\,\mathrm{erg}}\right)\left(\frac{0.29}{\mathrm{n_0\,cm^{-3}}}\right)^{-0.43}$$

Cioffi + 1998
Blondin + 1998

Phase fluctuations and thermal response is mostly adiabatic

Consider the adiabatic limit

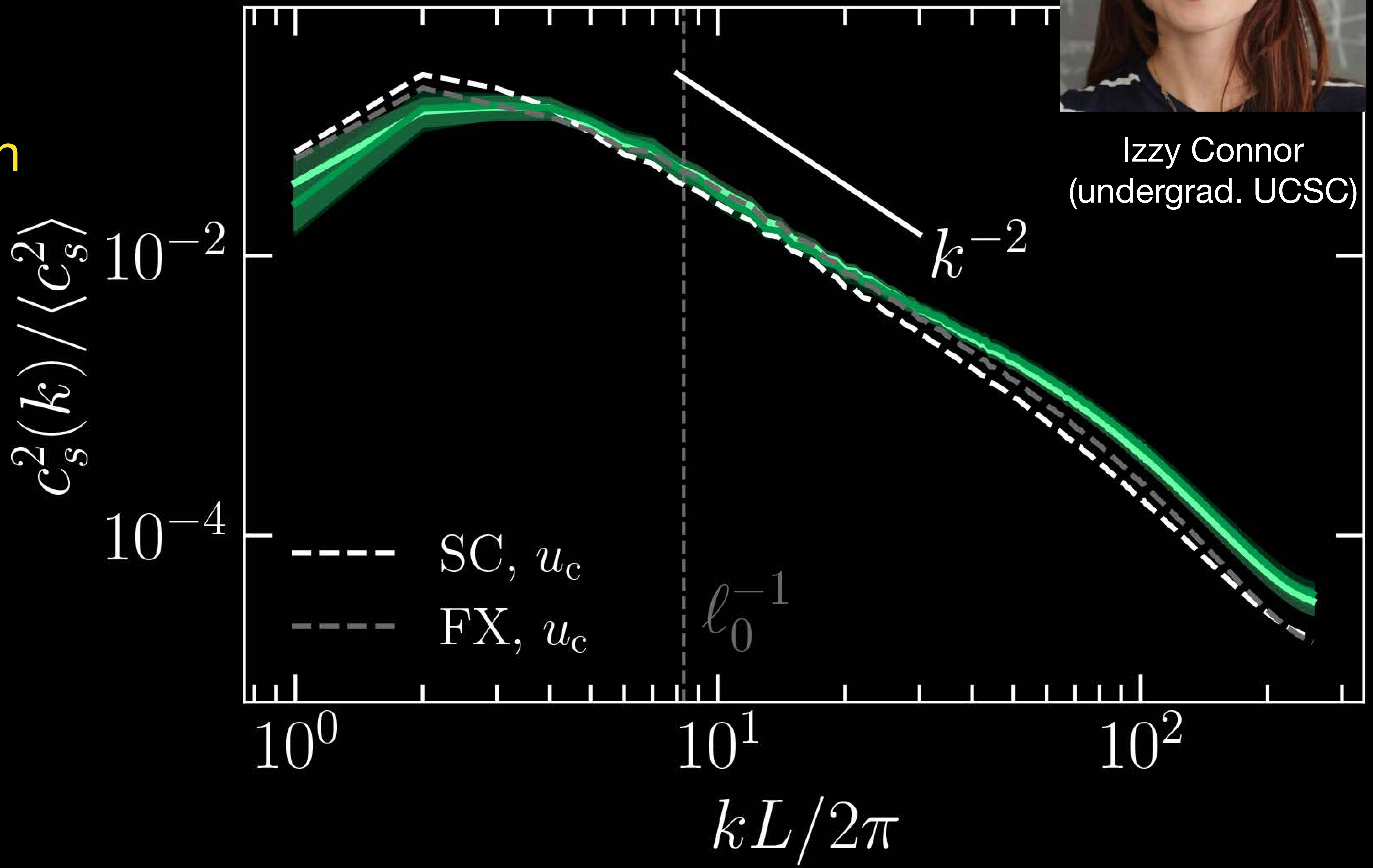
$$t_{\rm turb} \ll t_{\rm cool}$$

via linearized continuity equation

$$\tilde{\delta c_s} \sim c_{s,0}\frac{\gamma - 1}{2\omega}\mathbf{k}\cdot\tilde{\mathbf{u}}_c$$

$$\omega \sim c_{s,0}k$$

$$\tilde{\delta c_s} \sim \frac{\gamma - 1}{2}\tilde{u}_s$$



Izzy Connor
(undergrad. UCSC)

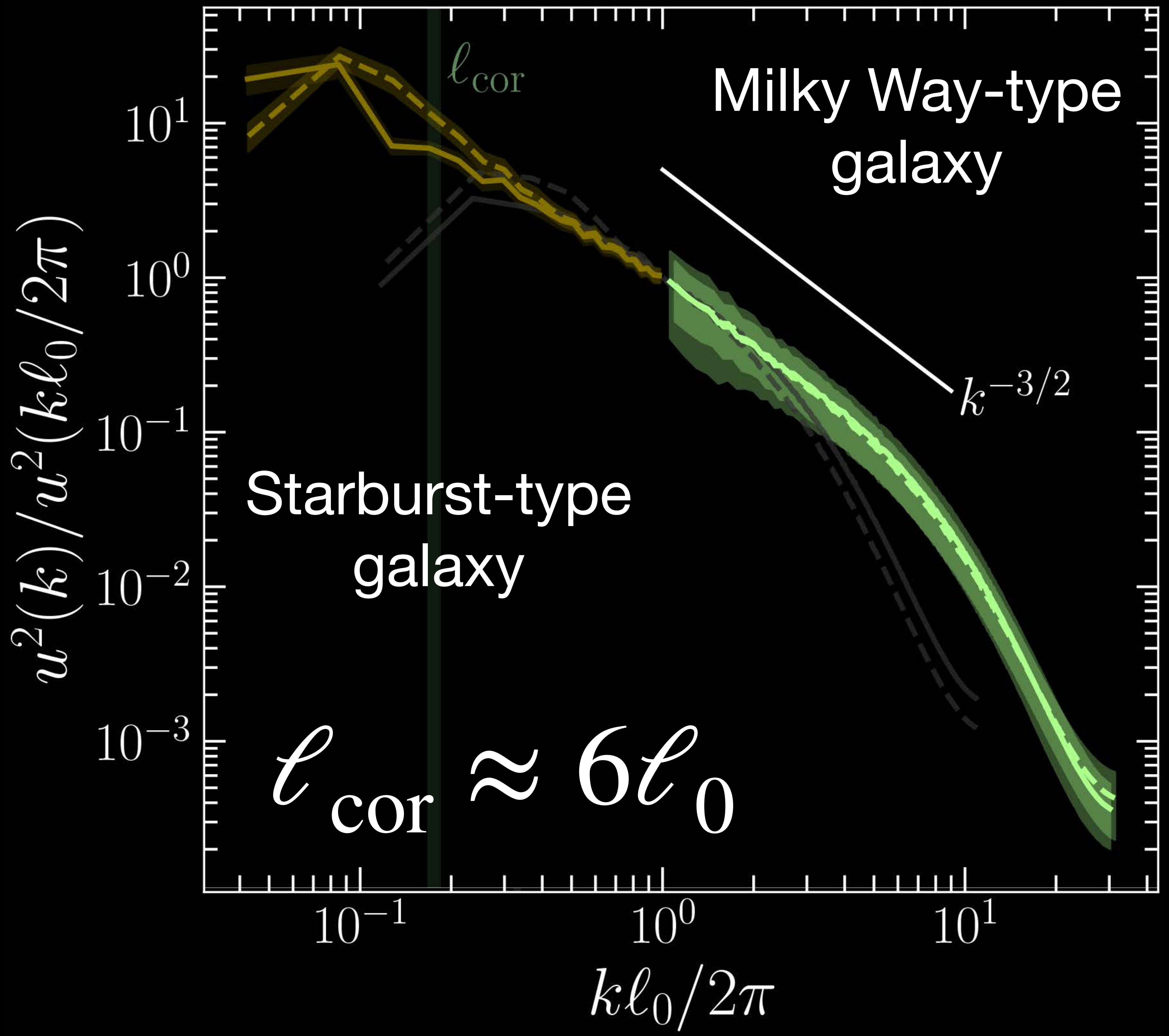
thermal response / phase fluctuations controlled by compressible modes

Connor, **Beattie** + (2025, submitted ApJ) Cascading from the winds to the disk: universality of supernovae-driven turbulence in different galactic ISMs

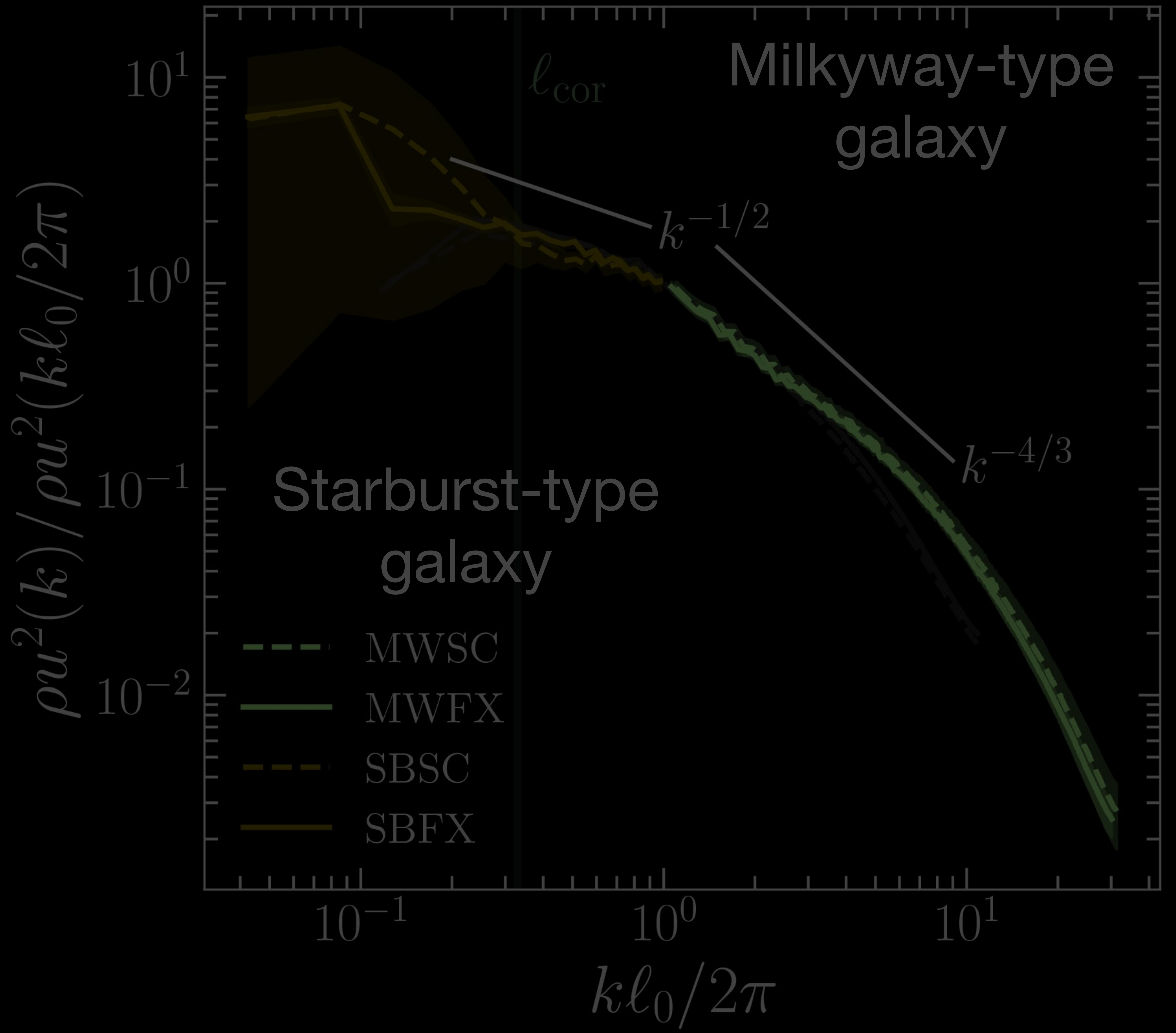
Universality between galactic models

$$\gamma_{\mathrm{SN_e,SB}} = 10\gamma_{\mathrm{SN_e,MW}}$$

Velocity spectrum



Kinetic energy spectrum



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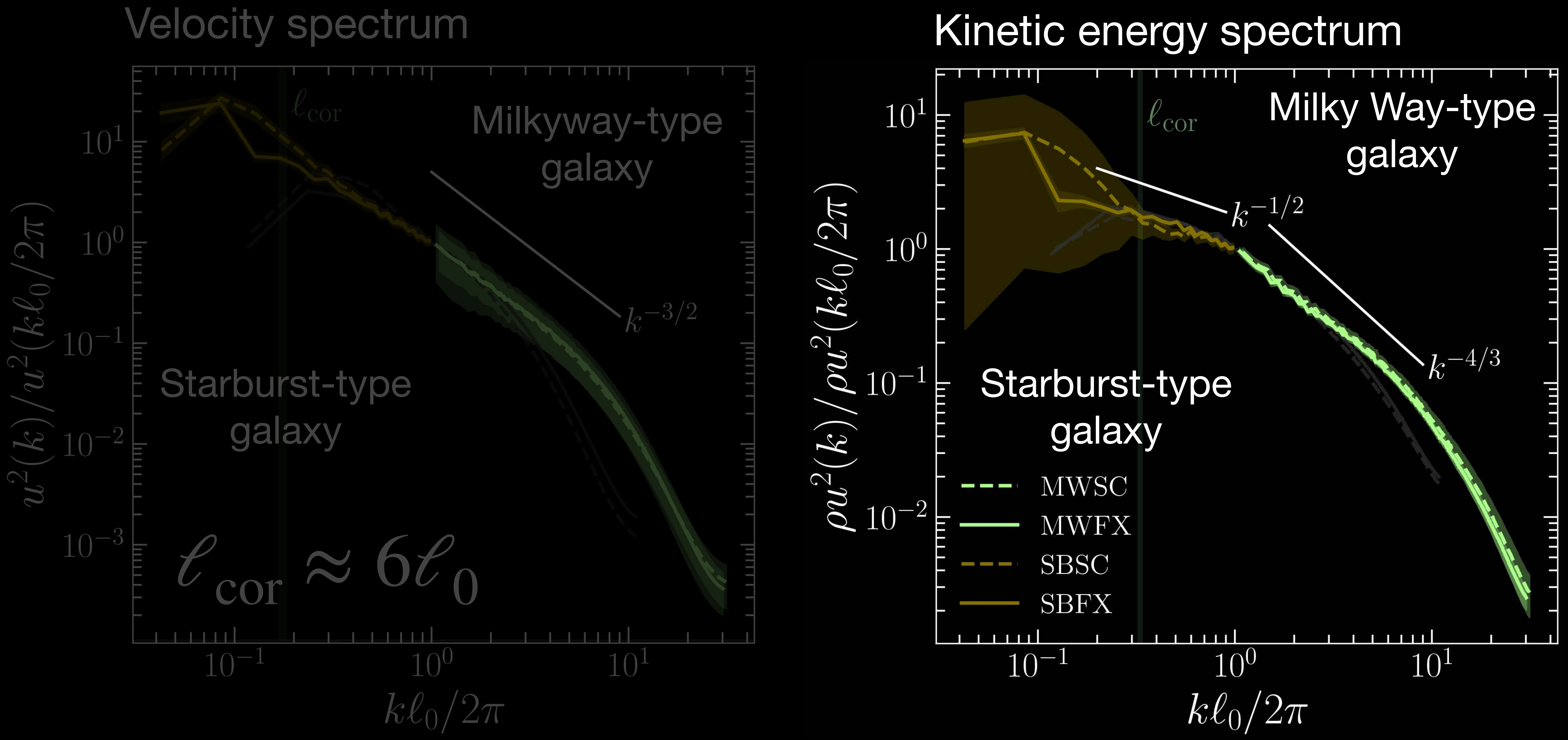

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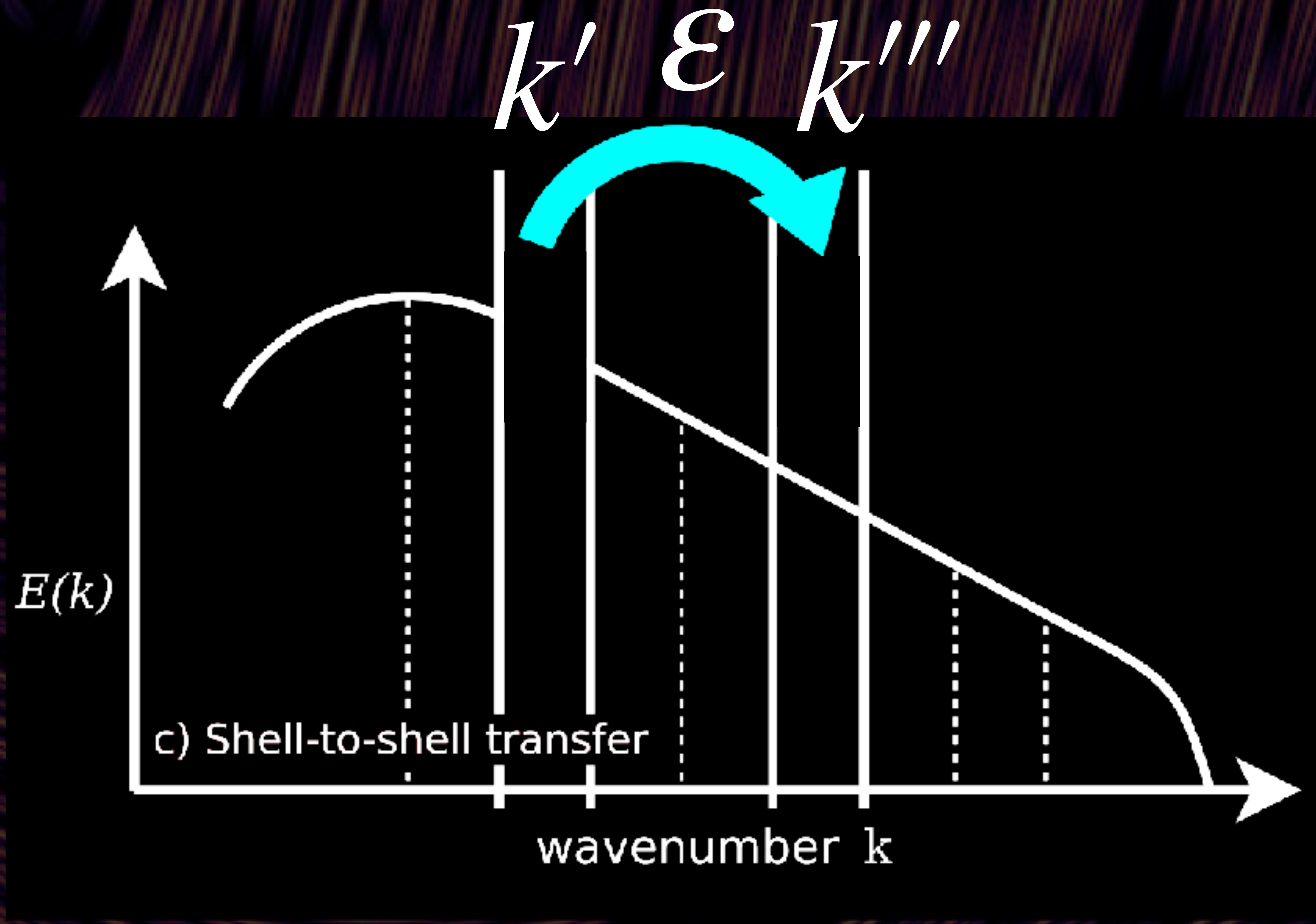
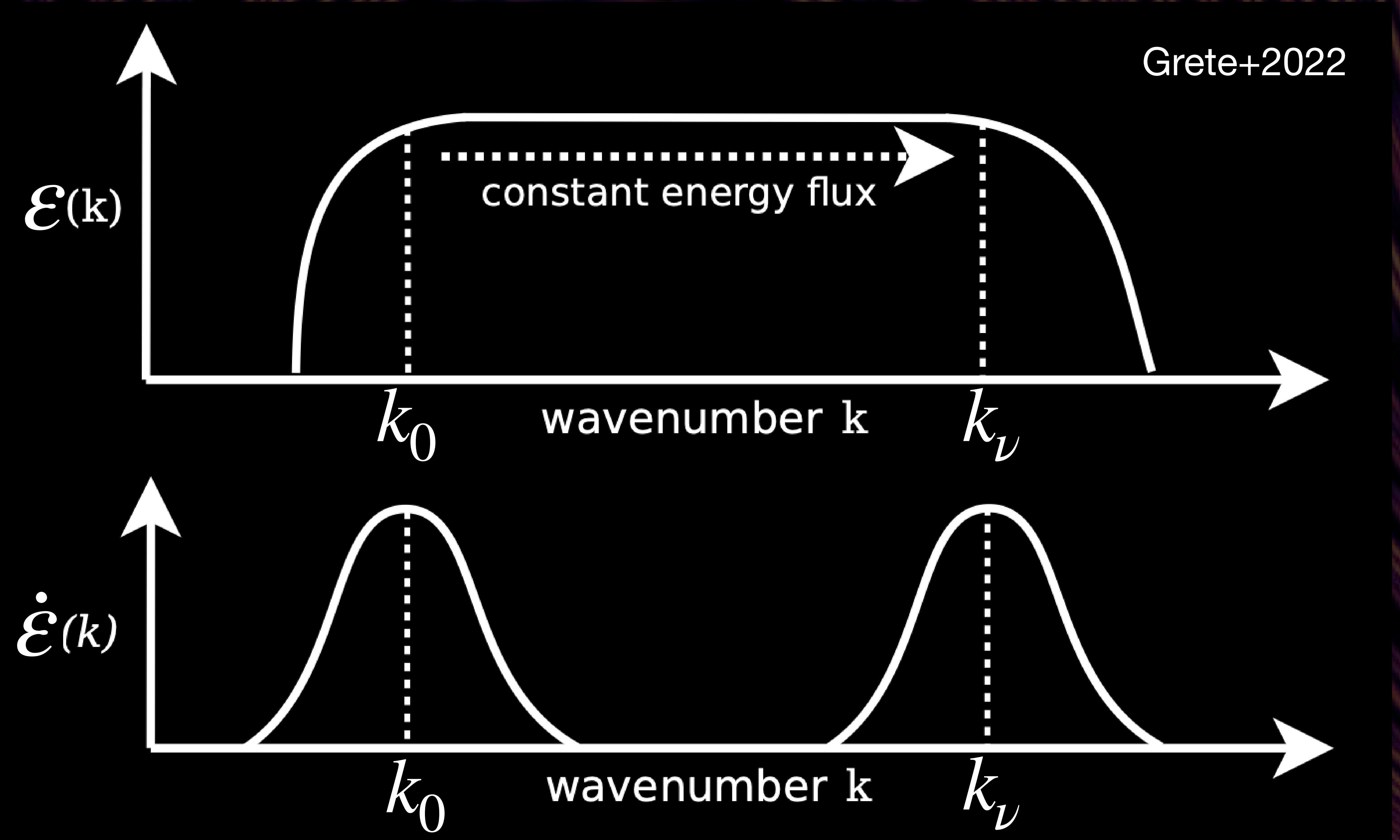
But to understand the turbulence we must understand the flux rate from mode to mode!





Grete+2022a,b,23

Dar+2001; Verma+2004; Mininni+2005; Alexakis+2005

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

But to understand the turbulence we must understand the flux rates from mode to mode!

Momentum conservation:

doner receiver

$$\mathbf{k}' + \mathbf{k}'' + \mathbf{k}''' = 0$$

mediator

doner receiver

$$\mathbf{k}' \xrightarrow{\mathbf{k}''} \mathbf{k}''' = -\mathbf{k}''' \xrightarrow{\mathbf{k}''} \mathbf{k}'$$

mediator

$$\mathbf{u}' = \mathbf{u}(\mathbf{r}') = \int \delta^3(\mathbf{k} - \mathbf{k}')\mathbf{u}(\mathbf{k})\exp\{2\pi i\mathbf{k}\cdot\mathbf{r}\}$$

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

But to understand the turbulence we must understand the flux from mode to mode!

$$\mathbf{k}' \xrightarrow{\mathbf{k}''} \mathbf{k}'''$$

$$\frac{1}{2}\partial_t \overbrace{\rho \mathbf{u}' \cdot \mathbf{u}'''}^{\text{kinetic energy density}} + \mathbf{u}''' \cdot \nabla \cdot \underbrace{\mathbb{F}_{\rho\mathbf{u}}(\mathbf{u}'', \mathbf{u}')}_{\substack{\text{energy flux rate density} \\ \text{from transport between } \mathbf{u}' \text{ and } \mathbf{u}'''}} = 0$$

$$\mathbf{u}''' \cdot \nabla \cdot \mathbb{F}(\mathbf{u}'', \mathbf{u}')_{\rho\mathbf{u}} = \mathbf{u}''' \otimes \mathbf{u}'' : \nabla \otimes \mathbf{u}' + \dots$$

$$\varepsilon \sim u^3/\ell$$

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

But to understand the turbulence we must understand the flux from mode to mode!

$$\mathbf{u}''' \cdot \nabla \cdot \mathbb{F}(\mathbf{u}'', \mathbf{u}')_{\rho\mathbf{u}} = \mathbf{u}''' \otimes \mathbf{u}'' : \nabla \otimes \mathbf{u}' + \dots$$

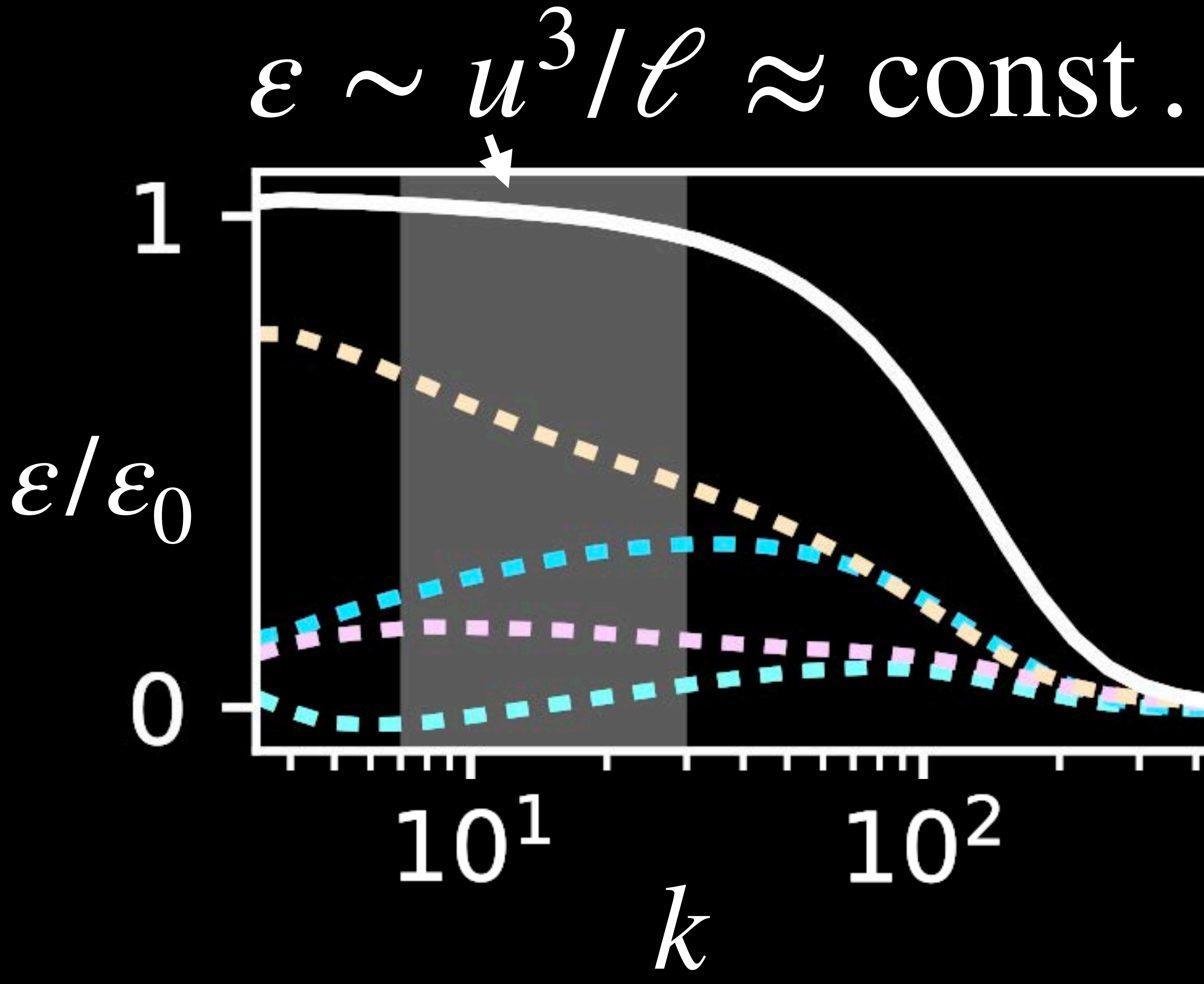
$$\mathbf{k}' \xrightarrow{\mathbf{k}''} \mathbf{k}'''$$

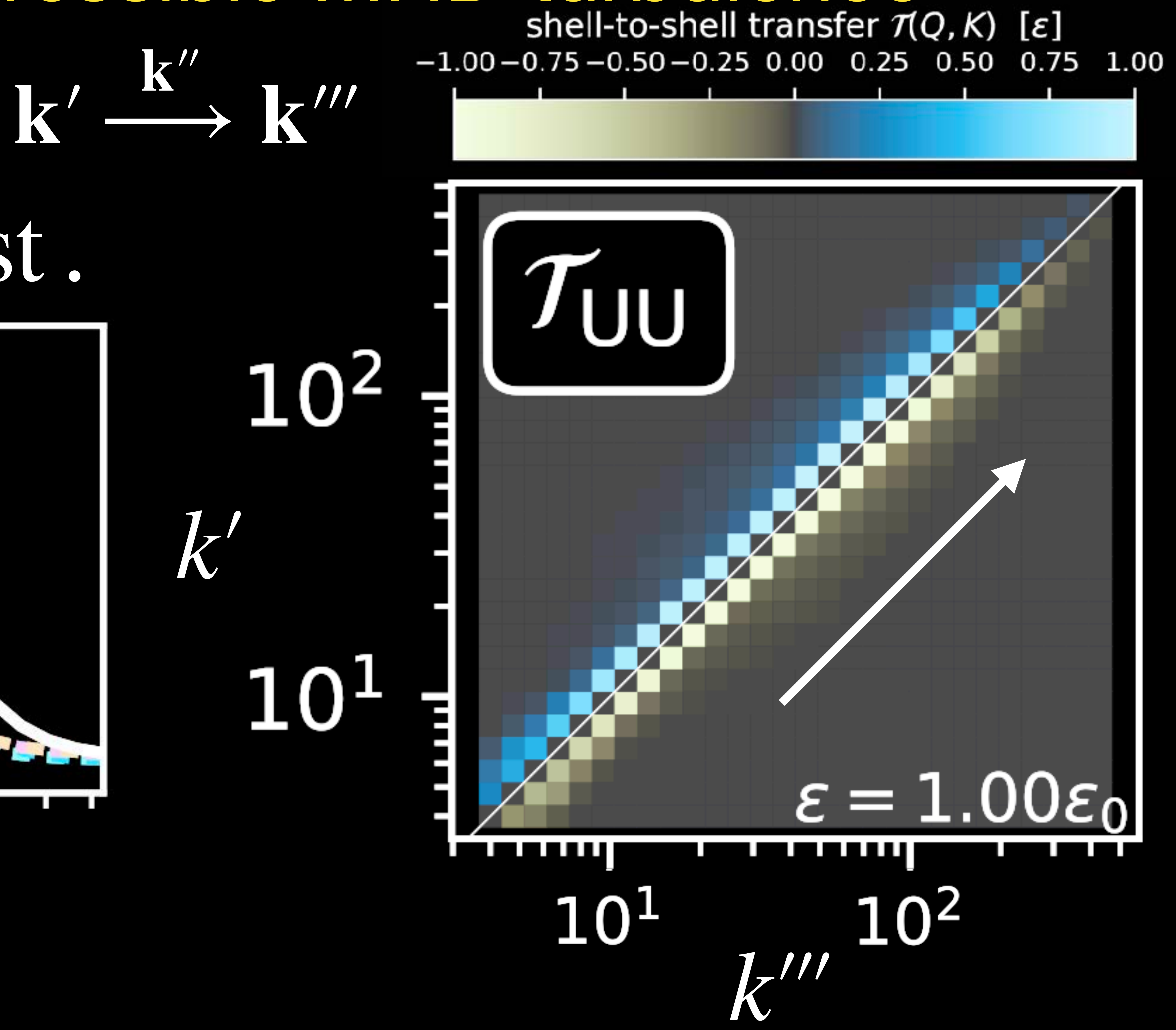
$$\mathcal{T}_{uu}(k', k''' | k'') = - \int dV \, \mathbf{u}''' \otimes \mathbf{u}'' : \nabla \otimes \mathbf{u}'$$

$$\varepsilon \sim u^3/\ell$$

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Expectations from compressible MHD turbulence in a box





Grete+2022

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Compressible modes and solenoidal modes live very different lives!
We should treat them differently!

$$\mathbf{k}' \xrightarrow{\mathbf{k}''} \mathbf{k}'''$$

$$\mathscr{T}^{c}_{cc}(k', k''') = -\int dV\, \mathbf{u}_c''' \otimes \mathbf{u}_c'' : \nabla \otimes \mathbf{u}_c'$$

cascade
transfers

$$\mathscr{T}^{s}_{ss}(k', k''') = -\int dV\, \mathbf{u}_s''' \otimes \mathbf{u}_s'' : \nabla \otimes \mathbf{u}_s'$$

$$u_s \cdot u_c = 0$$

$$\nabla \cdot u_s = 0$$

$$|\nabla \times u_c| = 0$$

$$u = u_c + u_s$$

$$\vdots$$

$$\mathscr{T}^{s}_{cs}(k', k''') = -\int dV\, \mathbf{u}_s''' \otimes \mathbf{u}_s'' : \nabla \otimes \mathbf{u}_c'$$

interaction
transfers

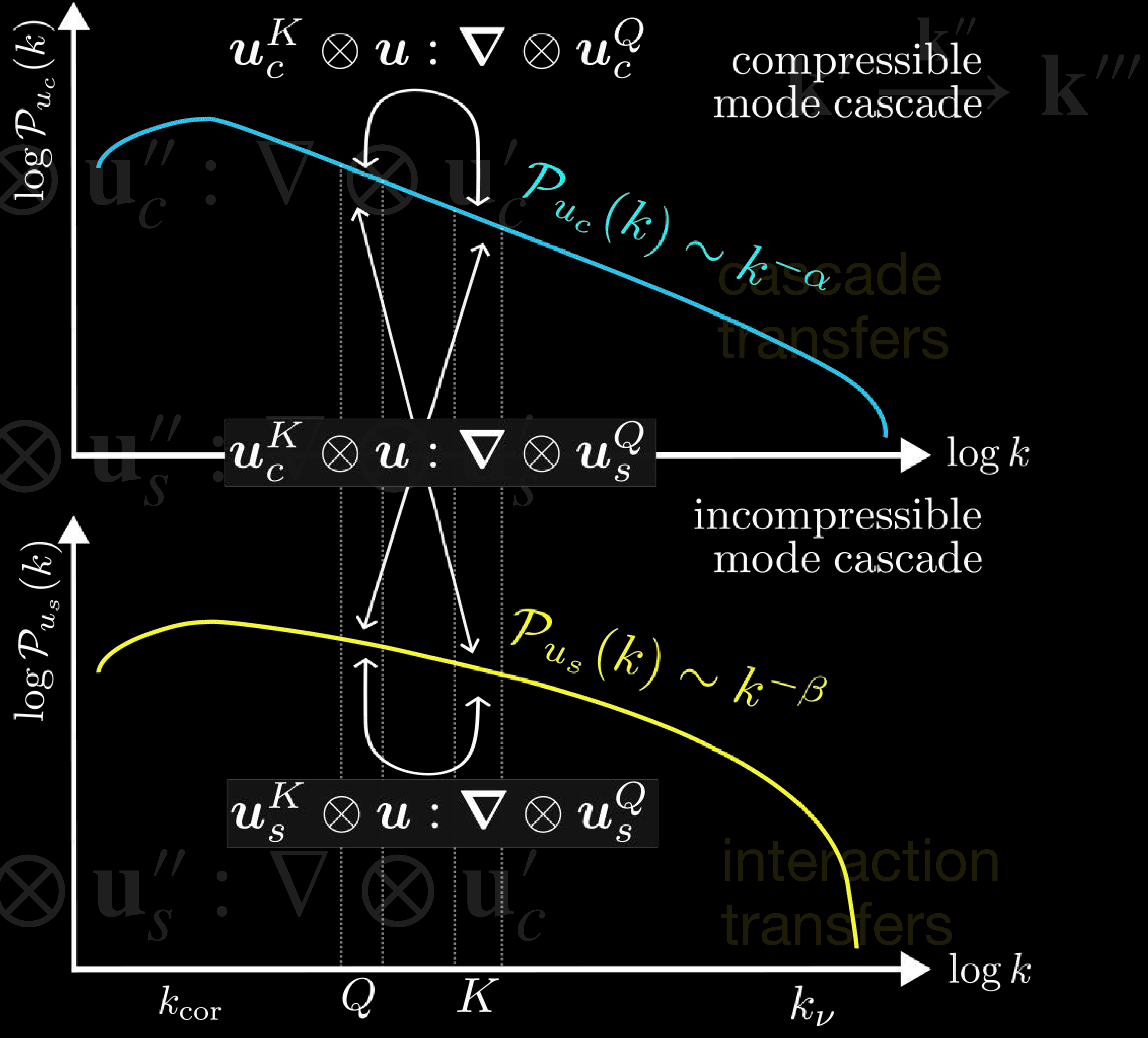
Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Compressible modes and solenoidal modes live very different lives! We should treat them differently!

$$\mathscr{T}^{c}_{cc}(k', k''') = -\int dV\, \mathbf{u}'''_c \otimes \mathbf{u}''_c : \nabla \otimes \mathbf{u}'_c$$

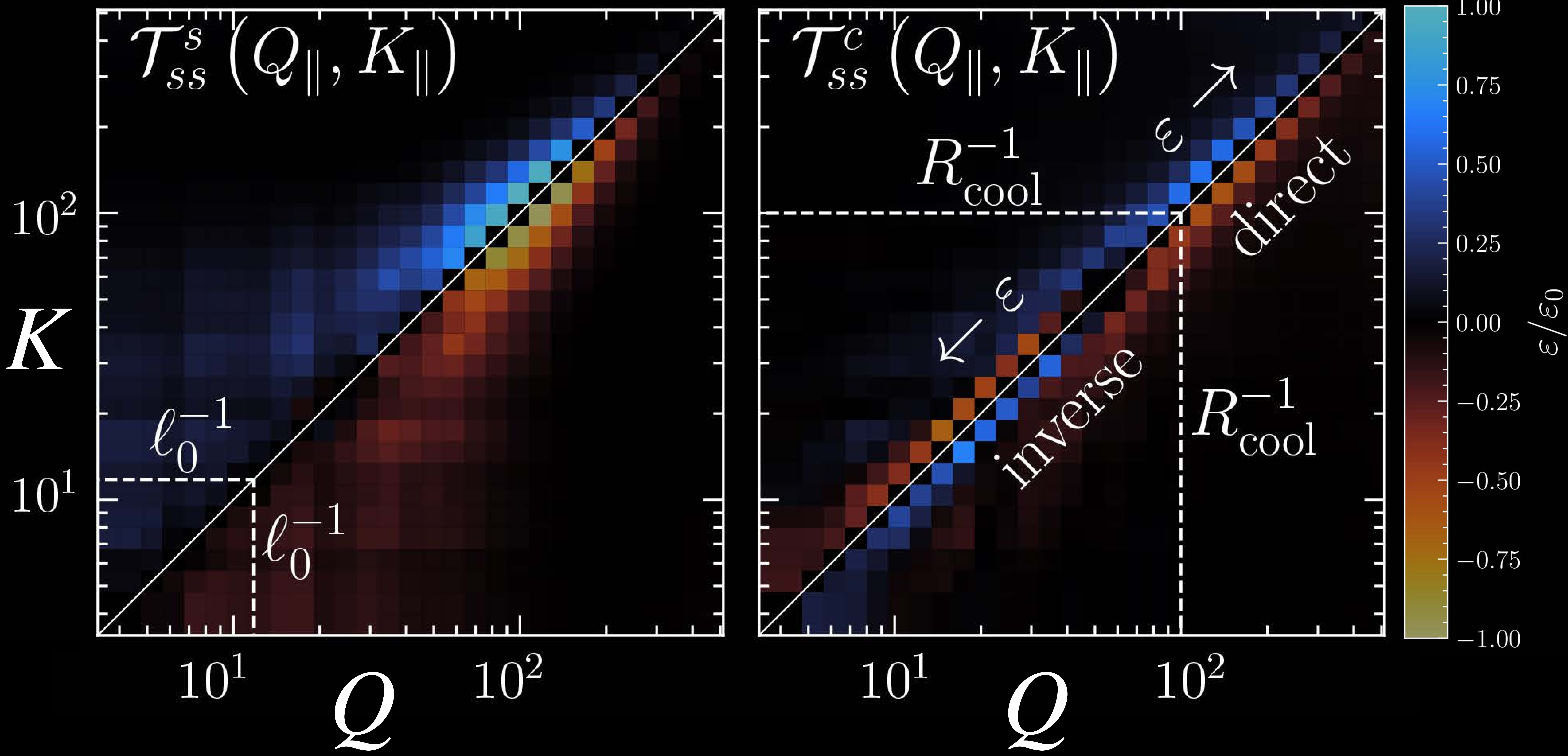
$$\mathscr{T}^{s}_{ss}(k', k''') = -\int dV\, \mathbf{u}'''_s \otimes \mathbf{u}''_s : \nabla \otimes \mathbf{u}'_s$$

$$\mathscr{T}^{s}_{cs}(k', k''') = -\int dV\, \mathbf{u}'''_s \otimes \mathbf{u}''_s : \nabla \otimes \mathbf{u}'_c$$

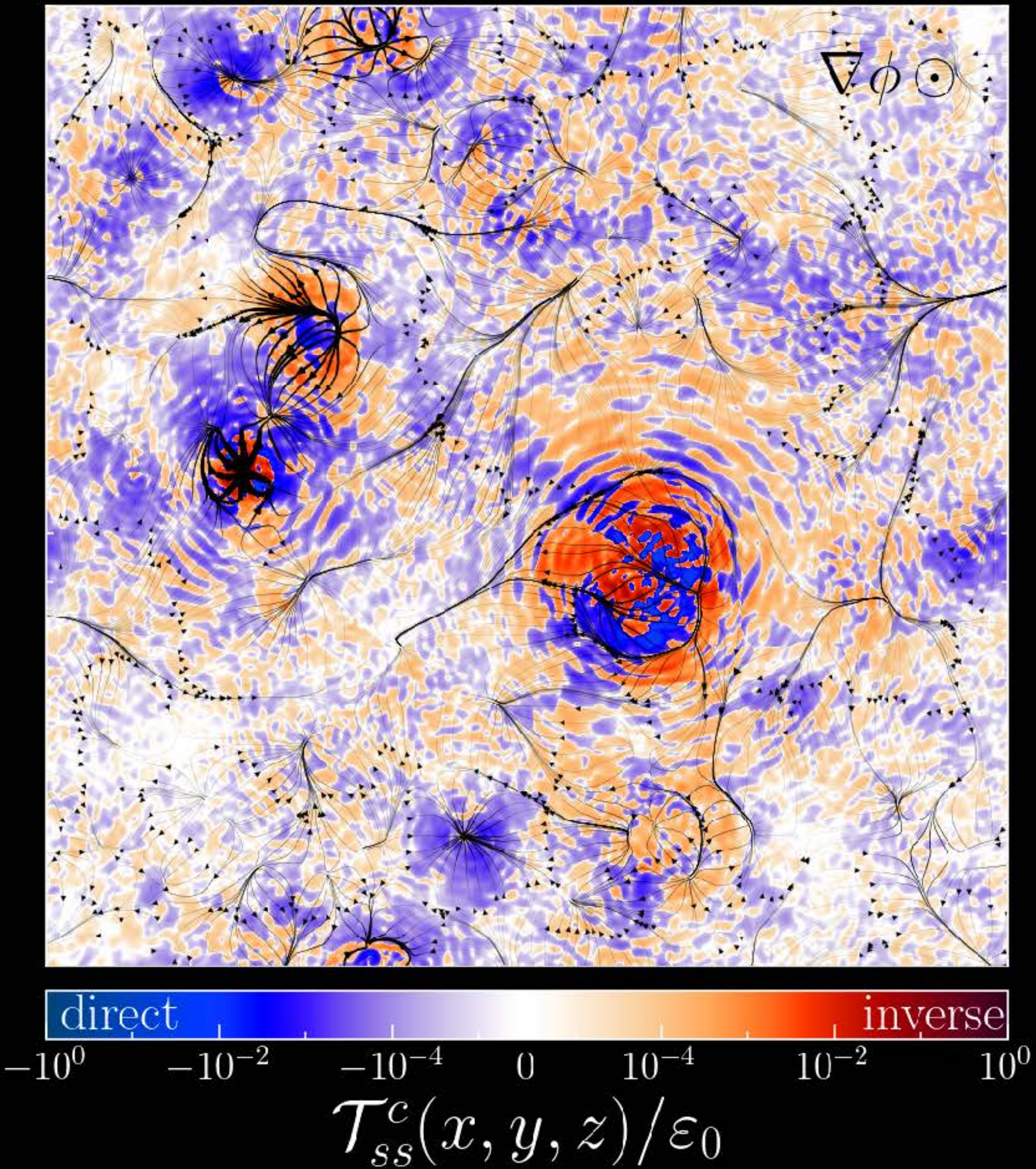


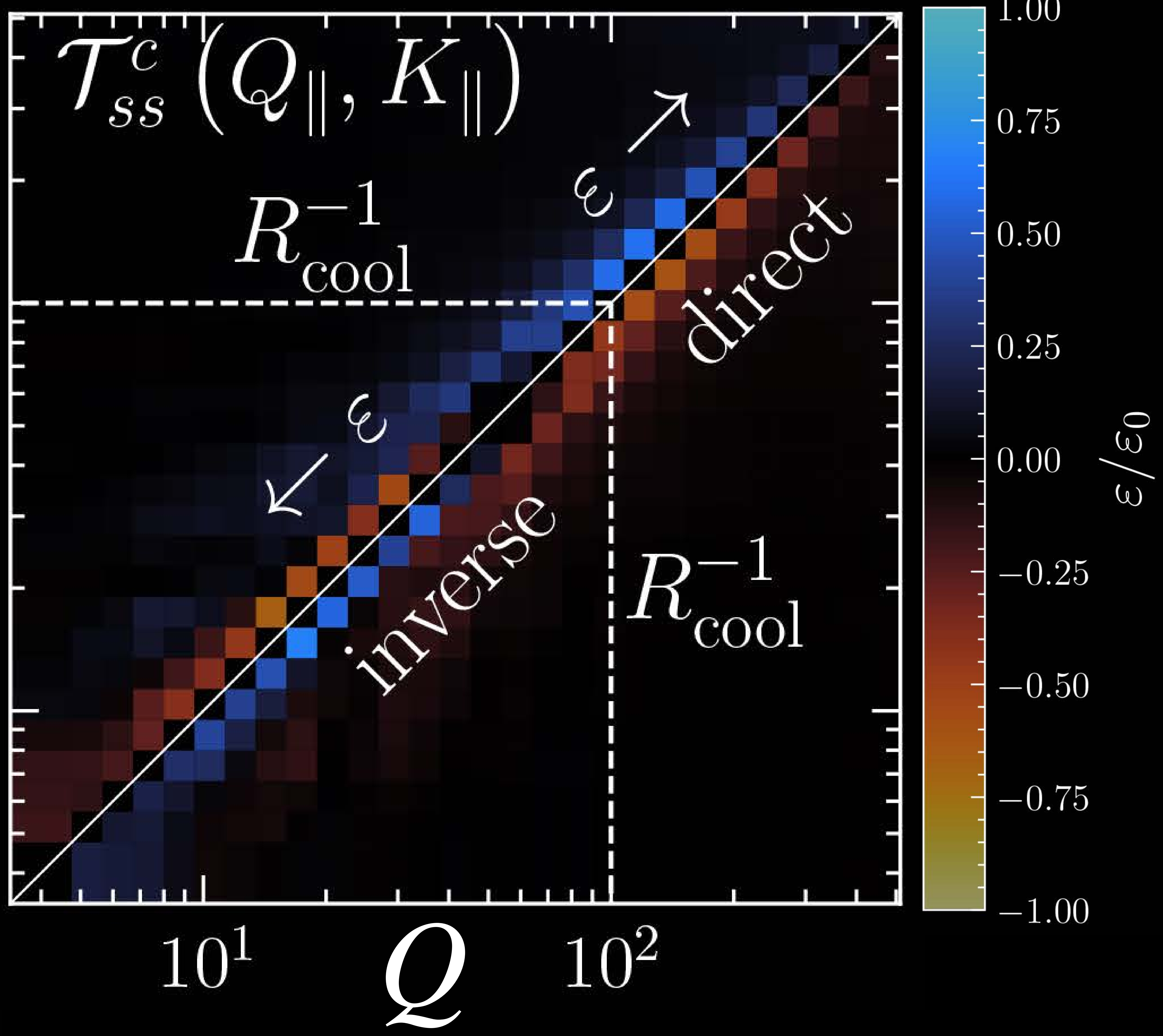
Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

The incompressible turbulence ($u_s \to u_s$ cascade)

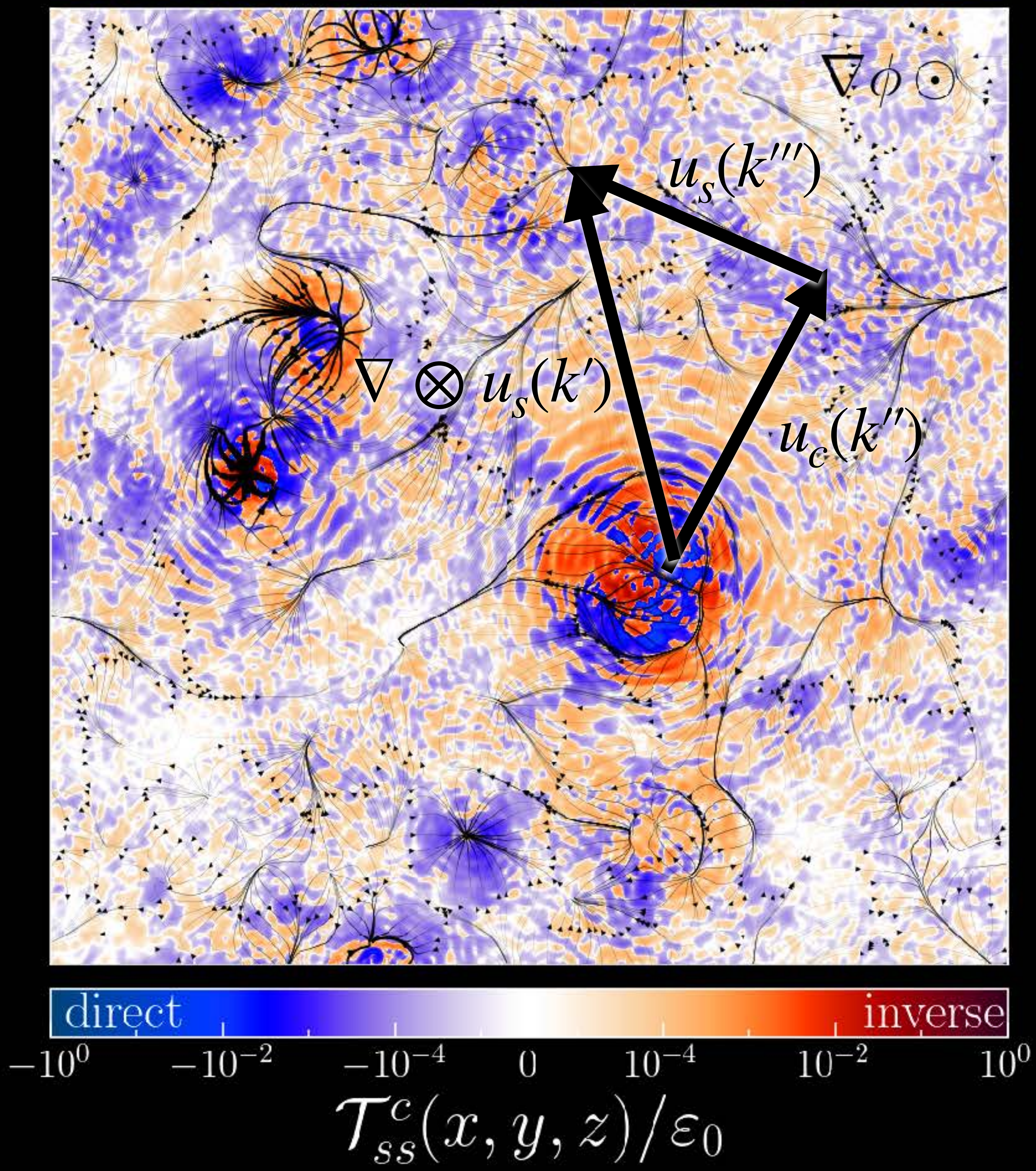


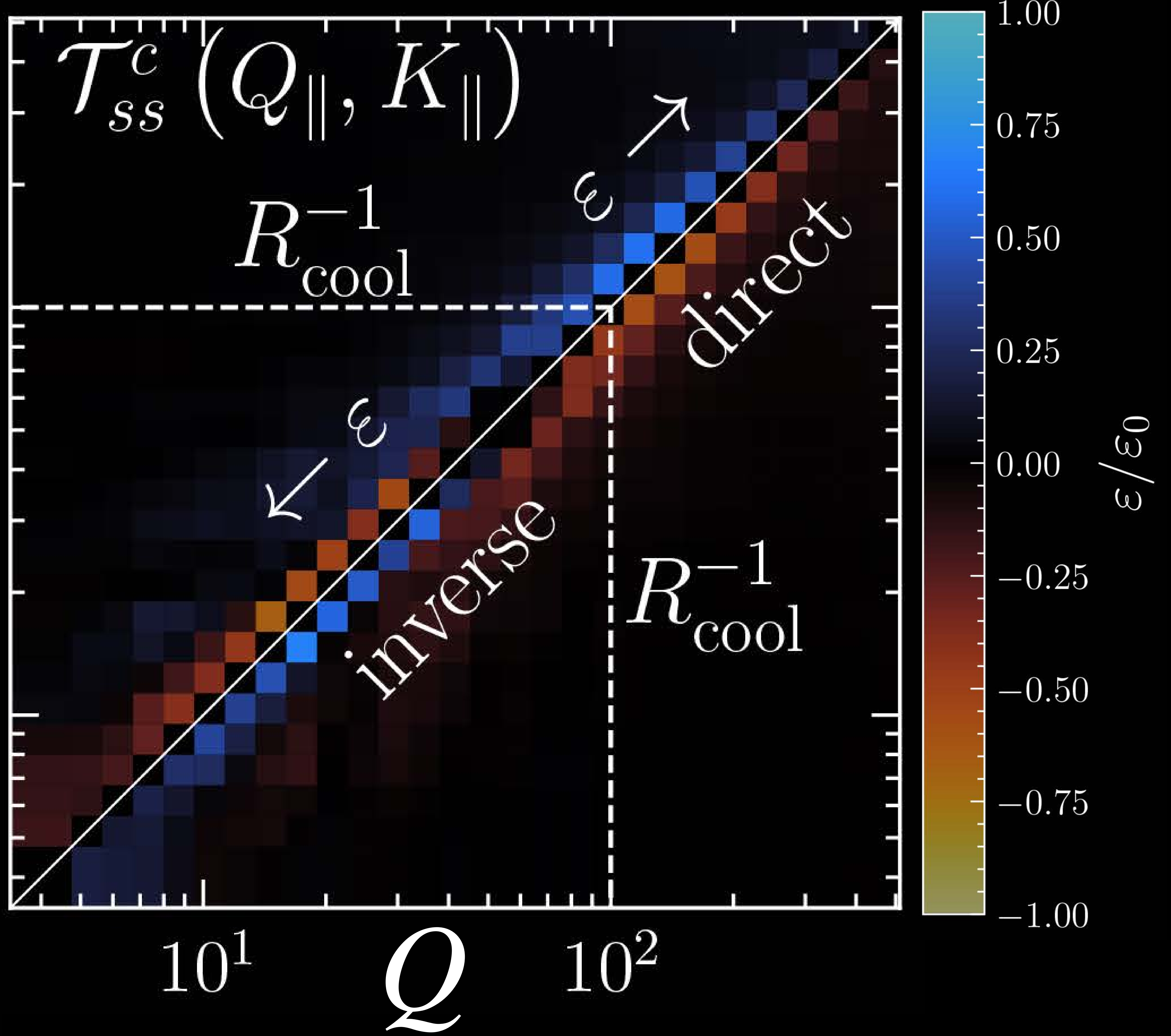
The incompressible turbulence ($u_s \rightarrow u_s$ cascade)



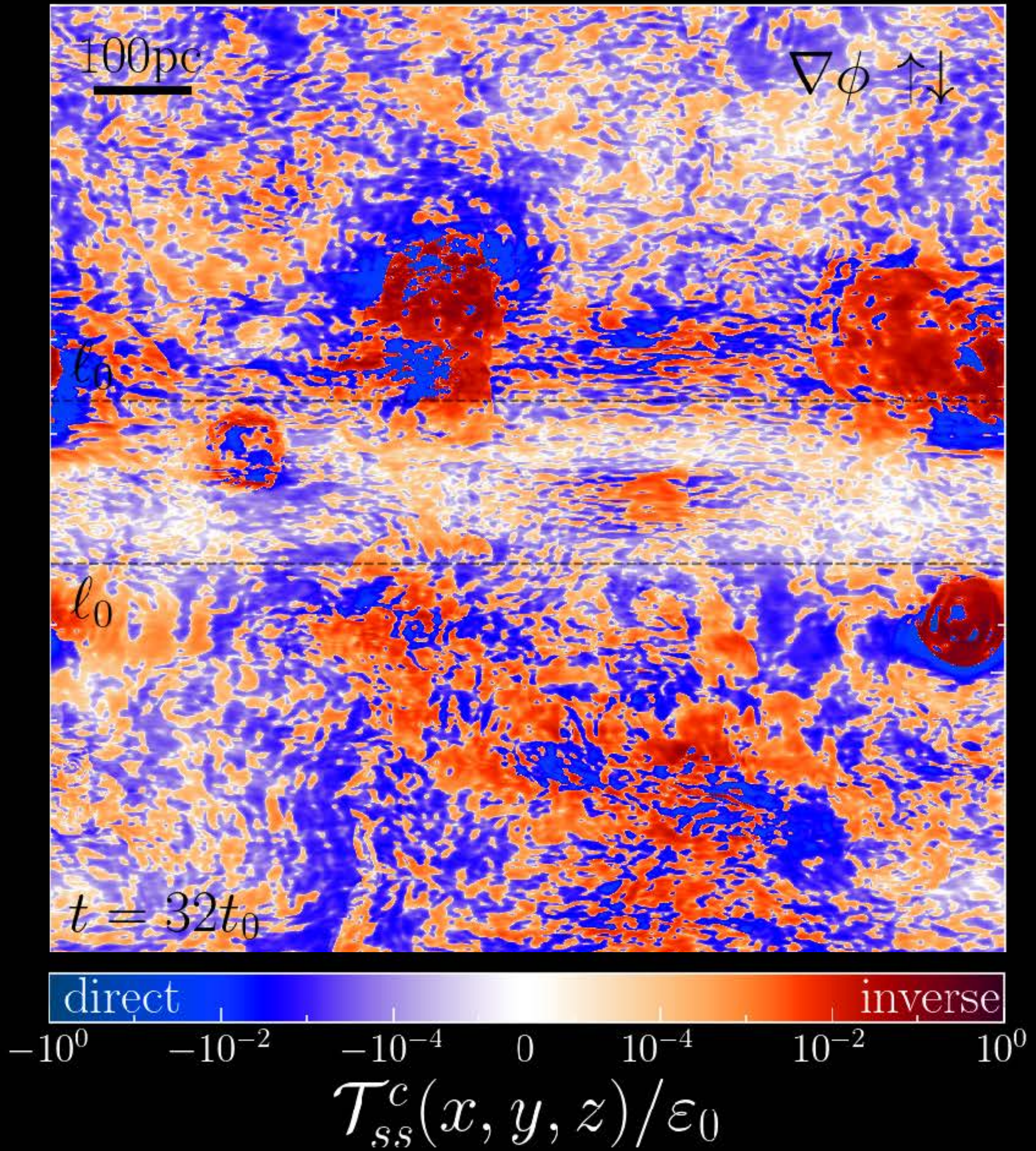


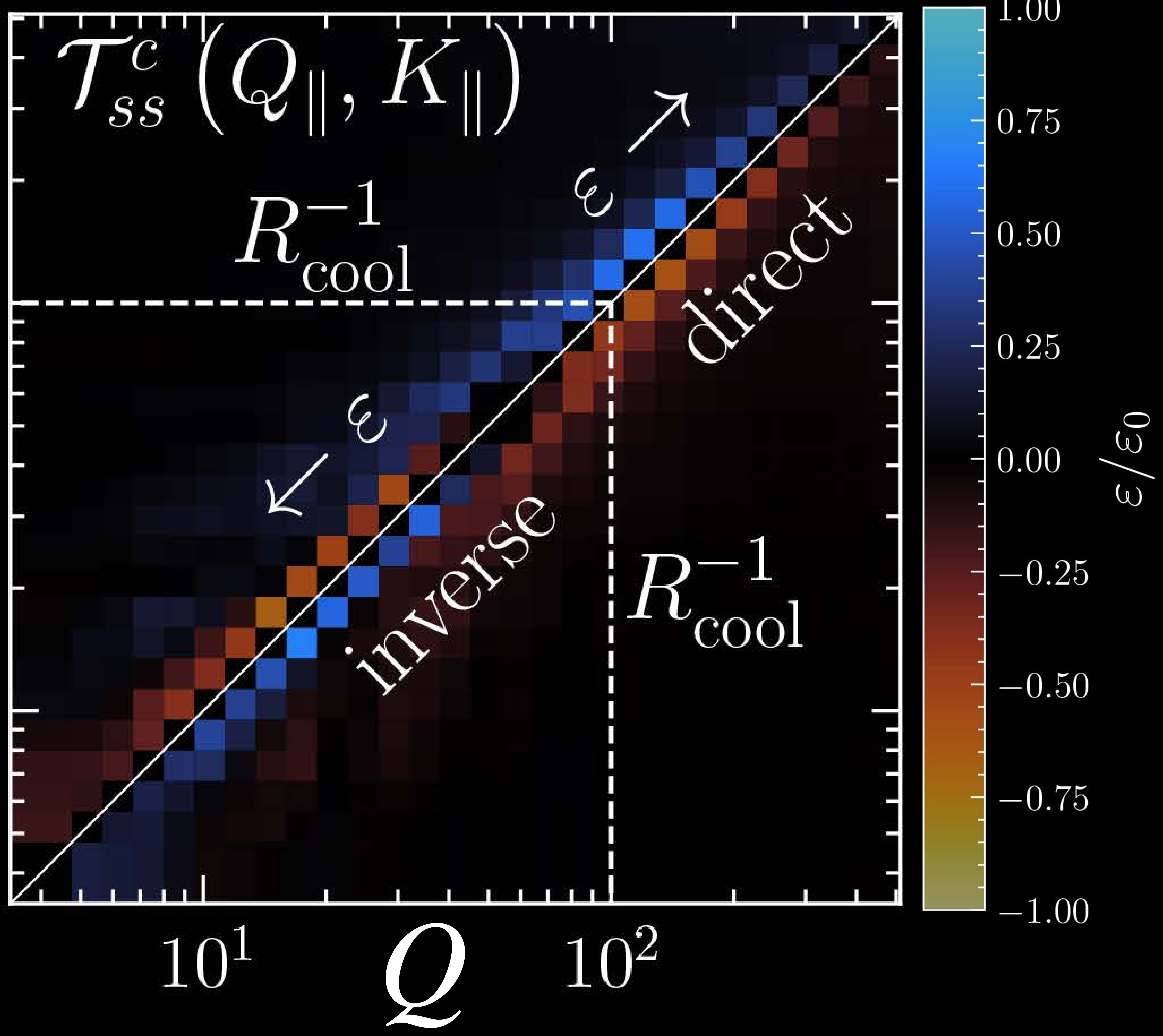
The incompressible turbulence ($u_s \to u_s$ cascade)



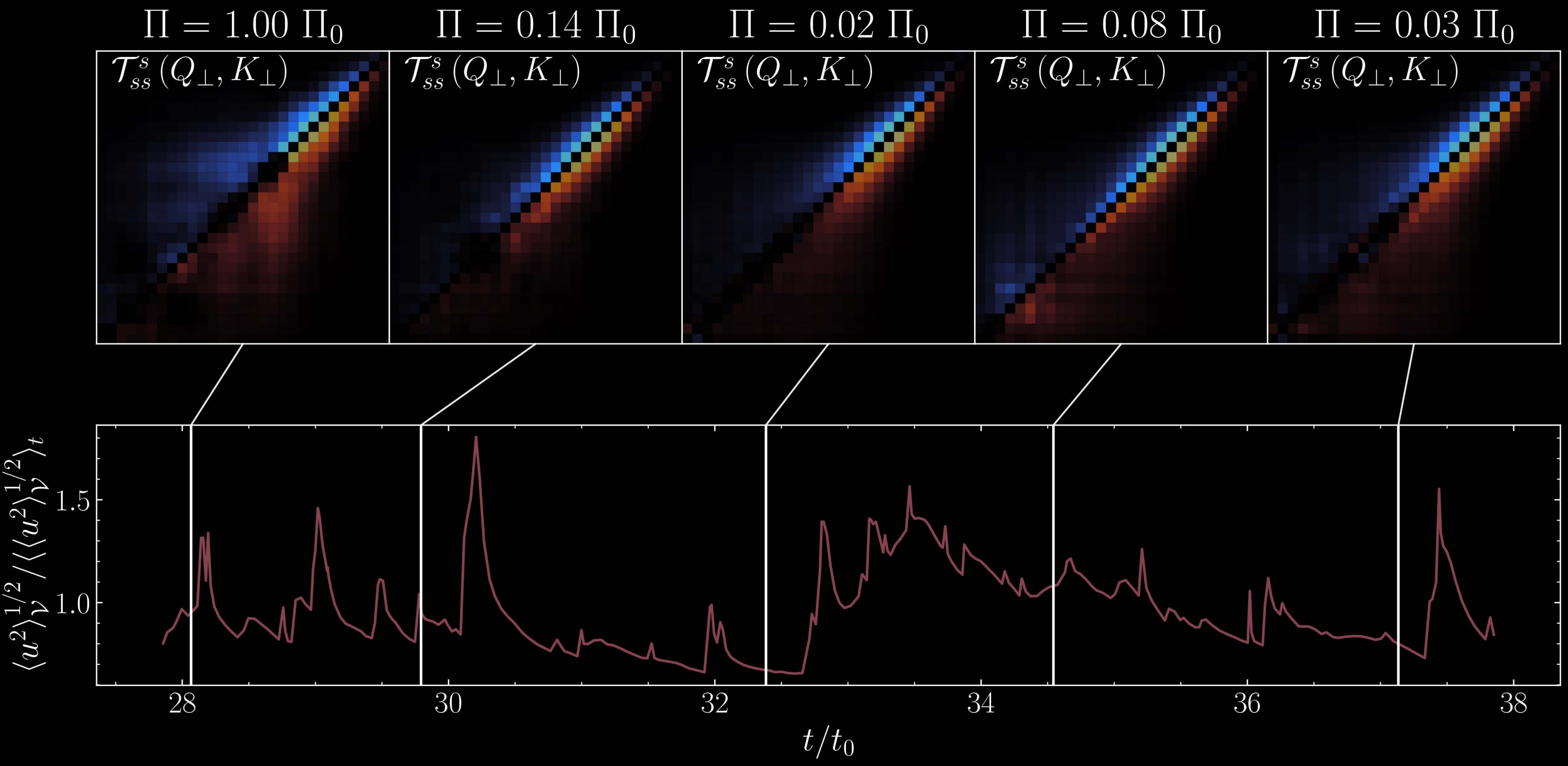


The incompressible turbulence ($u_s \to u_s$ cascade)

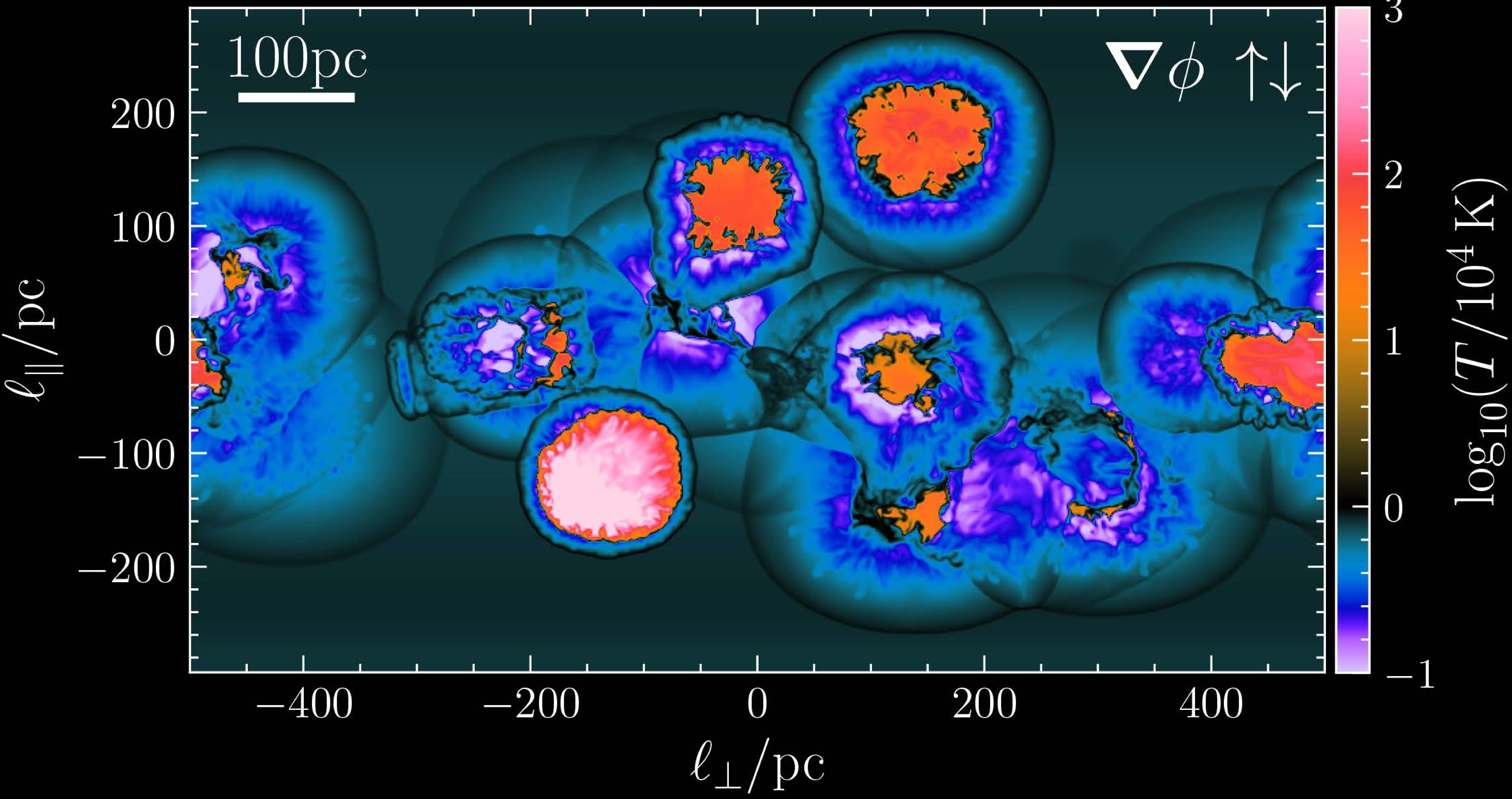




The entire incompressible cascade comes and goes



Where does the incompressible turbulence come from?



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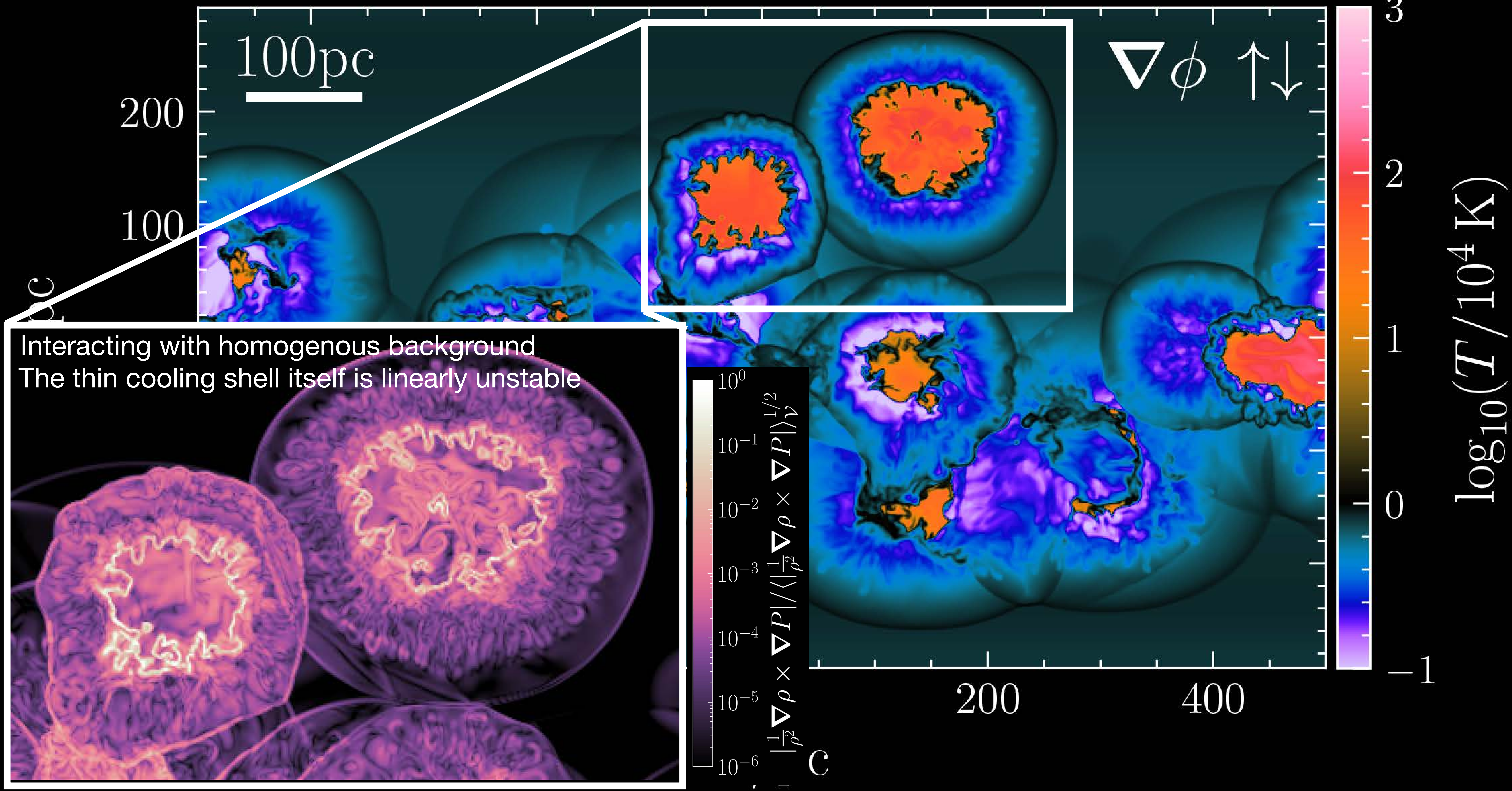
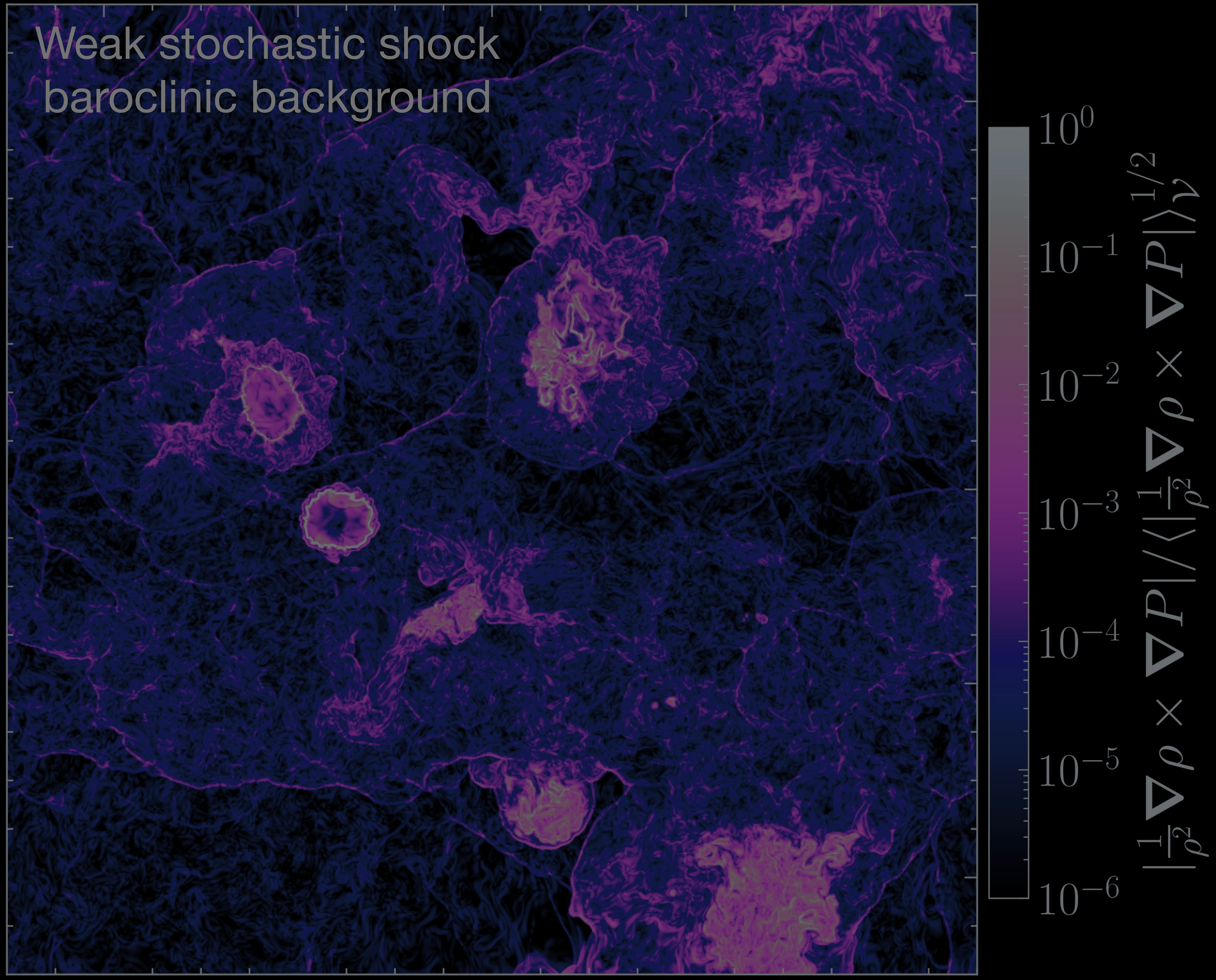
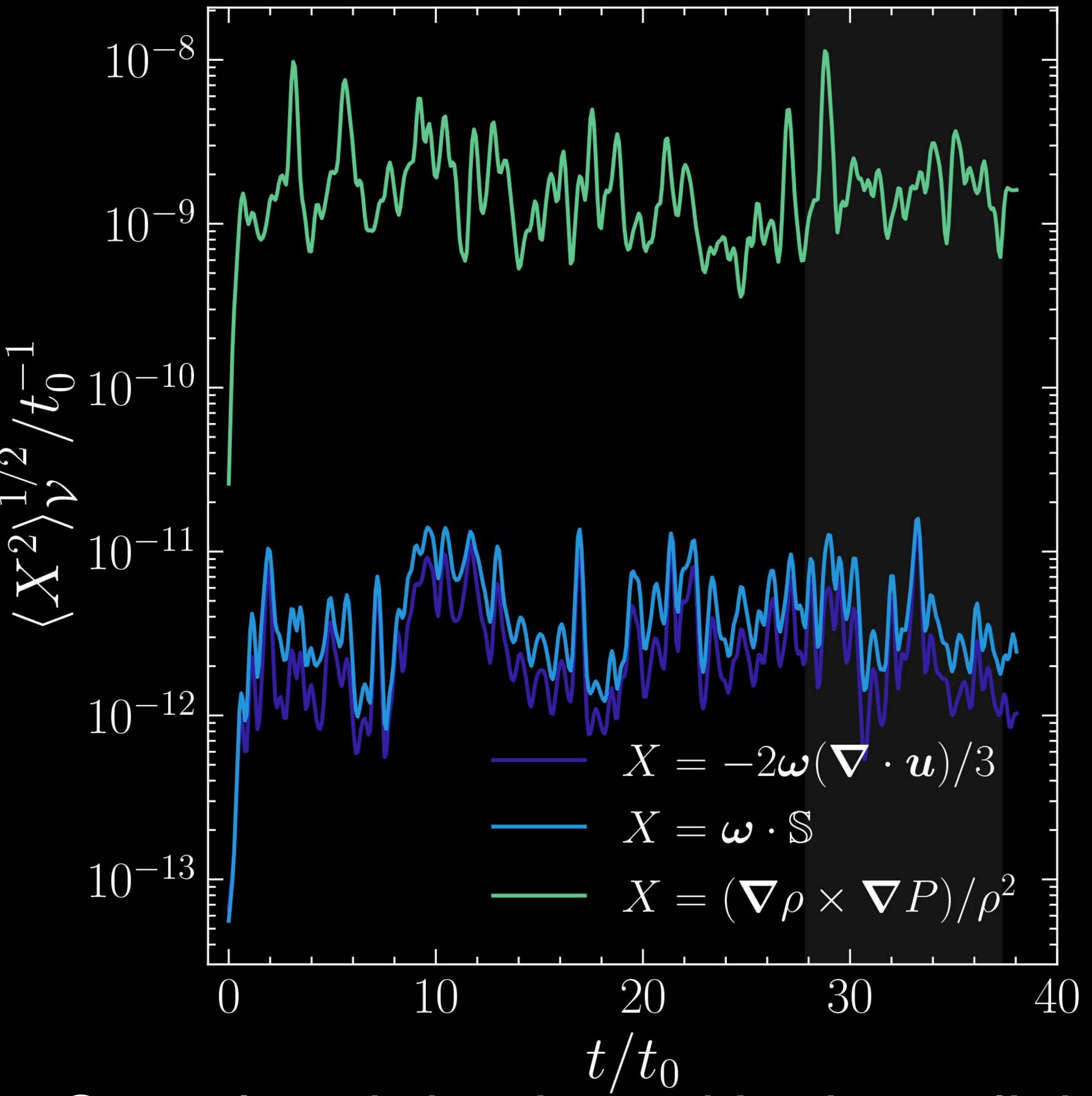


Where does the incompressible turbulence come from?

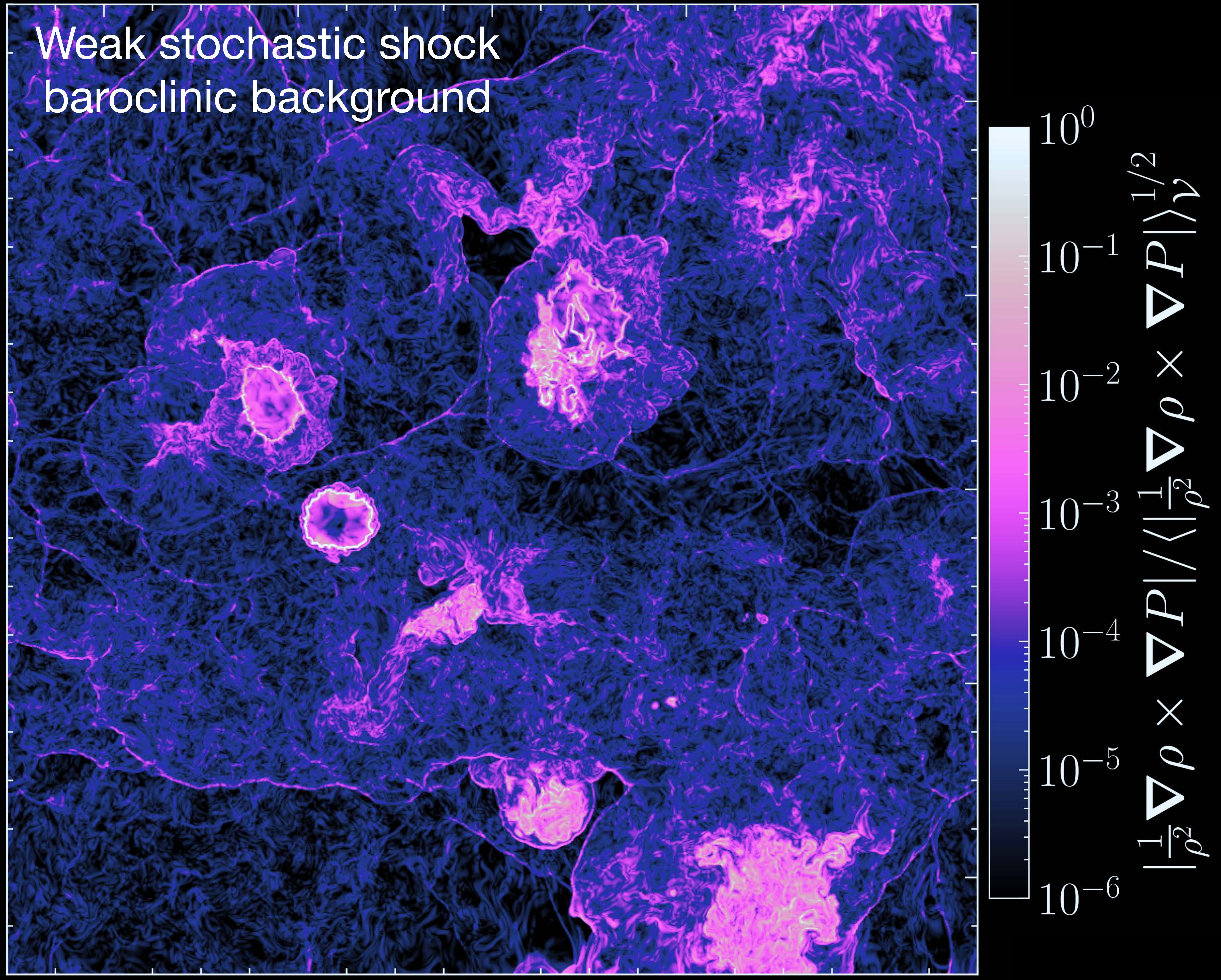


Vorticity sources

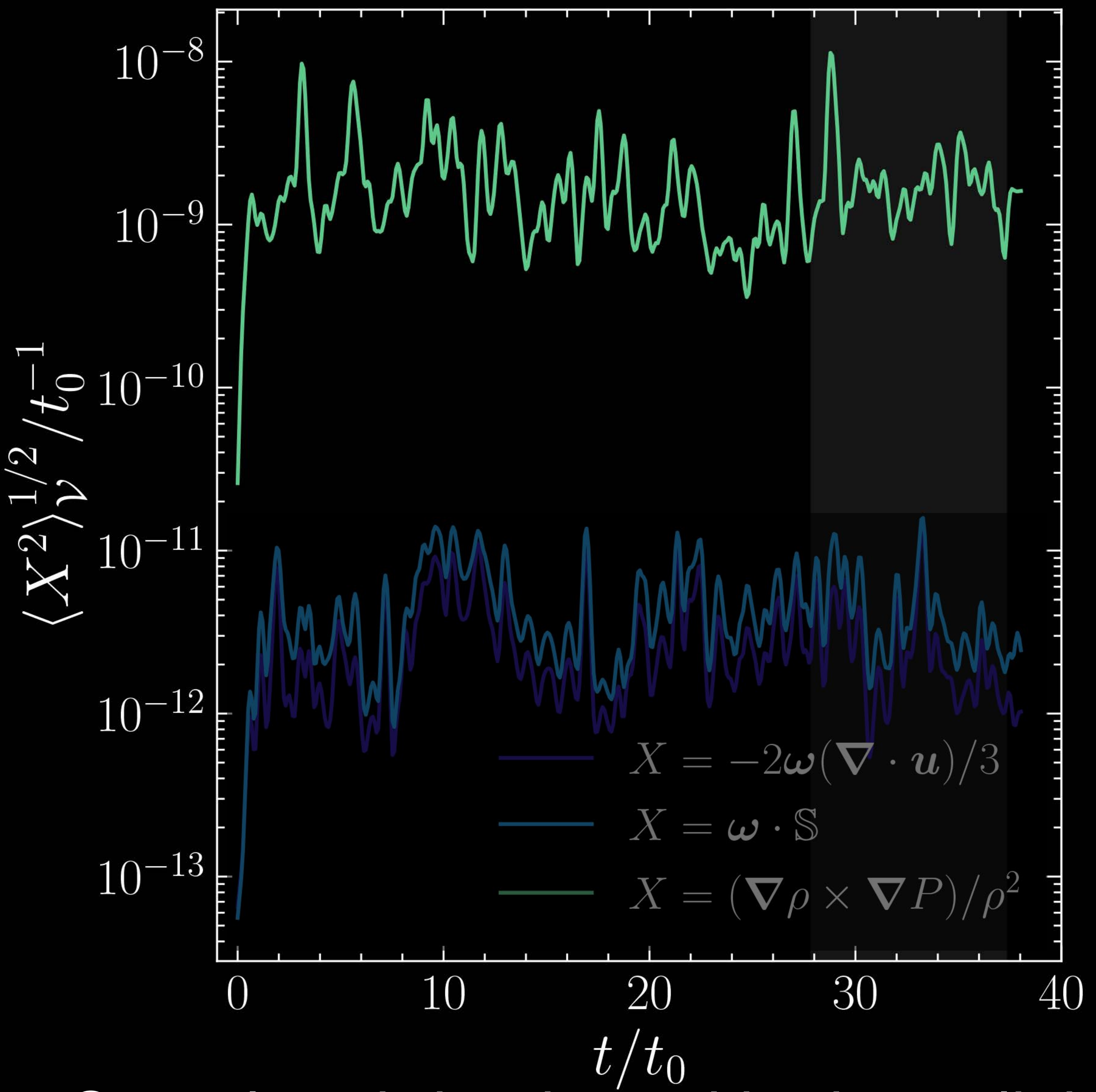


Completed dominated by baroclinic battery at all times.

Where does the incompressible turbulence come from?



Vorticity sources



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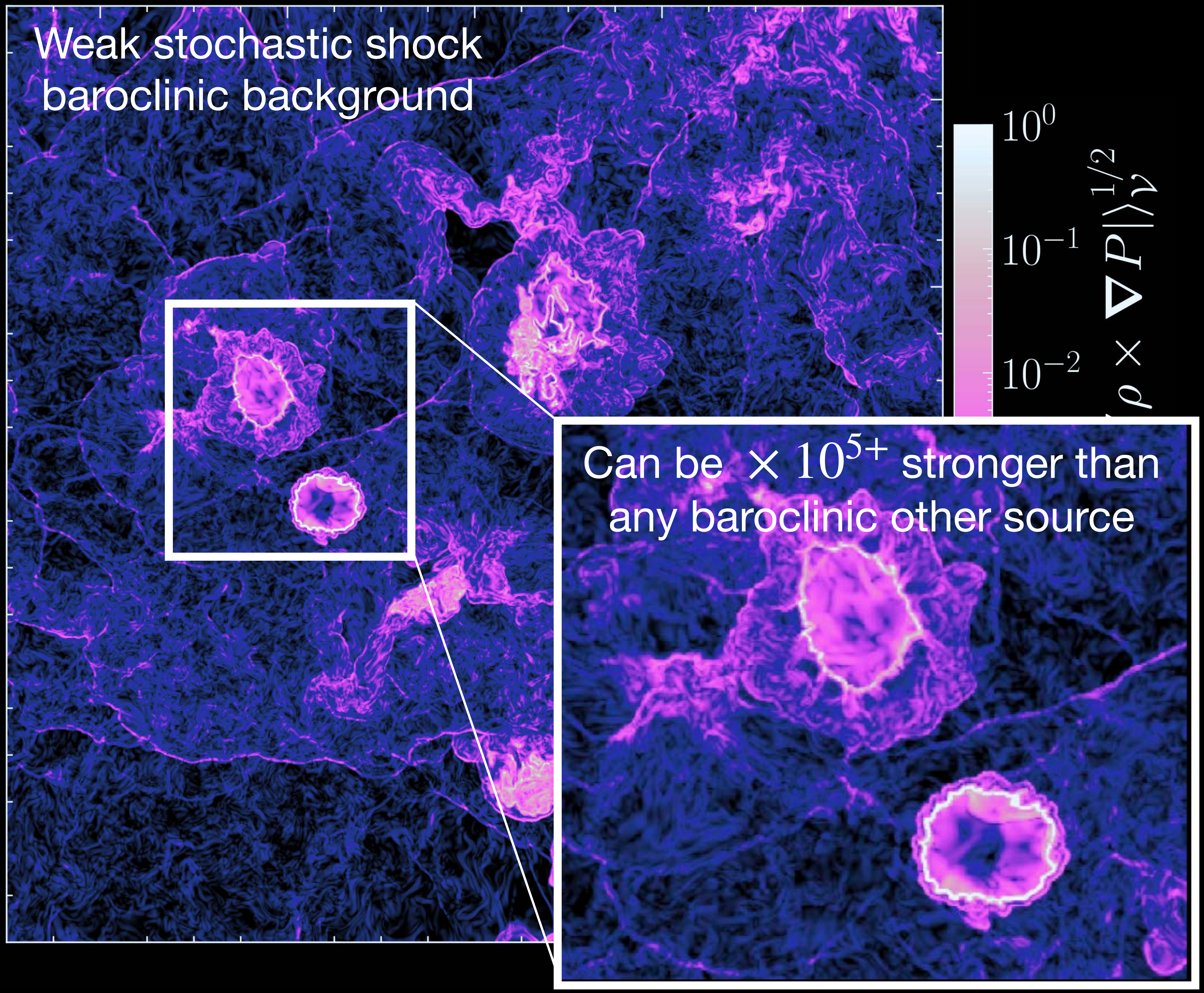
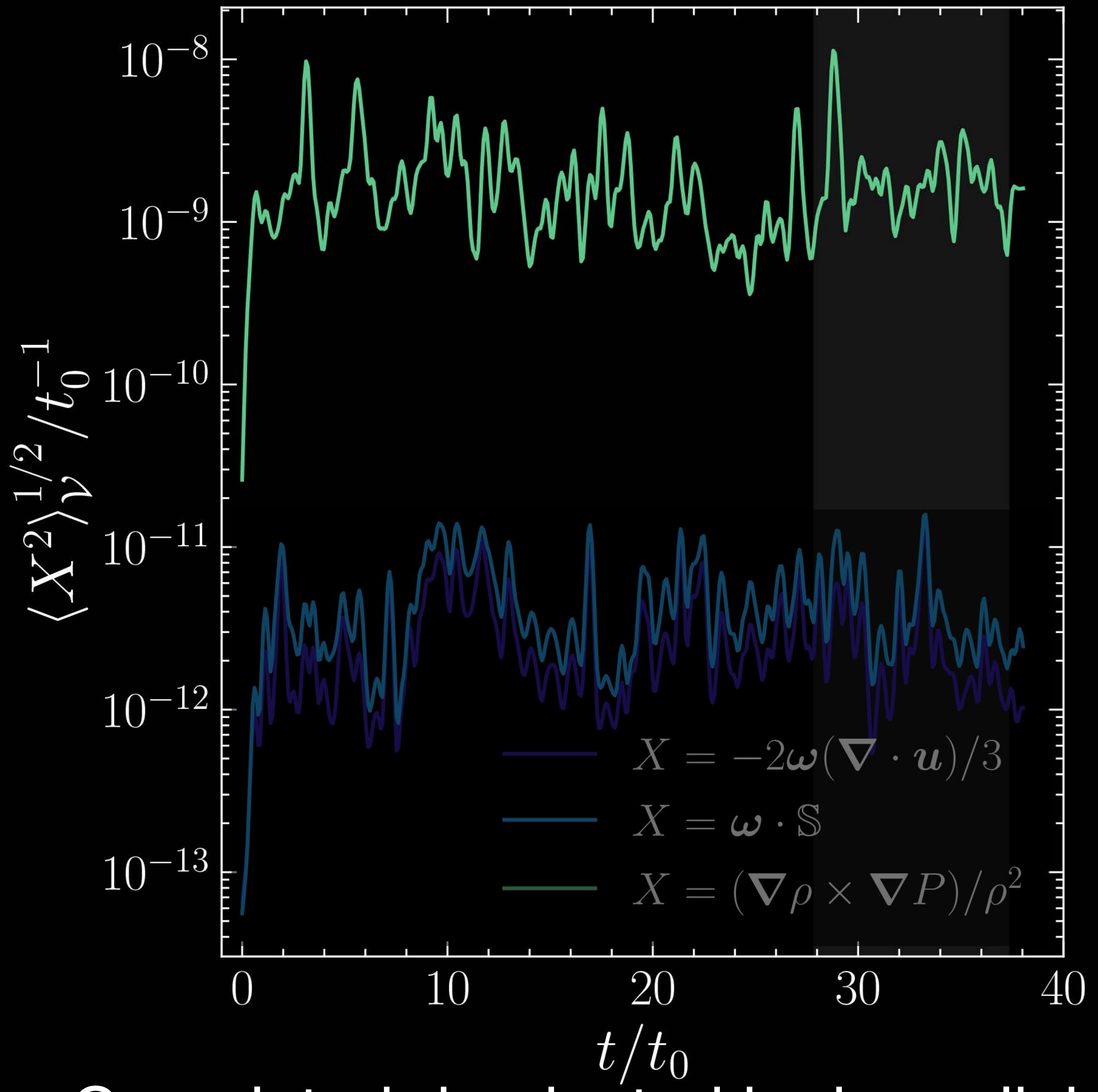


Vorticity sources



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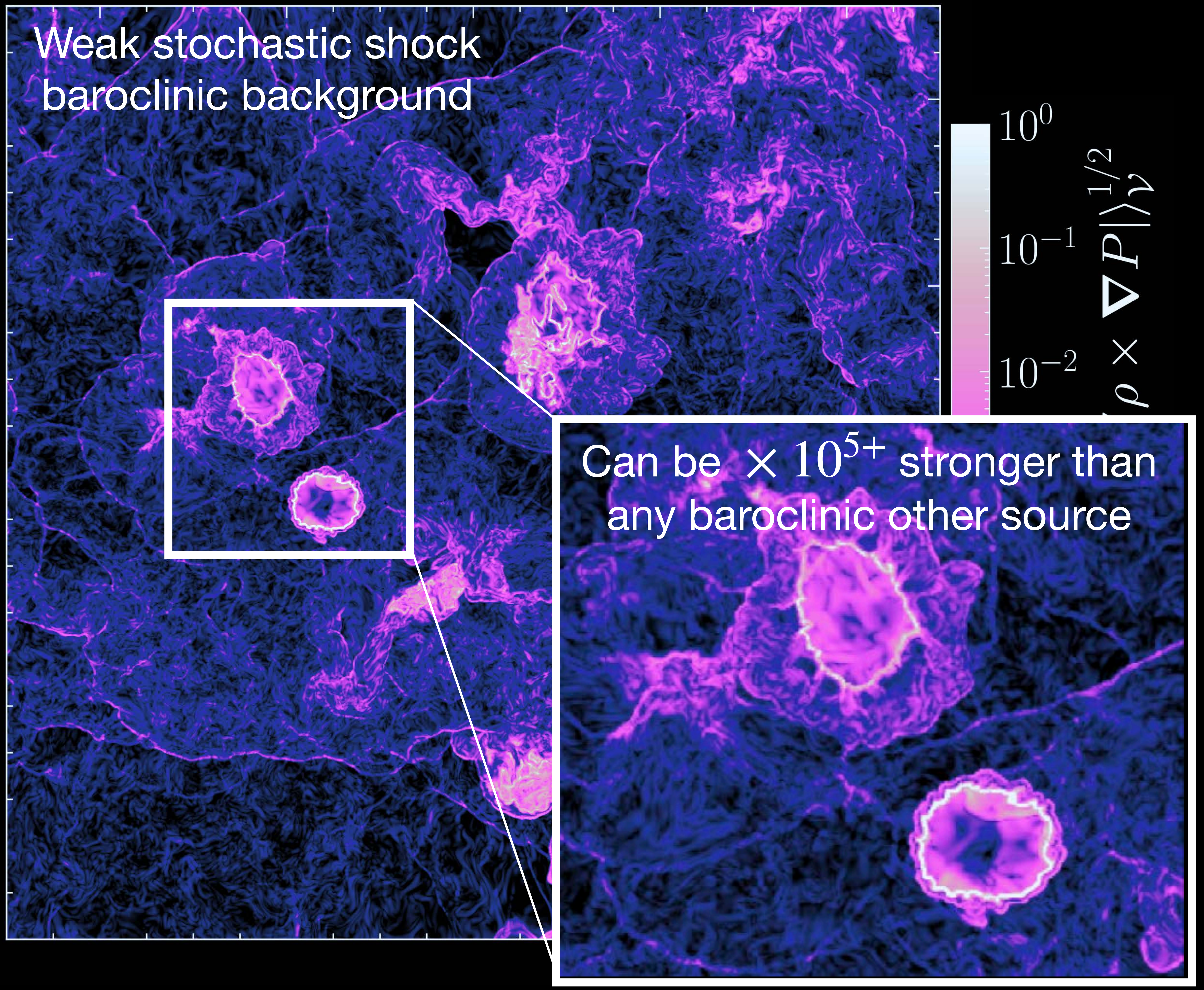
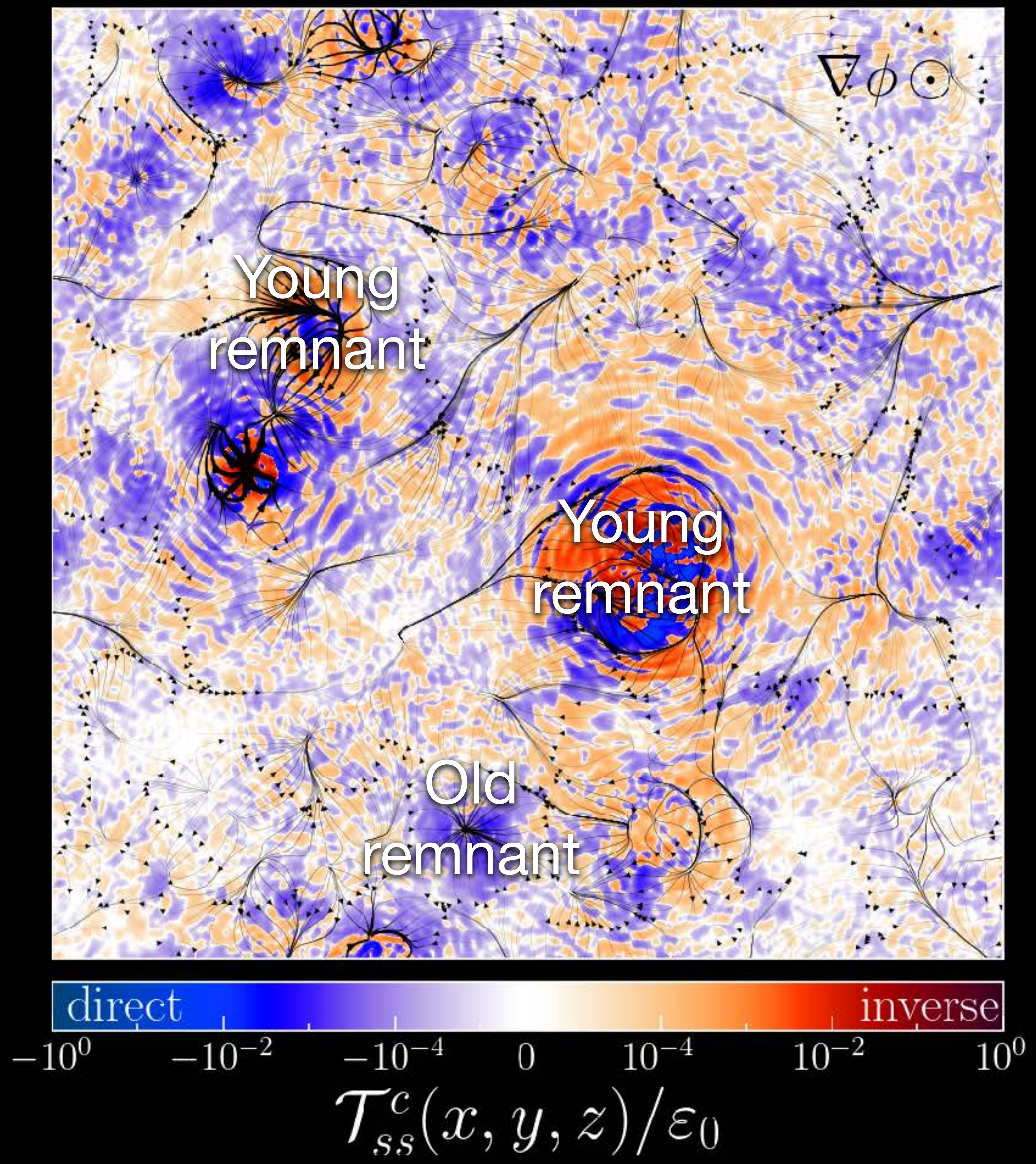
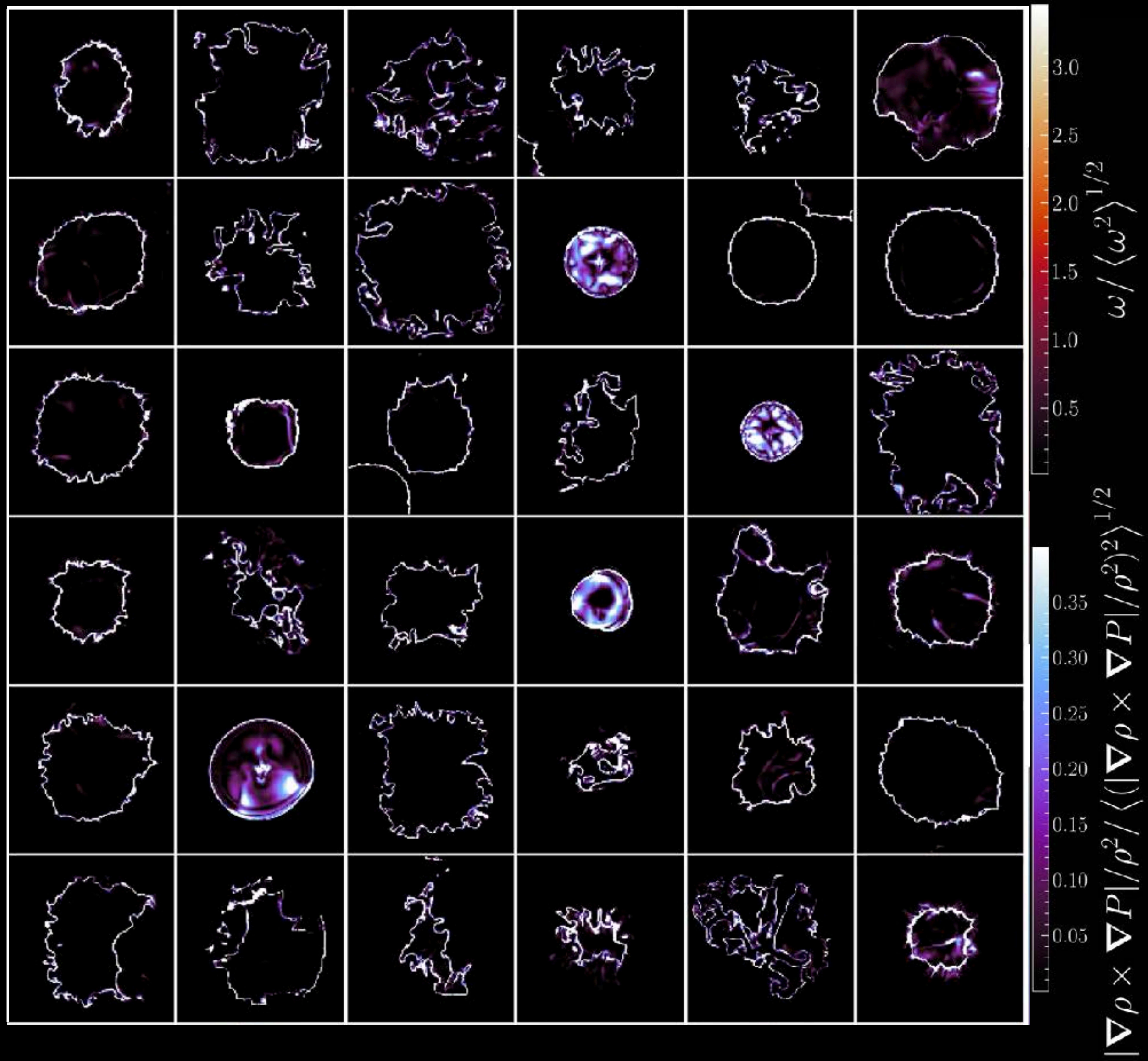




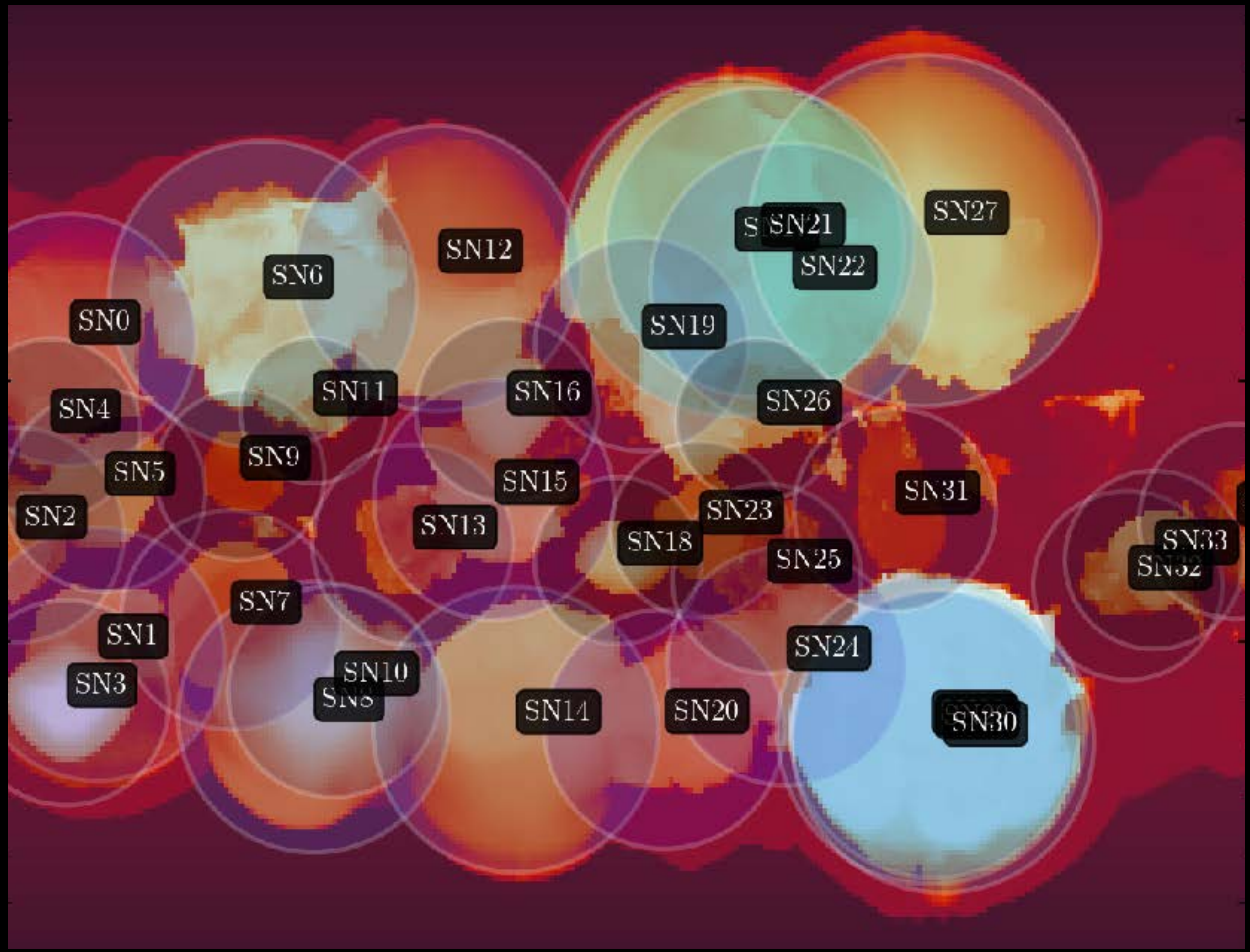
Completed dominated by baroclinic battery at all times.

Where does the incompressible turbulence come from?

Beattie (2025; submitted ApJL) Supernovae drive large-scale incompressible turbulence from small-scale instabilities



Cluster all SNR in 3D to extract local statistics



Where does the incompressible turbulence come from?

Assuming isotropic, stationary, high-Re

$$\mathscr{P}_{\omega\mathrm{B}}(k) \approx -\frac{d\,\Pi_\omega(k)}{dk}$$

where

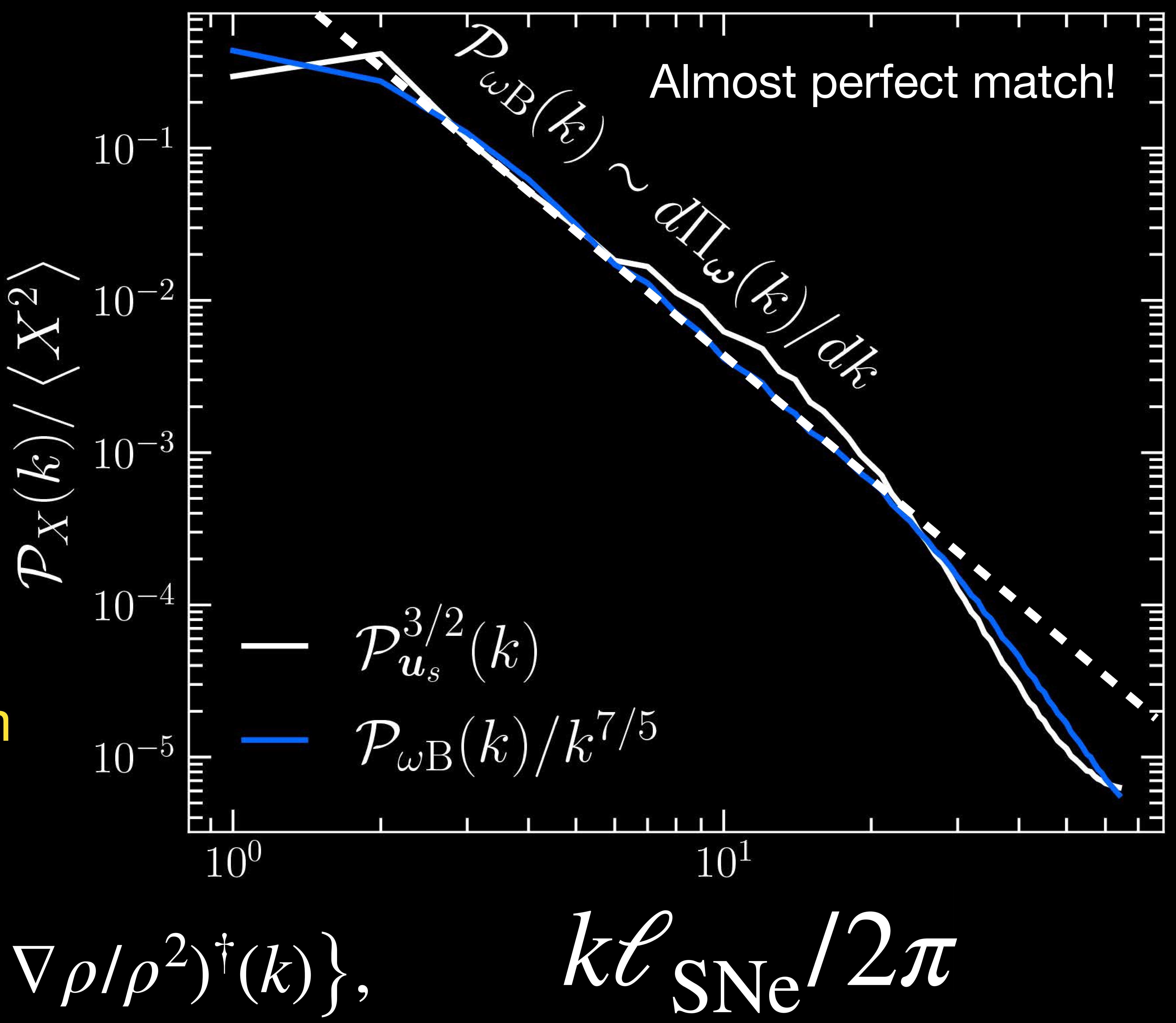
$$\frac{d\,\Pi_\omega(k)}{dk} \sim \frac{1}{t_{\mathrm{turb}}}\int d\Omega_k\,k^2\,\omega(k)\cdot\omega^\dagger(k)$$

$$\sim \frac{d\,\Pi_\omega(k)}{dk} \sim \frac{k^2\mathscr{P}_{u_s}(k)}{t_{\mathrm{turb}}}$$

relation between incompressible mode spectrum and baroclinic source spectrum

$$\mathscr{P}_{\omega\mathrm{B}}(k) \sim k^{7/5}\mathscr{P}^{3/2}_{u_s}(k)$$

$$\mathscr{P}_{\omega\mathrm{B}}(k) = \int k^2 d\Omega_k\,\mathrm{Re}\left\{\omega(k)\cdot(\nabla P\times\nabla\rho/\rho^2)^\dagger(k)\right\},$$



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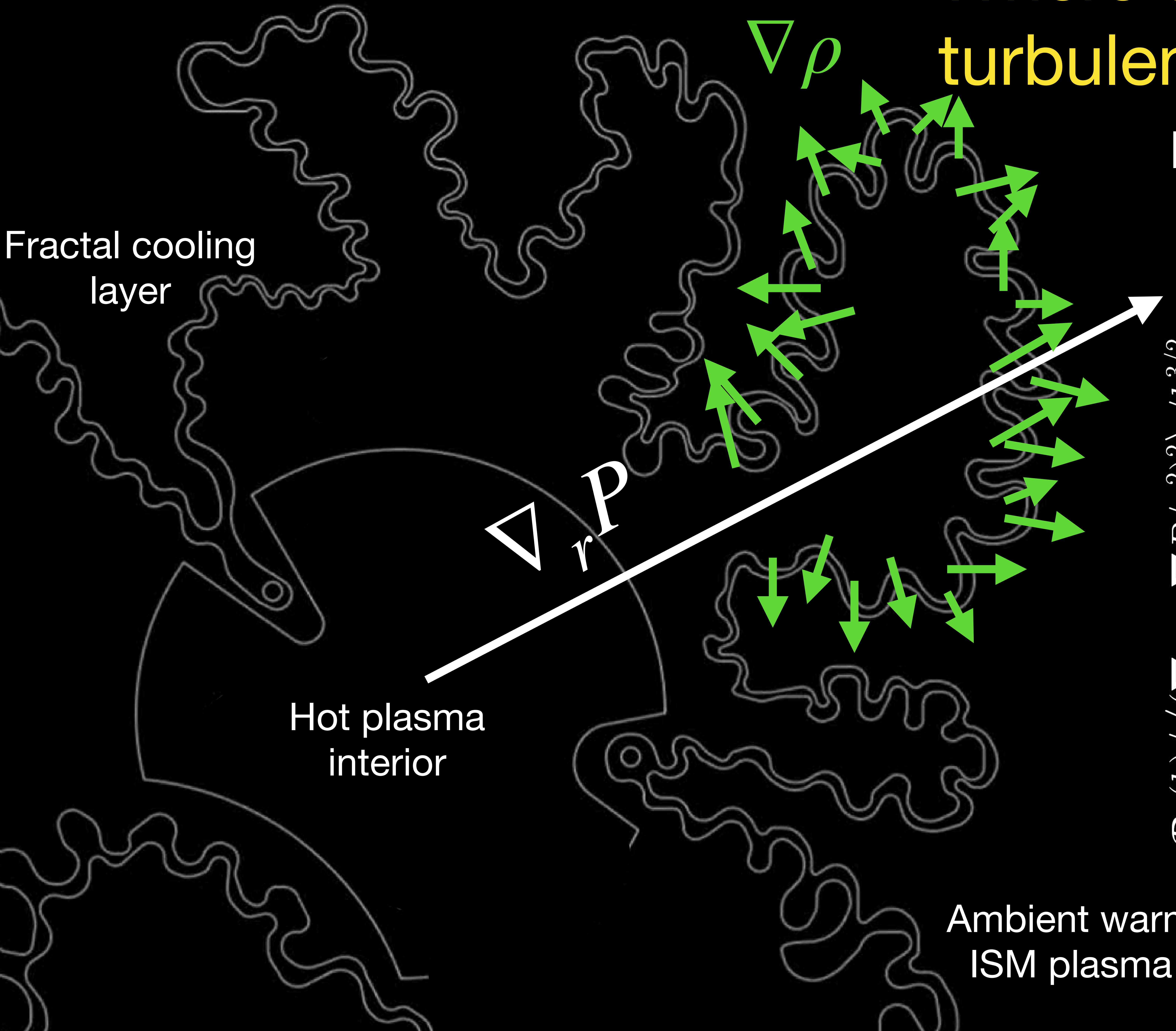
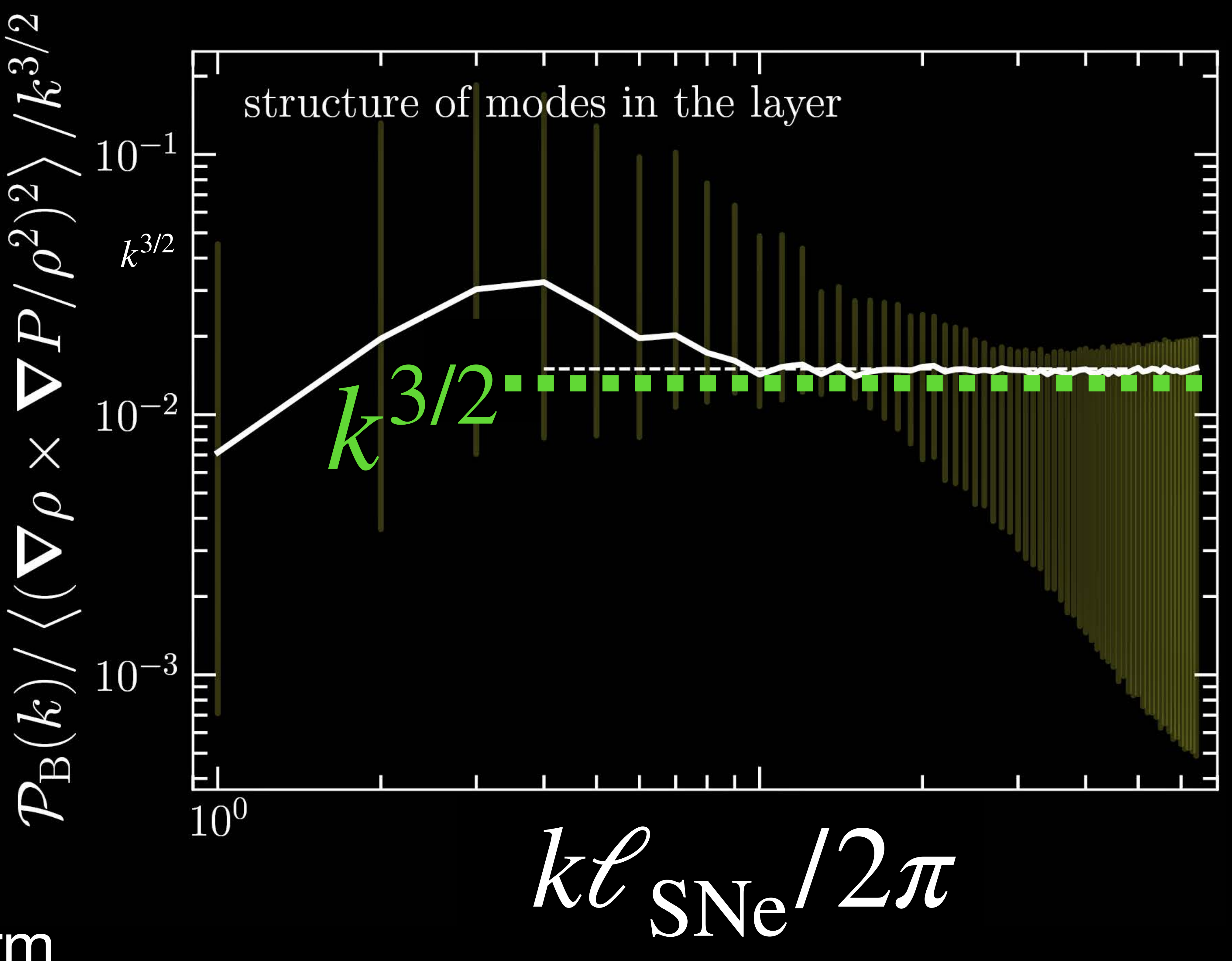


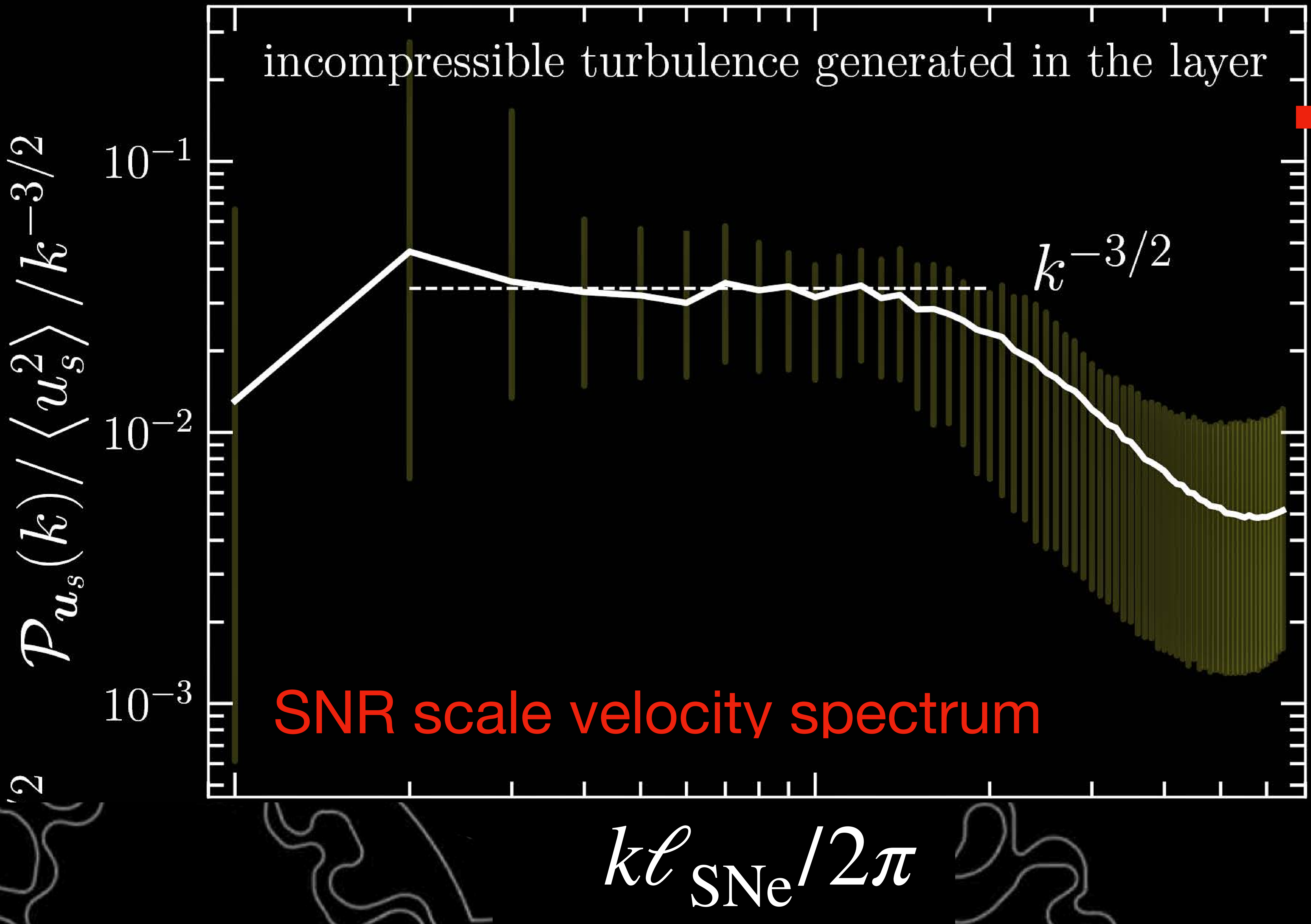
Barcoclinic structure admits to a folded field spectrum $\sim k^{3/2}$ (Kazanstev, 1968)



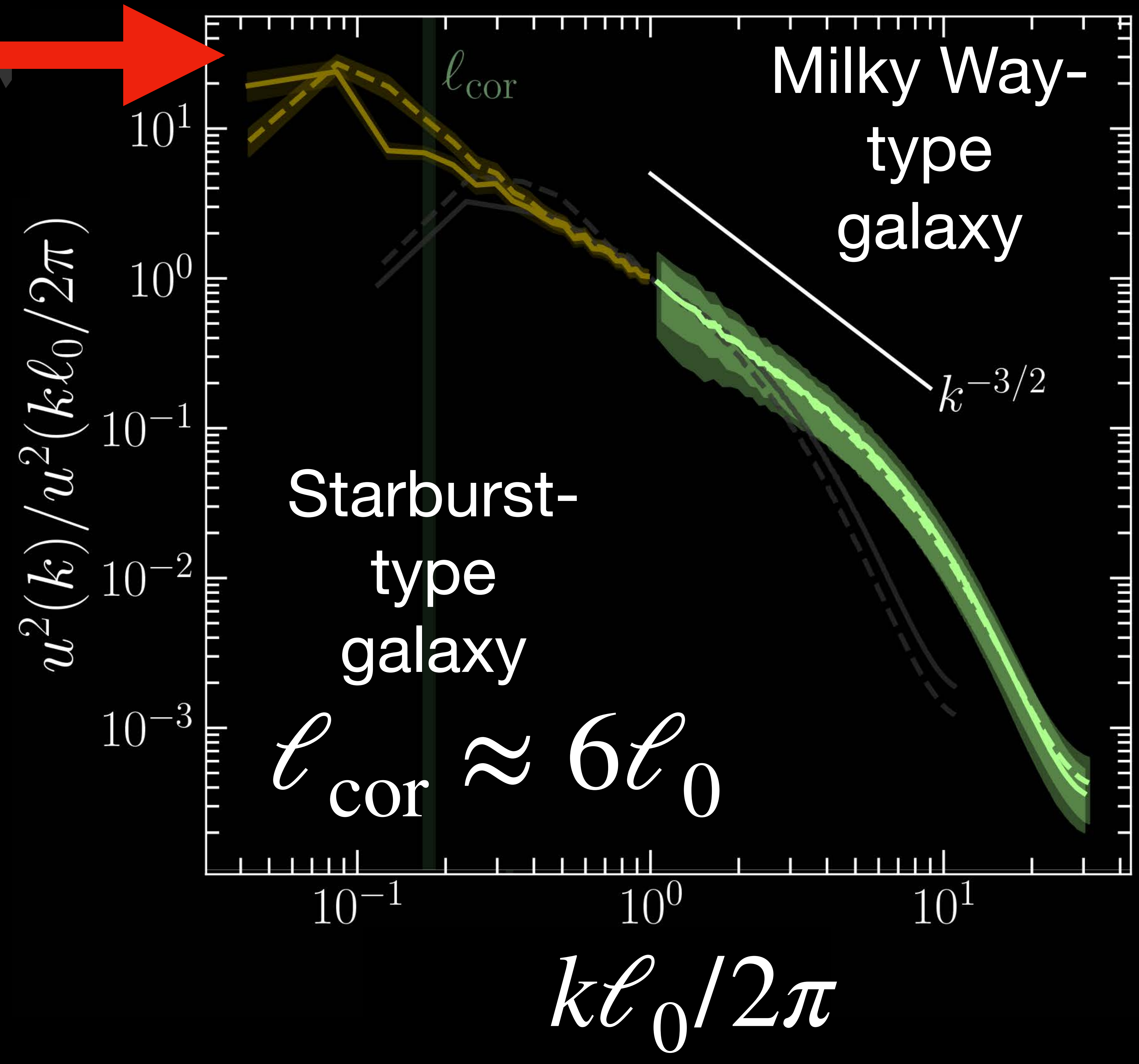
Modified schematic from Lancaster+2021

Incompressible modes generated on SNR scale imprinted on all scales

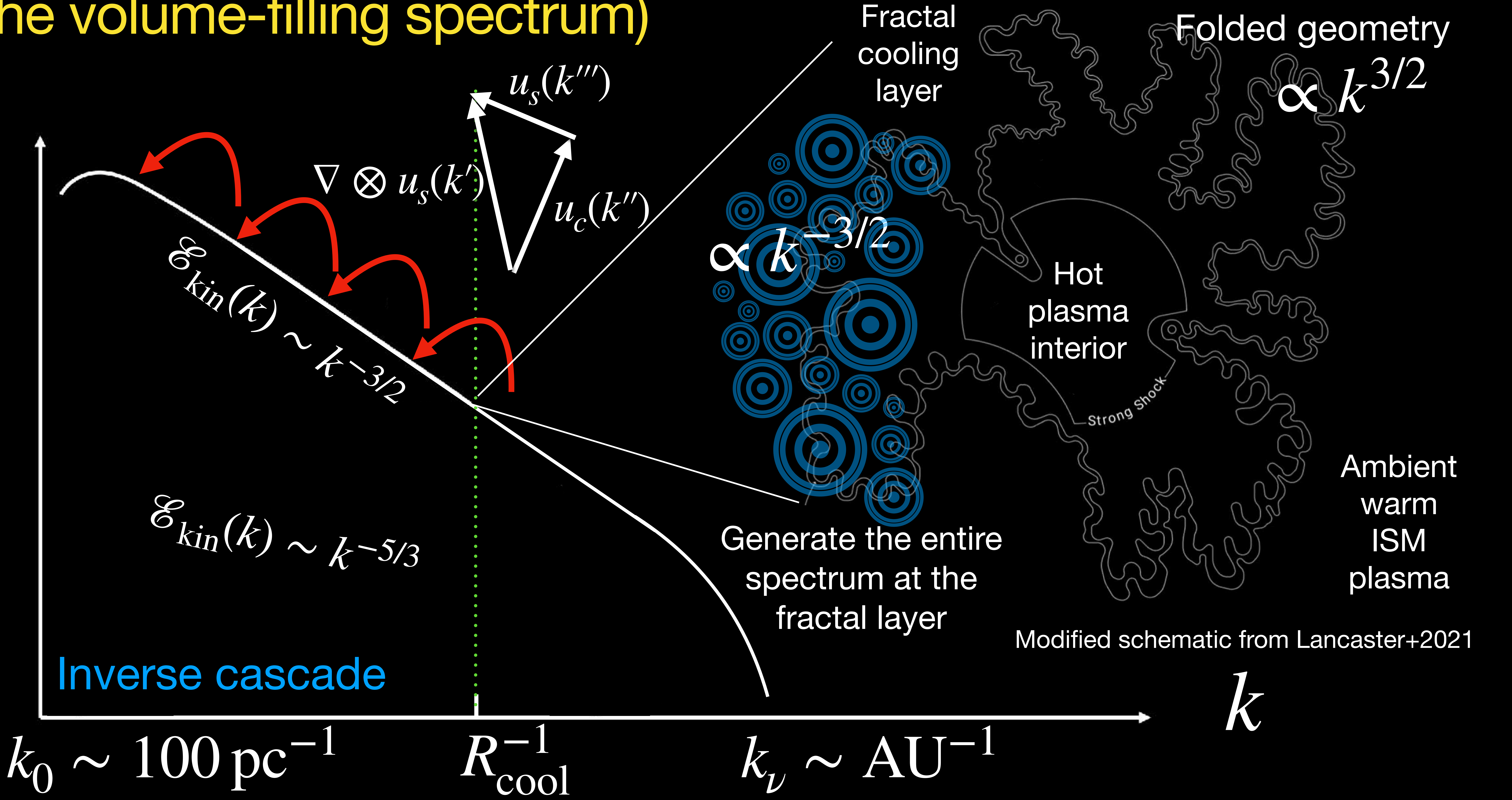
(scales larger than gaseous scale height!!!)



Global velocity spectrum



The SNe-driven warm ionized medium spectrum (the volume-filling spectrum)



Thanks, questions?

james.beattie@princeton. @astro_magnetism

Come chat to me about magnetized turbulence and dynamo!

Beattie+ (*Nature Astronomy, 2025*). The spectrum of magnetized turbulence in the interstellar medium

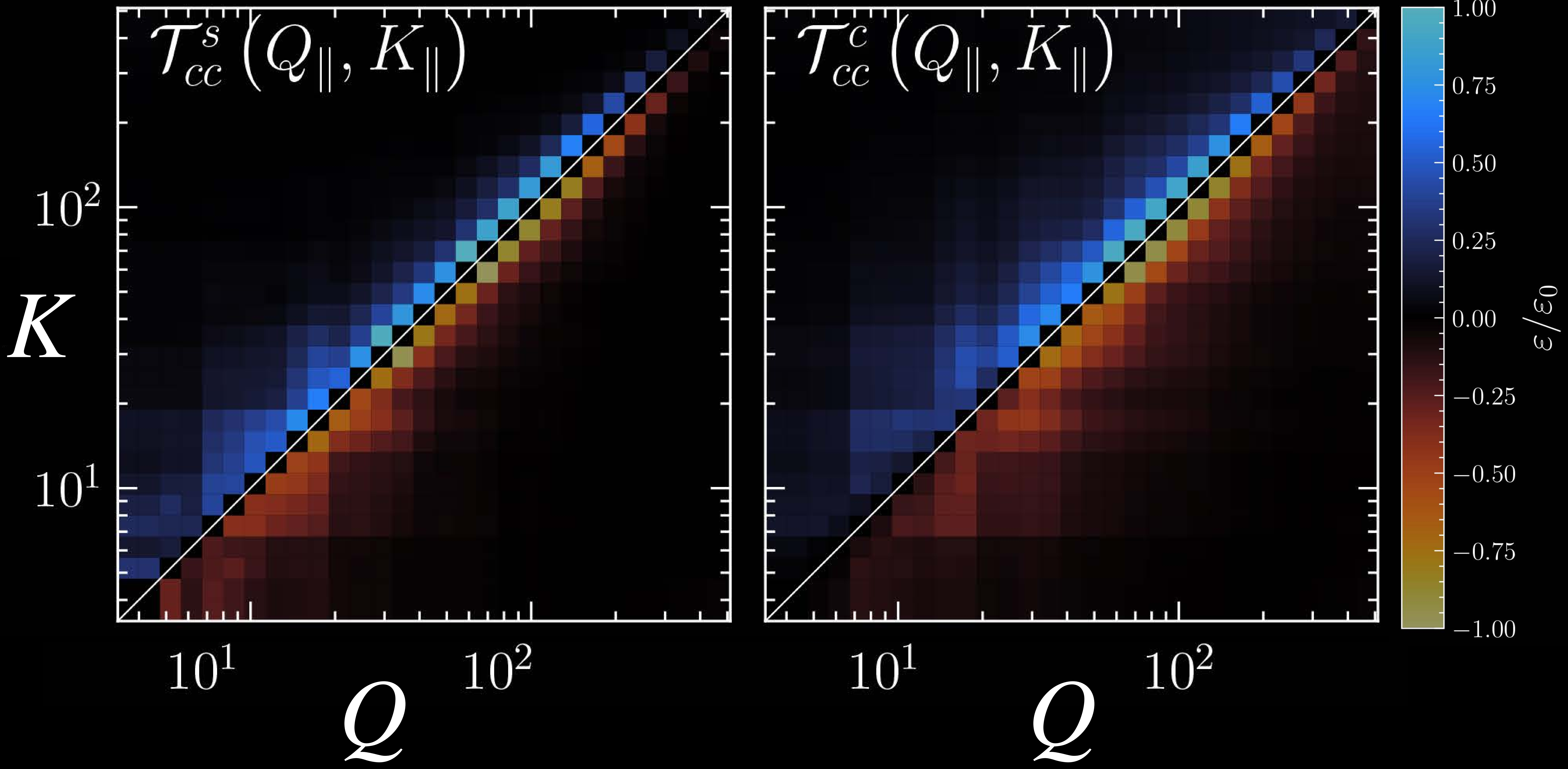
Beattie & *Bhattacharjee* (*submitted PRL, 2025*). Scale-dependent alignment in compressible magnetohydrodynamic turbulence

Beattie+ (MNRAS, *2025*). Taking control of compressible modes: bulk viscosity and the turbulent dynamo

Kriel, **Beattie**+ (*submitted PRL, 2025*). Scale-dependent alignment in compressible magnetohydrodynamic turbulence

Sampson, **Beattie+** (*submitted ApJL, 2025*). Cosmic ray and plasma coupling for isothermal supersonic turbulence in the magnetized interstellar medium

The compressible modes ($u_c \to u_c$ cascade)



The incompressible turbulence ($u_s \to u_s$ cascade)