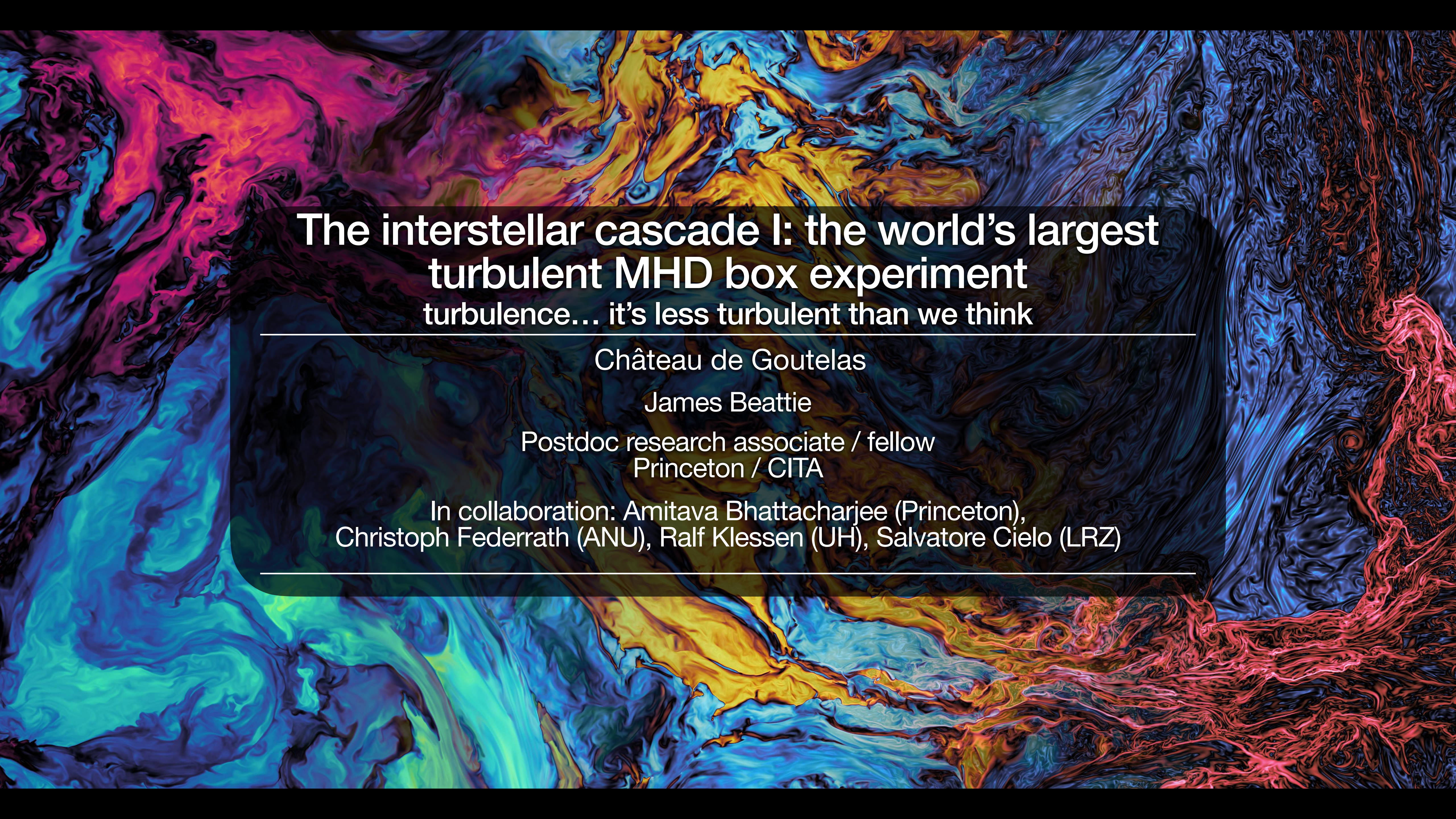




The interstellar cascade I: the world's largest turbulent MHD box experiment

turbulence... it's less turbulent than we think

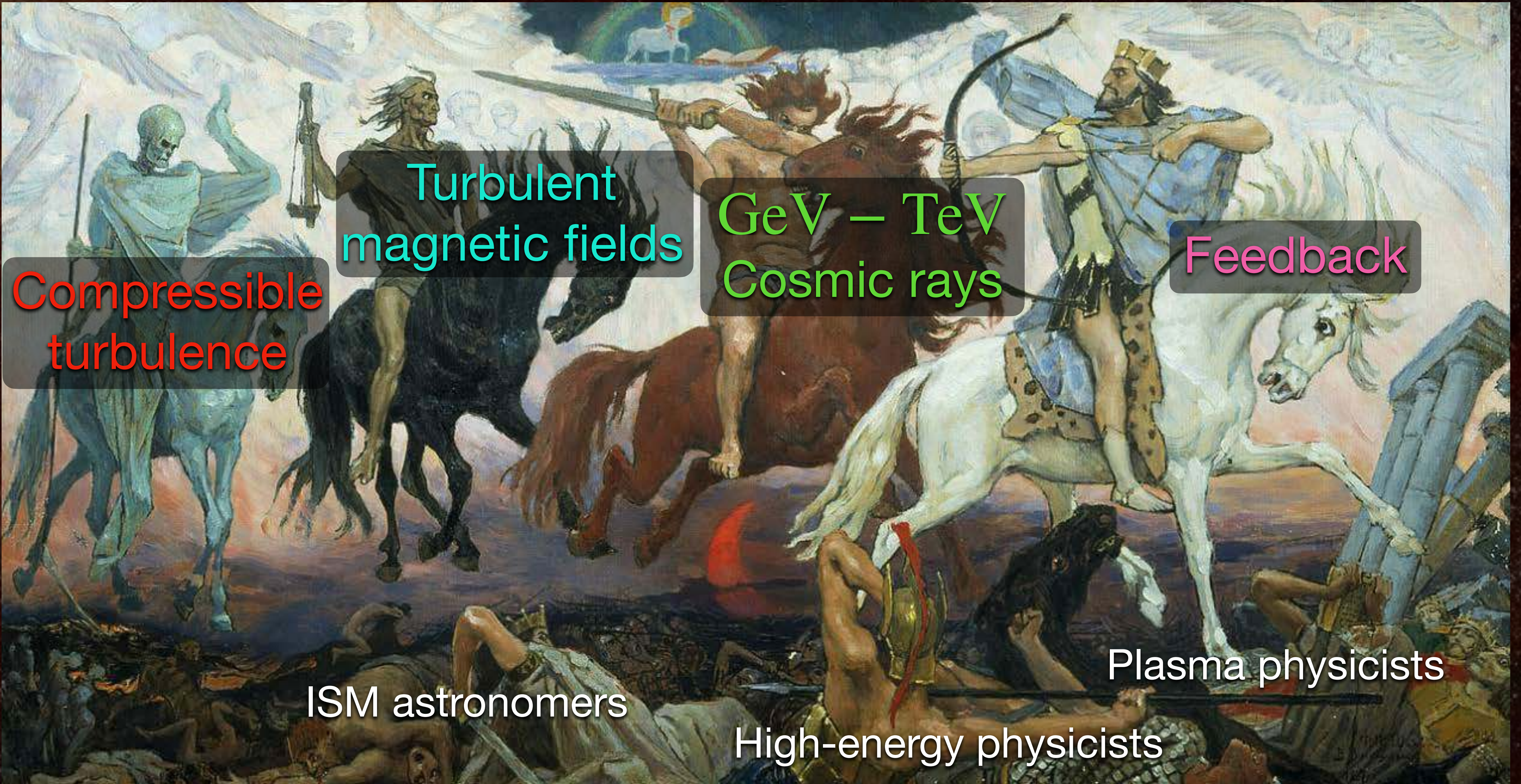
Château de Goutelas

James Beattie

Postdoc research associate / fellow
Princeton / CITA

In collaboration: Amitava Bhattacharjee (Princeton),
Christoph Federrath (ANU), Ralf Klessen (UH), Salvatore Cielo (LRZ)

My four horsemen of ISM physics



Compressible
turbulence

Turbulent
magnetic fields

GeV – TeV
Cosmic rays

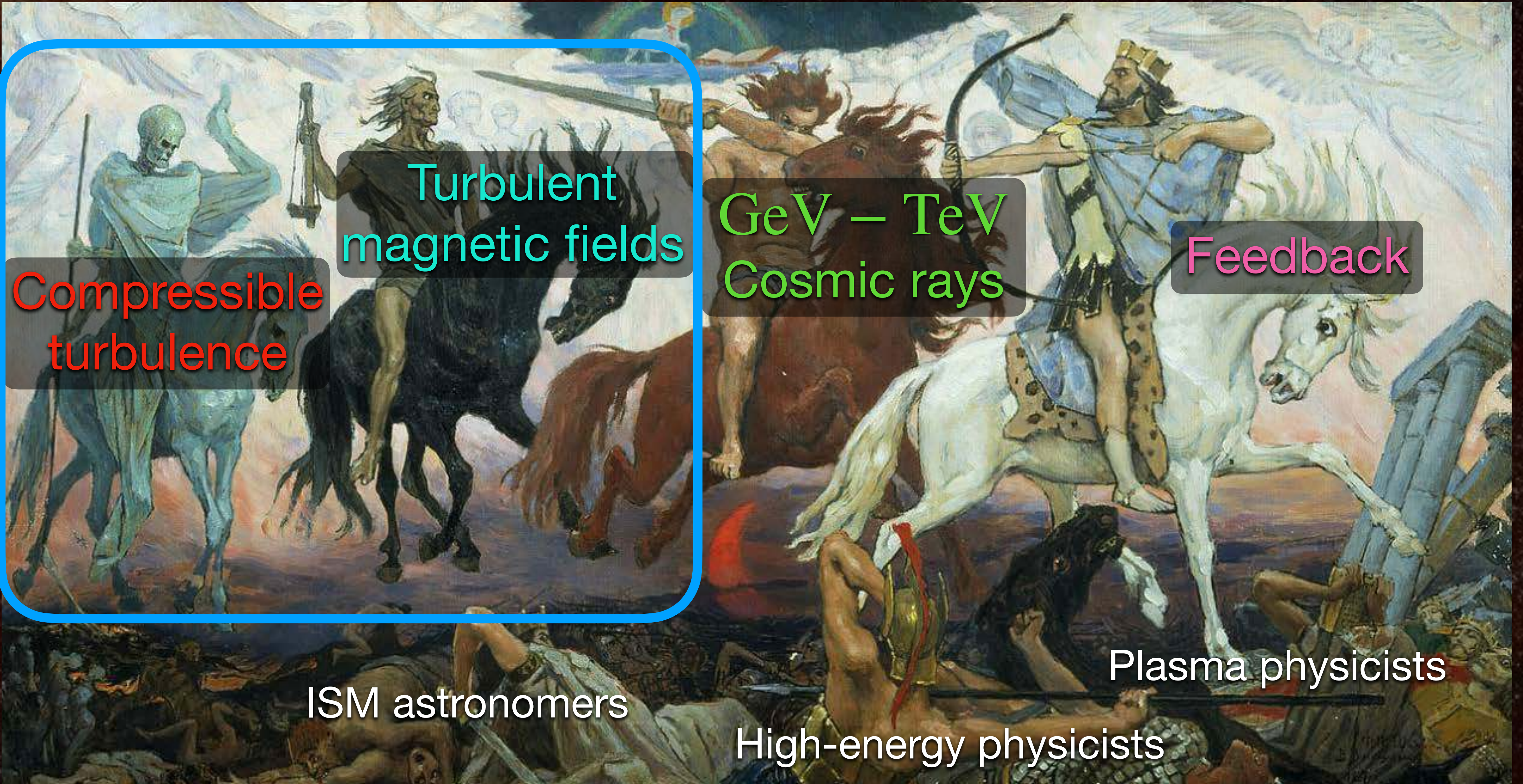
Feedback

ISM astronomers

High-energy physicists

Plasma physicists

My four horsemen of ISM physics



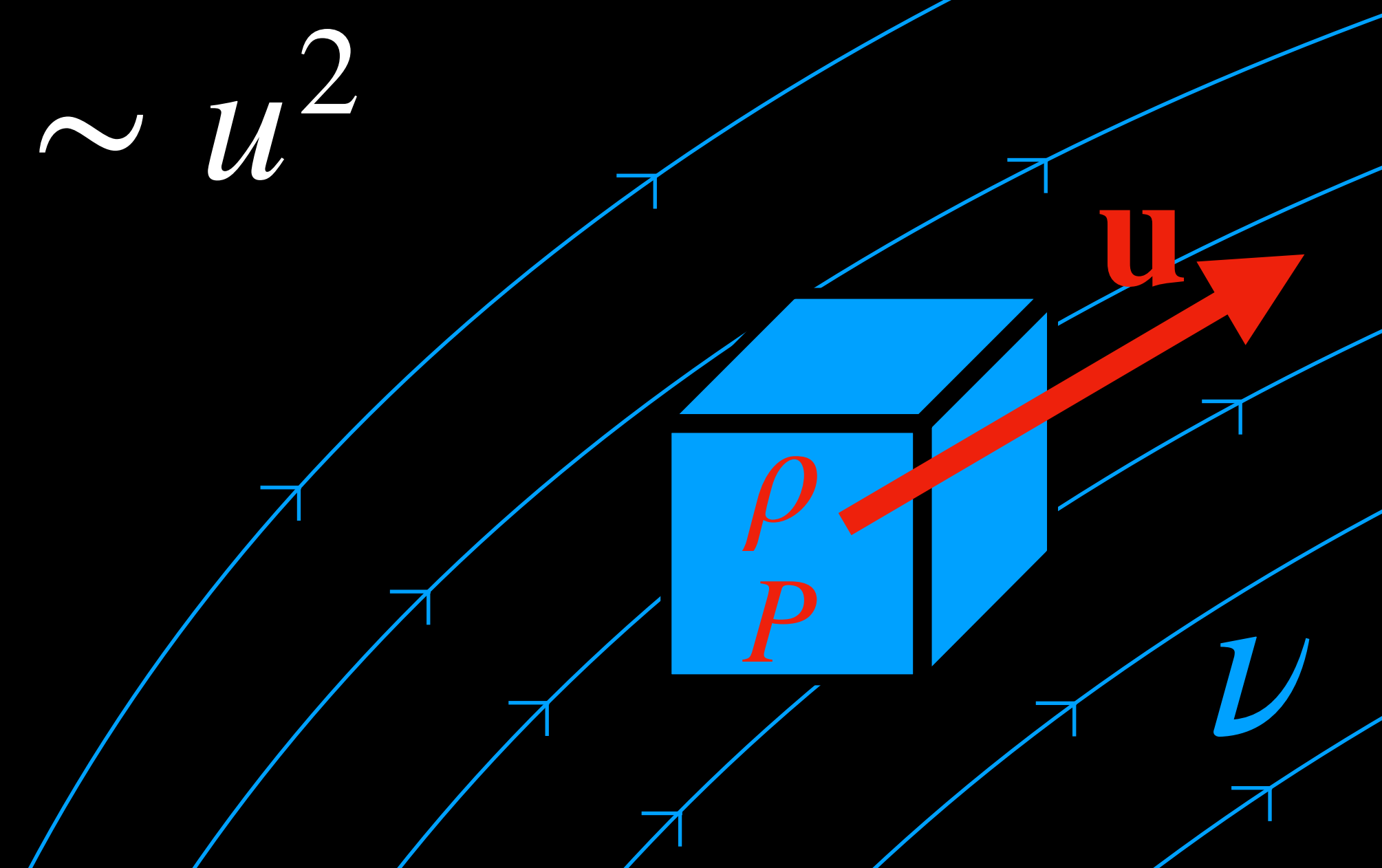
Turbulence

What is it (hand wavy)?

Momentum conservation for a hydrodynamical fluid element

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) = -\nabla \cdot P\mathbb{I} + 2\nu \nabla \cdot (\rho \mathcal{S})$$
$$\mathbf{u} \otimes \mathbf{u} = u_i u_j \sim u^2$$

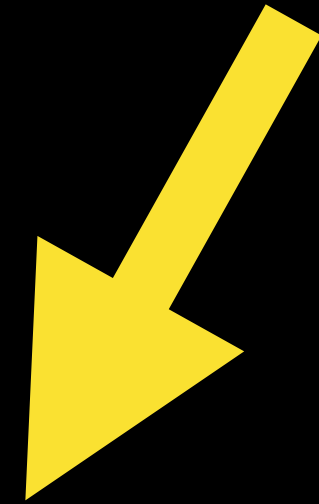
$$\lambda_{\text{mfp}}/L \ll 1$$



Turbulence

What is it (hand wavy)?

Reynolds stress



$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) = -\nabla \cdot P\mathbb{I} + 2\nu \nabla \cdot (\rho \mathcal{S})$$
$$\mathbf{u} \otimes \mathbf{u} = u_i u_j \sim u^2$$

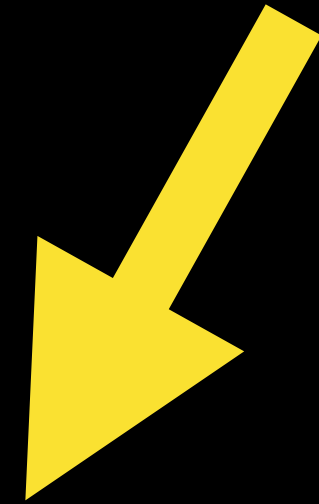


Viscous stress

Turbulence

What is it (hand wavey)?

quadratic nonlinearity



$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) = -\nabla \cdot P\mathbb{I} + 2\nu \nabla \cdot (\rho \mathcal{S})$$
$$\mathbf{u} \otimes \mathbf{u} = u_i u_j \sim u^2$$



Smooths out nonlinear things in the fluid

Turbulence

What is it (hand wavy)?

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) = - \nabla \cdot P \mathbb{I} + 2\nu \nabla \cdot (\rho \mathcal{S})$$

Creating nonlinear things in the fluid

$$\text{Re} = \frac{|\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})|}{|2\nu \nabla \cdot (\rho \mathcal{S})|} \sim \frac{UL}{\nu}$$

Smoothing out nonlinear things in the fluid

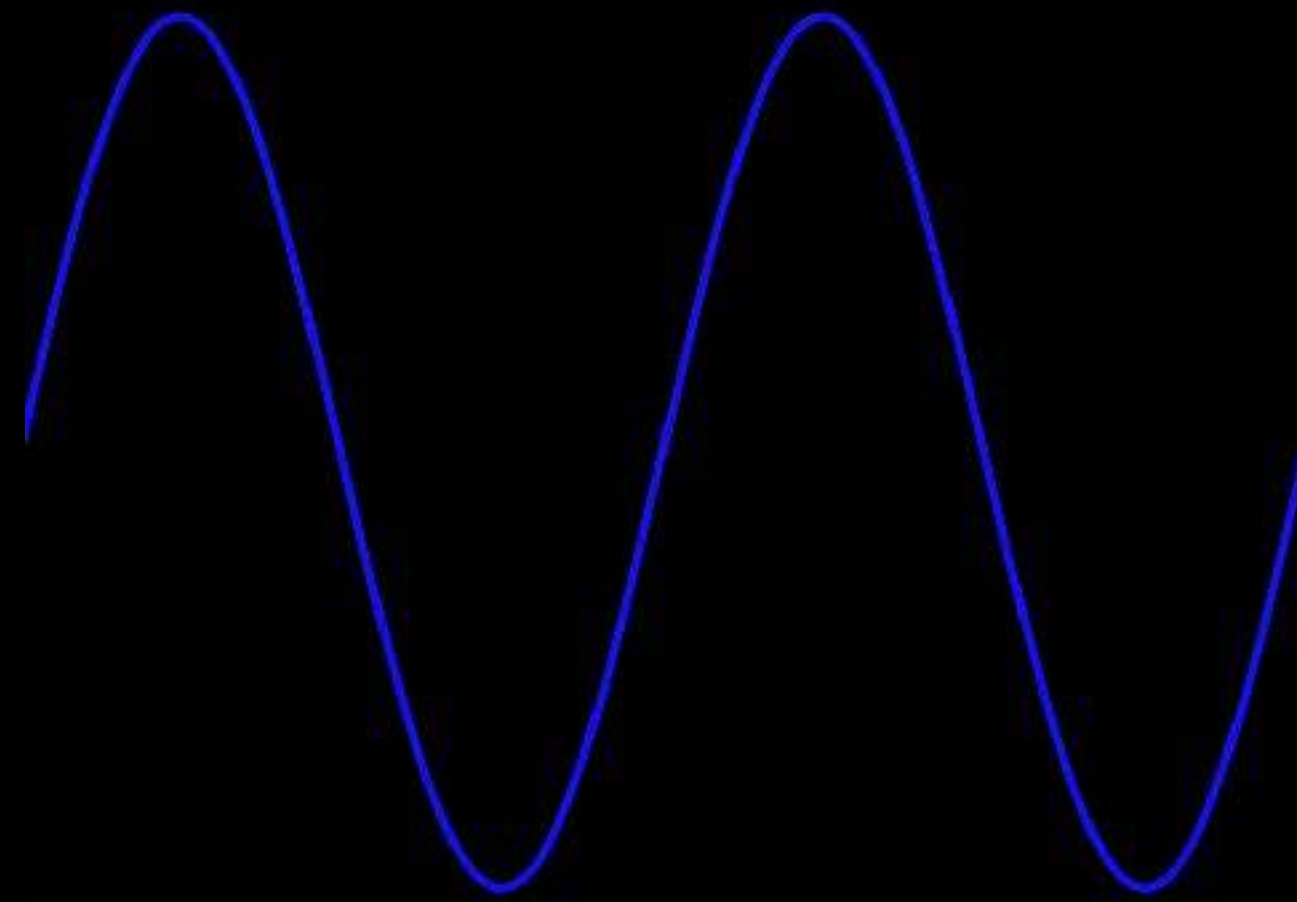
Turbulence

What does $\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})$ want to do?

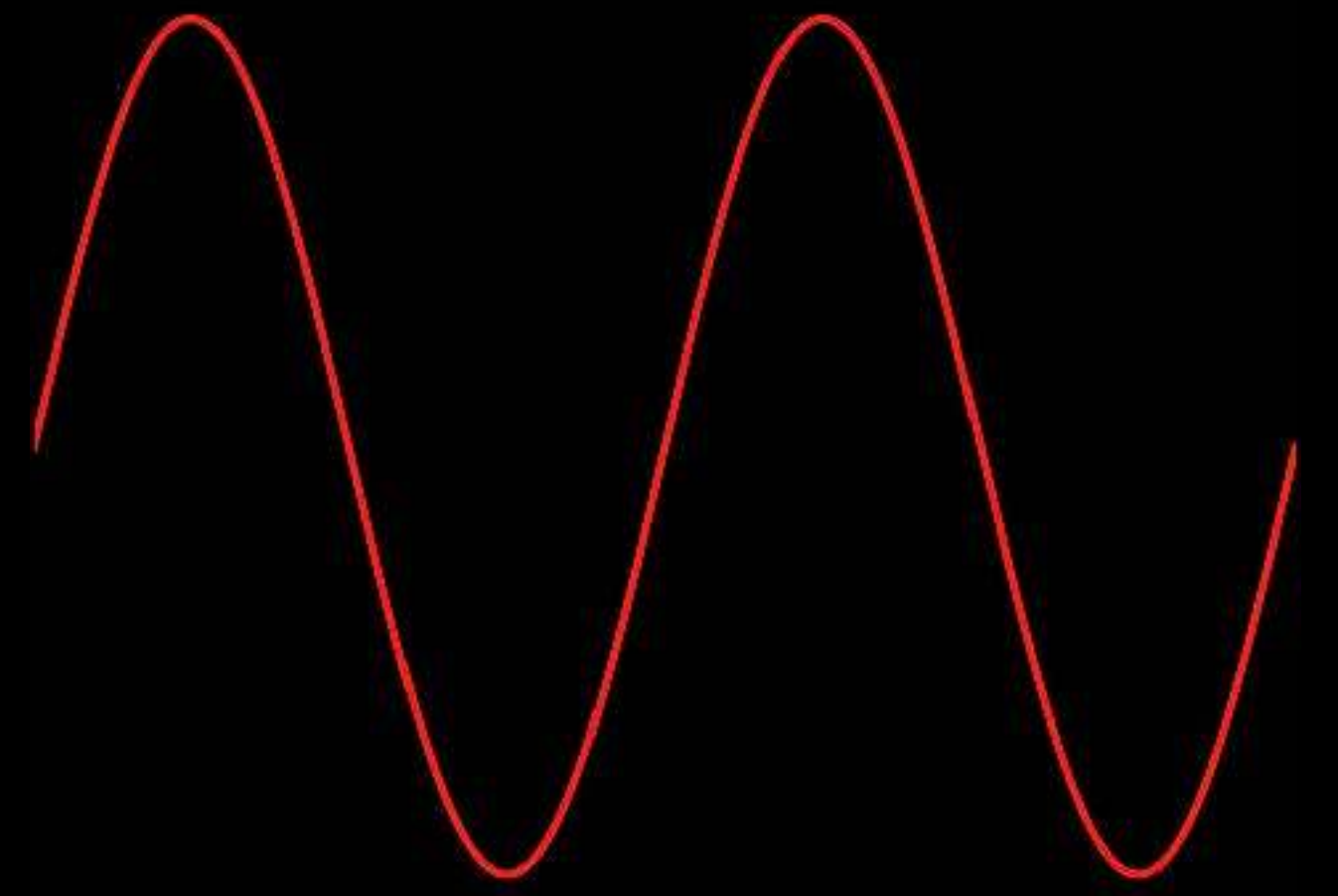
Consider two waves

$$u_1(x) = \sin(k_1 x)$$

$$u_2(x) = \sin(k_2 x)$$



$u_1(x)$



$u_2(x)$

$$k_1 = k_2$$

Turbulence

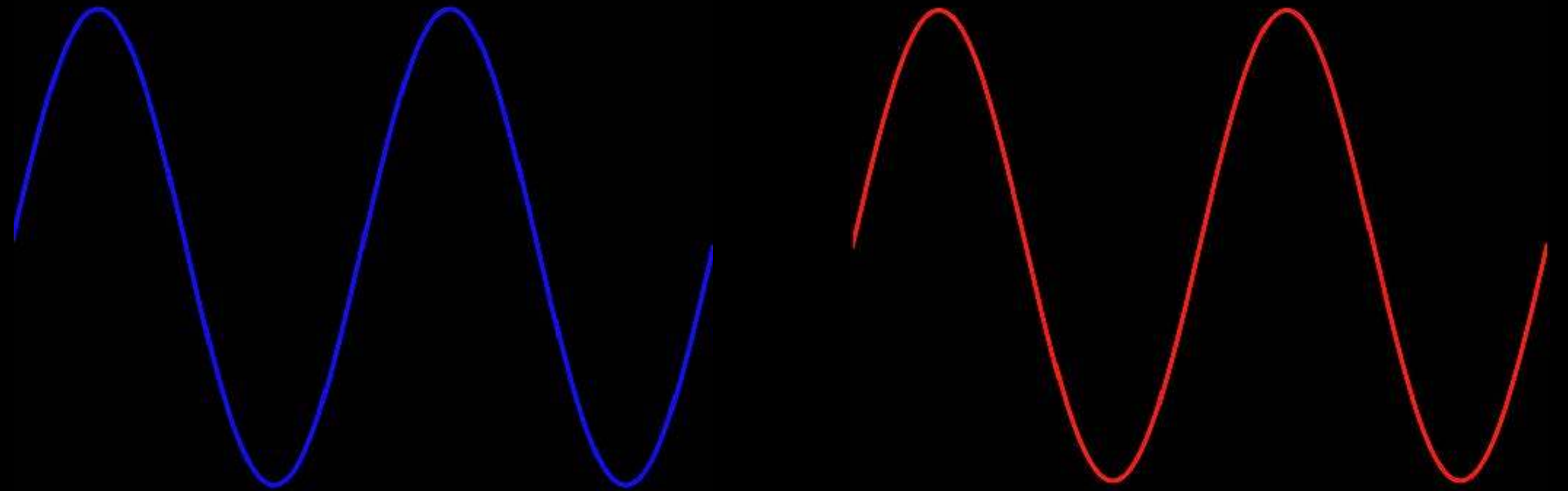
What does $\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})$ want to do?

$$k_1 = k_2$$

Consider two waves

$$u_1(x) = \sin(k_1 x)$$

$$u_2(x) = \sin(k_2 x)$$



The nonlinear term creates a new wave

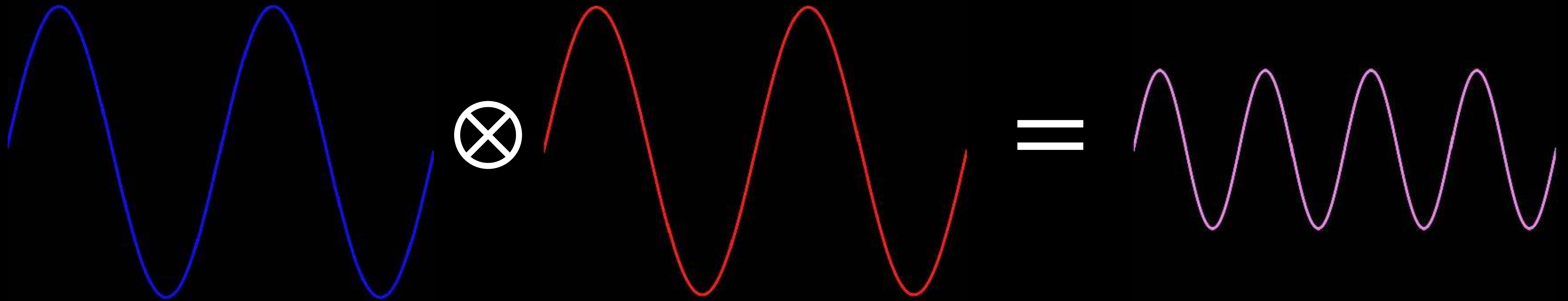
$$\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) \sim u_1 \partial_x u_2 = k_2 \sin(k_1 x) \cos(k_2 x) \propto \sin(k_3 x)$$

$$k_1 + k_2 = k_3 \quad (\text{momentum conservation})$$

Turbulence

What does $\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})$ want to do — cascade!!!

$$\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})$$

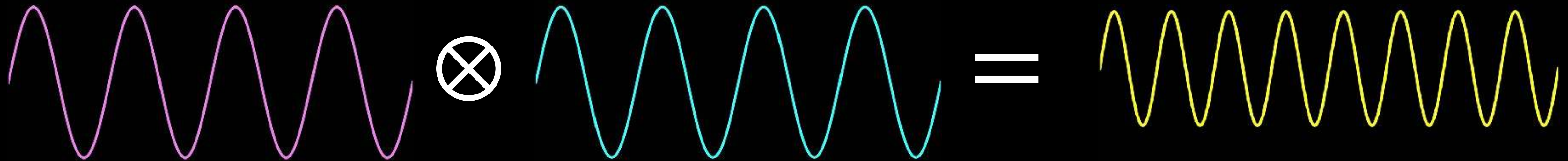


$$k_1 + k_2 = k_3$$

Turbulence

What does $\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})$ want to do — cascade!!!

$$\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})$$

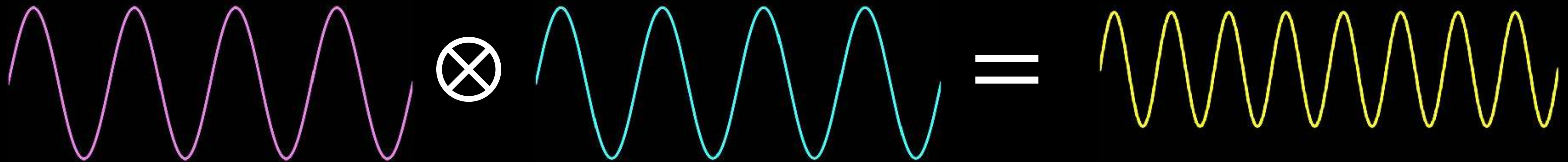


$$k_3 + k_4 = k_5$$

Turbulence

What does $\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})$ want to do — cascade!!!

$$\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})$$



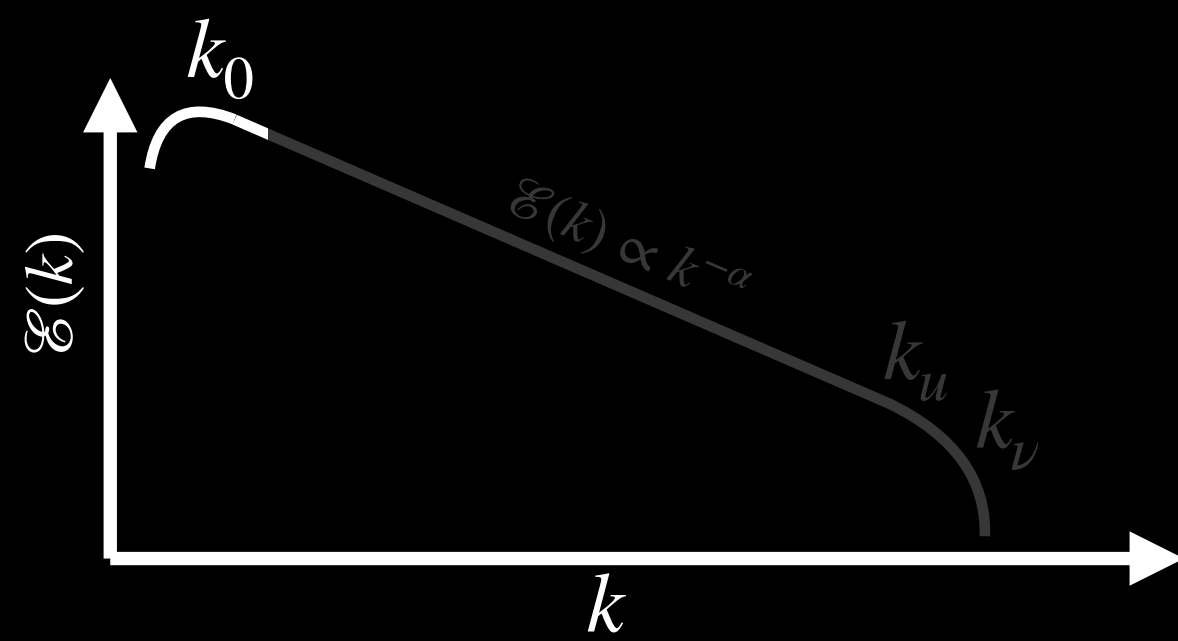
The diagram shows a purple sine wave and a cyan sine wave separated by a circled 'X' symbol, followed by an equals sign and a yellow sine wave. The yellow sine wave has a frequency that is the sum of the frequencies of the purple and cyan waves, illustrating the cascade process where energy is transferred to smaller scales.

$$k_3 + k_4 = k_5$$

New waves modes created on
nonlinear timescales:

$$t_{nl} \sim [\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})]^{-1/2} \sim \ell / u$$

Turbulence Cascade



Credit: NASA, ESA, S. Beckwith (STScI) and the Hubble Heritage Team (STScI/AURA)

inspired by Shukurov, 2011

$$\begin{aligned} \text{WIM: } \text{Re} &\sim 10^7 & \lambda_{\text{mfp}} &\sim 5 R_{\oplus} \\ \text{WNM: } \text{Re} &\sim 10^7 & \lambda_{\text{mfp}} &\sim 1.3 \text{ AU} \\ \text{CNM: } \text{Re} &\sim 10^{10} & \lambda_{\text{mfp}} &\sim 0.3 \text{ pc} \end{aligned}$$

Ferrière, 2020; Plasma Physics and Controlled Fusion

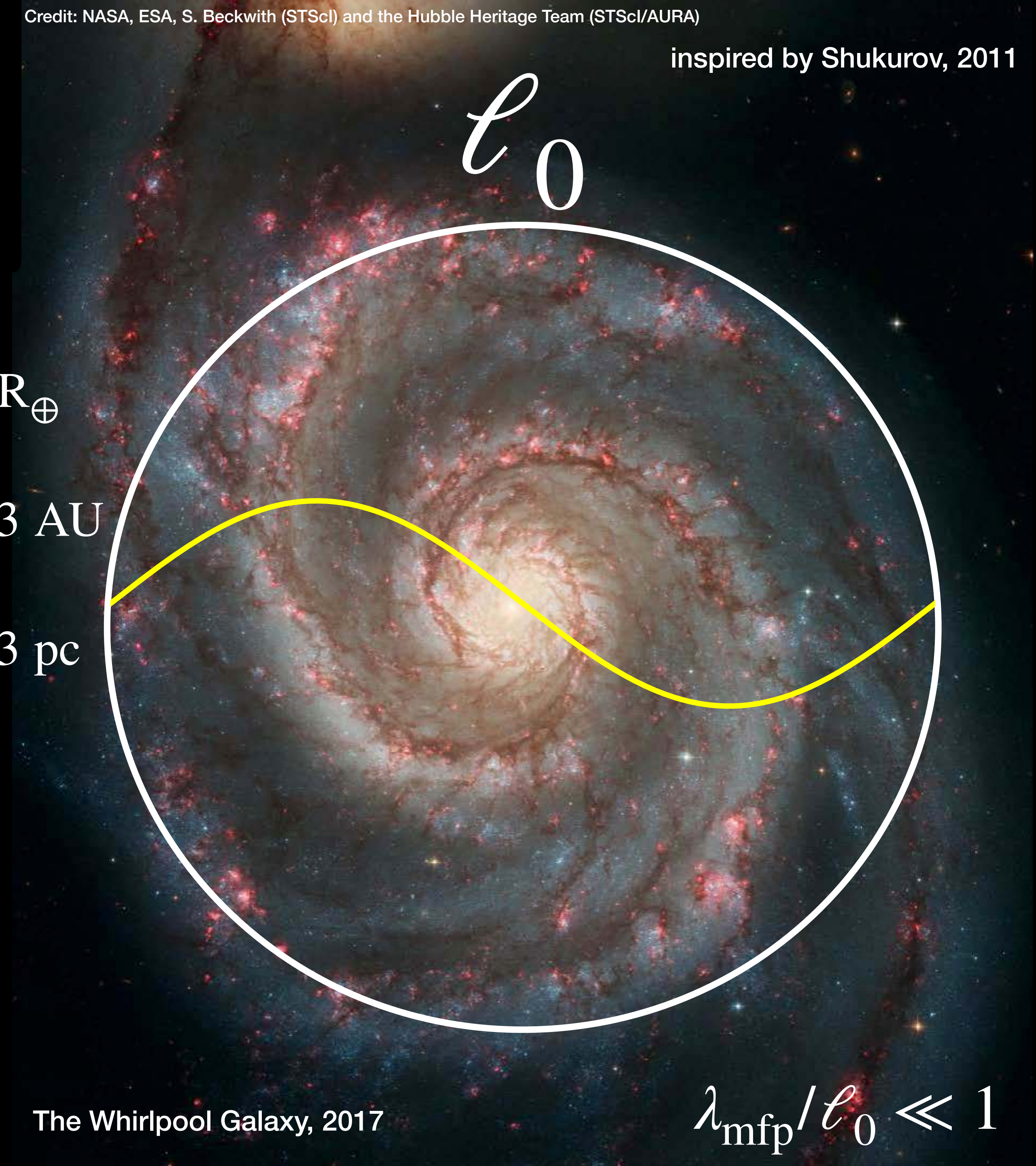
The quadratic nonlinear term

$$|\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})|$$

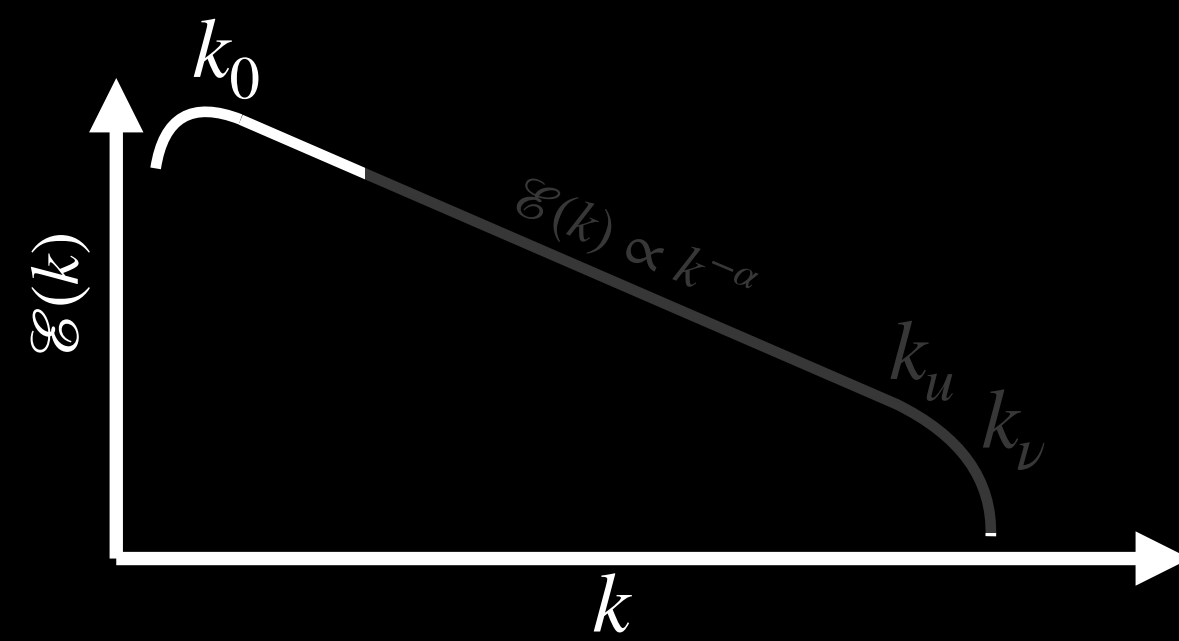
dominates on large scales, ℓ

The Whirlpool Galaxy, 2017

$$\lambda_{\text{mfp}} / \ell_0 \ll 1$$



Turbulence Cascade



The quadratic nonlinear term

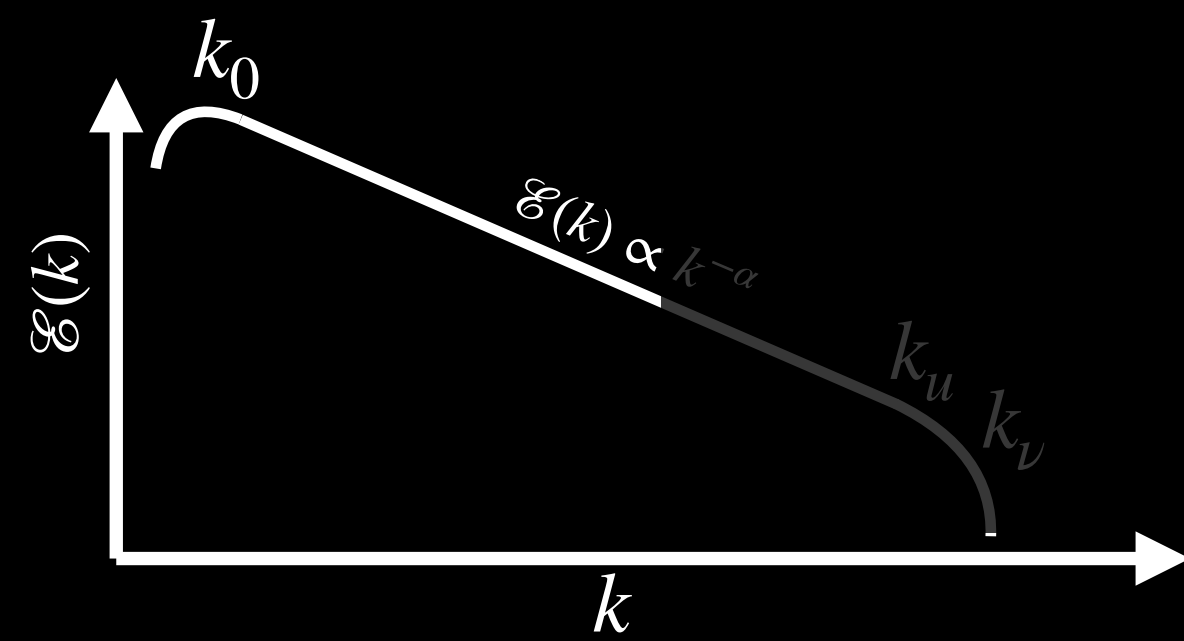
$$|\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})|$$

dominates on large scales, ℓ

Creates new modes on

$$t_{\text{nl}} \sim [\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})]^{-1/2} \sim \mathcal{O}(\text{Gyr})$$

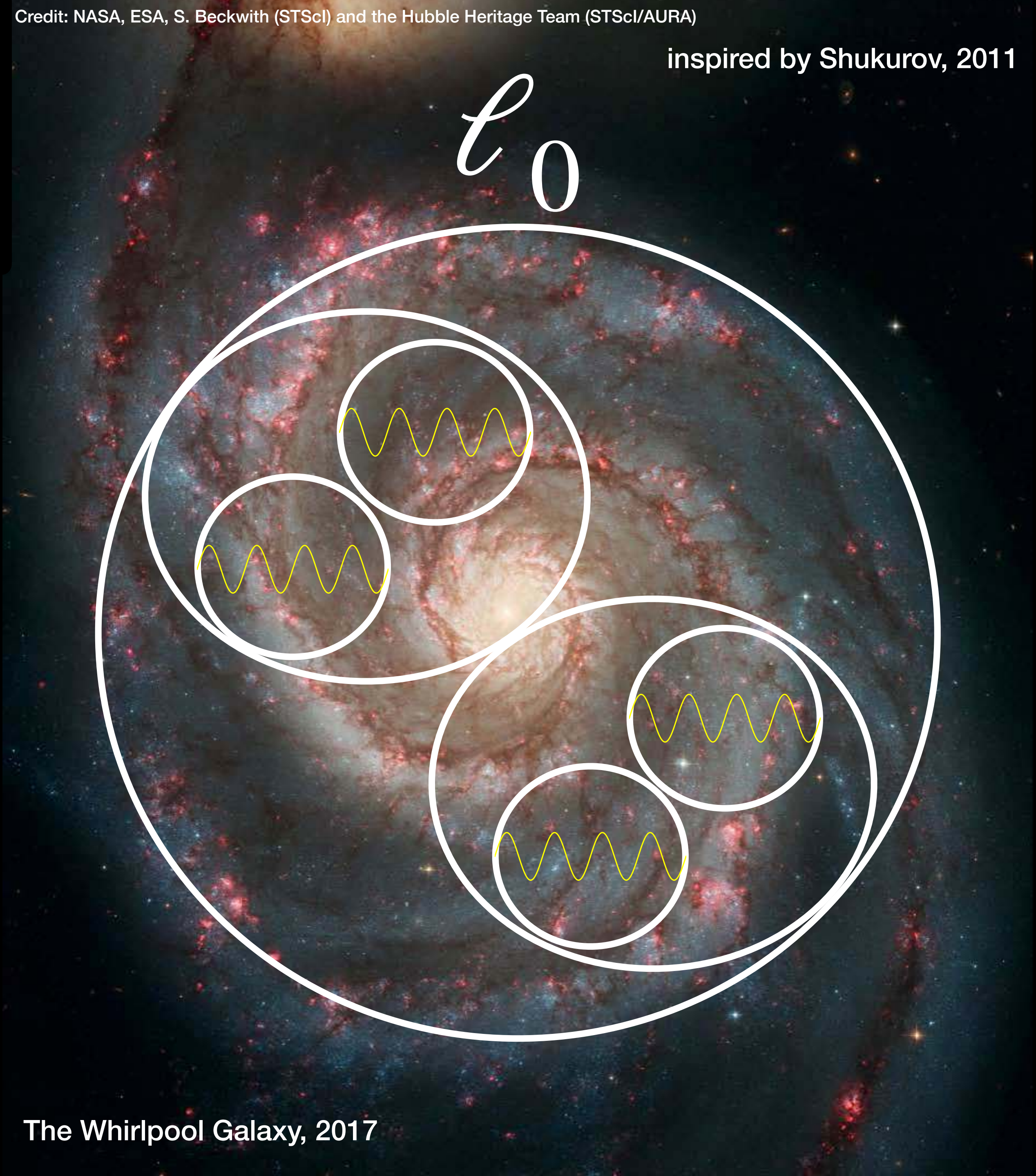
Turbulence Cascade



inside of the cascade

Credit: NASA, ESA, S. Beckwith (STScI) and the Hubble Heritage Team (STScI/AURA)

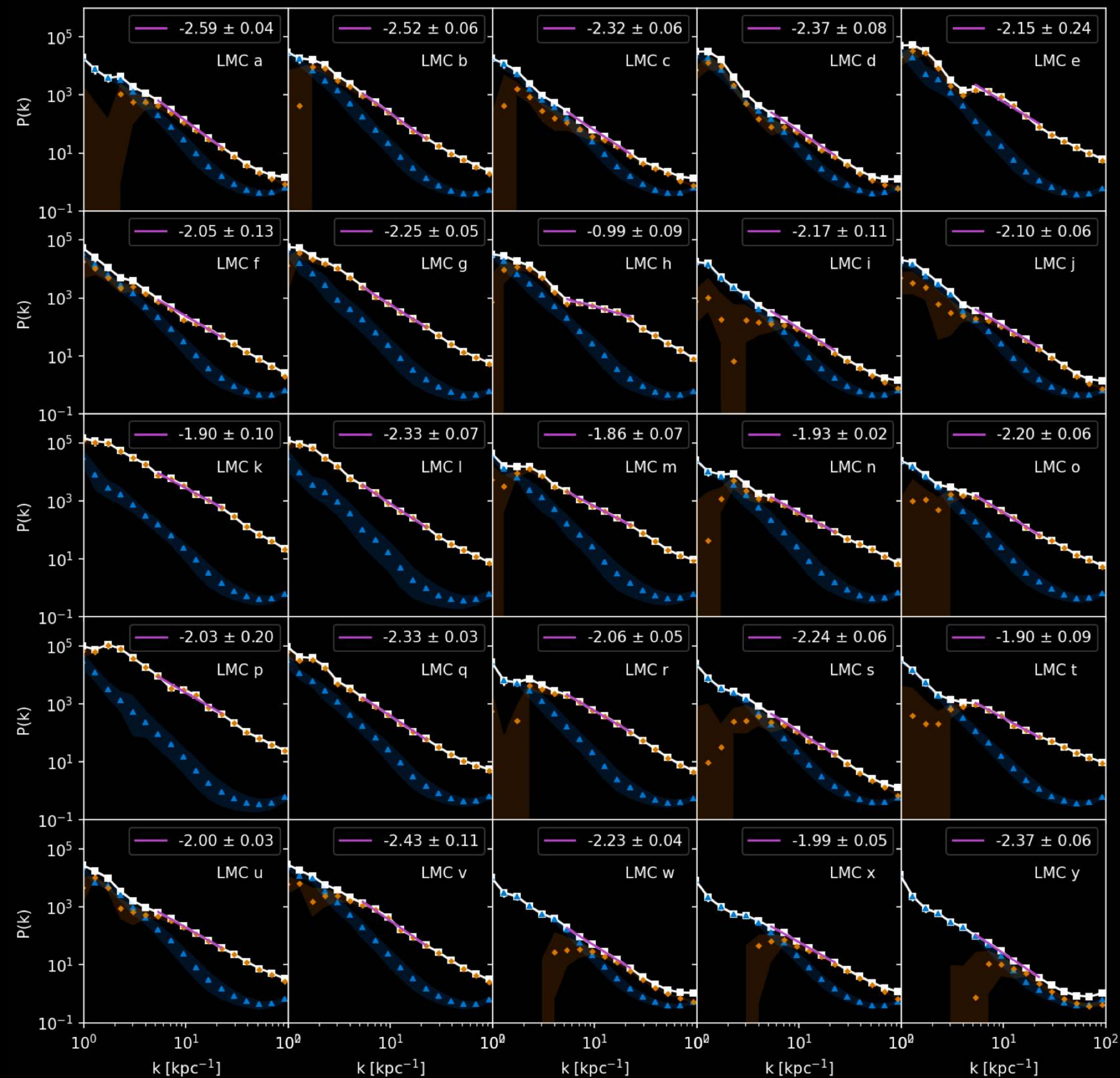
inspired by Shukurov, 2011



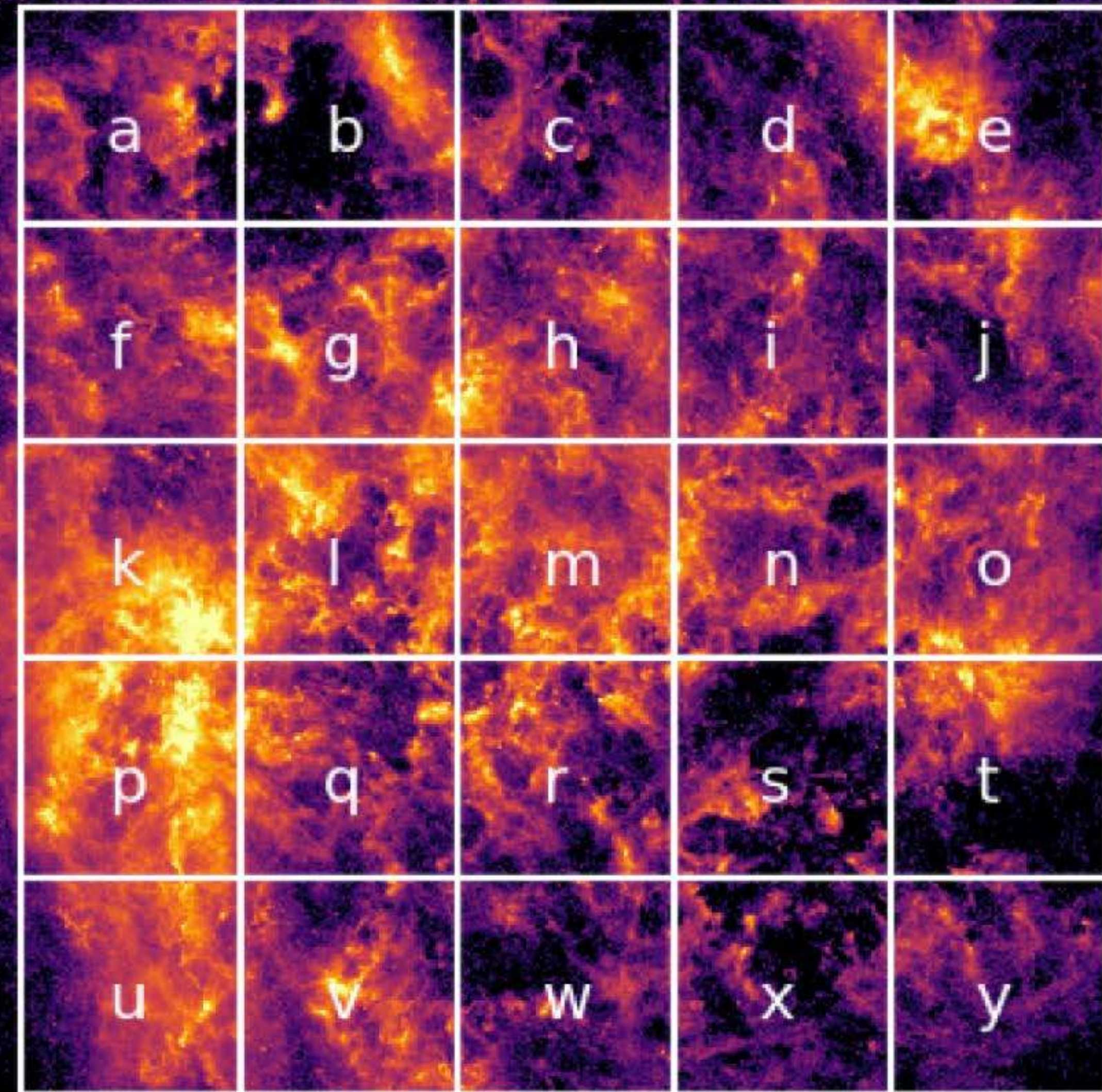
The Whirlpool Galaxy, 2017

Turbulence

LMC: $500\mu\text{m}$, *Herschel* (processing Gordon et al. 2014)



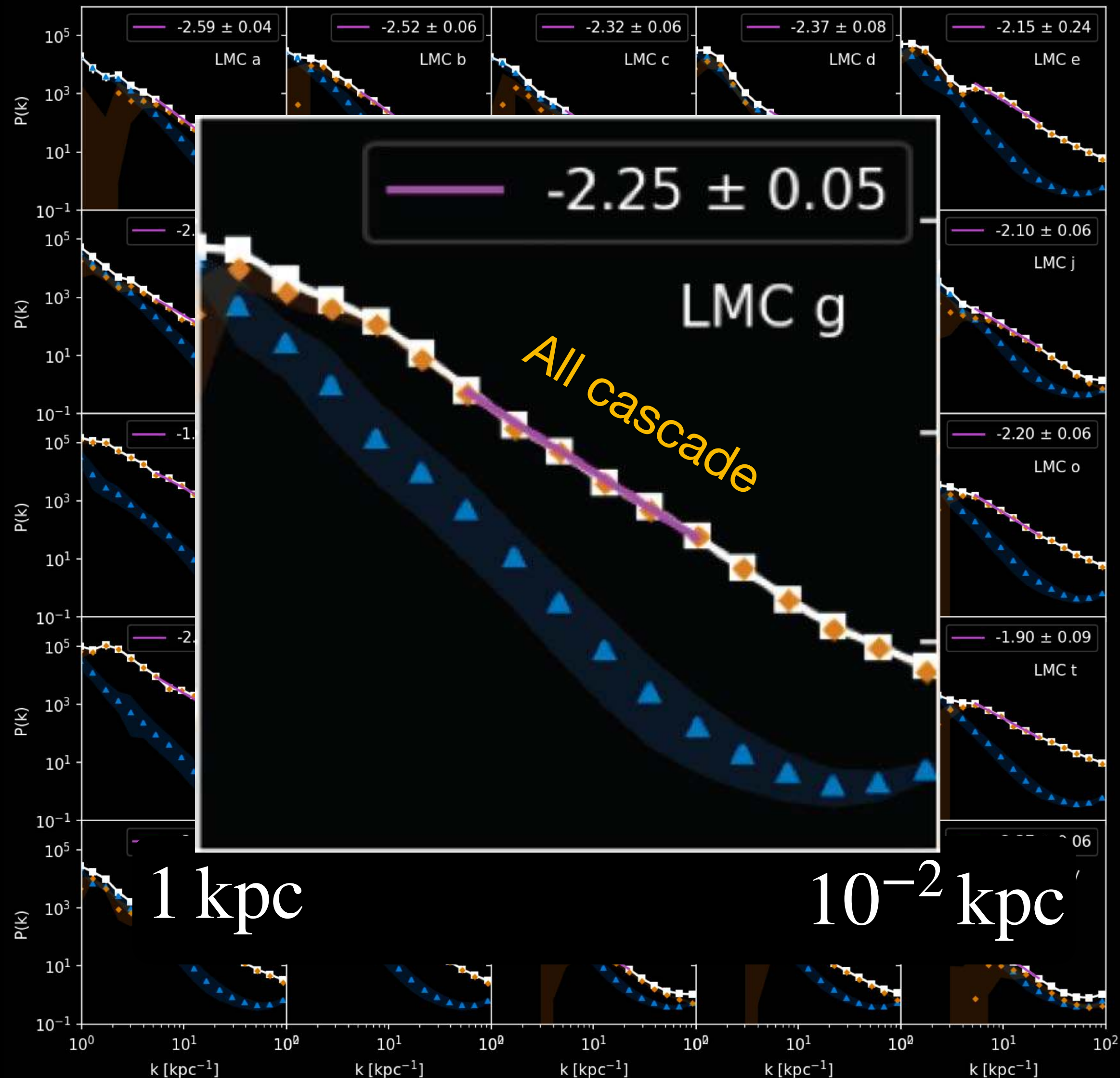
1 kpc



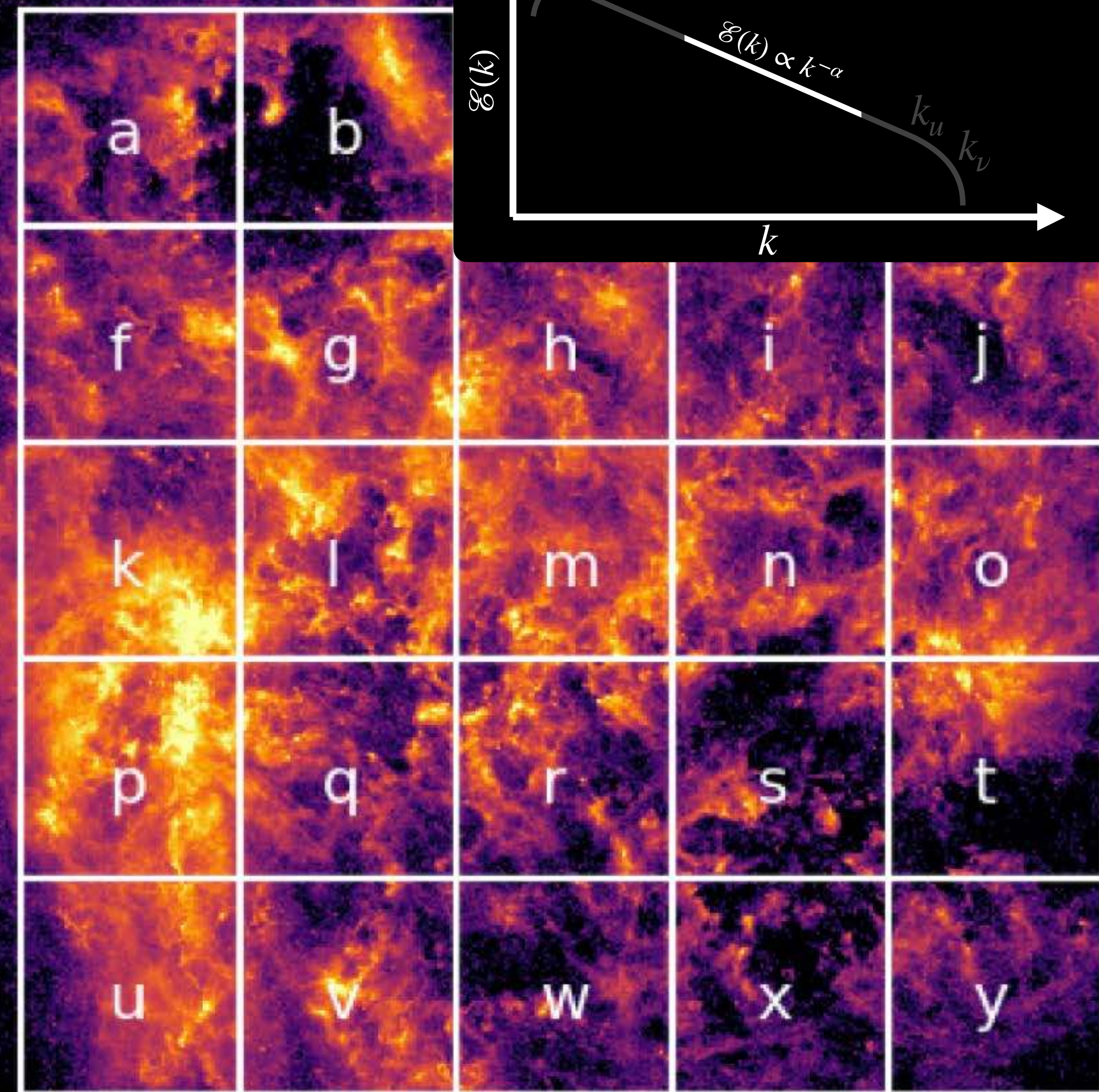
Colman, et al., 2022, MNRAS

Turbulence

LMC: 500 μ m, *Herschel* (processing Gordon et al. 2014)

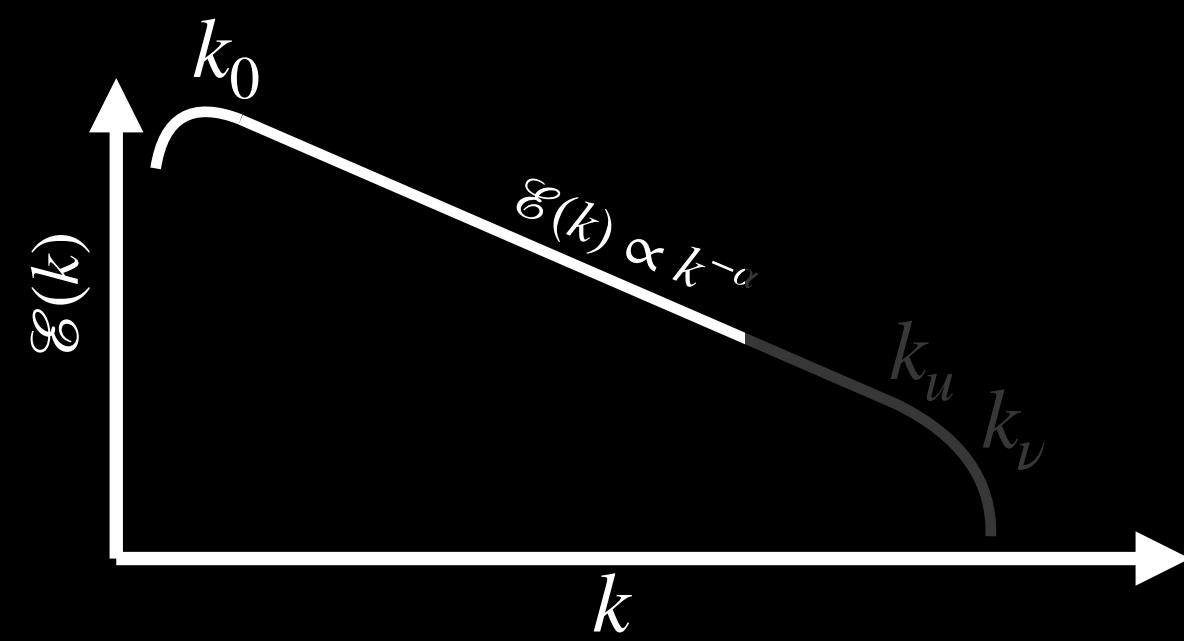


1 kpc



Colman, et al., 2022, MNRAS

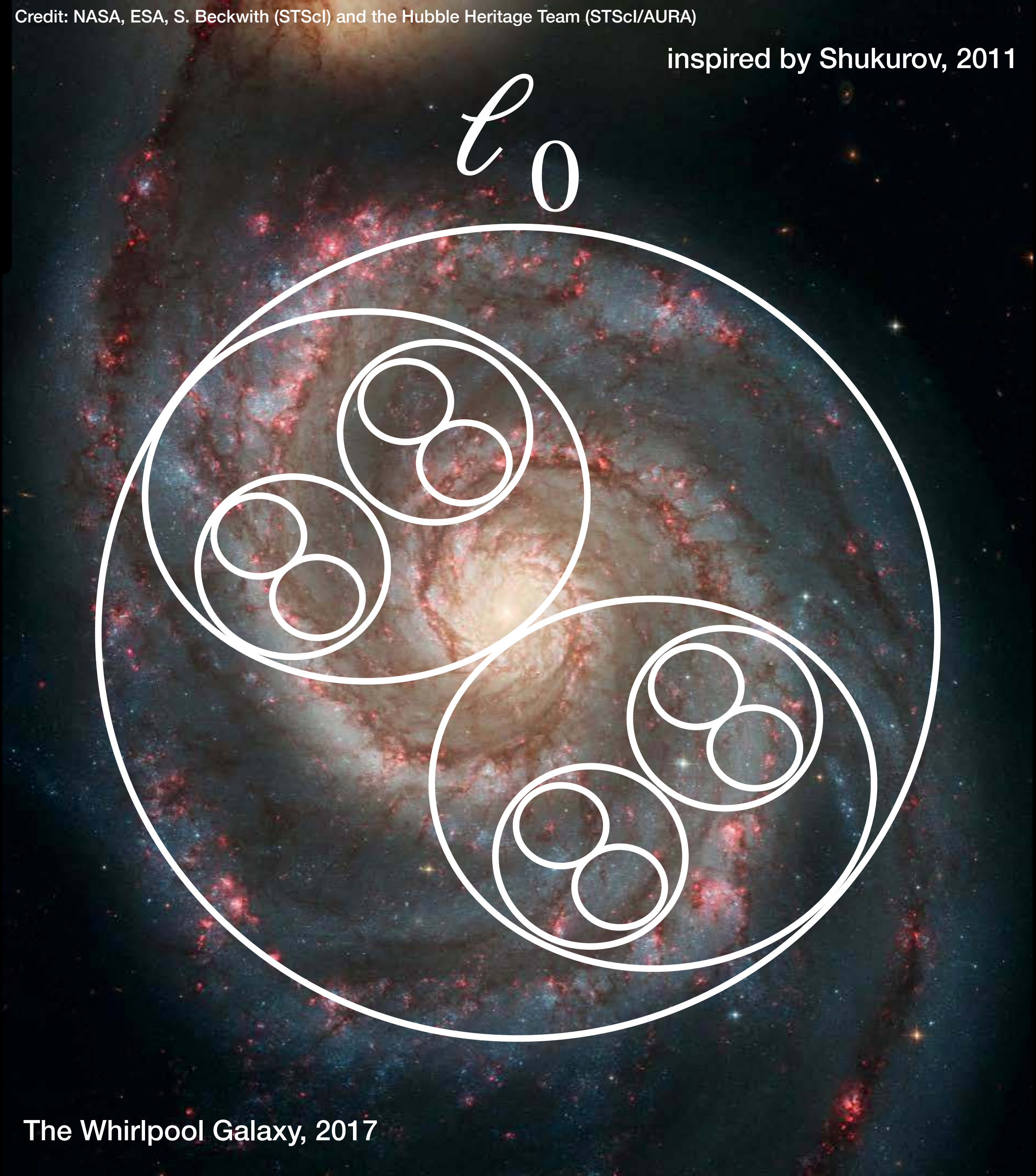
Turbulence Cascade



deeper into
the cascade

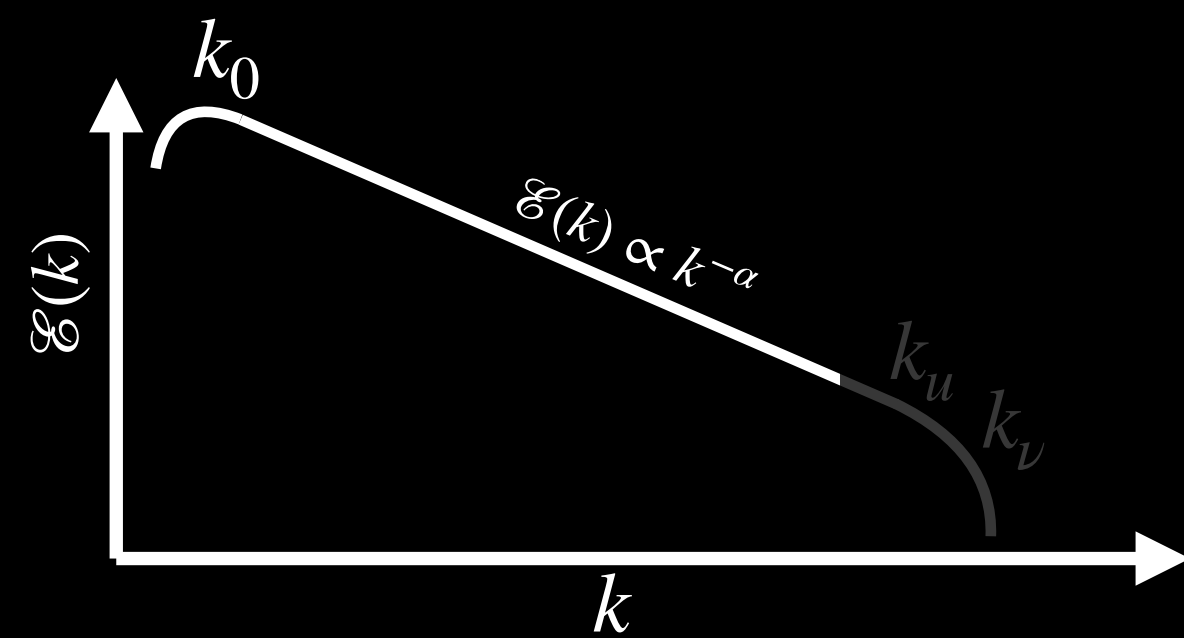
Credit: NASA, ESA, S. Beckwith (STScI) and the Hubble Heritage Team (STScI/AURA)

inspired by Shukurov, 2011



The Whirlpool Galaxy, 2017

Turbulence Cascade

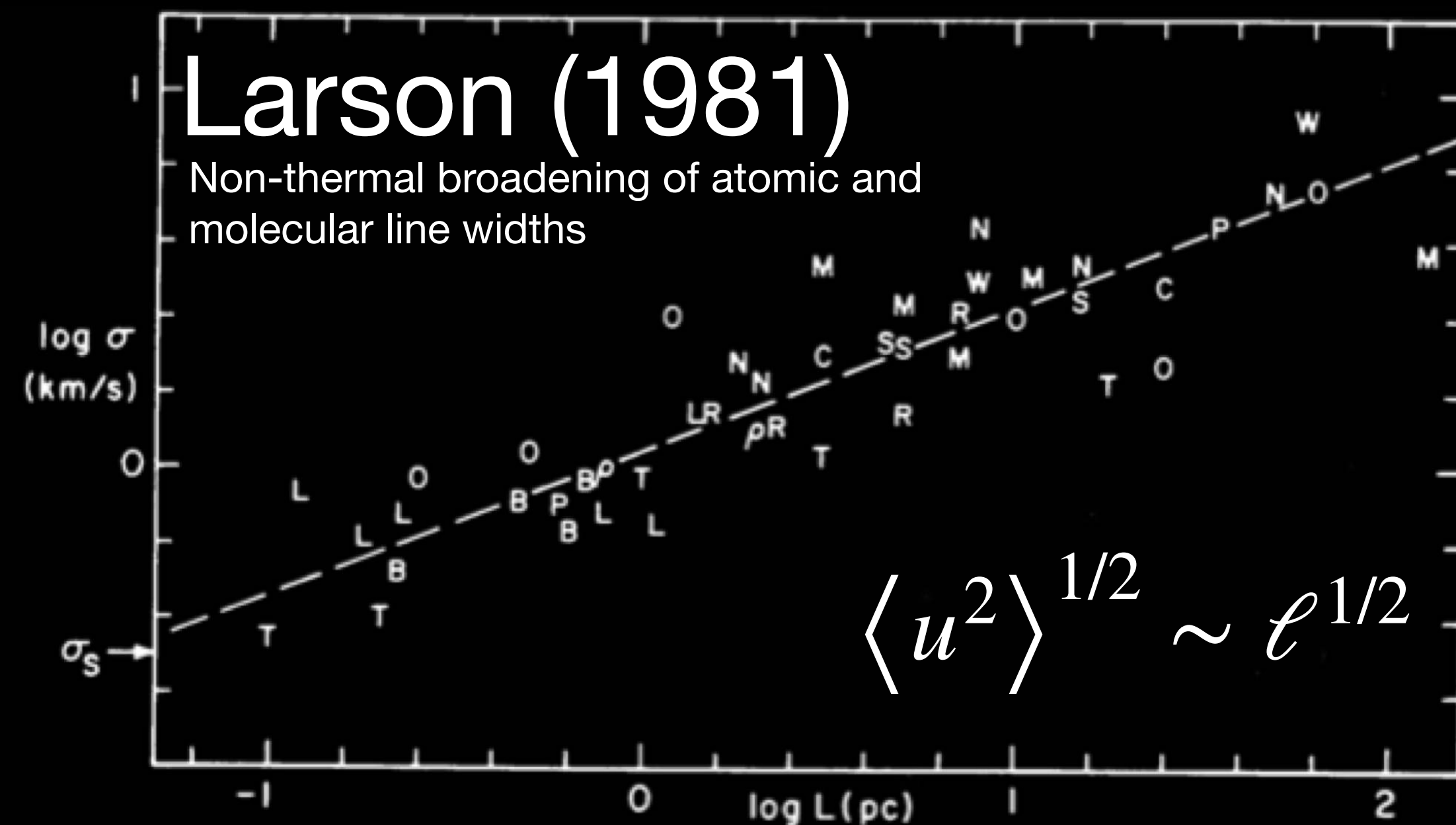


Credit: NASA, ESA, S. Beckwith (STScI) and the Hubble Heritage Team (STScI/AURA)

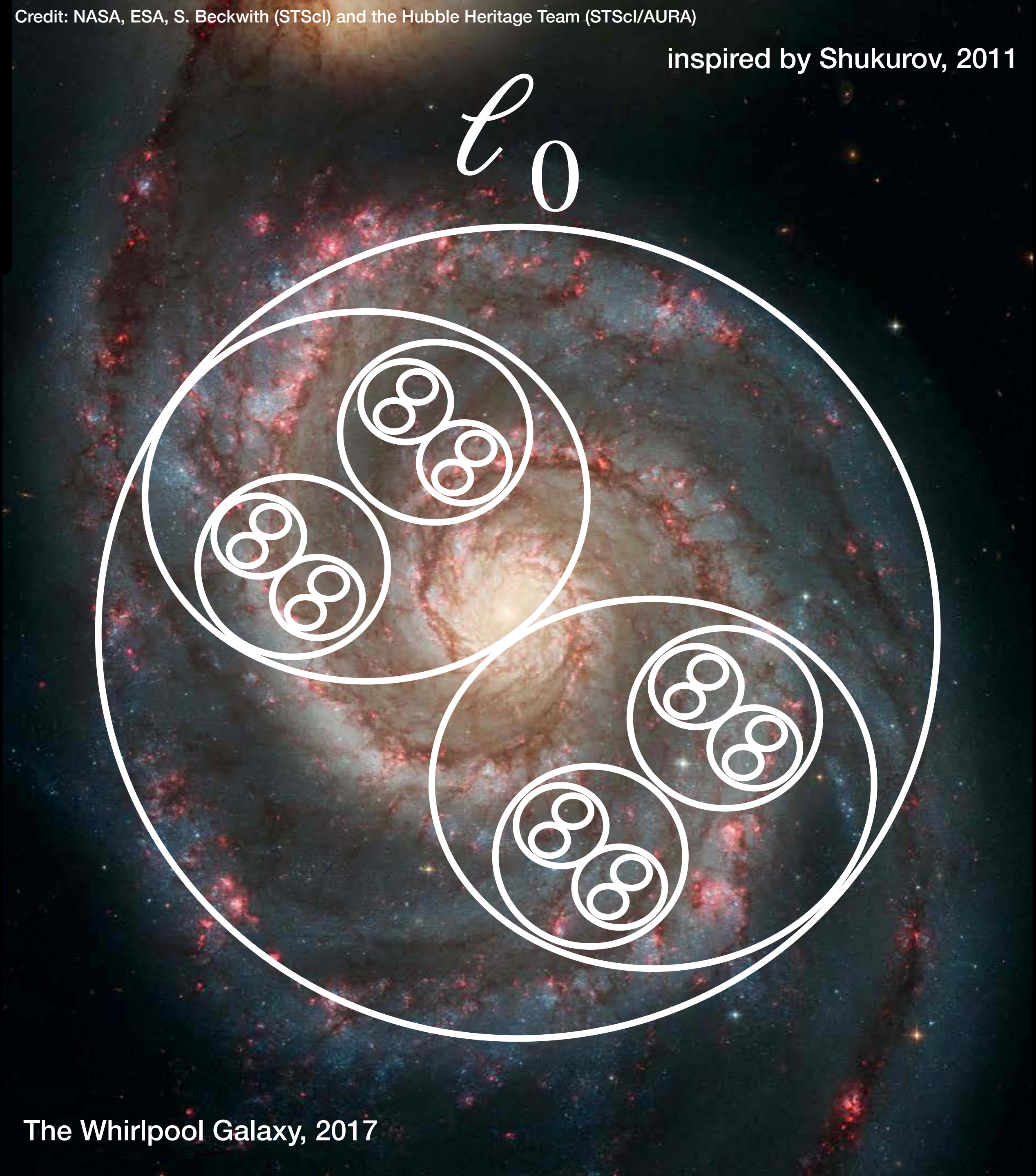
inspired by Shukurov, 2011

Reynolds number is shrinking

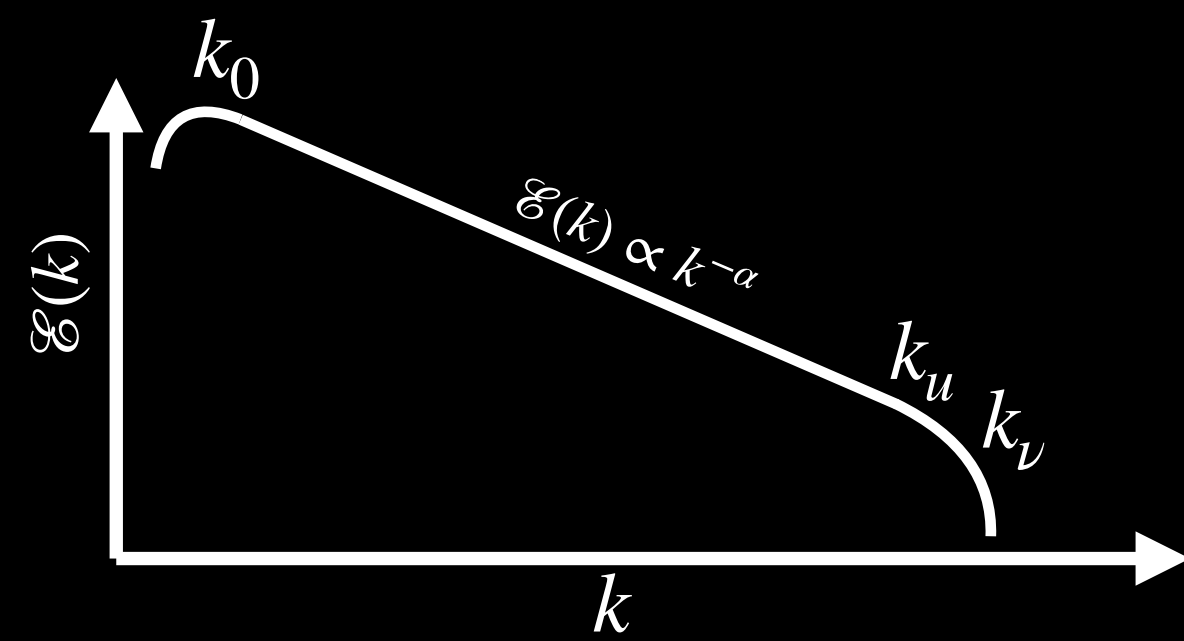
$$\text{Re} \sim \langle u^2 \rangle^{1/2} \ell / \nu \sim \ell^{3/2} / \nu$$



The Whirlpool Galaxy, 2017



Turbulence Cascade



until

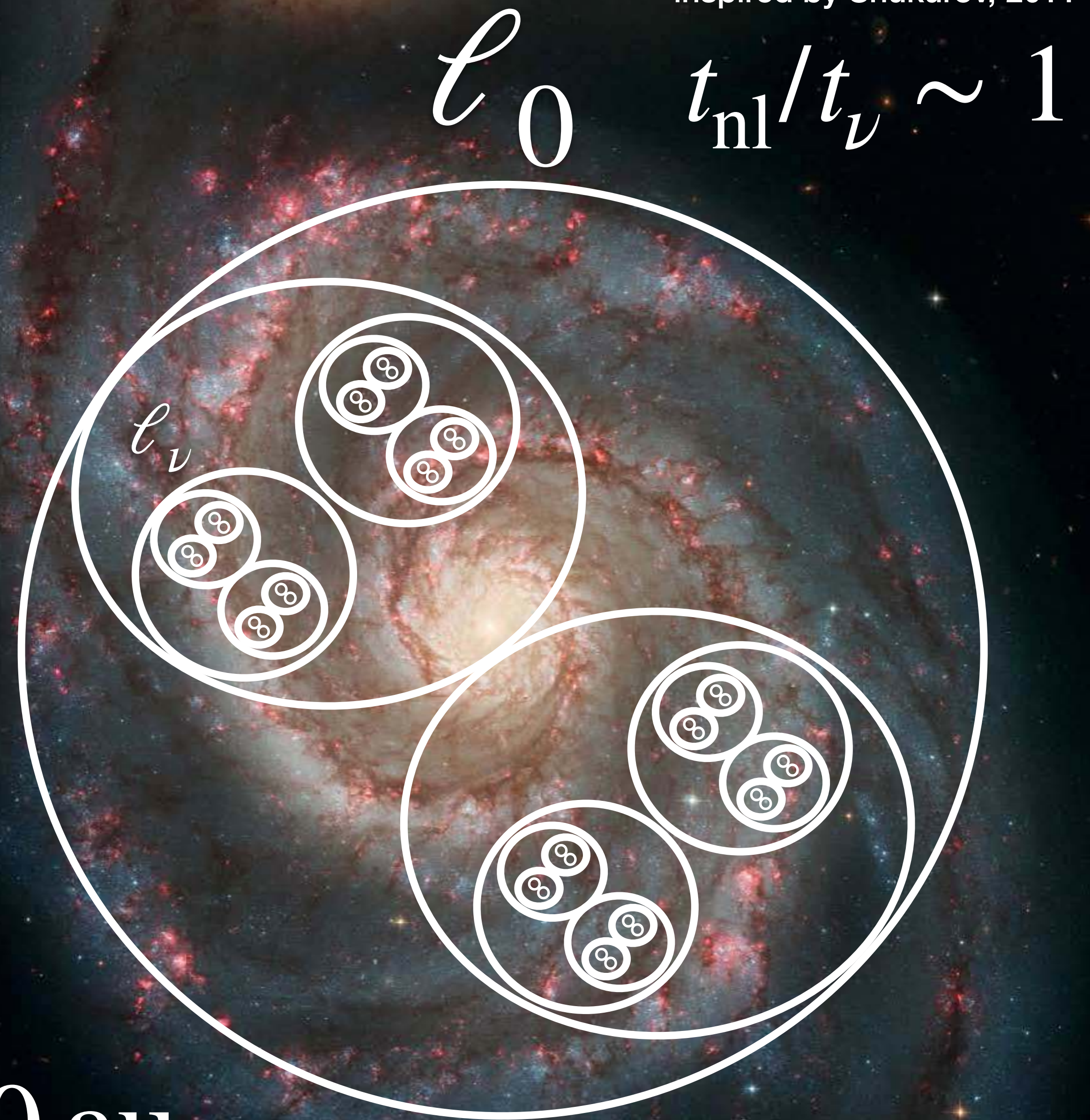
$$\text{Re} = \frac{|\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})|}{|2\nu \nabla \cdot (\rho \mathbf{S})|} = 1$$

dissipation scales for WIM cascade:

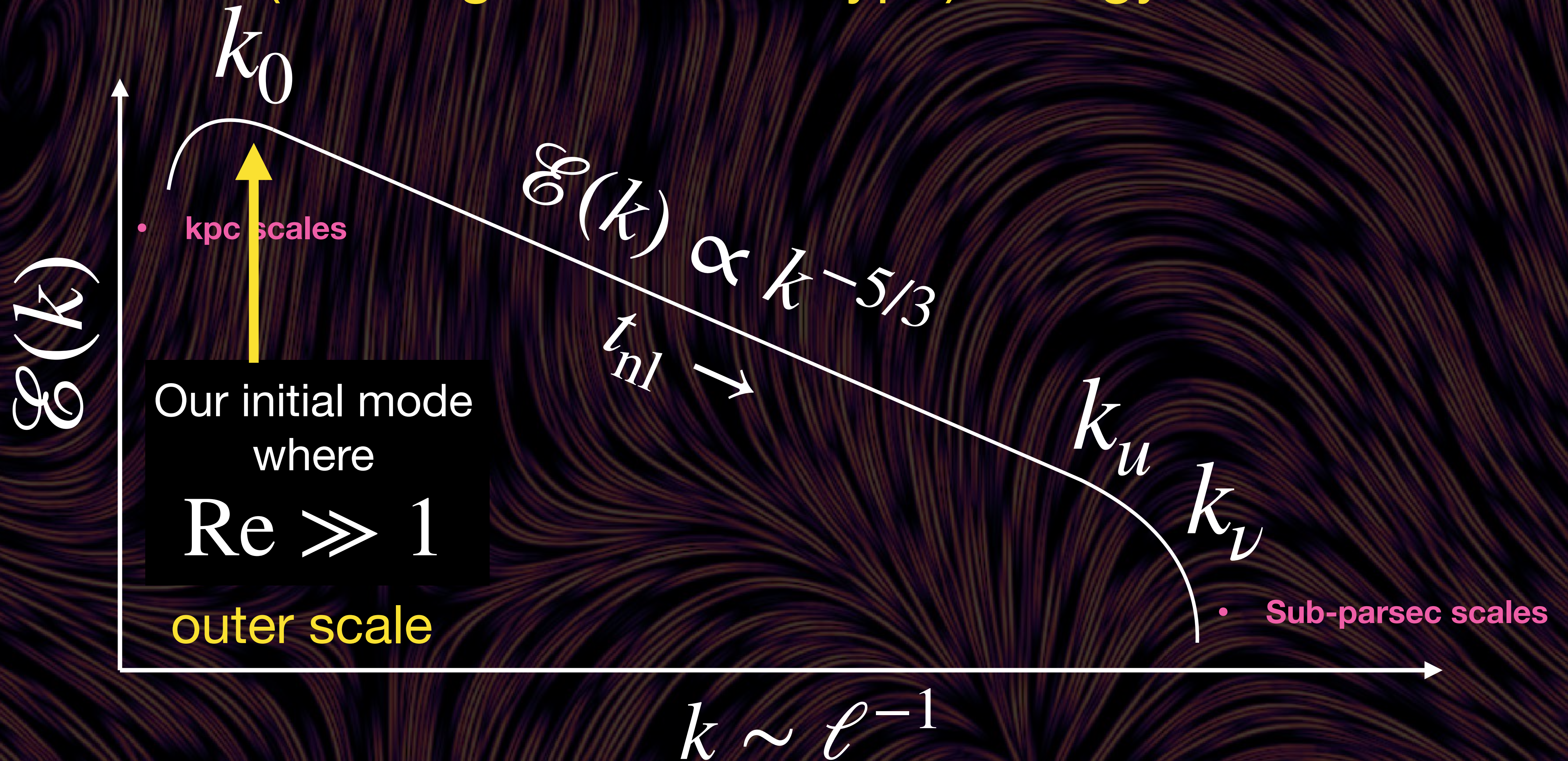
$$\ell_\nu \sim \text{Re}^{-3/4} \ell_0$$

$$\text{WIM: } \text{Re} \sim 10^7$$

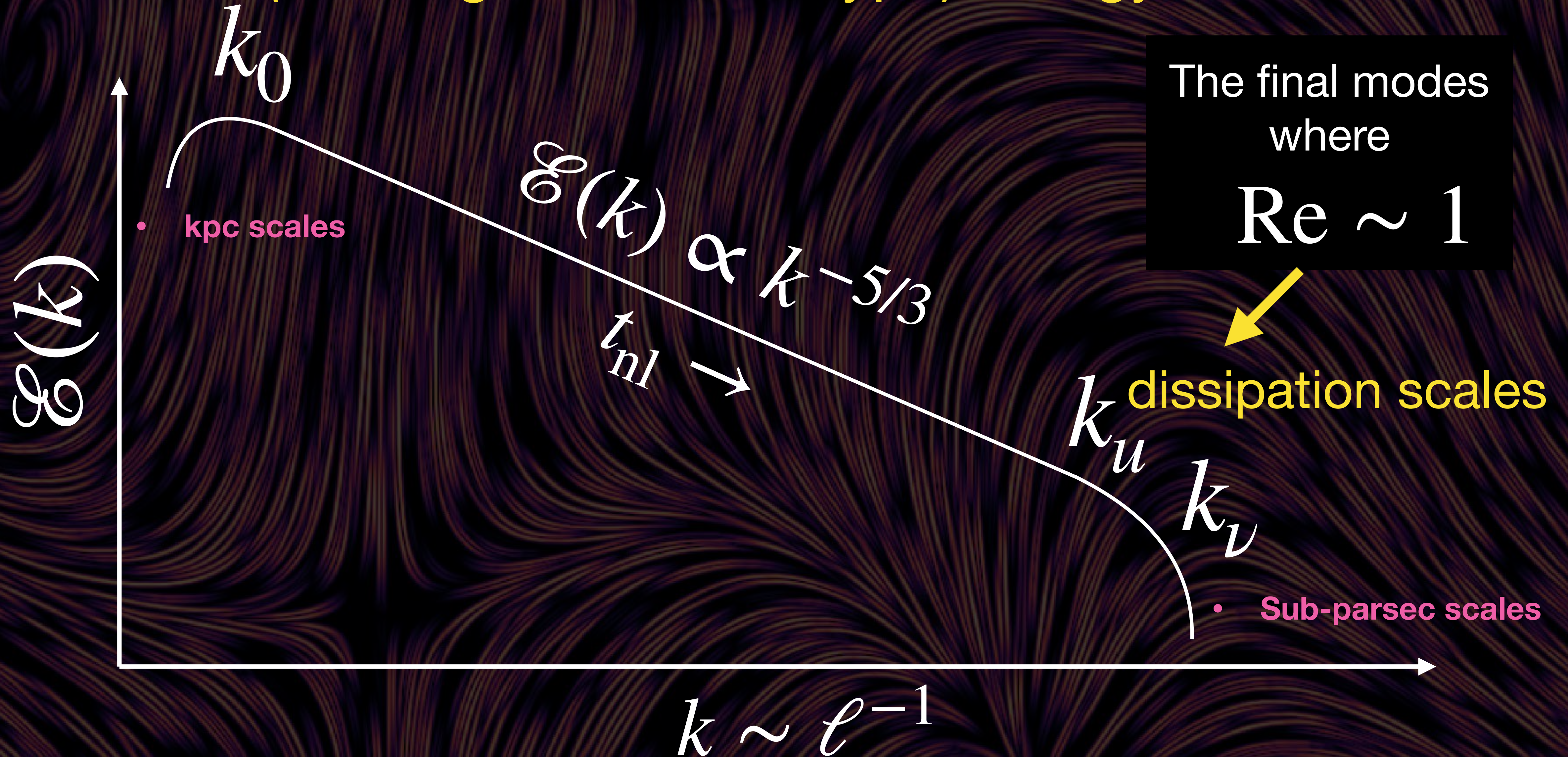
$$\ell_0 \sim 1 \text{ kpc} \implies \ell_\nu \sim 1000 \text{ au}$$



The (Kolmogorov, 1941 -type) energy cascade

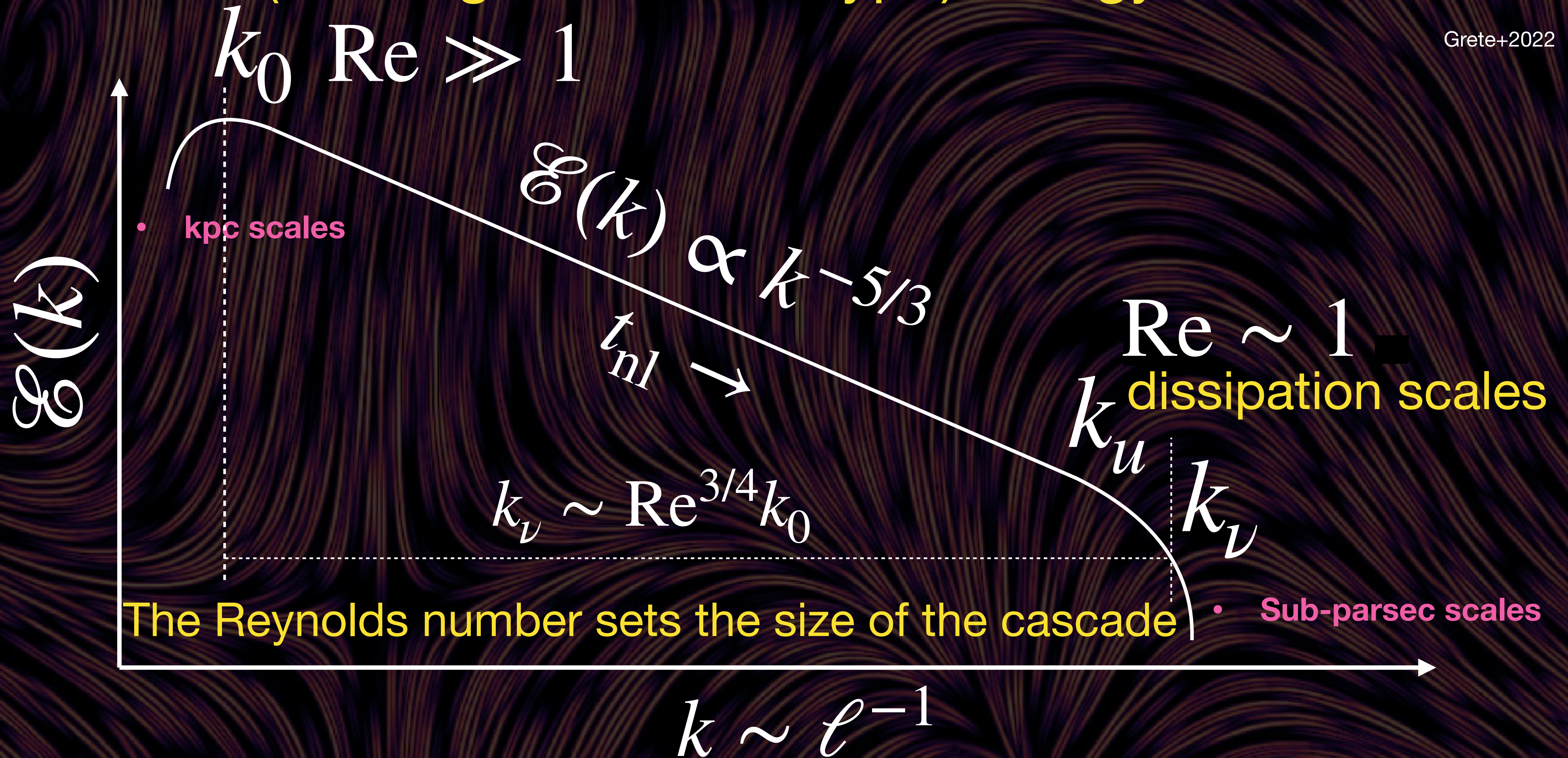


The (Kolmogorov, 1941 -type) energy cascade

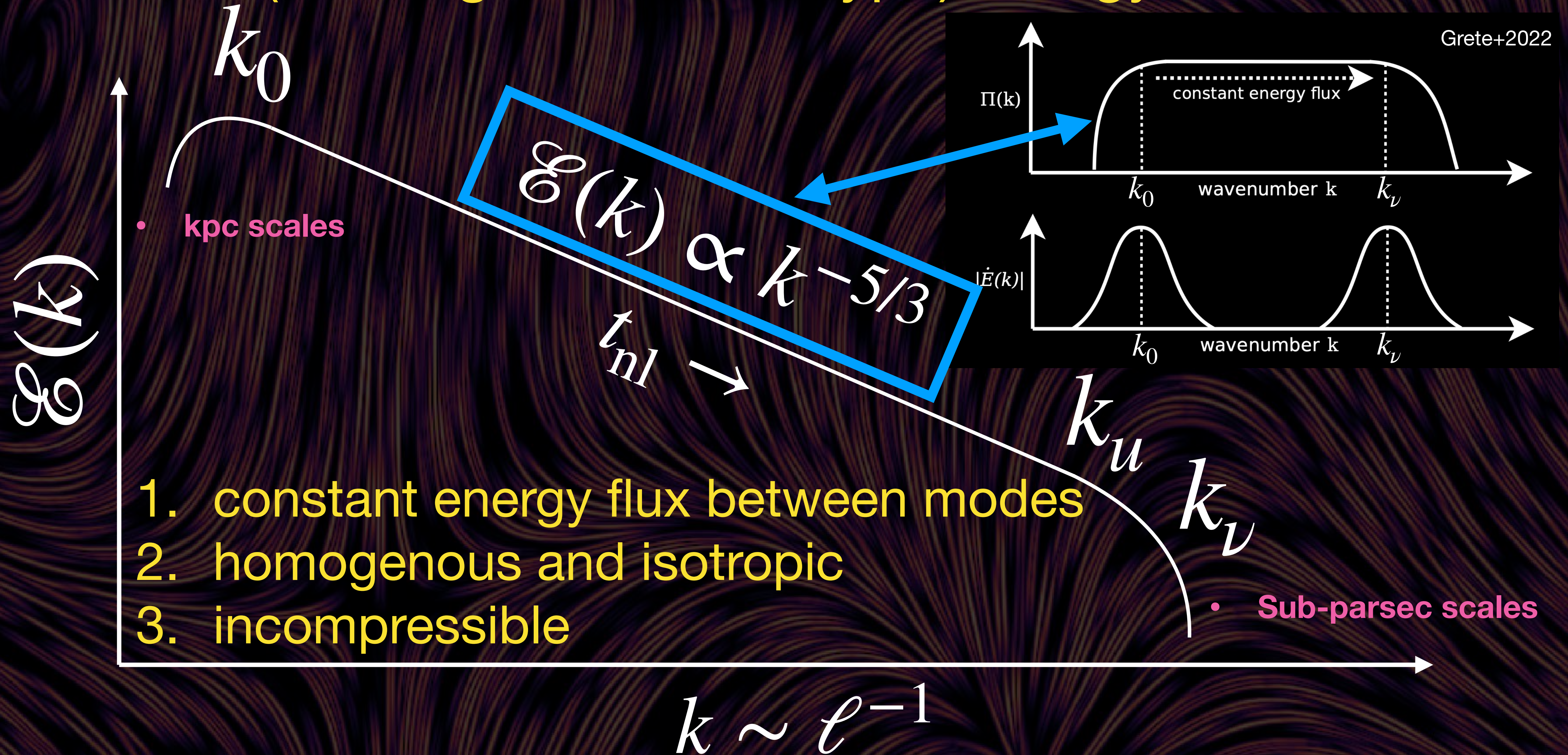


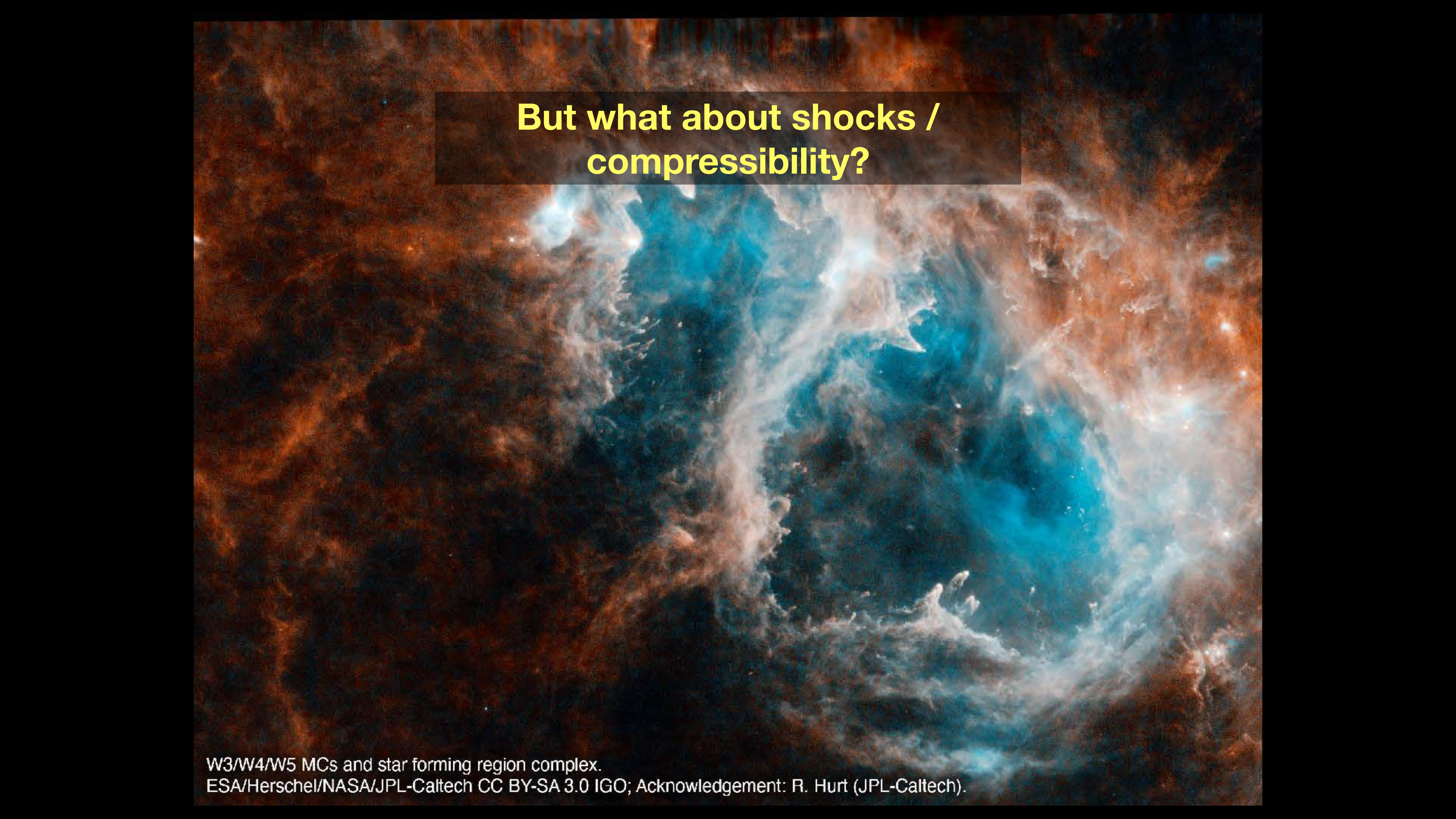
The (Kolmogorov, 1941 -type) energy cascade

Grete+2022



The (Kolmogorov, 1941 -type) energy cascade





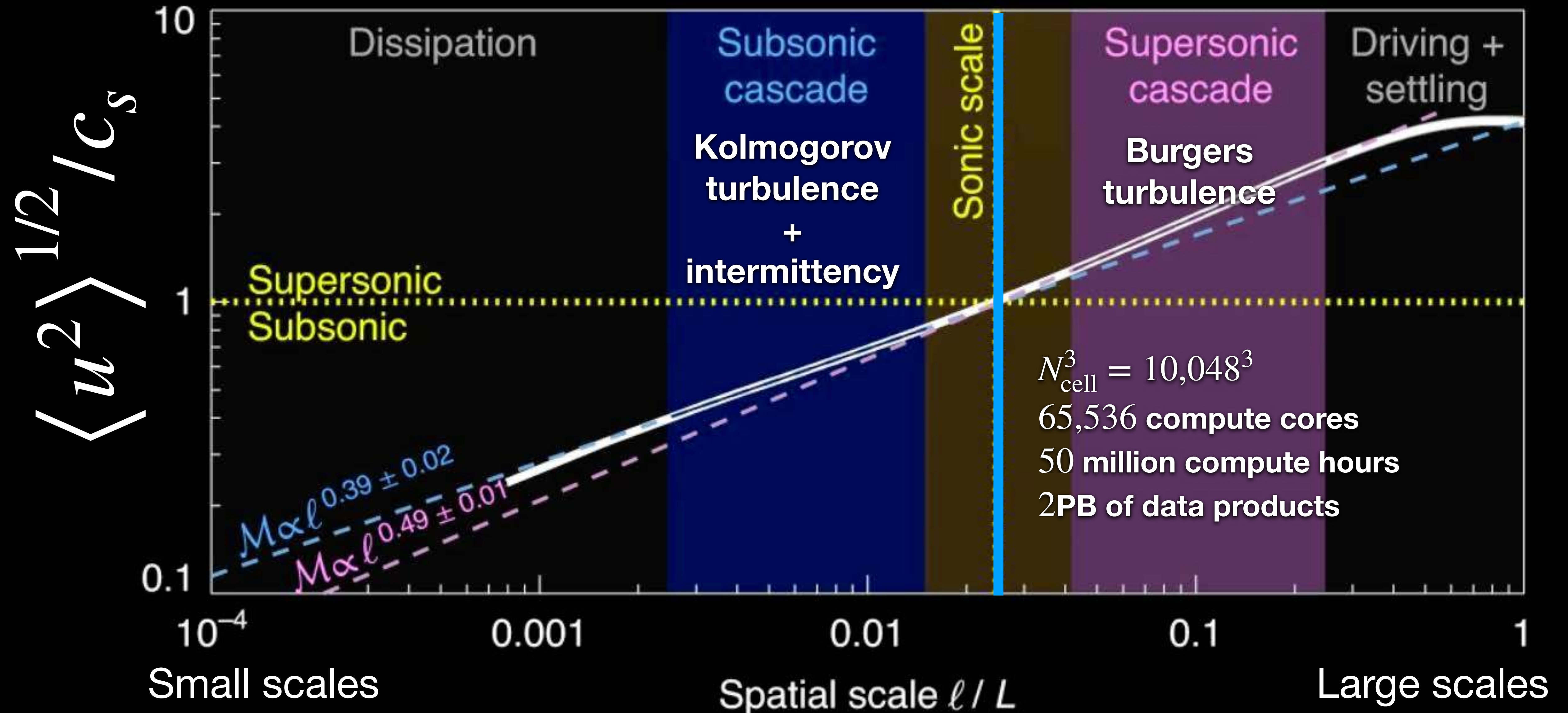
**But what about shocks /
compressibility?**

W3/W4/W5 MCs and star forming region complex.
ESA/Herschel/NASA/JPL-Caltech CC BY-SA 3.0 IGO; Acknowledgement: R. Hurt (JPL-Caltech).

The energy cascade in supersonic hydrodynamical turbulence

Federrath, Klessen, Ipachio & Beattie (2021)

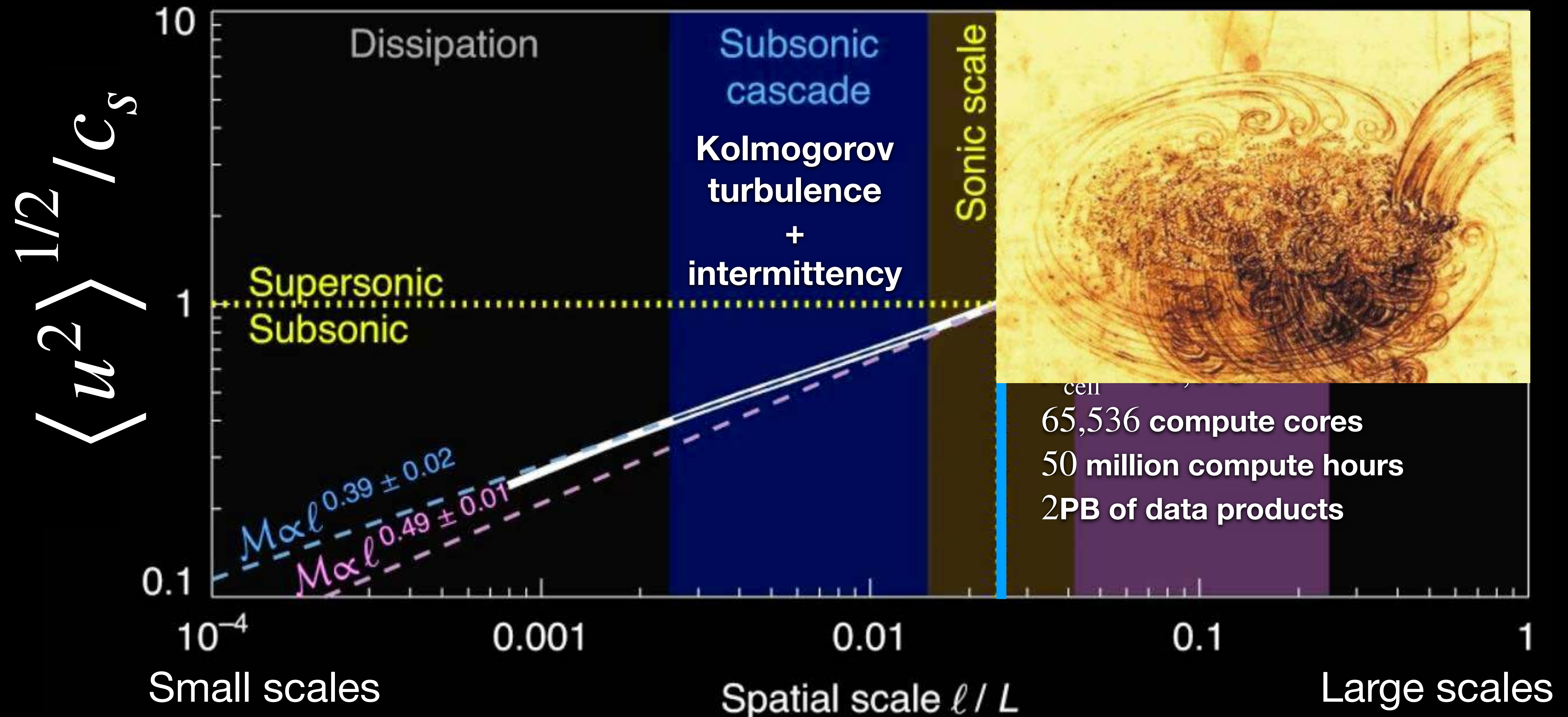
Nature Astronomy



The energy cascade in supersonic hydrodynamical turbulence

Federrath, Klessen, Ipachio & Beattie (2021)
Nature Astronomy

$$\mathcal{E}(k) \sim k^{-5/3}$$

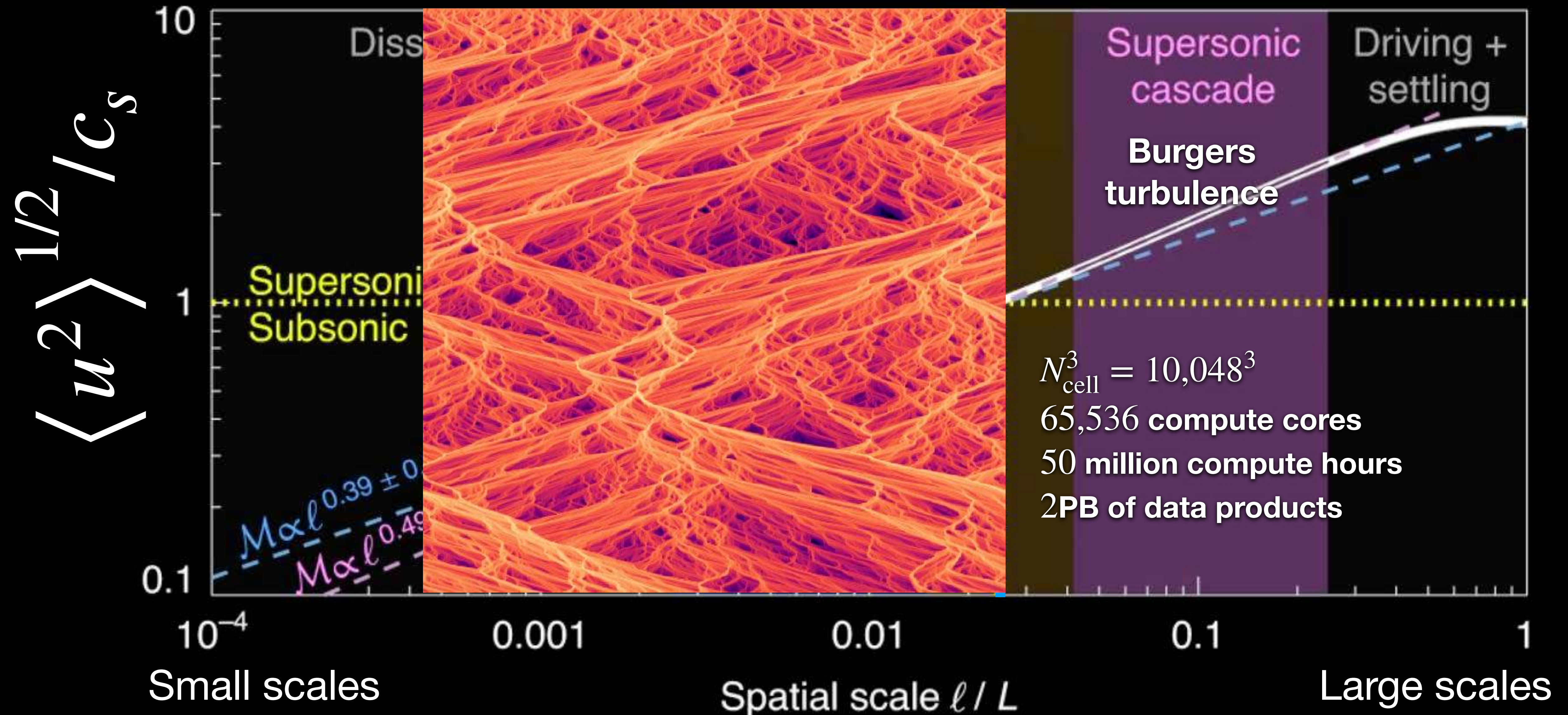


The energy cascade in supersonic hydrodynamical turbulence

Federrath, Klessen, Ipachio & Beattie (2021)

Nature Astronomy

$$\mathcal{E}(k) \sim k^{-2}$$



The energy cascade in supersonic hydrodynamical turbulence

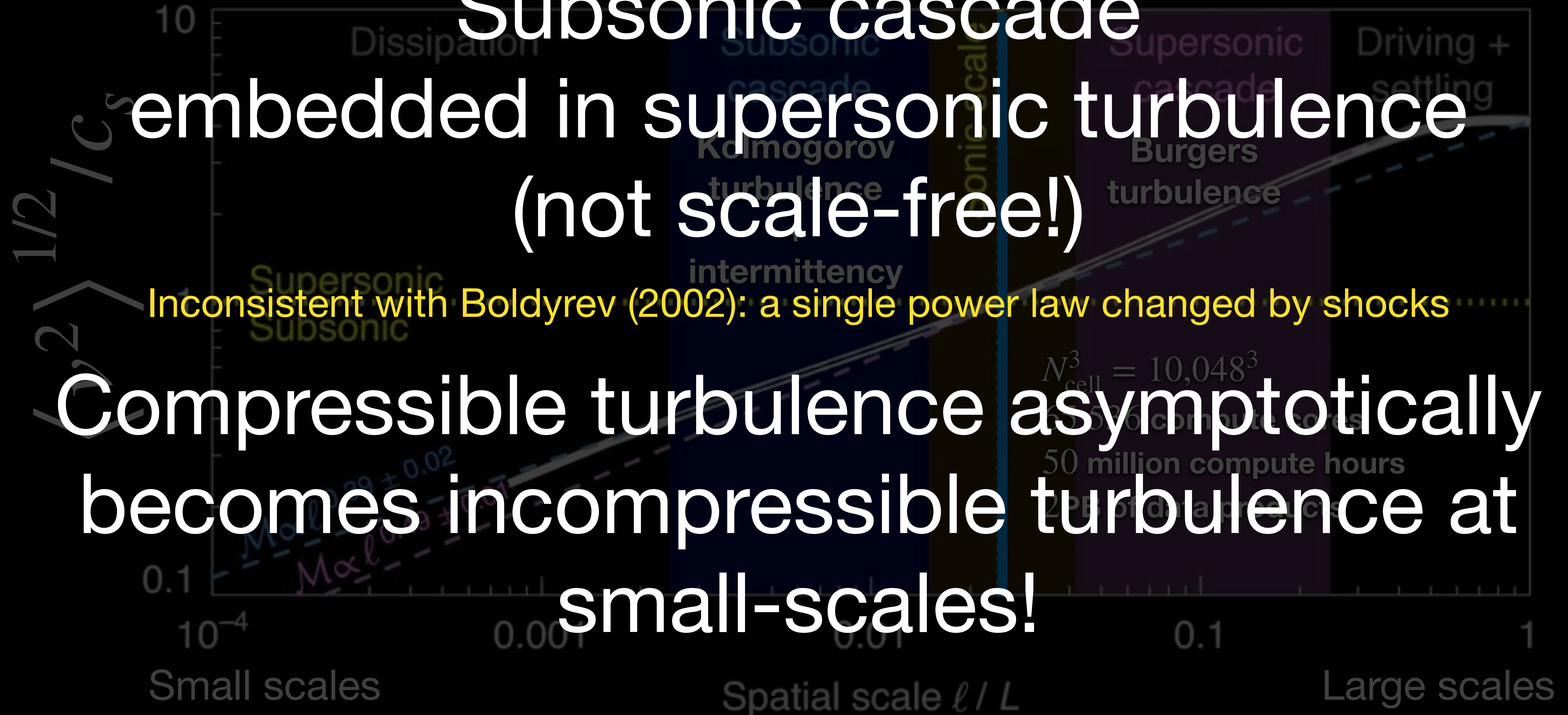
Federrath, Klessen, Ipachio & Beattie (2021)

Nature Astronomy

Subsonic cascade
embedded in supersonic turbulence
(not scale-free!)

Inconsistent with Boldyrev (2002): a single power law changed by shocks

Compressible turbulence asymptotically
becomes incompressible turbulence at
small-scales!

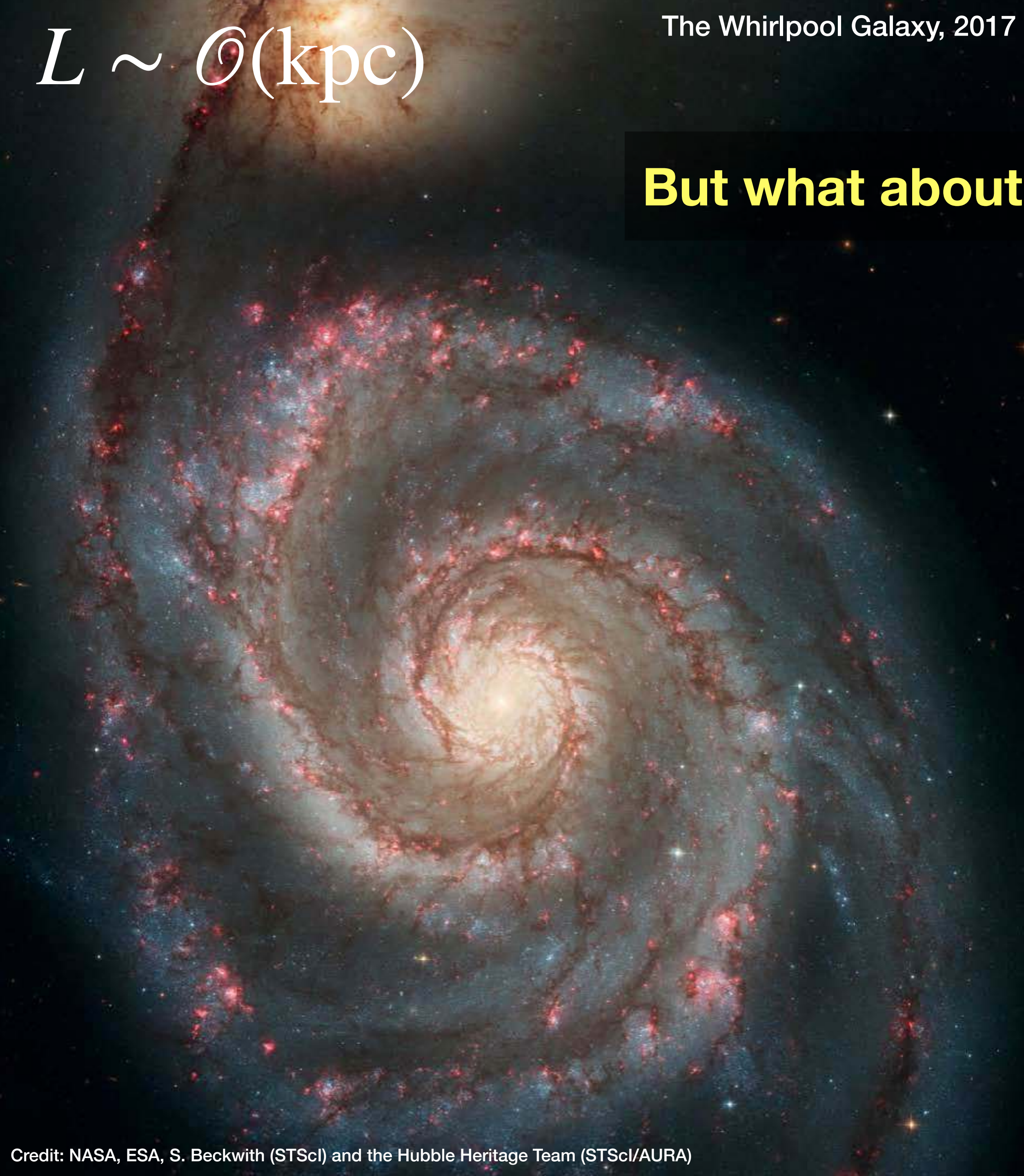


$$L \sim \mathcal{O}(\text{kpc})$$

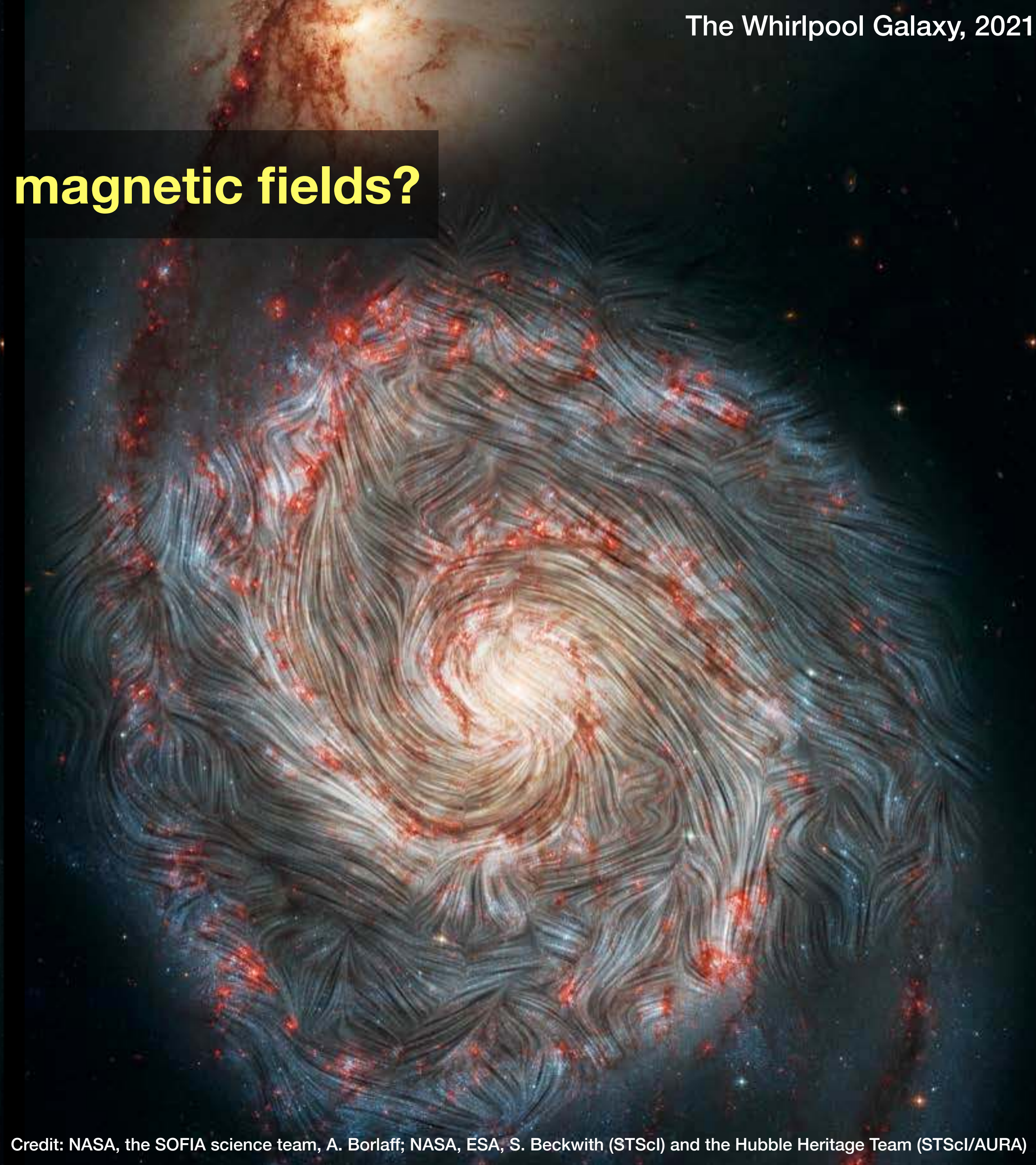
The Whirlpool Galaxy, 2017

The Whirlpool Galaxy, 2021

But what about magnetic fields?



Credit: NASA, ESA, S. Beckwith (STScI) and the Hubble Heritage Team (STScI/AURA)



Credit: NASA, the SOFIA science team, A. Borlaff; NASA, ESA, S. Beckwith (STScI) and the Hubble Heritage Team (STScI/AURA)

$$L \sim \mathcal{O}(\text{pc})$$

Soler, 2019, A&A

Irreducible — add magnetic flux to the cascade on all scales

straight, strong fields ($\kappa \propto B^{-1/2}$; Schekochihin+2004) penetrating through Orion A
Orion A in infrared; ESA/Herschel/Planck; J. D. Soler, MPIA

Magnetic Reynolds Number

What is it?

Maxwell's equations + fluid = induction equation

$$\frac{\partial \mathbf{b}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{b} - \eta \mathbf{j}), \quad \mathbf{j} = \frac{1}{4\pi} \nabla \times \mathbf{b}$$

Creating nonlinear magnetic things in the fluid

$$R_m = \frac{|\nabla \times (\mathbf{u} \times \mathbf{b})|}{|\nabla \times \eta \mathbf{j}|} \sim \frac{UL}{\eta}$$

Dissipating nonlinear magnetic things in the fluid

Re & Rm Landscape Rincon (2019)

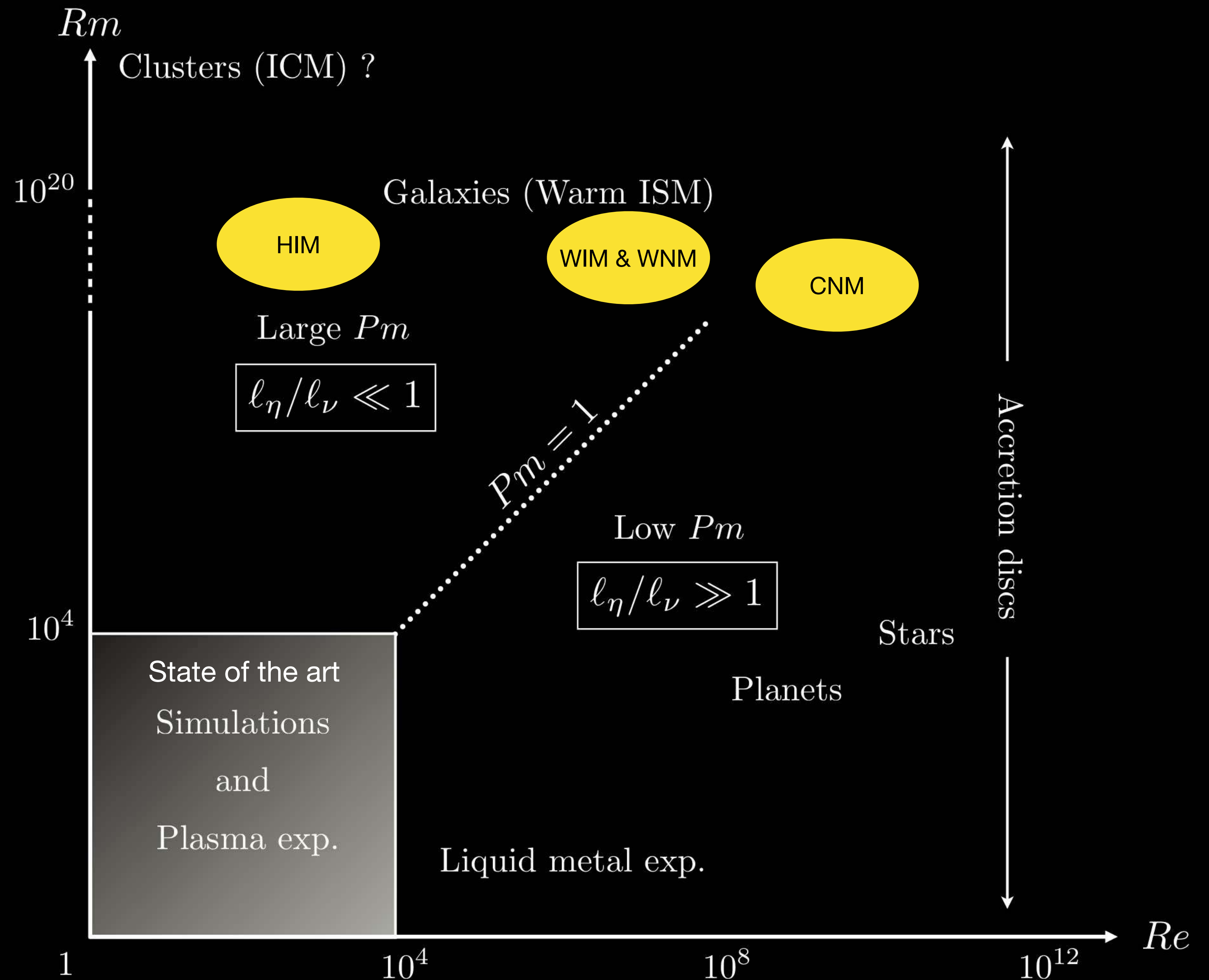
WIM: $Rm \sim 10^{18}$

WNM: $Rm \sim 10^{18}$

CNM: $Rm \sim 10^{15}$

Ferrière, 2020; Plasma Physics and
Controlled Fusion

$$Pm = \frac{Rm}{Re}$$



Creating the simplest compressible MHD turbulence simulation at $\text{Re} \gtrsim 10^6$

10,080³ magnetised interstellar medium turbulence simulation

Beattie, Federrath, Mocz, Klessen, Cielo & Bhattacharjee

1. What is the scaling of the energy cascade in compressible MHD turbulence with no net b flux?
2. How are the characteristic scales organised in the ISM turbulence?
3. What are the saturation physics of the turbulent dynamo?

PI of a 100million core-hour project on superMUC-NG

ILES of compressible MHD turbulence

Turbulence: $\sigma_V/c_s \approx 4$, $\ell_0 = L/2$

Magnetic fields: $B = b_{\text{turb}}$, $\mathcal{M}_A \approx 2$

Three experiments for convergence tests:

- **LOW-RES:** 2,520³ (0.3Mcore-h, 8,640cores)
- **MID-RES:** 5,040³ (4.0Mcore-h, 34,560cores)
- **HIGH-RES:** 10,080³ (80.0Mcore-h, 148,240cores)
 - Resolving 10pc down to ~200au everywhere on the grid
 - Factor of 4 higher linear grid resolution than IllustrisTNG

3.45PB in data products

Broad Code details:

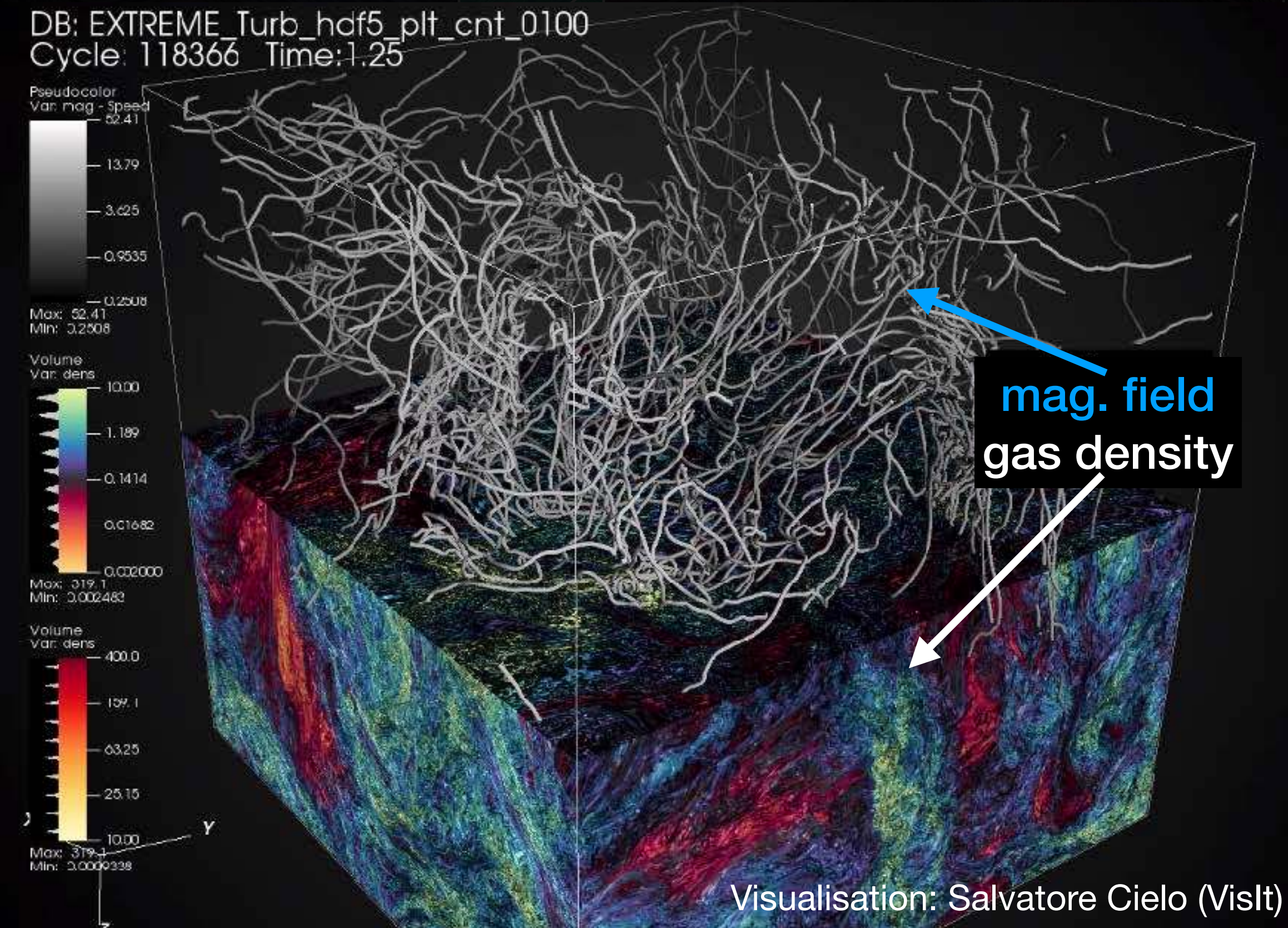
- Highly-modified version of finite volume code *FLASH*, second-order in space approximate Riemann (PPM) solver with framework outlined in Bouchut+2010, tested in *FLASH* in Waagen+2011.
- Ideal (ILES) compressible non-helical, isothermal MHD turbulence with finite correlation time (OU process; Federrath2022).

DB: EXTREME_Turb_hdf5_plt_cnt_0100
Cycle: 118366 Time: 1.25

Pseudocolor
Var: mag - Speed
Max: 52.41
Min: 0.2508

Volume
Var: dens
Max: 319.1
Min: 0.002482

Volume
Var: dens
Max: 319.1
Min: 0.000938



Visualisation: Salvatore Cielo (VisIt)

Stochastically forced Compressible MHD equations (ILES)

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$\partial_t (\rho \mathbf{u}) + \nabla \cdot \left(\rho \mathbf{u} \otimes \mathbf{u} + p \mathbb{I} - \frac{1}{4\pi} \mathbf{b} \otimes \mathbf{b} \right) = \rho \mathbf{f} + \nabla \cdot \mathbb{D}_\nu(\rho \mathbf{u})$$

$$\partial_t \mathbf{b} + \nabla \cdot (\mathbf{u} \otimes \mathbf{b} - \mathbf{b} \otimes \mathbf{u}) = \nabla \cdot \mathbb{D}_\eta(\mathbf{b})$$

$$\nabla \cdot \mathbf{b} = 0$$

$$p = c_s^2 \rho + \frac{1}{8\pi} \mathbf{b} \cdot \mathbf{b} \quad 10,080^3 \implies \text{Re} \sim \text{Rm} \sim (1 - 3) \times 10^6$$

Computed for our solver using results from Lakshmi+2023

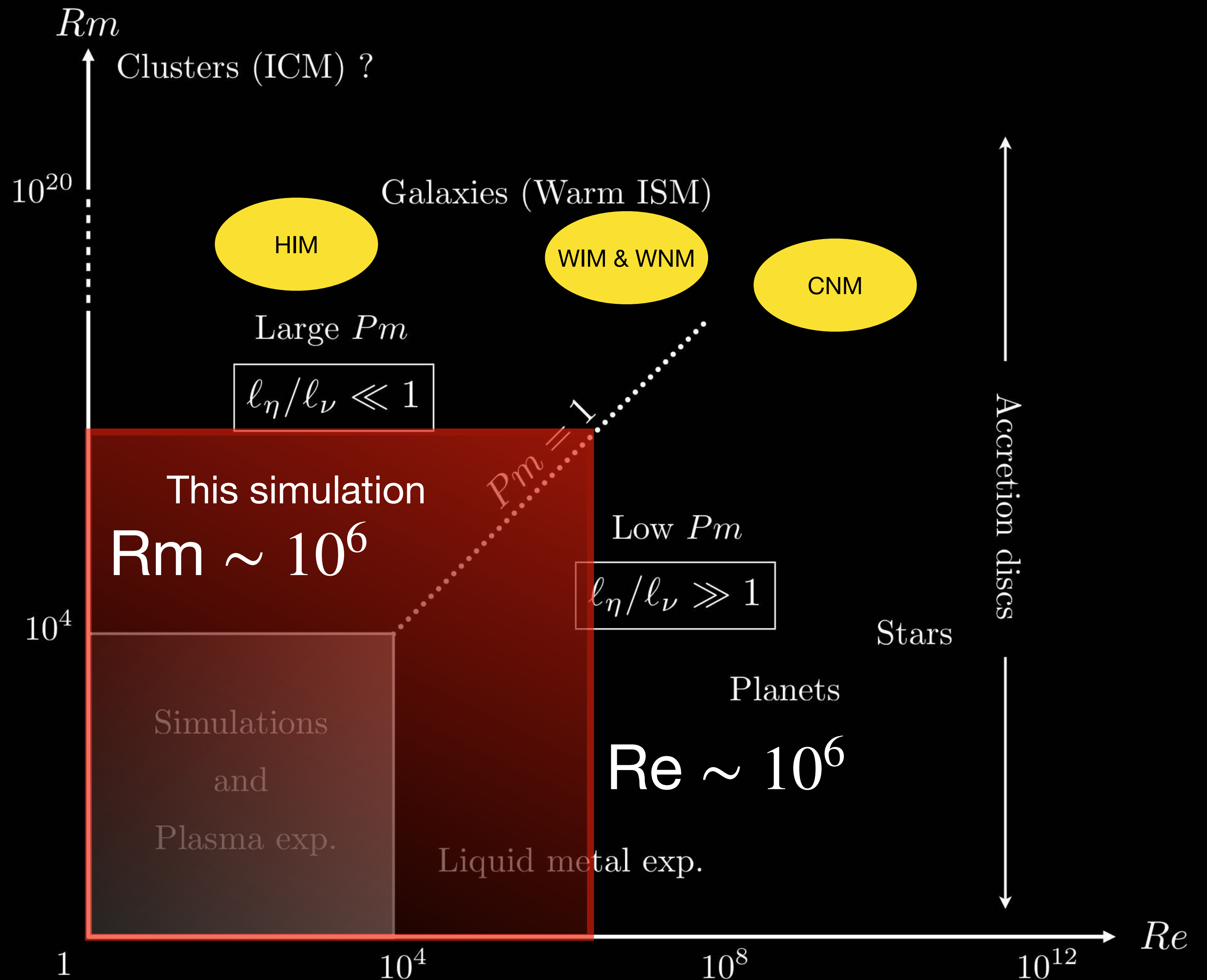
Re & Rm Landscape Rincon (2019)

WIM: $Rm \sim 10^{18}$

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CNM: $Rm \sim 10^{15}$

Ferrière, 2020; Plasma Physics and
Controlled Fusion



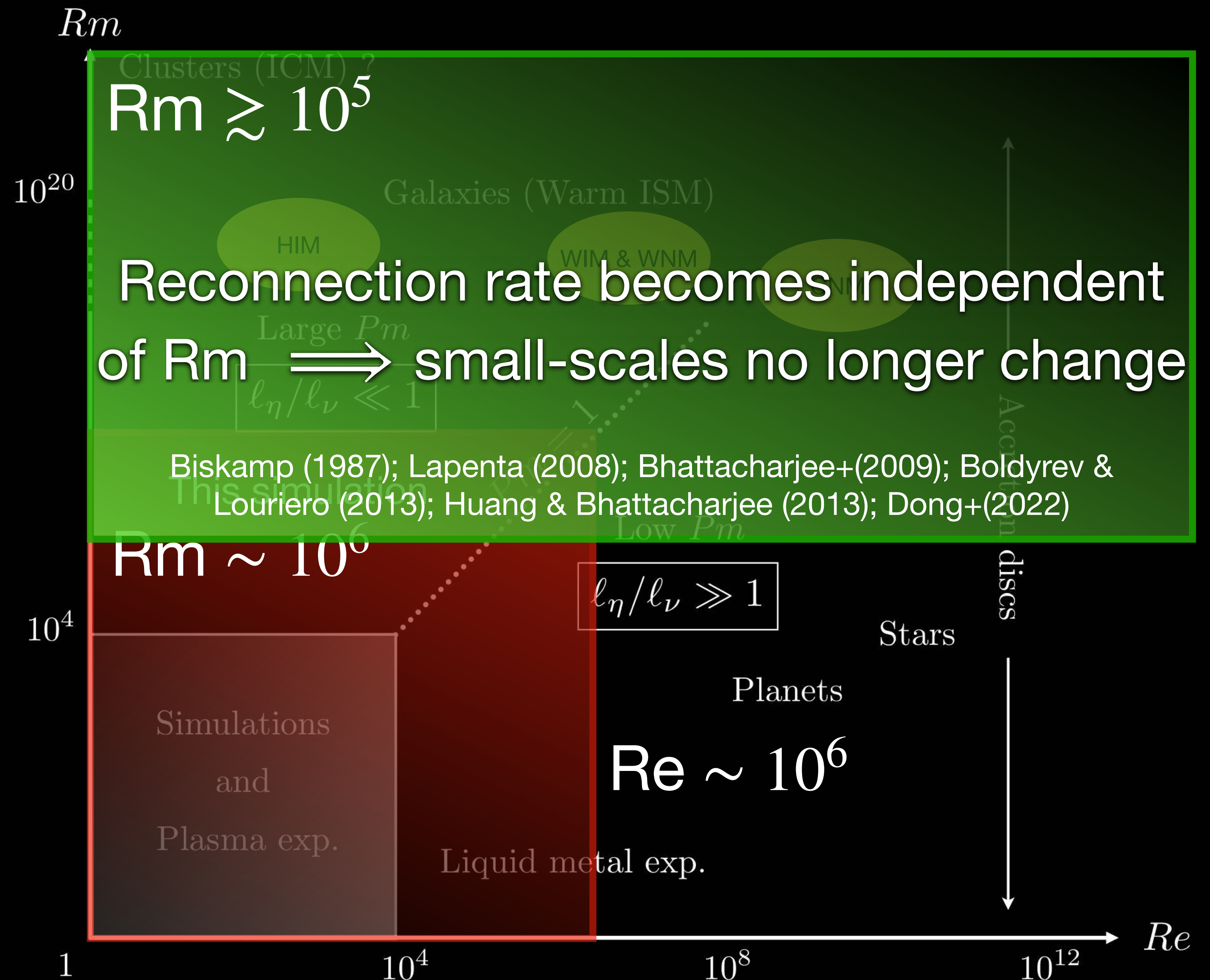
Re & Rm Landscape Rincon (2019)

WIM: $Rm \sim 10^{18}$

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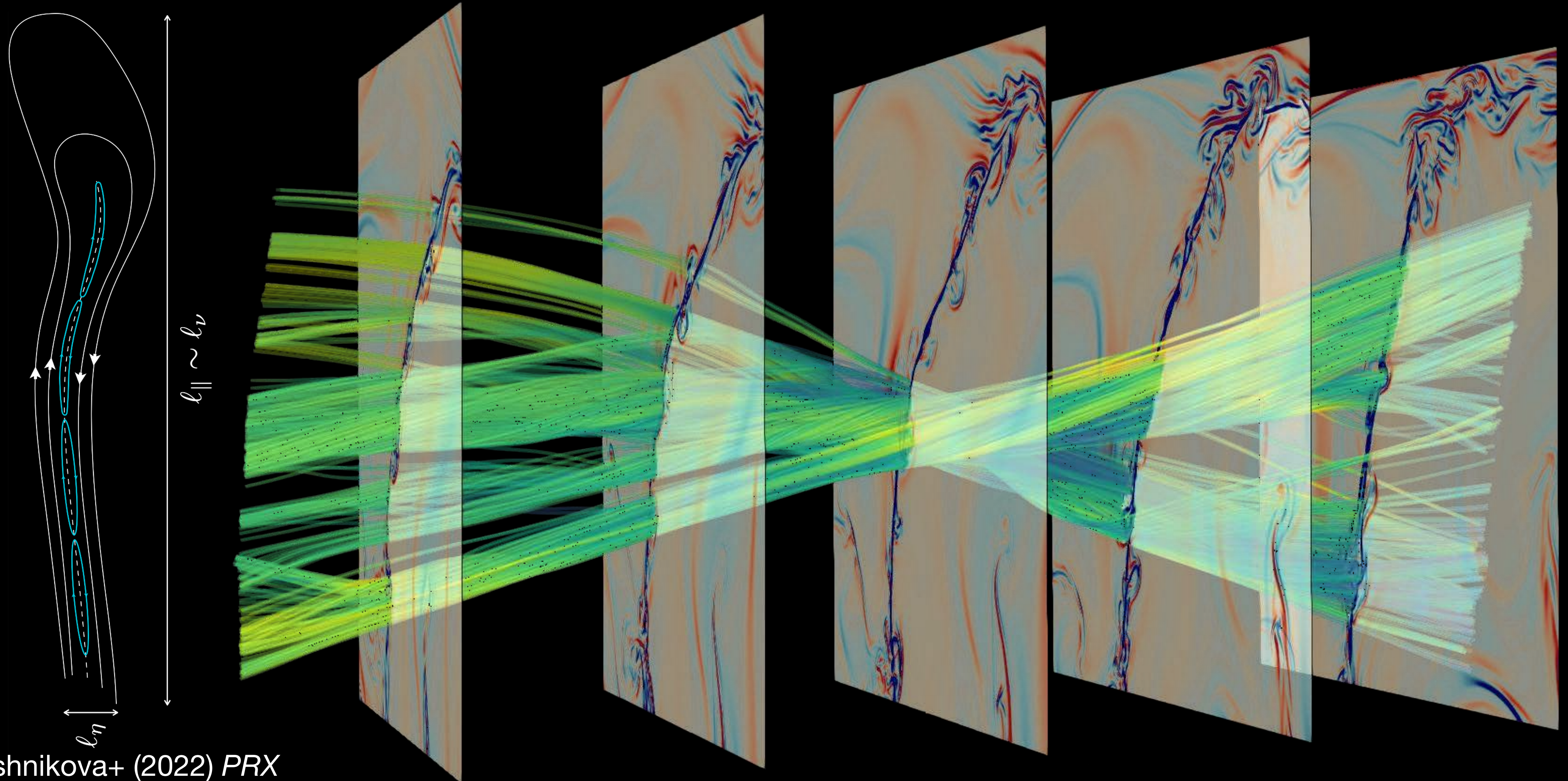
CNM: $Rm \sim 10^{15}$

Ferrière, 2020; Plasma Physics and
Controlled Fusion



The magnetic energy cascade

Dong+(2022) *Science Advances*

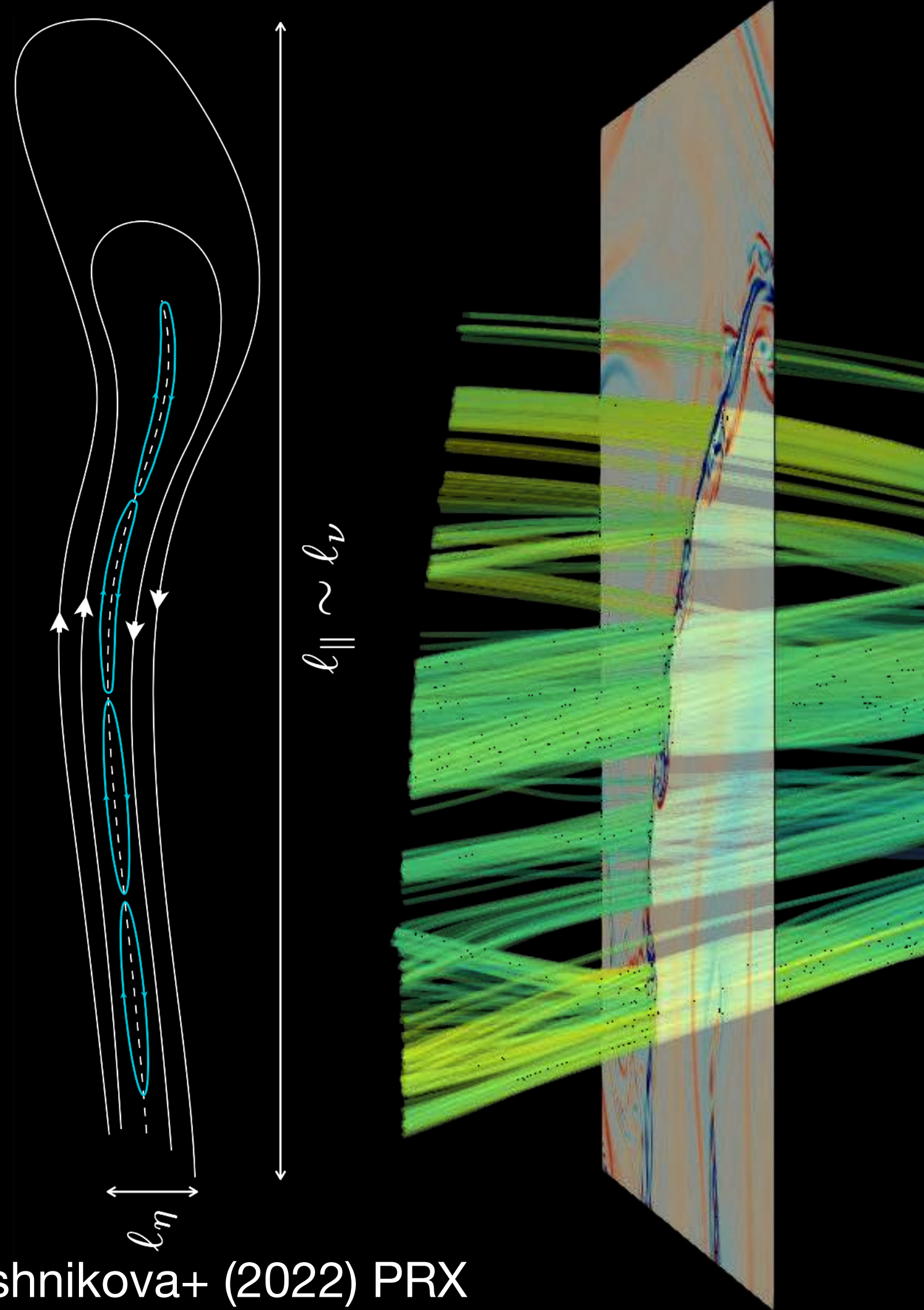


Galishnikova+ (2022) *PRX*

tearing instabilities growing inside a 3D current sheet

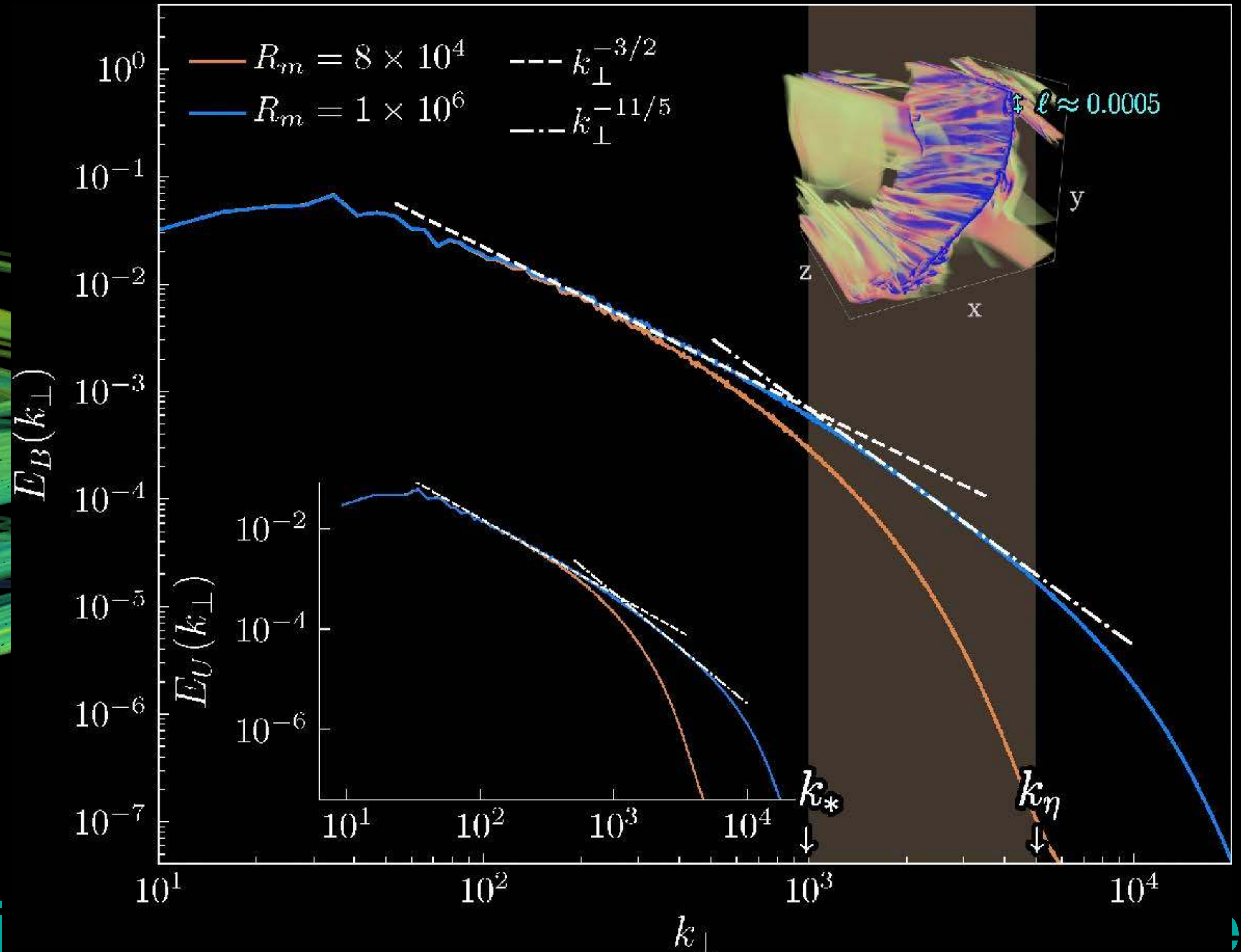
The magnetic energy cascade

Dong+(2022) *Science Advances*



Galishnikova+ (2022) PRX

plasmoid instabi



Stochastically forced Compressible MHD equations (ILES)

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$\partial_t (\rho \mathbf{u}) + \nabla \cdot \left(\rho \mathbf{u} \otimes \mathbf{u} + p \mathbb{I} - \frac{1}{4\pi} \mathbf{b} \otimes \mathbf{b} \right) = \boxed{\rho \mathbf{f}} + \nabla \cdot \mathbb{D}_\nu(\rho \mathbf{u})$$

$$\partial_t \mathbf{b} + \nabla \cdot (\mathbf{u} \otimes \mathbf{b} - \mathbf{b} \otimes \mathbf{u}) = \nabla \cdot \mathbb{D}_\eta(\mathbf{b})$$

$$\nabla \cdot \mathbf{b} = 0$$

the turbulence source function

$$p = c_s^2 \rho + \frac{1}{8\pi} \mathbf{b} \cdot \mathbf{b}$$

Stochastically forced Compressible MHD equations (ILES)

$$d\hat{\mathbf{f}}(\mathbf{k}, t) = f_0(\mathbf{k})\mathbb{P}(\mathbf{k}) \cdot d\mathbf{W}(t) - \hat{\mathbf{f}}(\mathbf{k}, t)\frac{dt}{t_0}$$

$d\mathbf{W}(t)$ Wiener process that draws delta correlated from $\sim \mathcal{N}(0,1)$

K space projection tensor

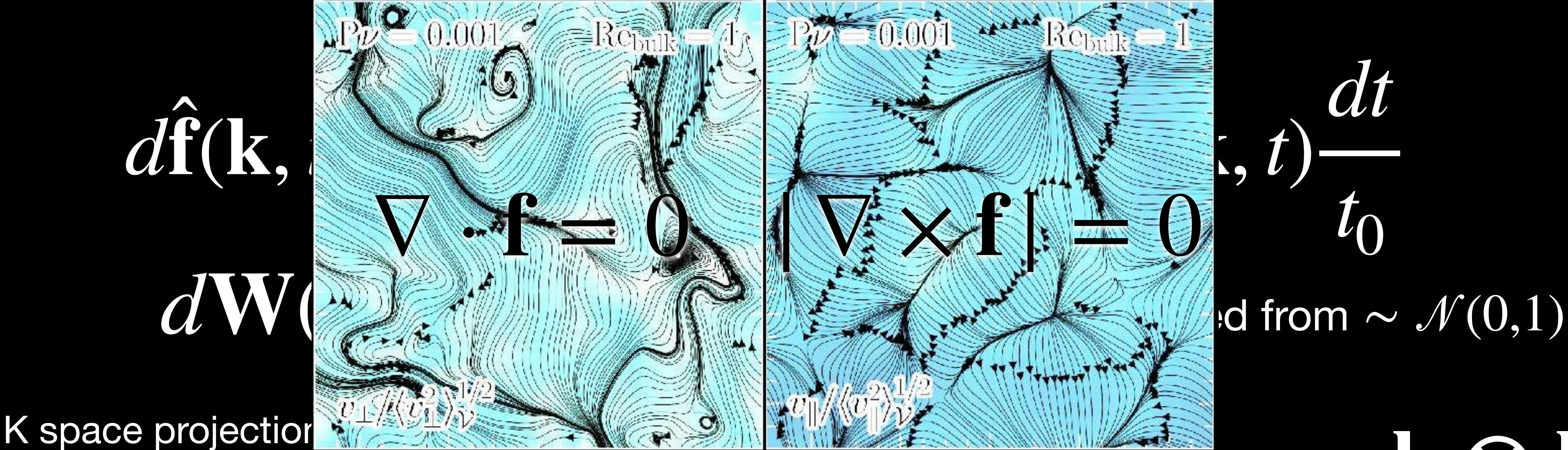
$$\mathbb{P} = \zeta\mathbb{P}^\perp + (1 - 2\zeta)\mathbb{P}^\parallel = \zeta\mathbb{I} + (1 - 2\zeta)\frac{\mathbf{k} \otimes \mathbf{k}}{|\mathbf{k}|^2}$$

$$\zeta = 1 \implies \nabla \cdot \mathbf{f} = 0$$

$$\zeta = 0 \implies \nabla \times \mathbf{f} = 0$$

t_0 e-folding time of the forcing / correlation time / outer-scale turbulent turnover time

Stochastically forced Compressible MHD equations (ILES)



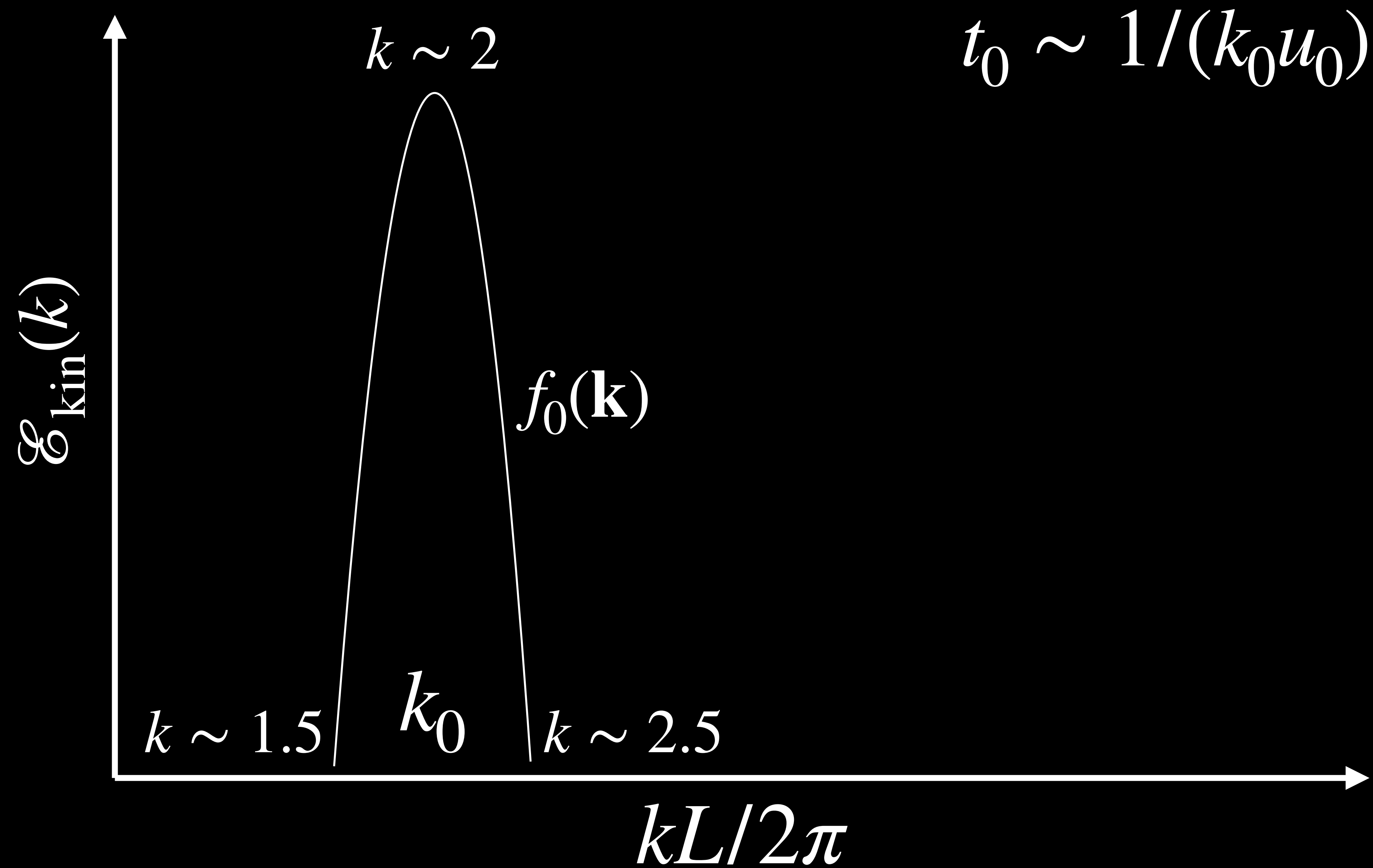
$$\mathbb{P} = \zeta \mathbb{P}^{\perp} + (1 - 2\zeta) \mathbb{P}^{\parallel} = \zeta \mathbb{I} + (1 - 2\zeta) \frac{\mathbf{k} \otimes \mathbf{k}}{|\mathbf{k}|^2}$$

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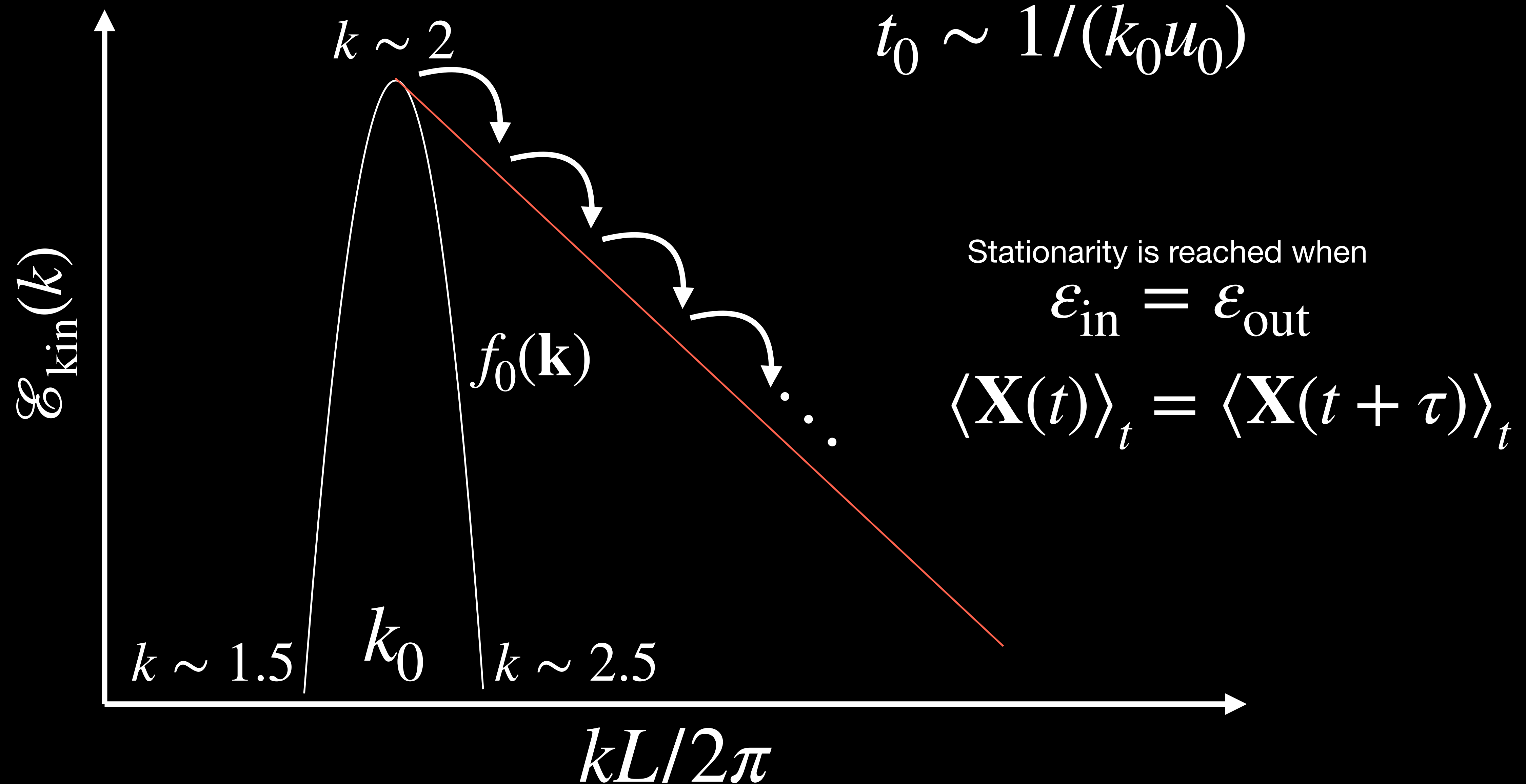
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Stochastically forced Compressible MHD equations (ILES)



Stochastically forced Compressible MHD equations (ILES)



10,080³ magnetised interstellar medium turbulence simulation

Beattie, Federrath, Mocz, Klessen &

1. What is the scaling of the energy cascade in compressible MHD turbulence with no magnetic field?
2. How are the characteristic scales organised in ISM turbulence?
3. What are the saturation physics of the turbulent dynamo?

PI of a 100million core-hour project on supercomputers

ILES of compressible MHD turbulence

Turbulence: $\sigma_V/c_s \approx 4$, $\ell_0 = L/2$

Magnetic fields: $B = b_{\text{turb}}$, $\mathcal{M}_A \approx 2$

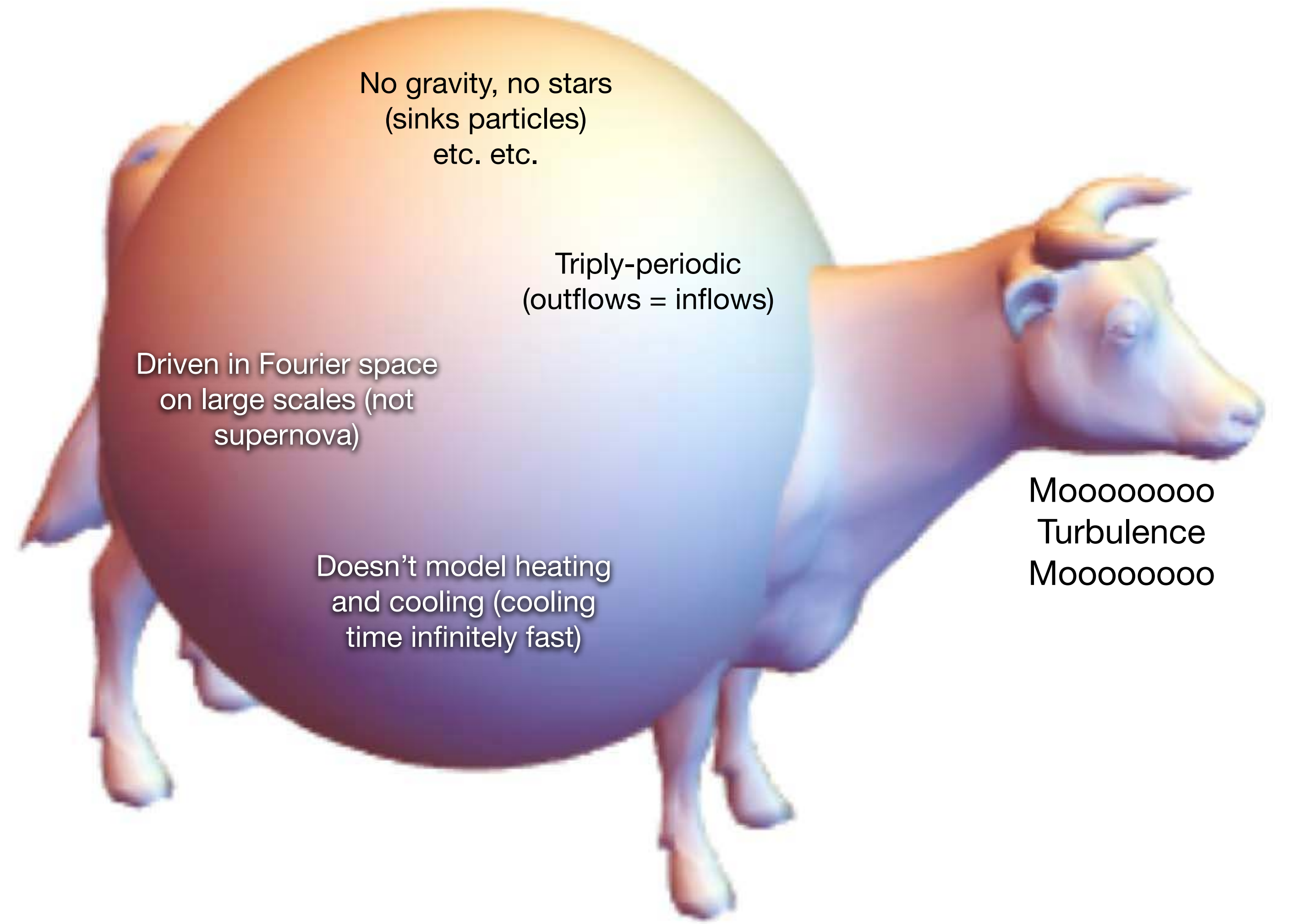
Three experiments for convergence tests:

- LOW-RES: 2,520³ (0.3Mcore-h, 8,640³)
- MID-RES: 5,040³ (4.0Mcore-h, 34,560³)
- **HIGH-RES: 10,080³ (80.0Mcore-h, 148,608³)**
 - Resolving 10pc down to ~200au everywhere on the grid
 - Factor of 4 higher linear grid resolution than IllustrisT100

3.45PB in data products

Broad Code details:

- Highly-modified version of finite volume code *FLASH*, second-order in space approximate Riemann (PPM)

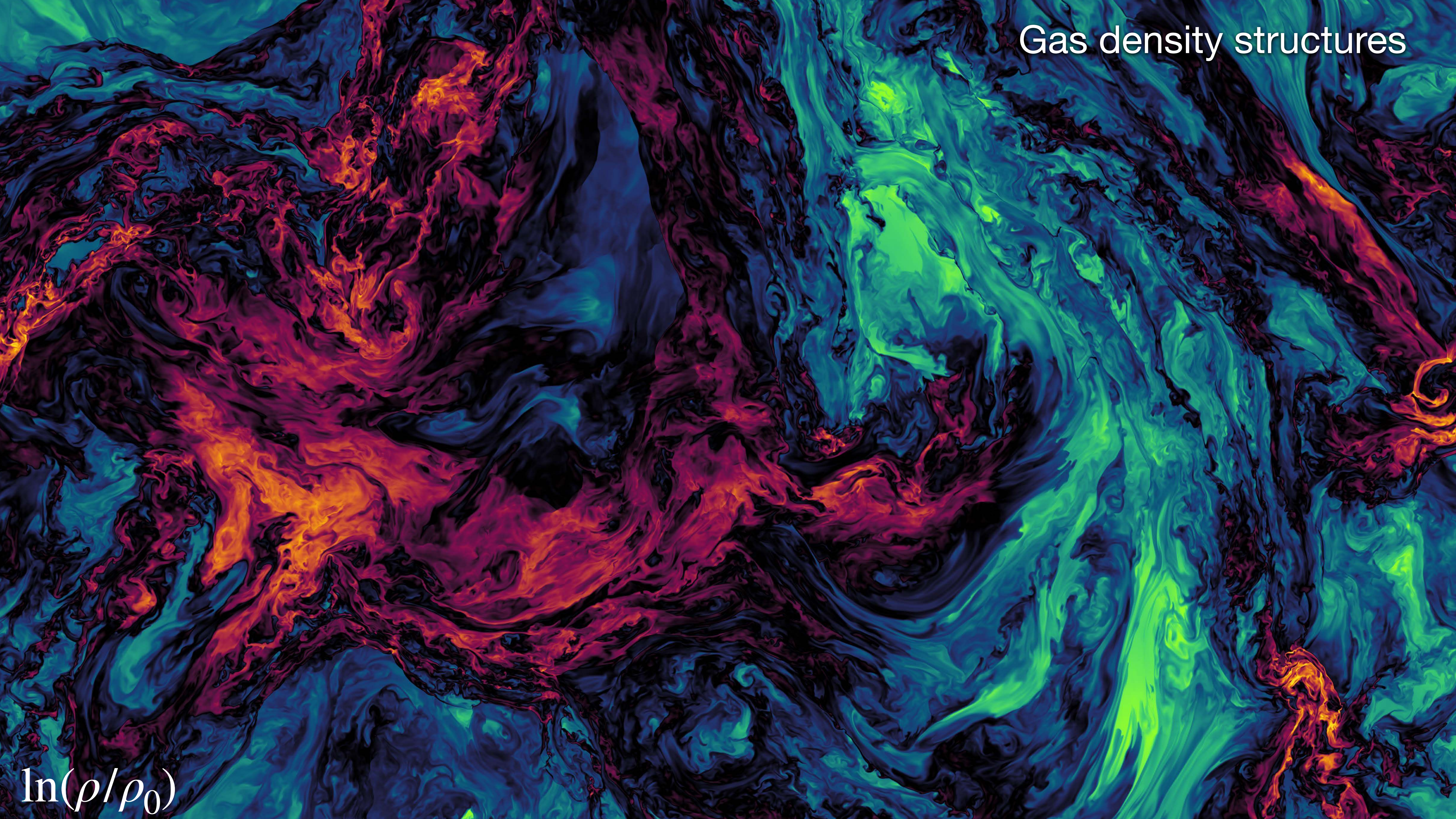


Visualisation: Salvatore Cielo (VisIt)

Volume integral statistics

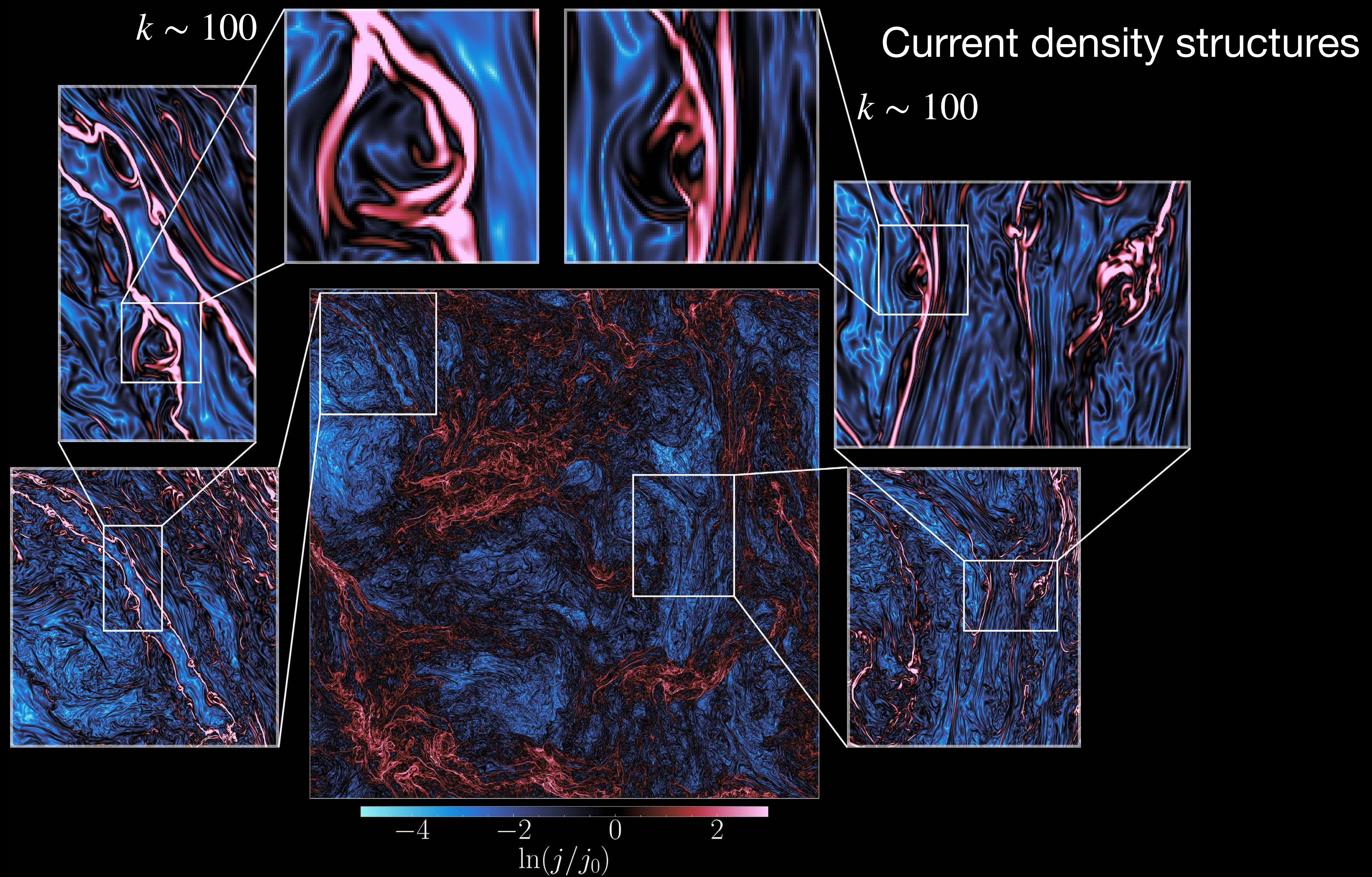
Gas density structures

$\ln(\rho/\rho_0)$



magnetic field structures



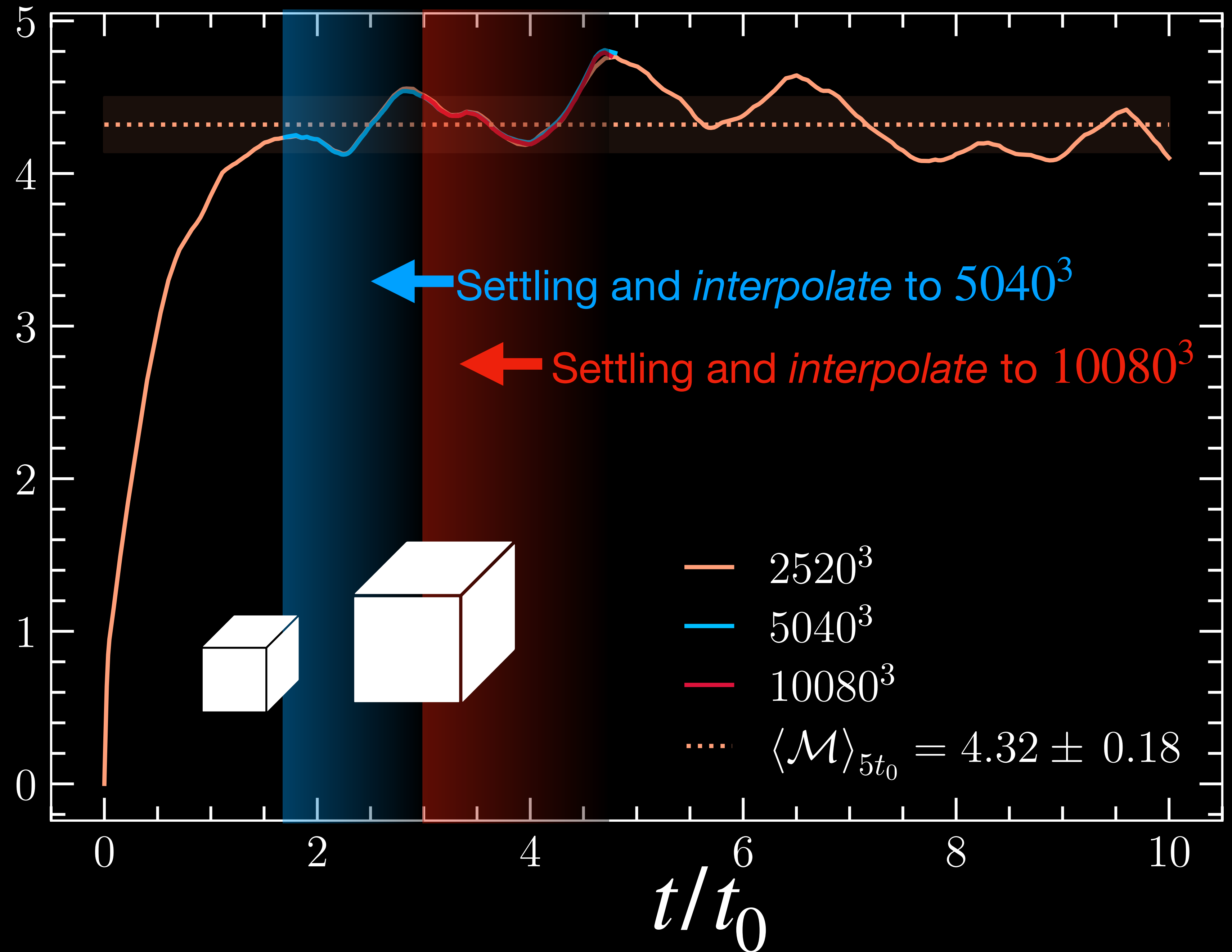


Volume integral Quantities

$$\mathcal{M} = \frac{\langle u^2 \rangle_{\mathcal{V}}^{1/2}}{c_s}$$

Interpolation to
generate ICs for
successively higher
resolution
experiments

$\mathcal{M}$

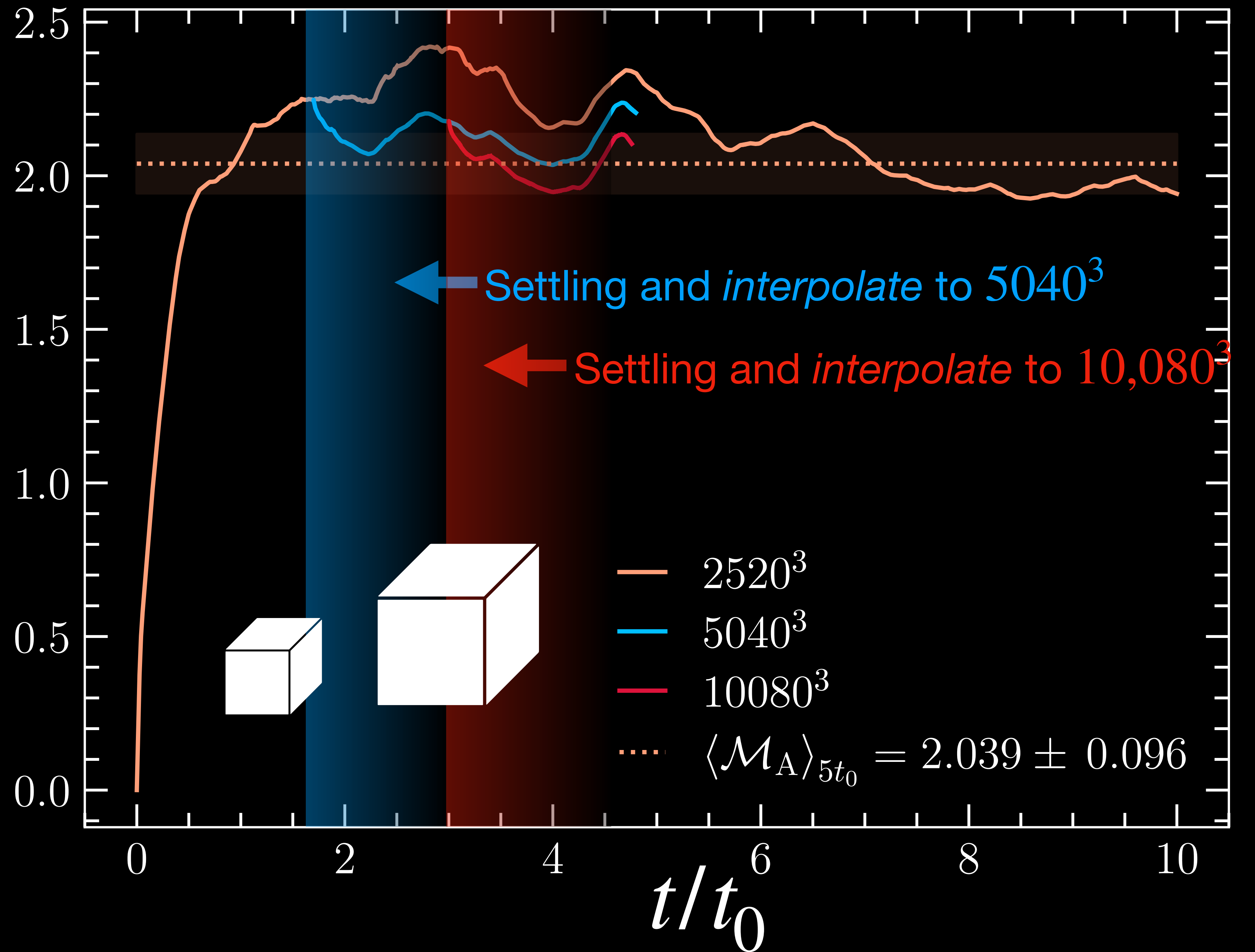


Volume integral Quantities

$$\mathcal{M}_A = \sqrt{\frac{E_{\text{kin}}}{E_{\text{mag}}}}$$

Interpolation to
generate ICs for
successively higher
resolution
experiments

$\mathcal{M}_A$



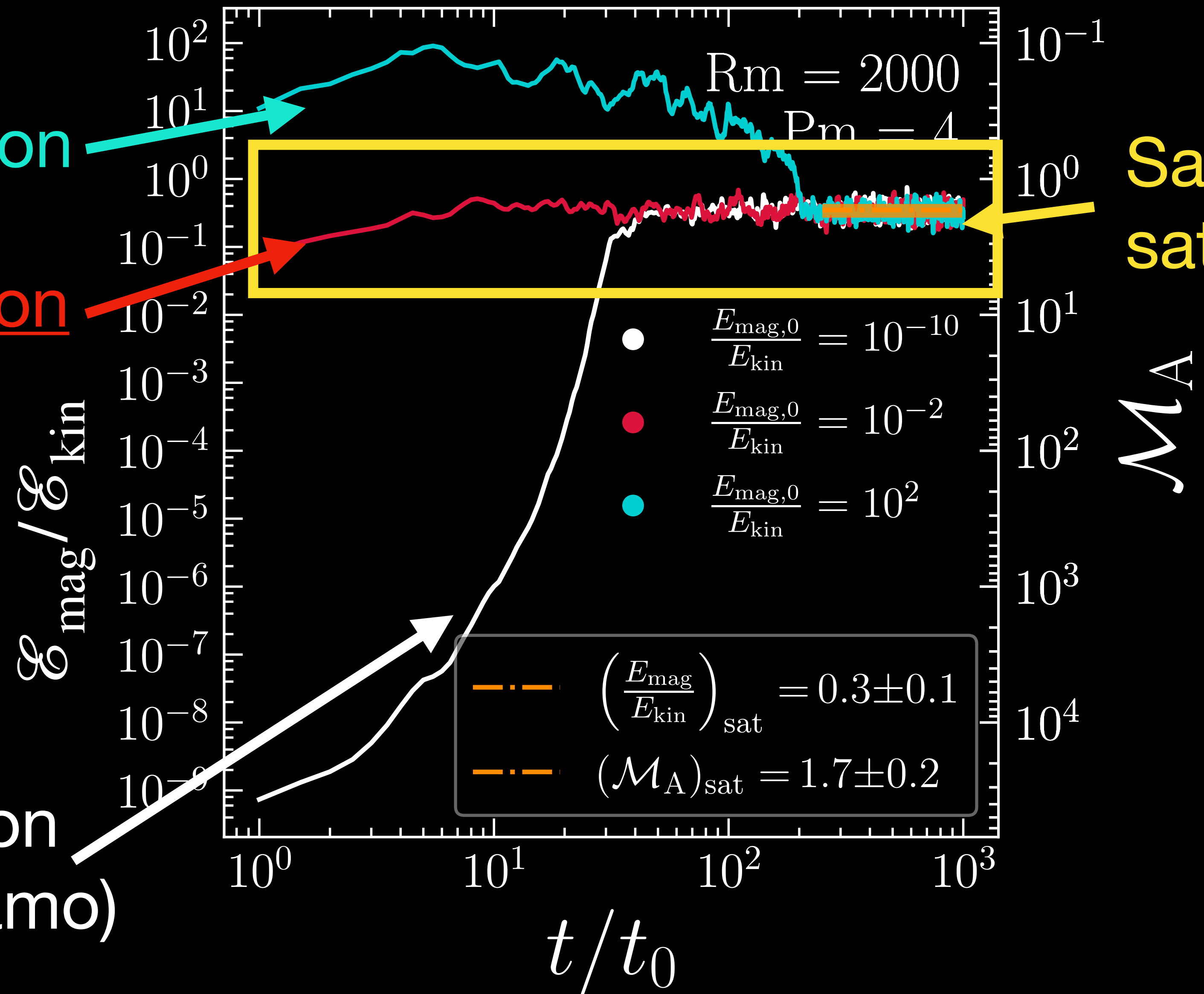
Magnetic ICs.

Decay into the saturation

Initialise in the saturation

$$\mathcal{M}_A = \sqrt{\frac{E_{\text{kin}}}{E_{\text{mag}}}}$$

Grow into the saturation
(classical turbulent dynamo)



Same
saturation

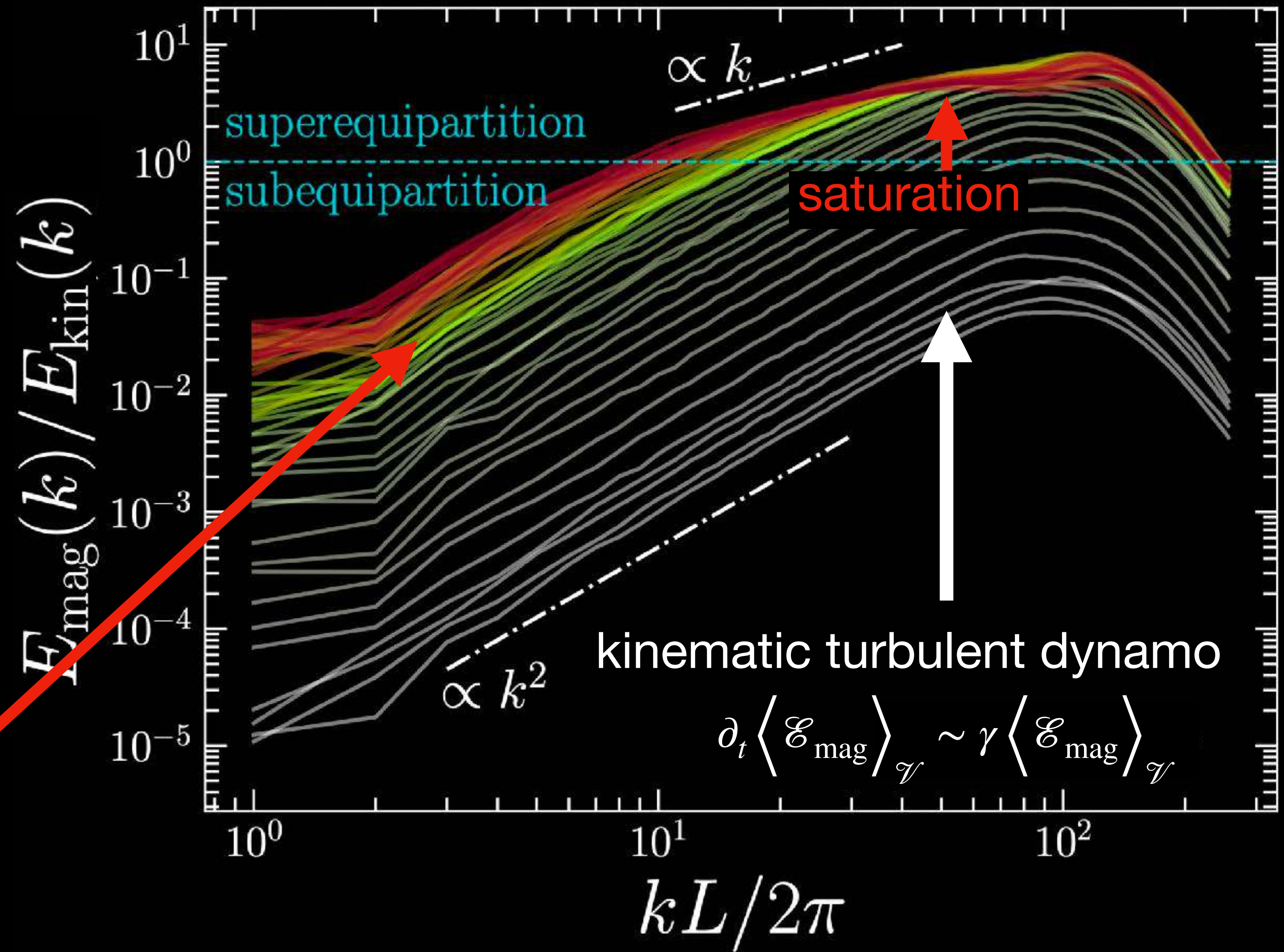
$\mathcal{M}_A$

Magnetic ICs.

$$\langle b \rangle = 0$$

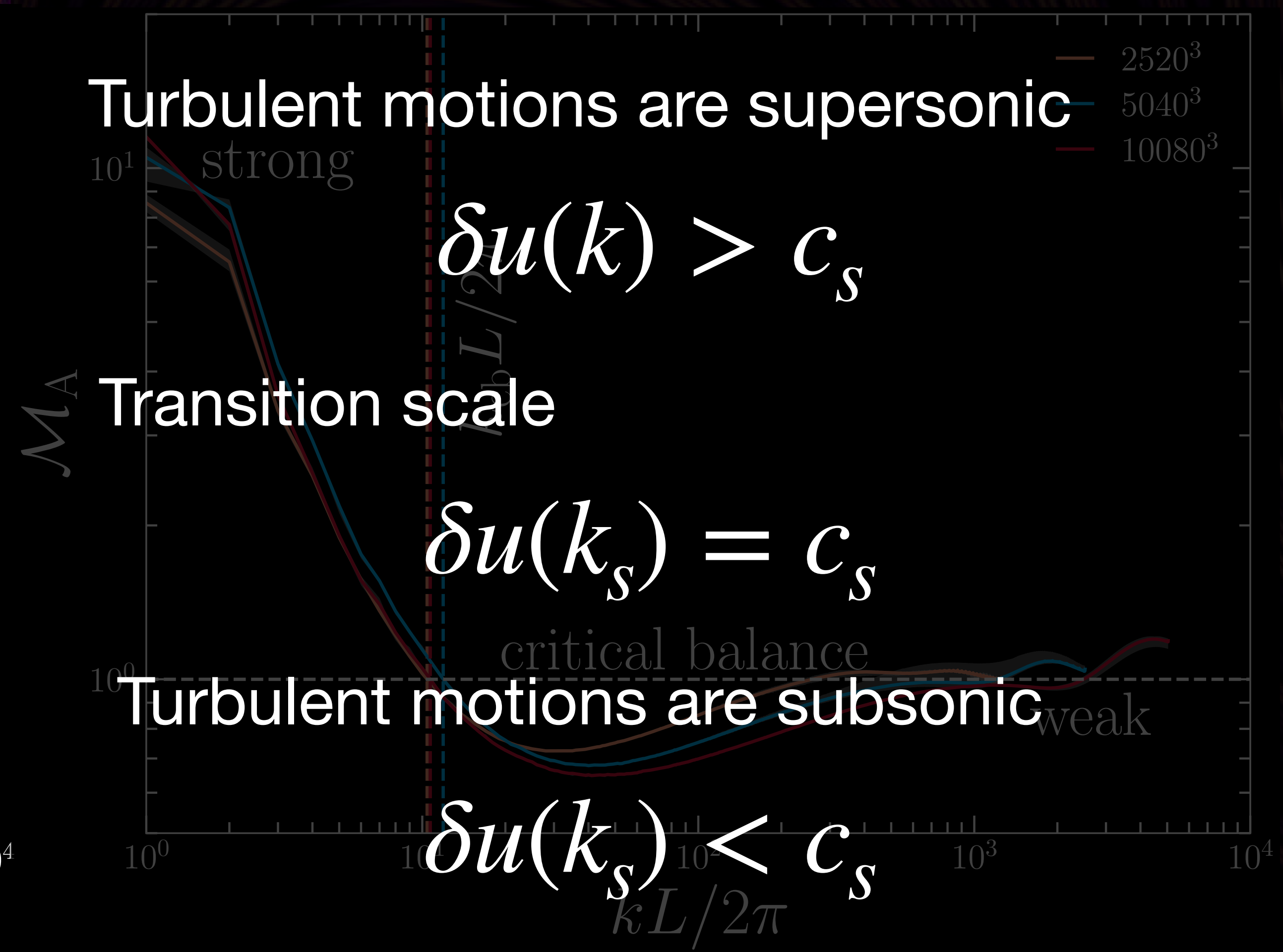
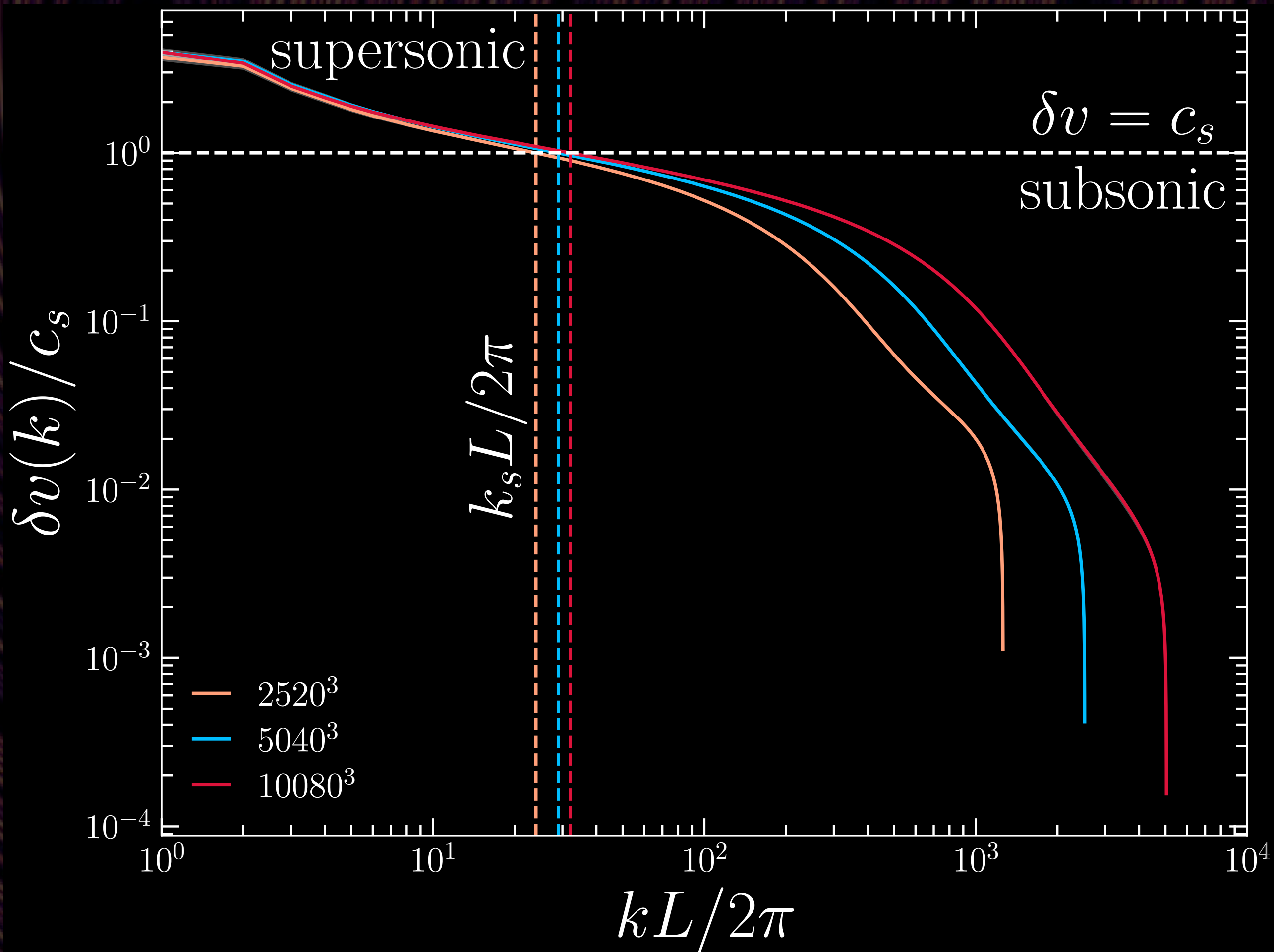
Completely turbulent
magnetic
field maintained by a
turbulent dynamo

ICs taken from a
saturated dynamo
experiment



Turbulent spectra and scales

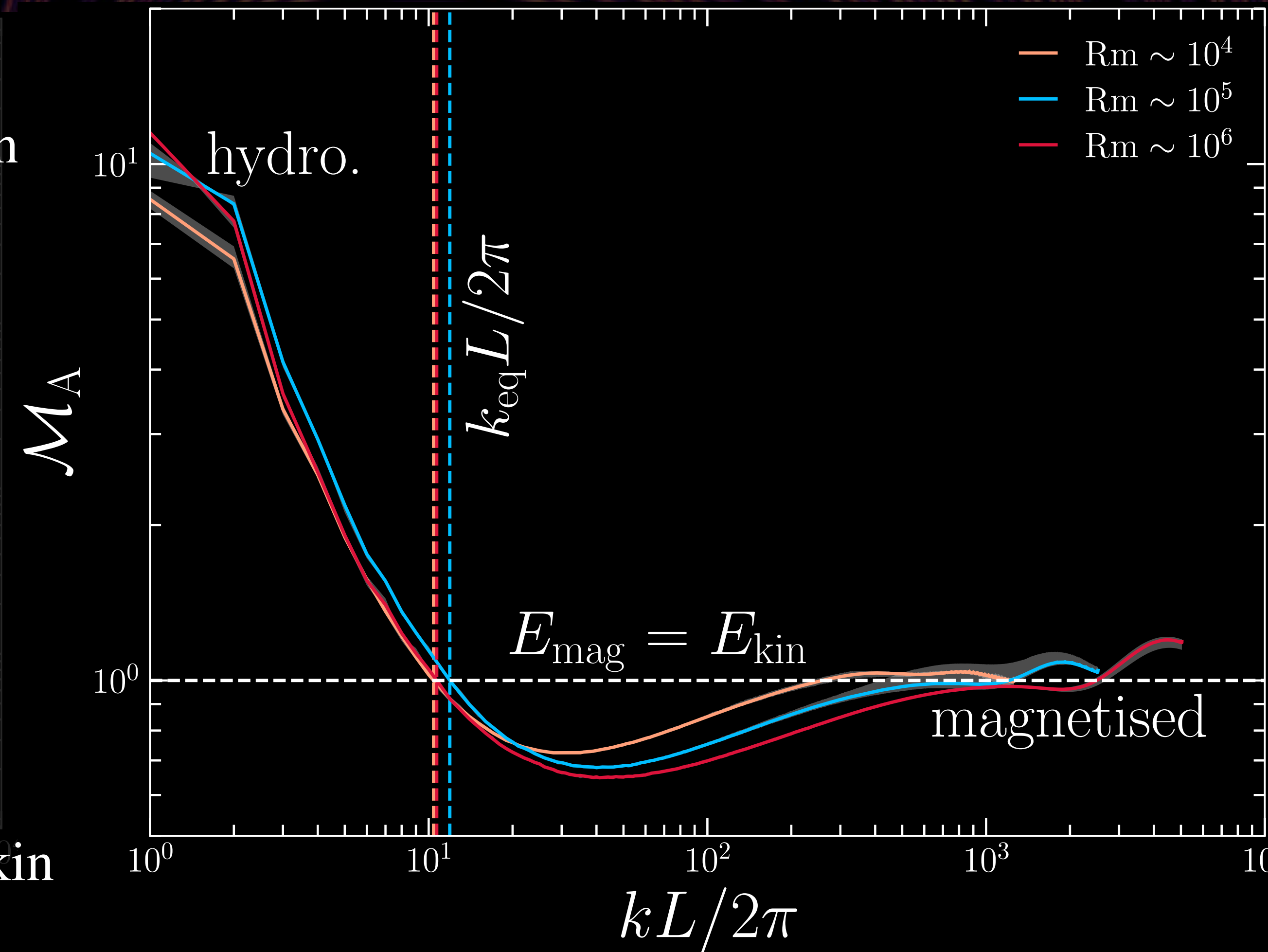
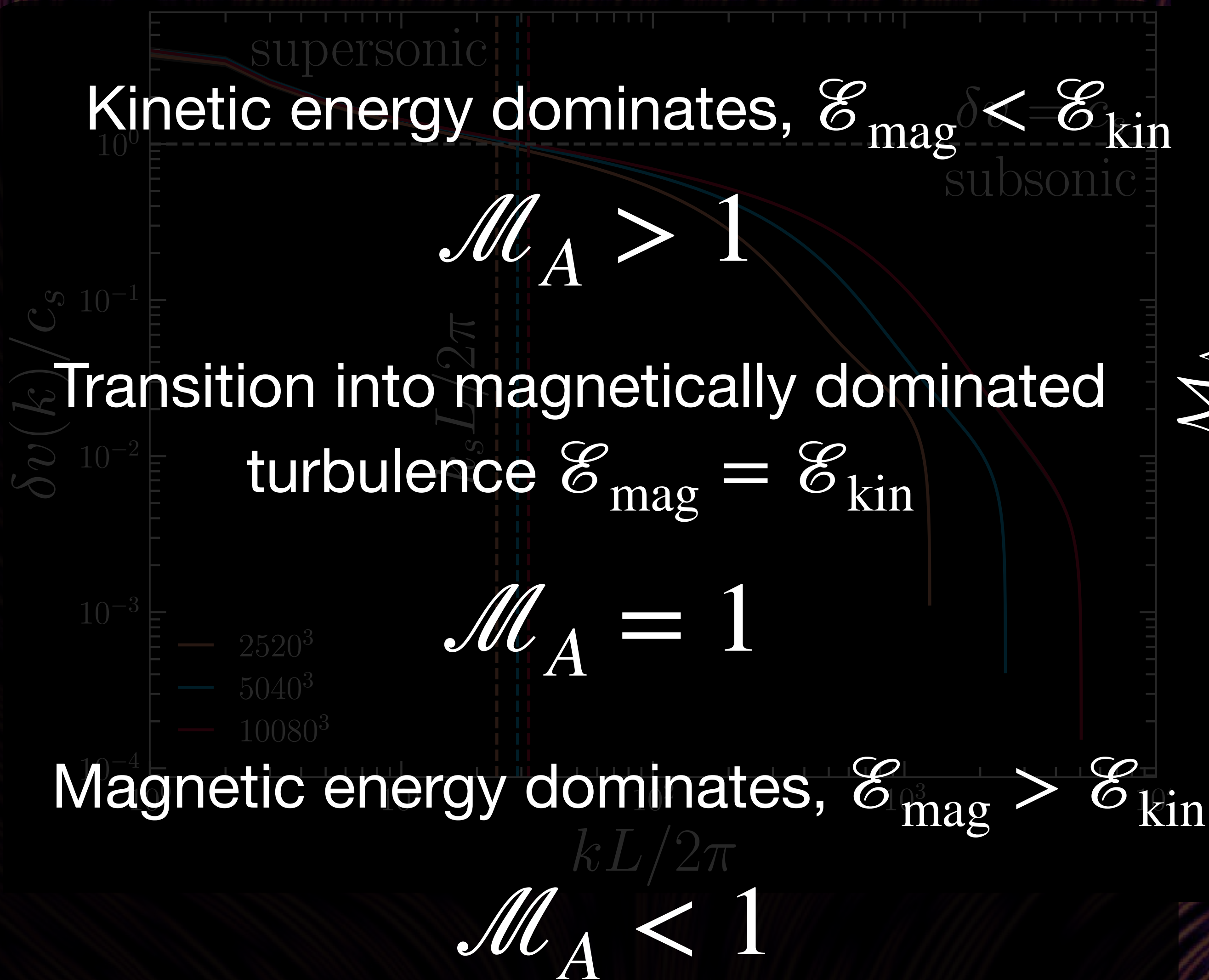
Two important scales



First measurement of magnetosonic scale

Two important scales

Energy equipartition scale



Two important scales

Kinetic energy dominates, $\mathcal{E}_{\text{mag}} < \mathcal{E}_{\text{kin}}$

$$\mathcal{M}_A > 1$$

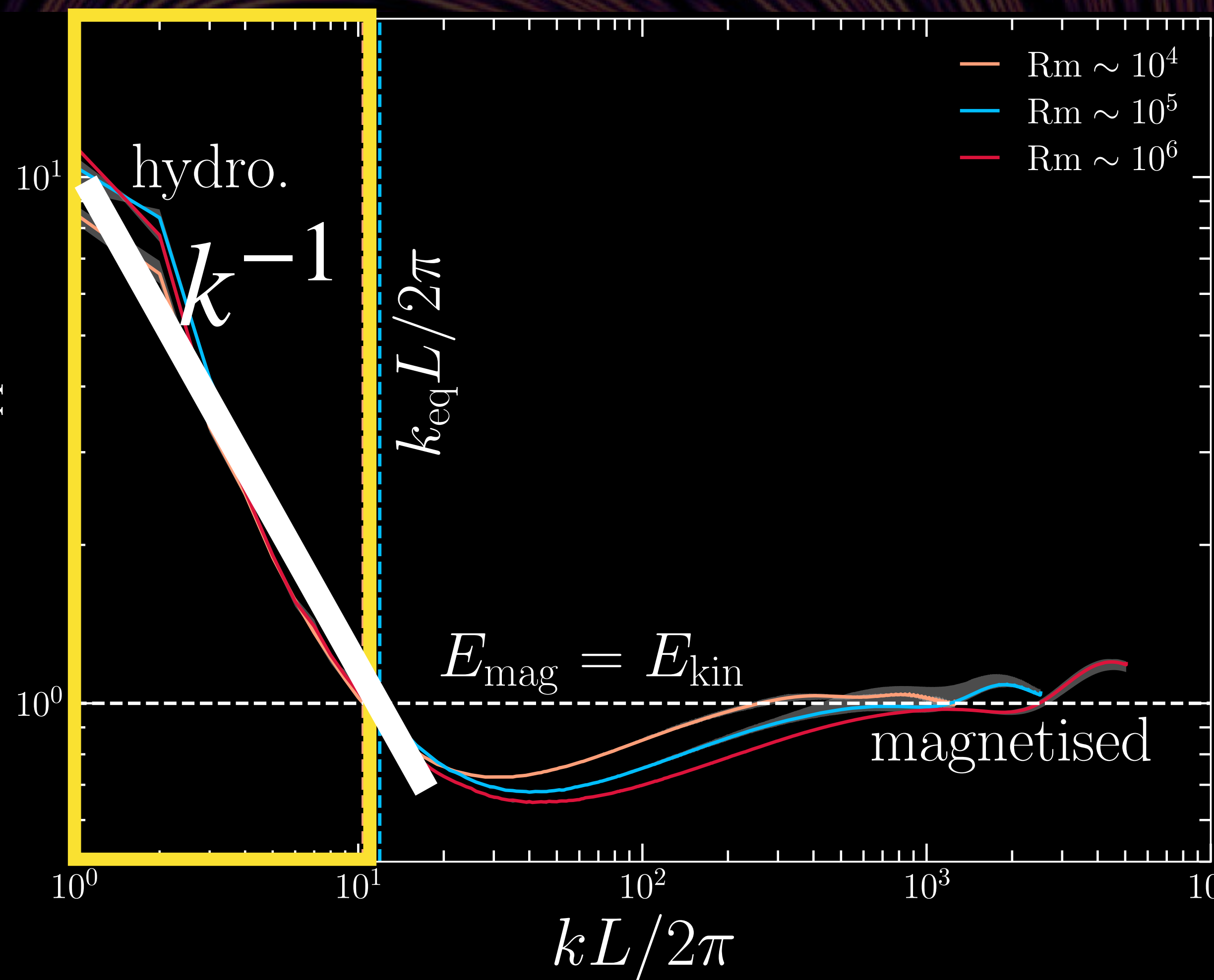
Transition into magnetically dominated turbulence $\mathcal{E}_{\text{mag}} = \mathcal{E}_{\text{kin}}$

$$\mathcal{M}_A = 1$$

Magnetic energy dominates, $\mathcal{E}_{\text{mag}} > \mathcal{E}_{\text{kin}}$

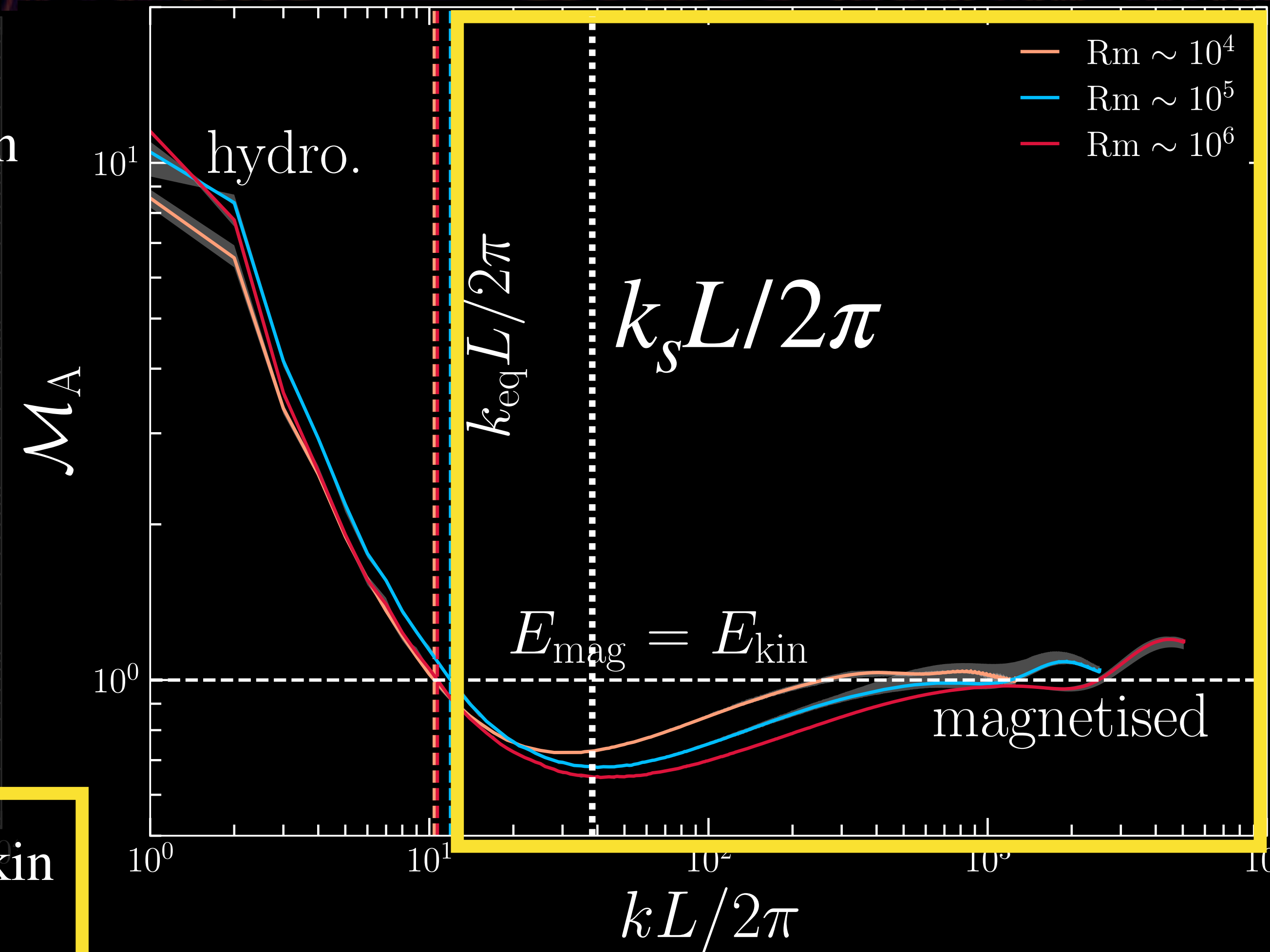
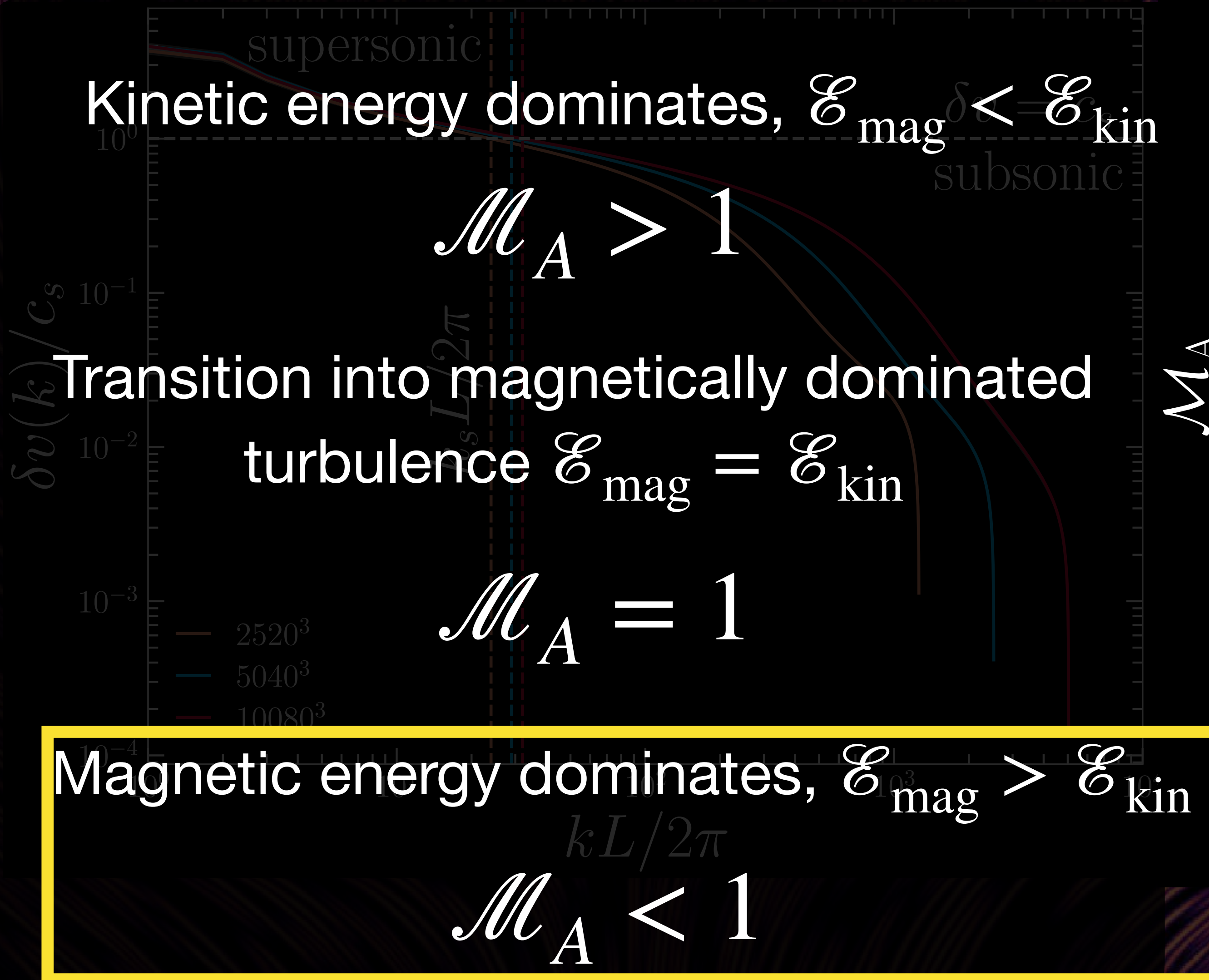
$$\mathcal{M}_A < 1$$

Energy equipartition scale



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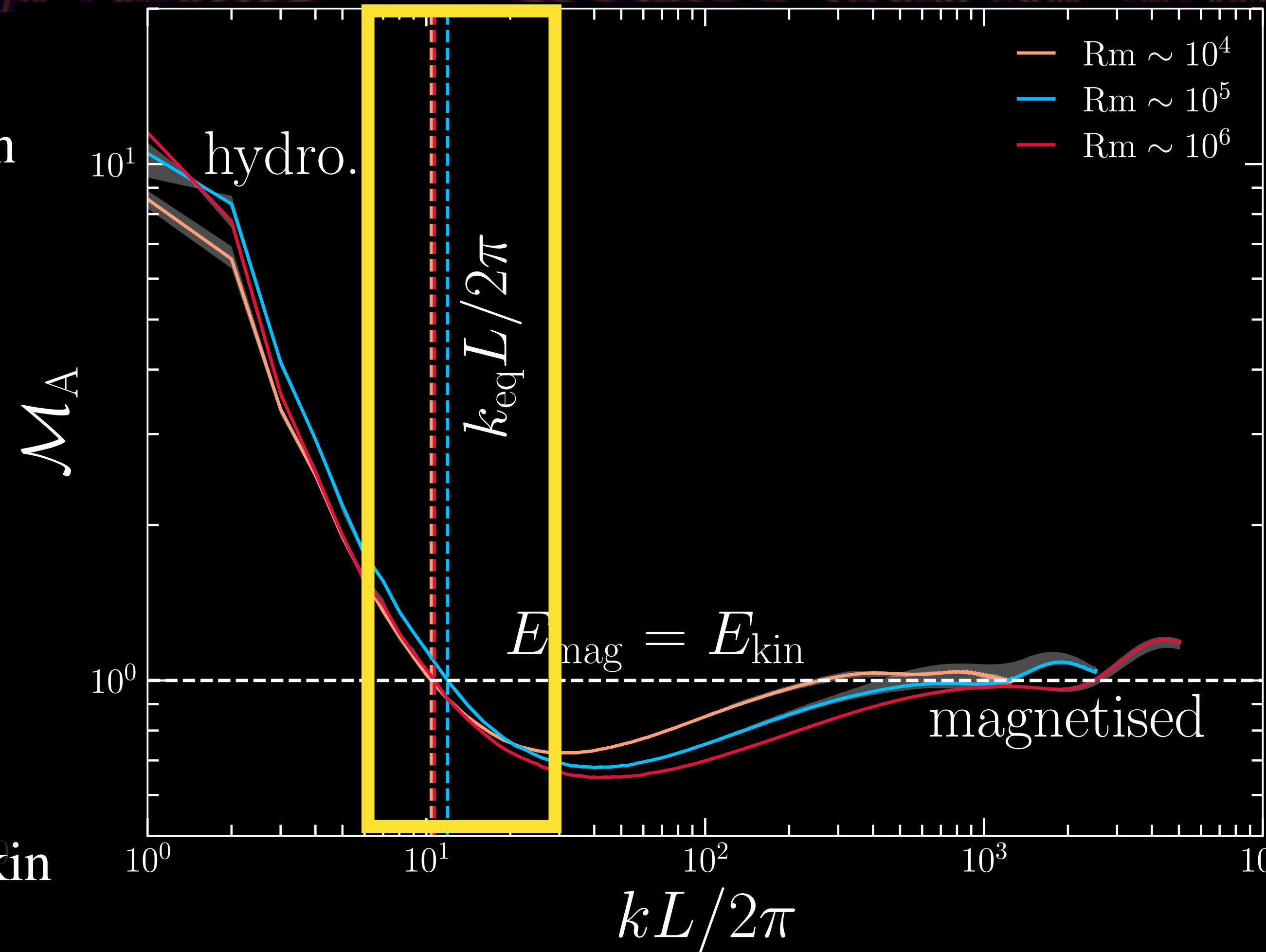
$$\mathcal{M}_A > 1$$

Transition into magnetically dominated turbulence $\mathcal{E}_{\text{mag}} = \mathcal{E}_{\text{kin}}$

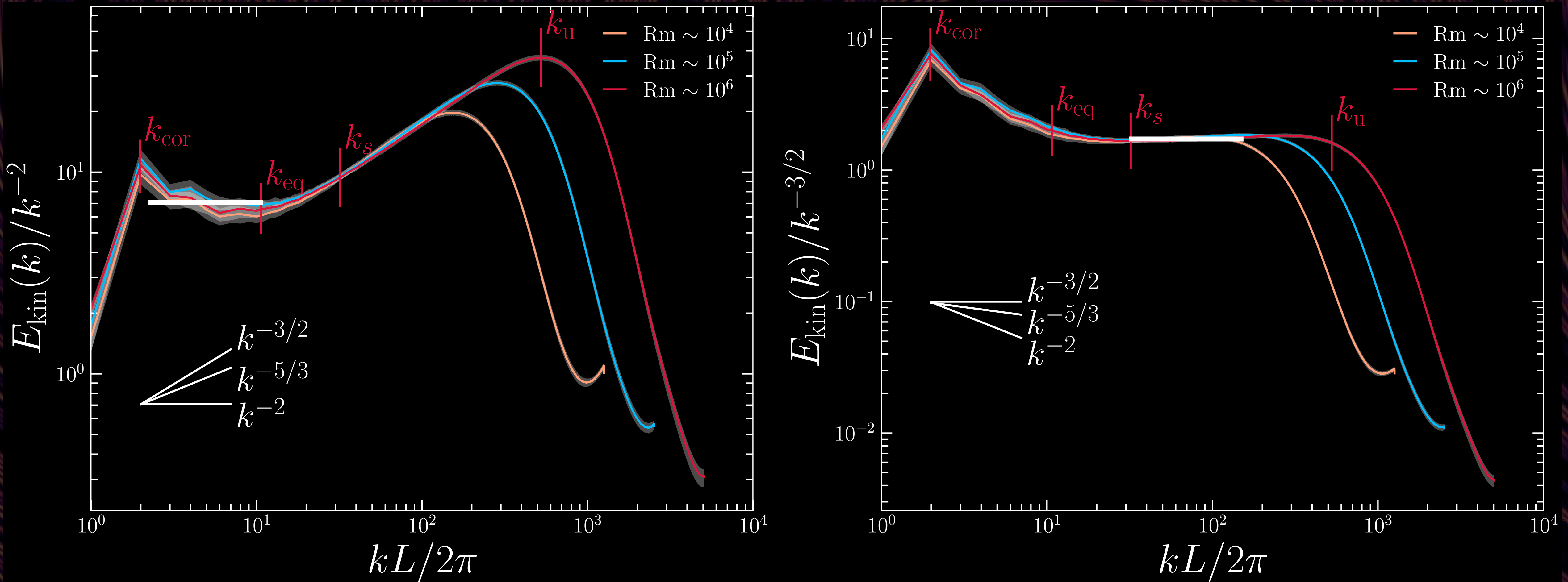
$$\mathcal{M}_A = 1$$

Magnetic energy dominates, $\mathcal{E}_{\text{mag}} > \mathcal{E}_{\text{kin}}$

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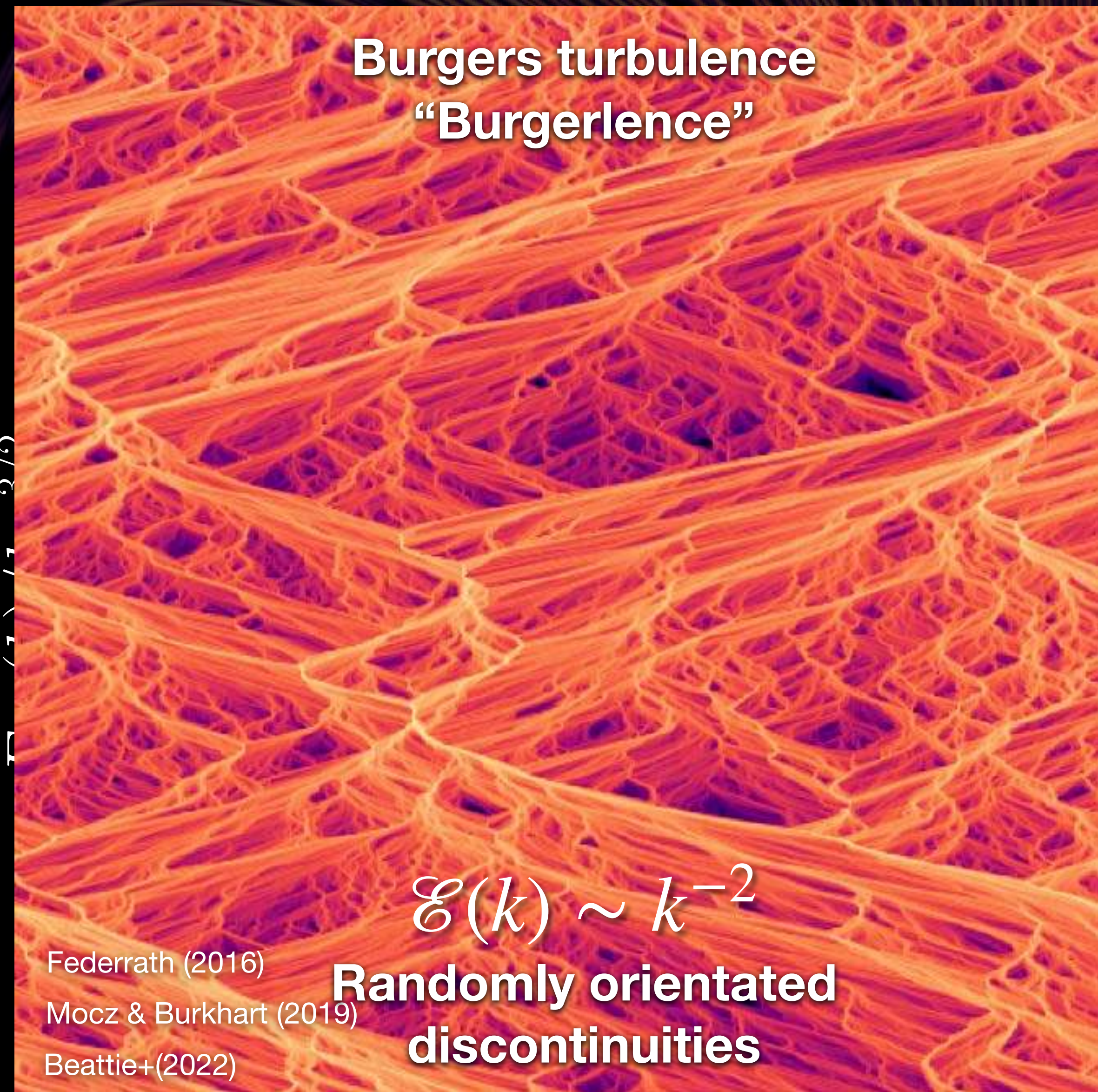
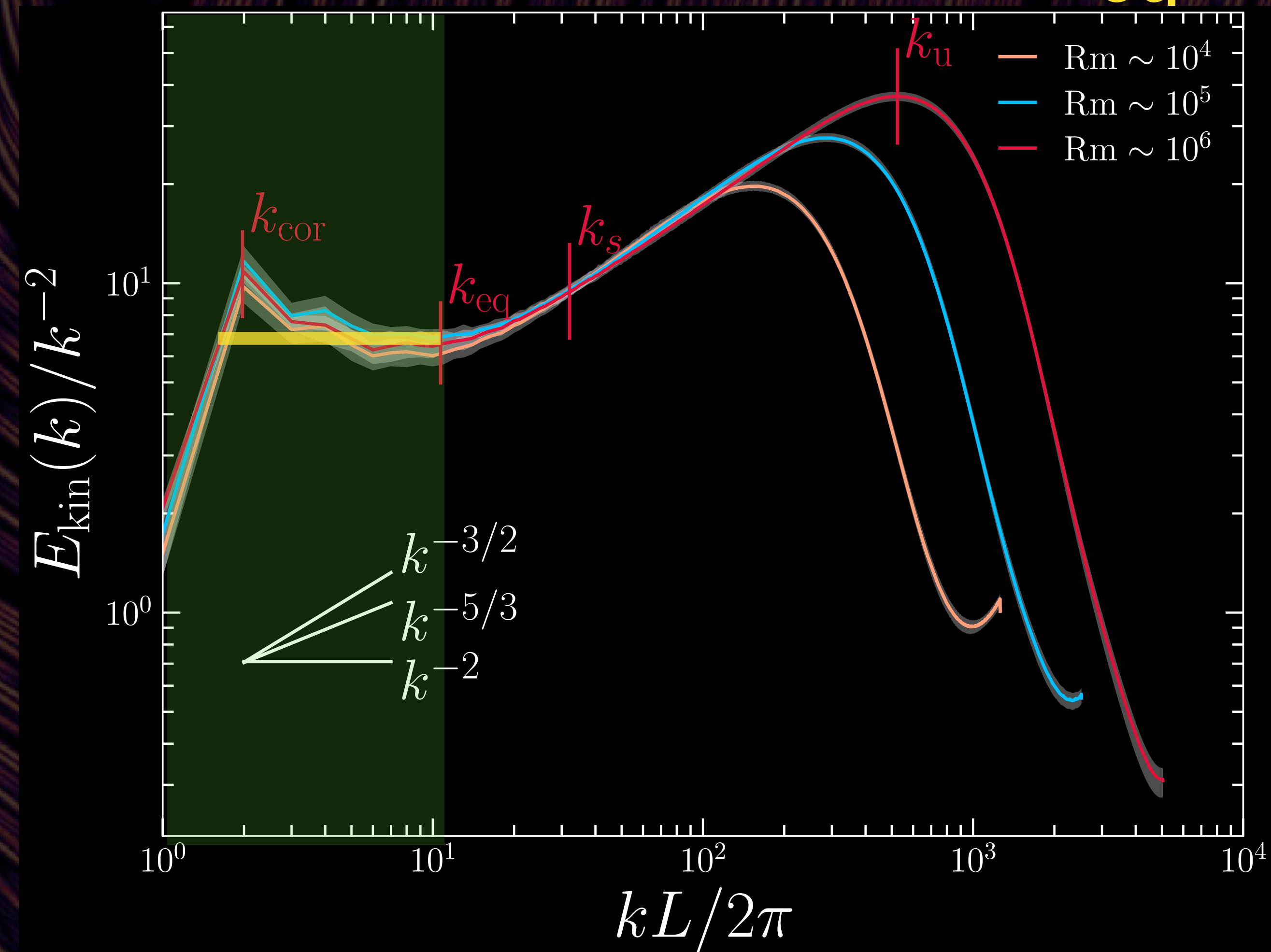


The kinetic energy cascade



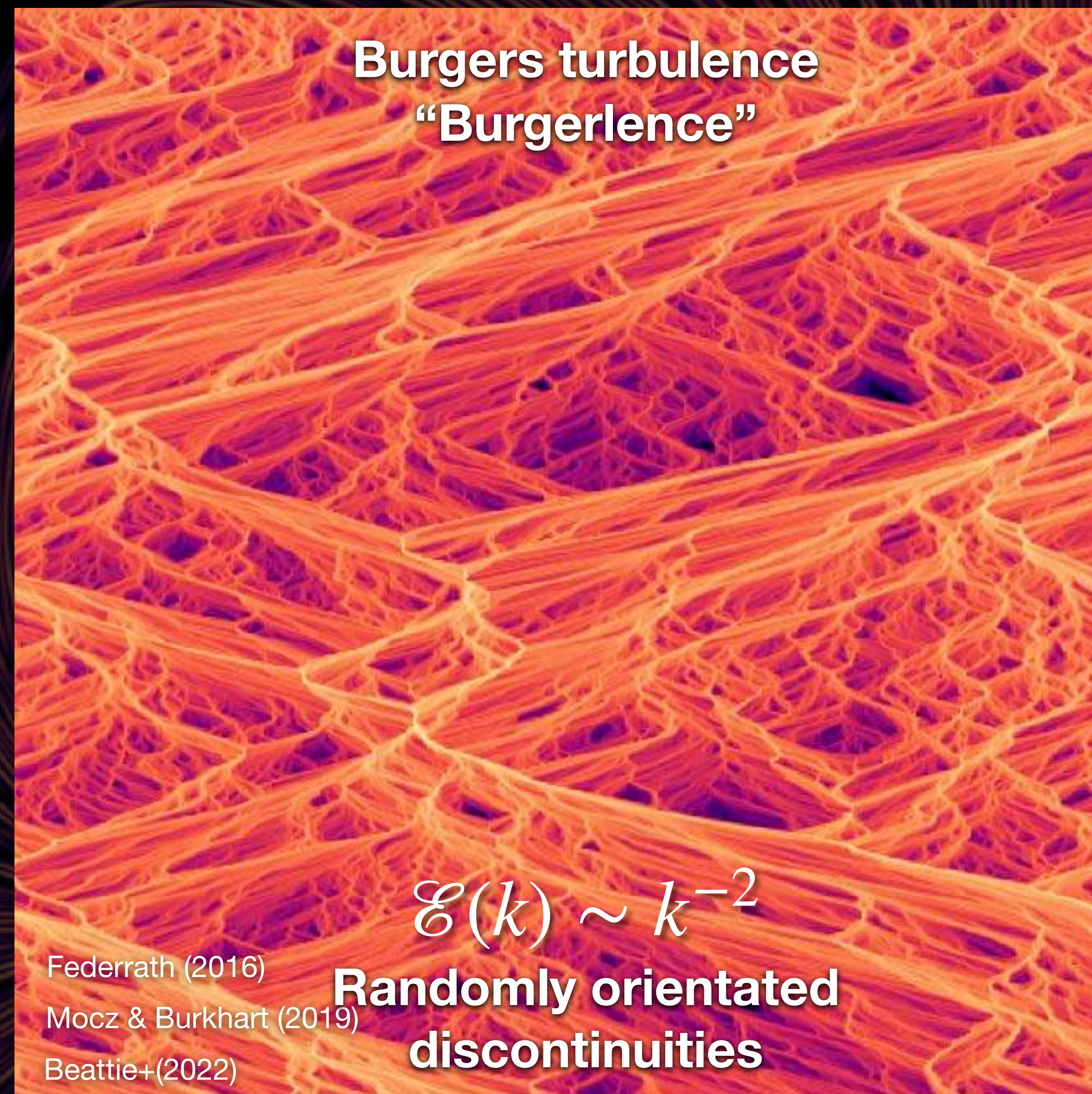
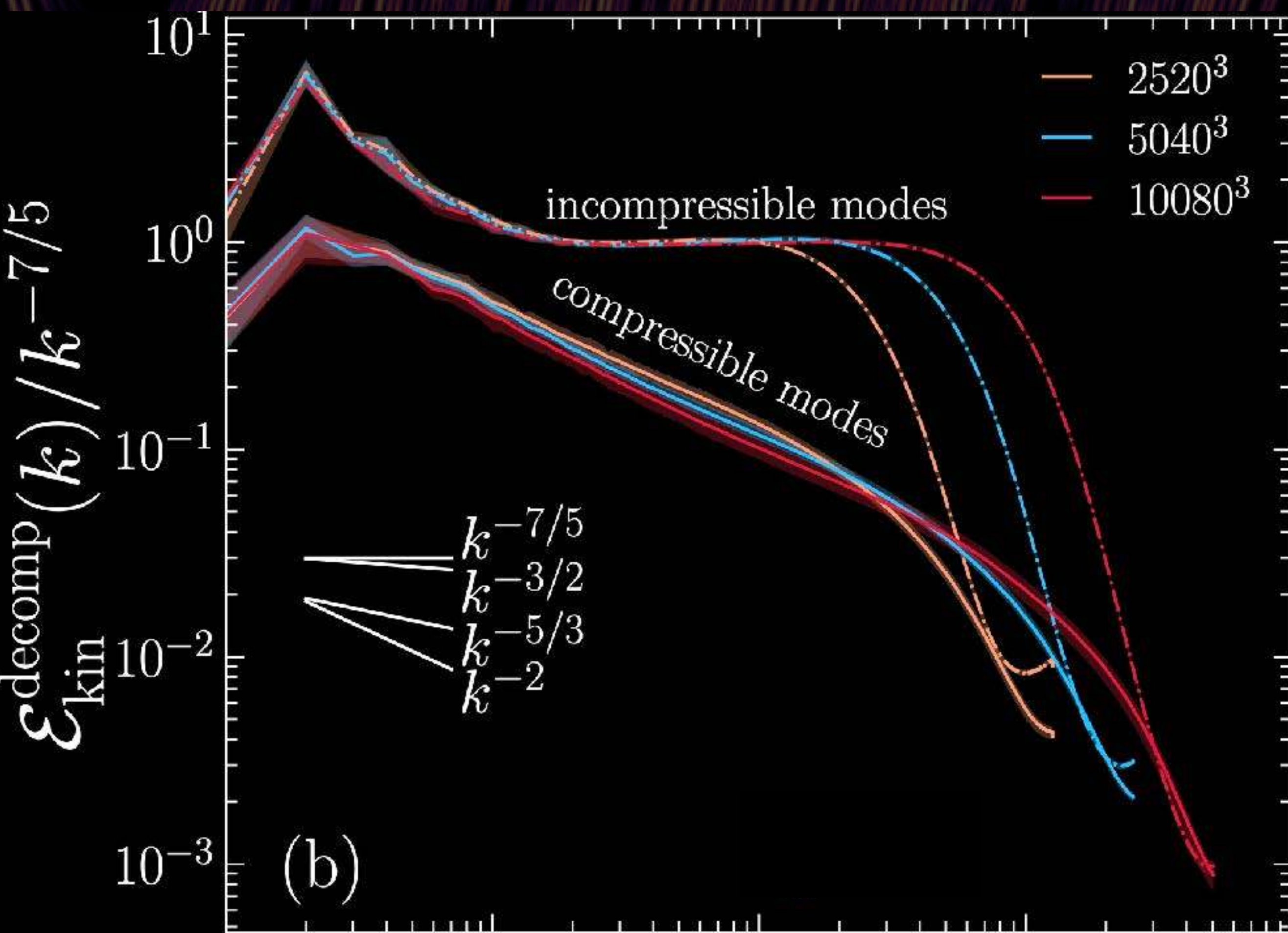
The supersonic energy cascade

$$\mathcal{E}(k) \sim k^{-2}, k \leq k_{\text{eq}}$$



The supersonic energy cascade

$$\mathcal{E}(k) \sim k^{-2}, k \leq k_{\text{eq}}$$



The subsonic energy cascade

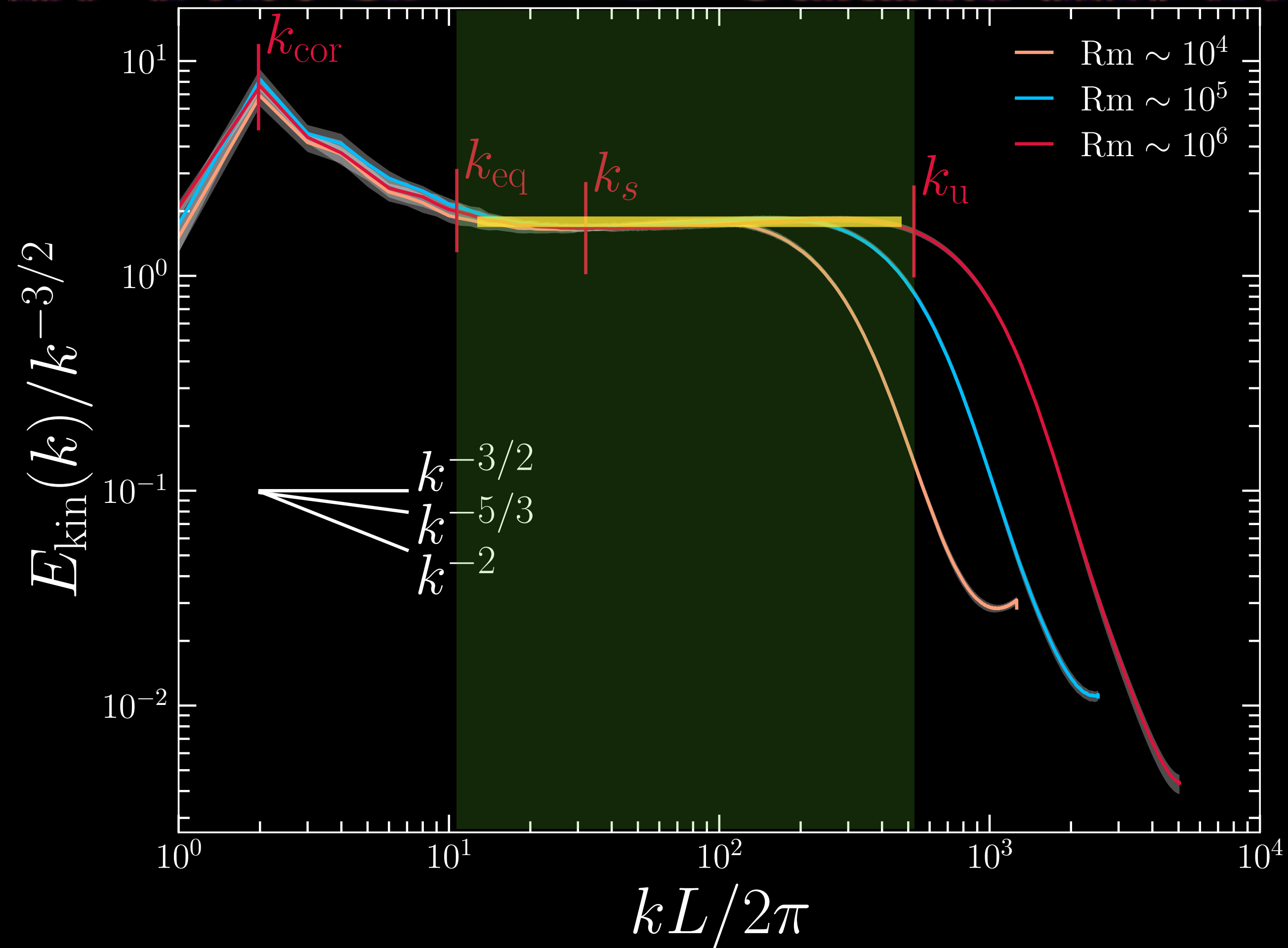
$$\mathcal{E}(k) \sim k^{-3/2}, \quad k > k_{\text{eq}}$$

IK-type turbulence (Kolmogorov + b flux)?

Dynamical alignment?

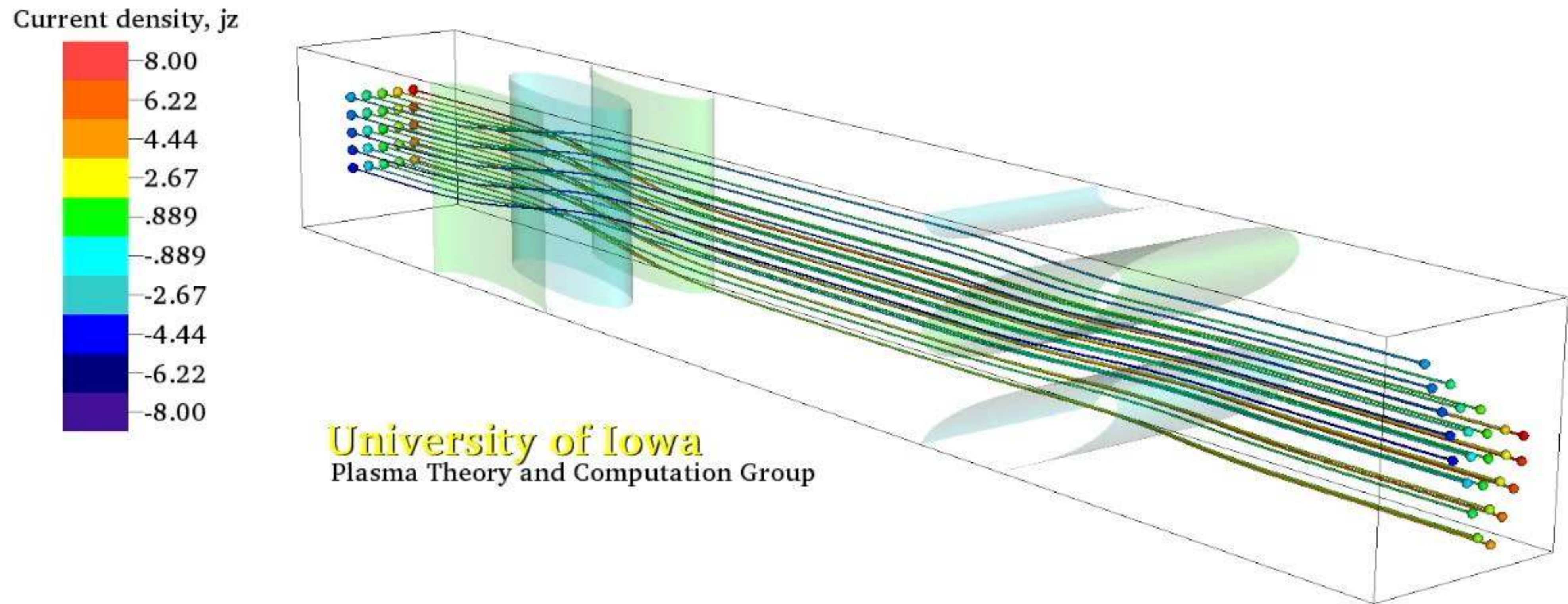
?

A mechanism for depleting nonlinearities
in the turbulence



The incompressible MHD energy cascade — new timescales

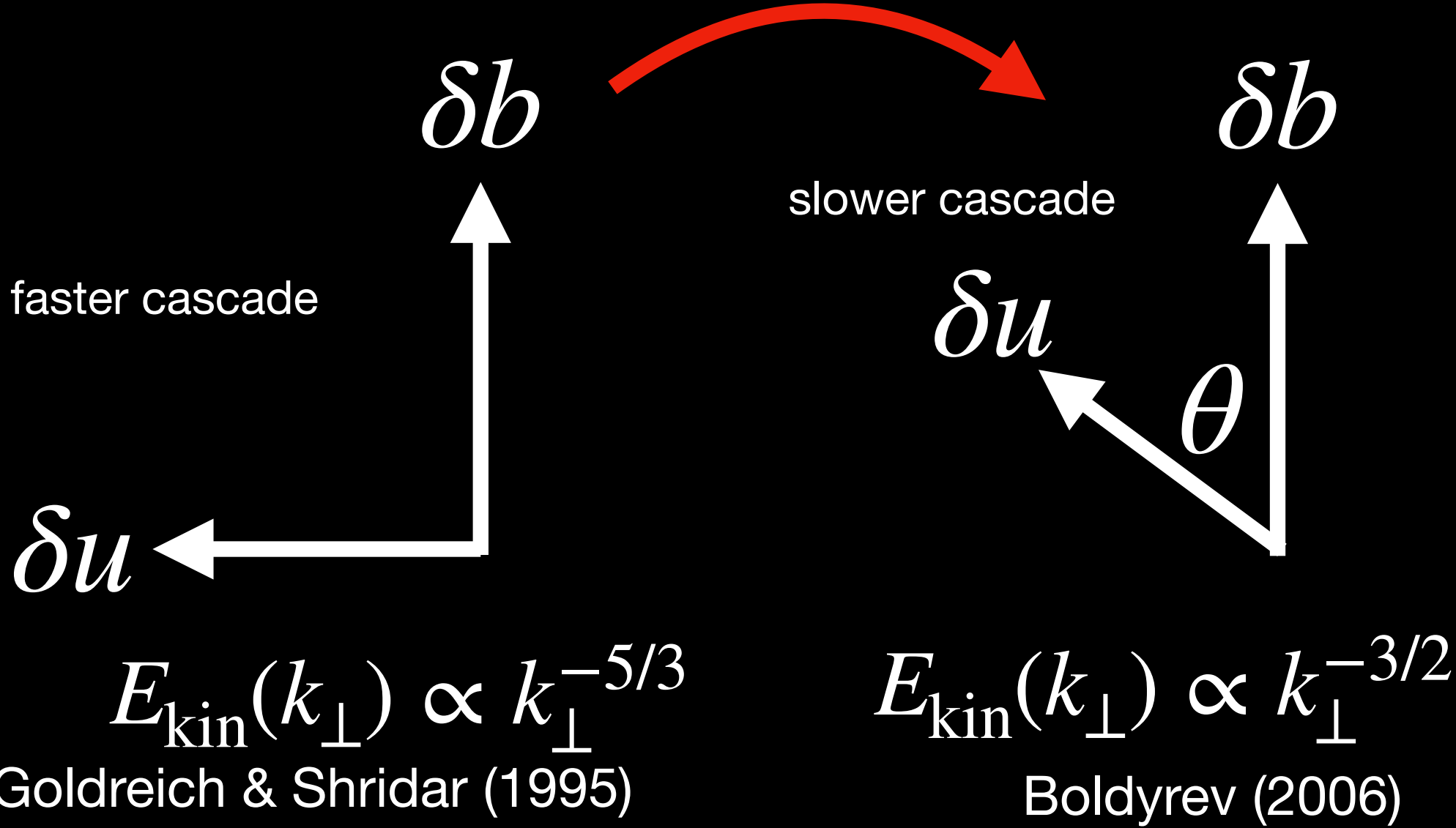
Interacting Alfvén waves (visualization from Greg G. Howes)





Dynamic alignment?

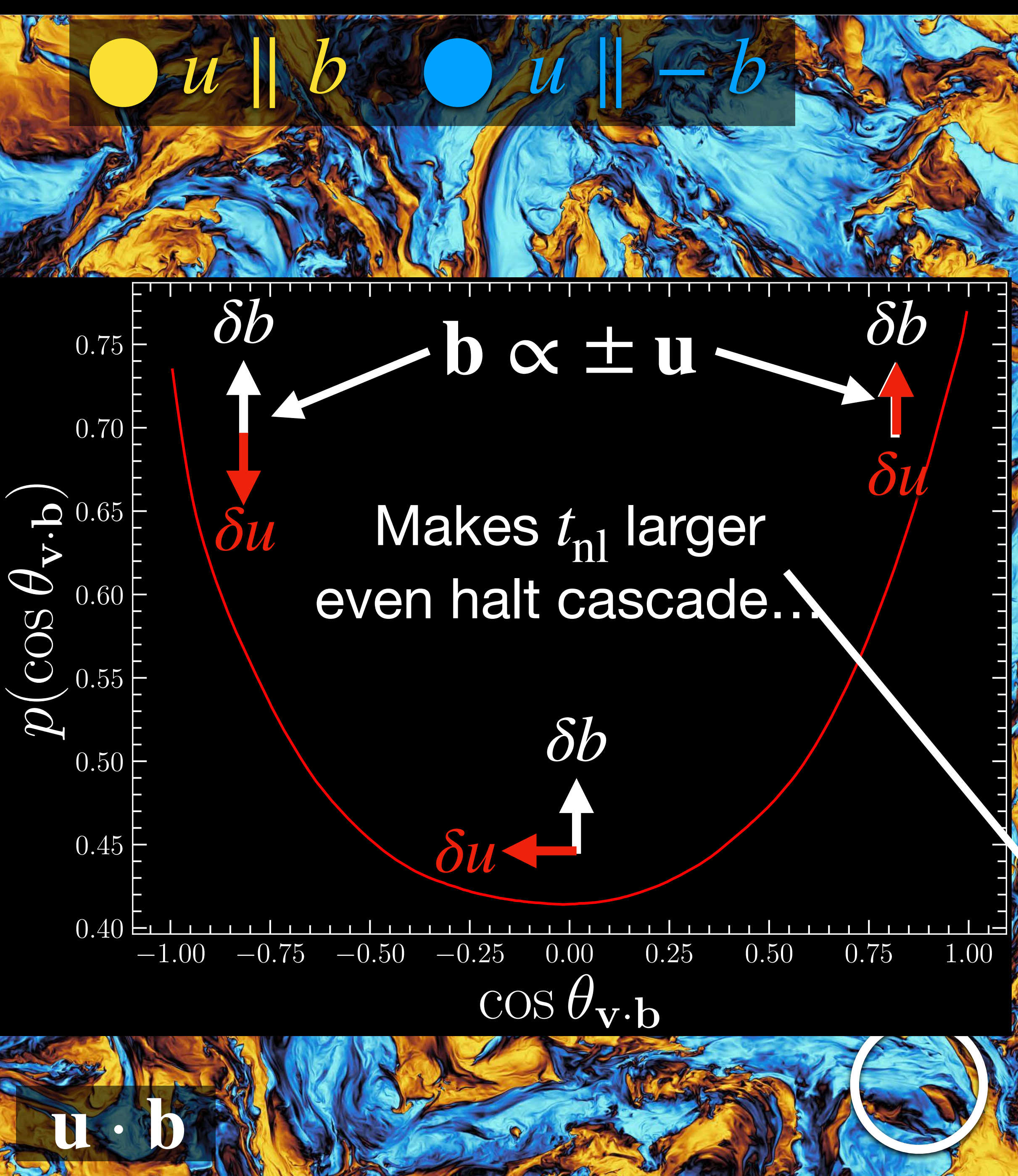
Shearing events between counter-propagating Alfvén waves



Modifies the cascade timescale

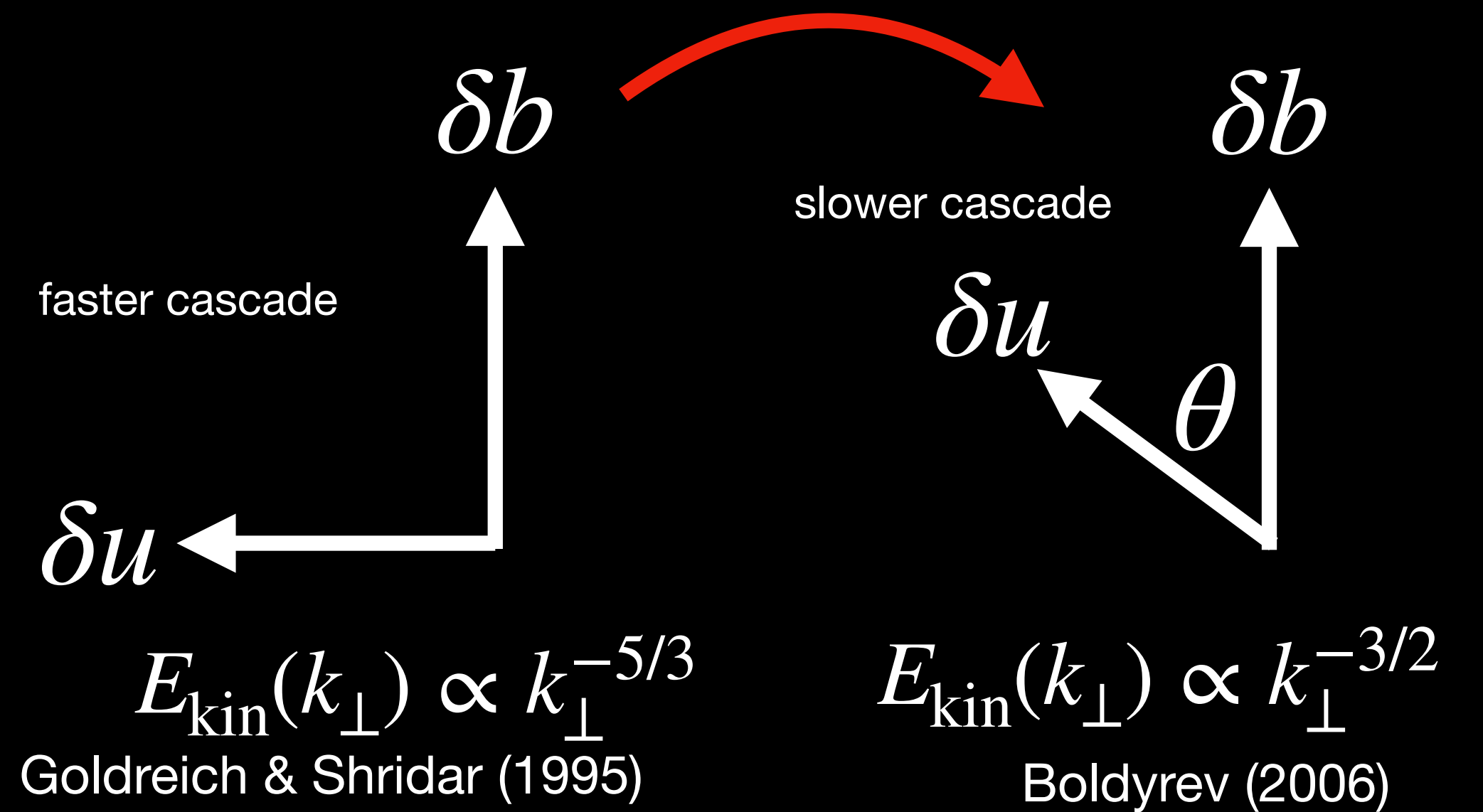
$$t_{nl} \sim \frac{\ell_{\perp}}{z^{\mp} \sin \theta_{z^{\mp}}}, \quad z^{\mp} = (u \mp b)$$

Boldyrev (2006)



Dynamic alignment?

Shearing events between counter-propagating Alfvén waves



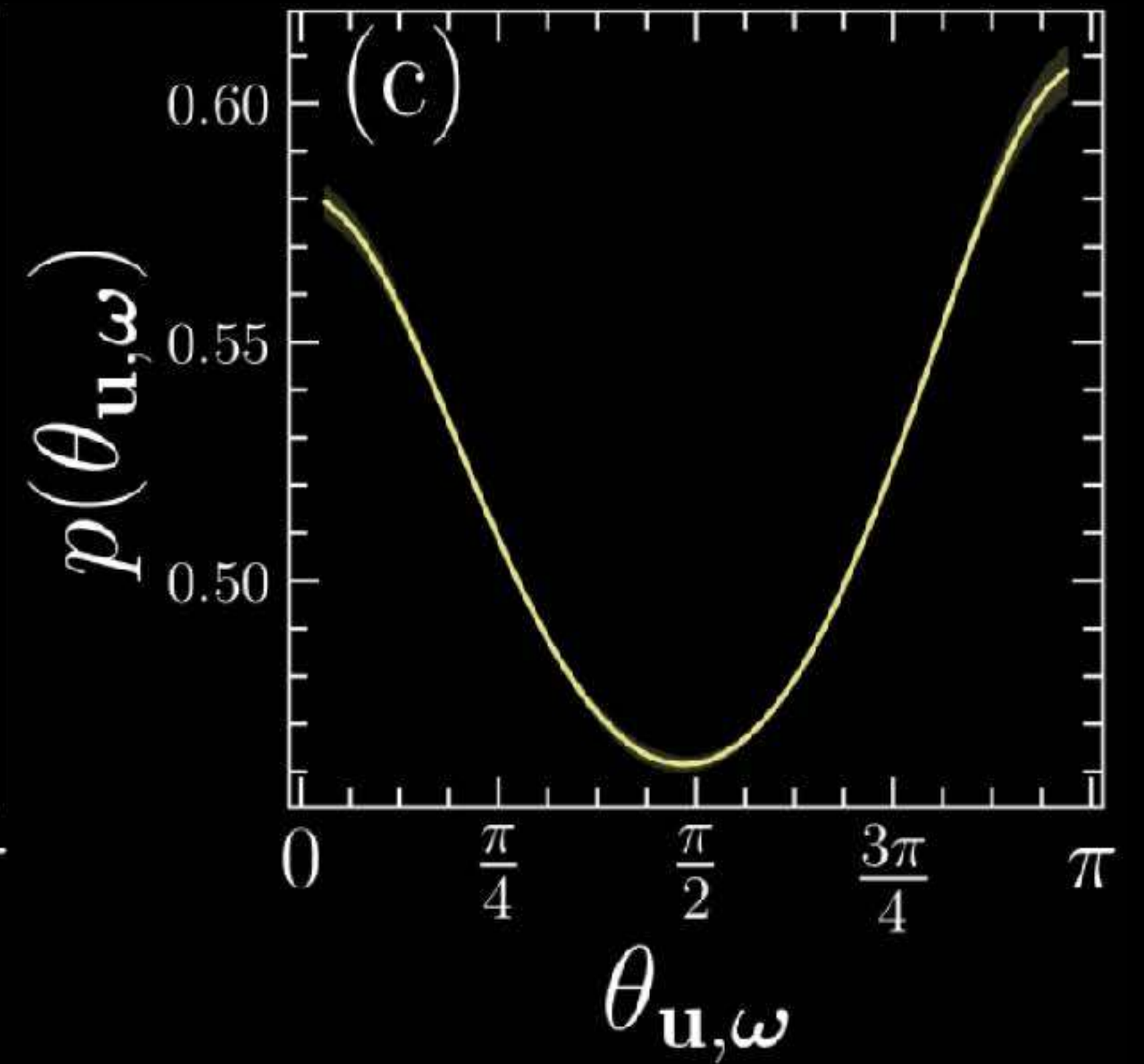
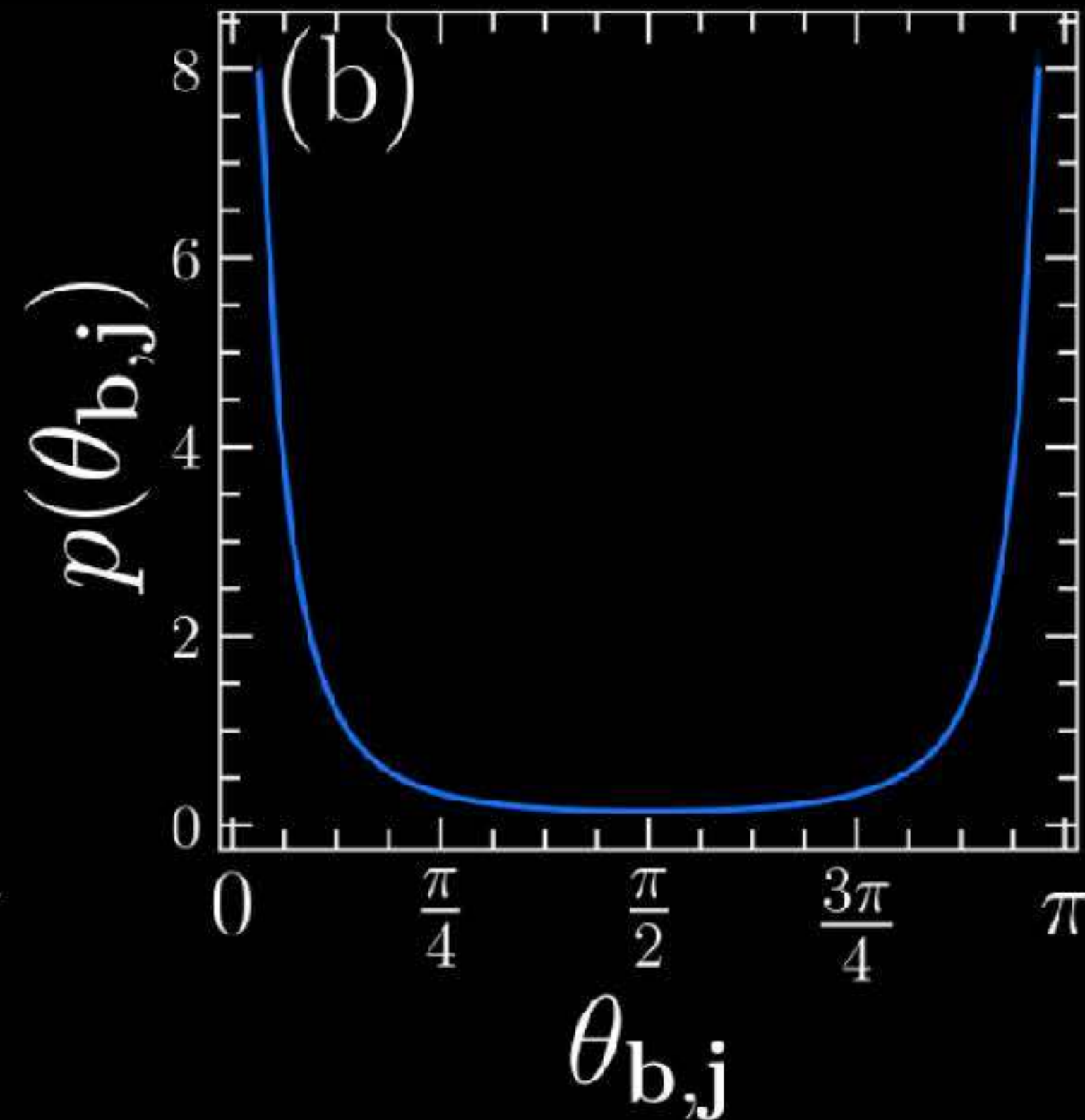
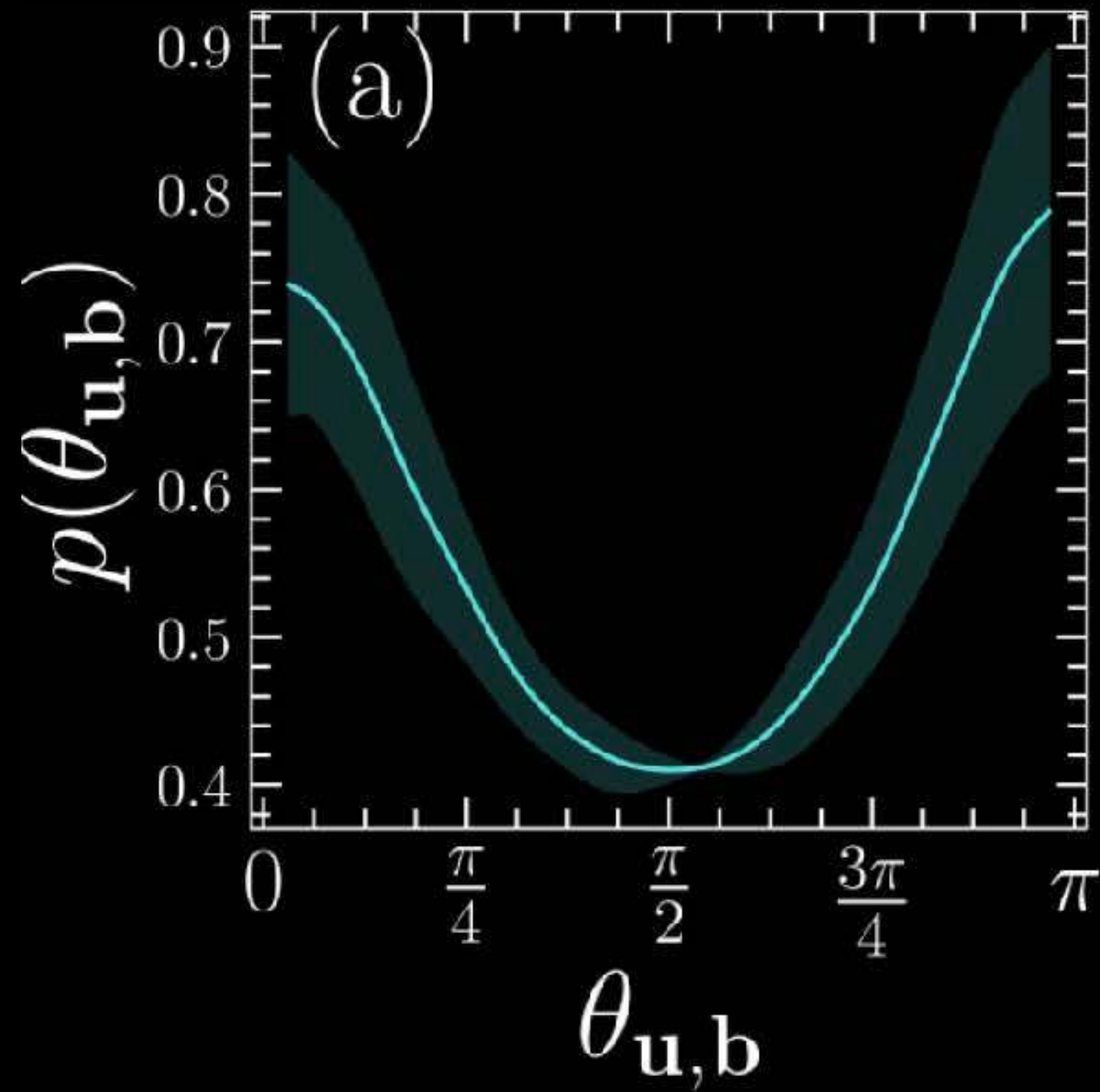
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$$t_{\text{nl}} \sim \frac{\ell_{\perp}}{z^{\mp} \sin \theta_{z^{\mp}}}, \quad z^{\mp} = (u \mp b)$$

Boldyrev (2006)

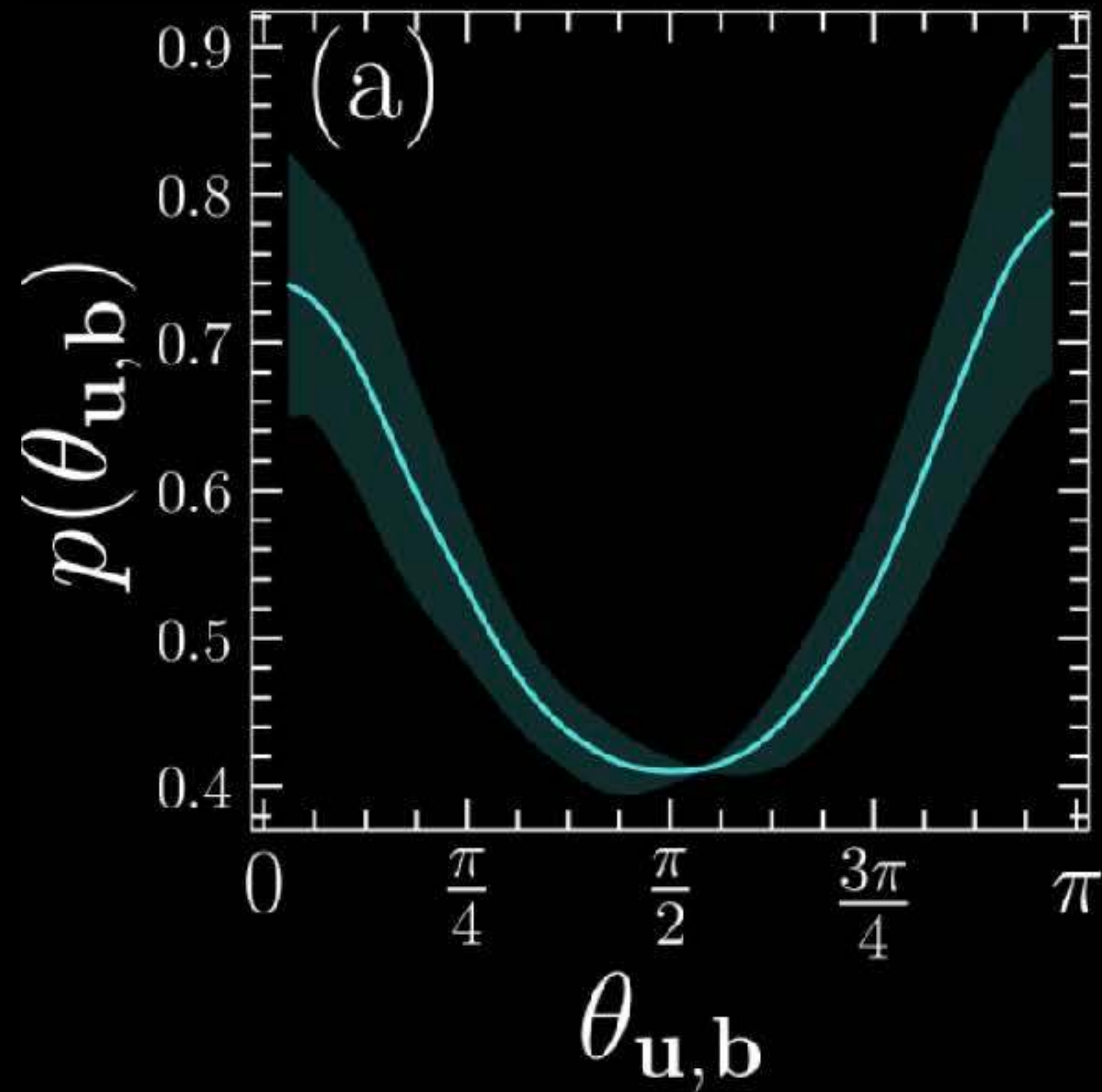
But even more alignment than just $\mathbf{u}$ and $\mathbf{b}$
 Searching to weaken the nonlinearities

$$\boldsymbol{\omega} = \nabla \times \mathbf{u}$$



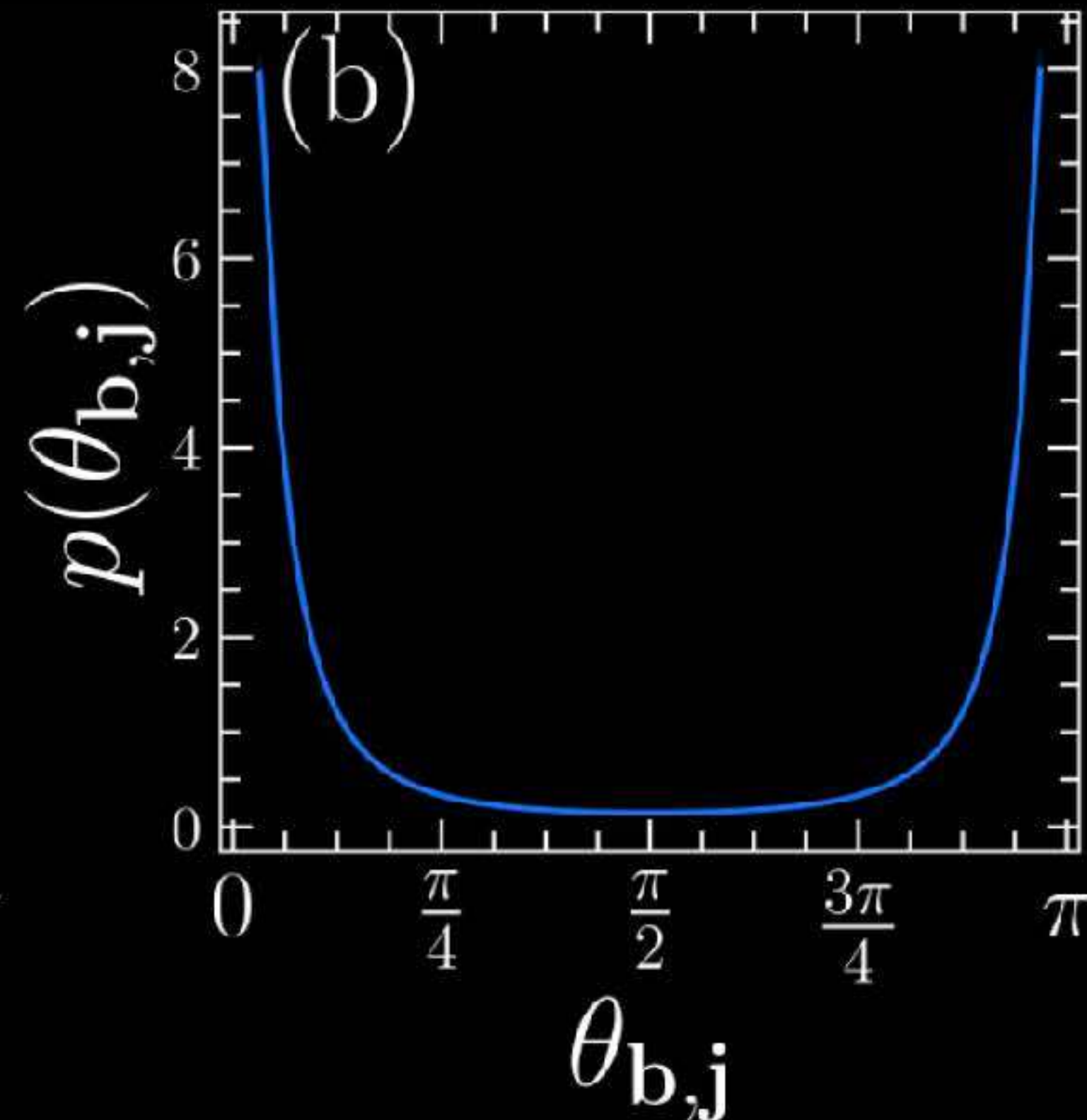
$$\mathbf{u} \propto \mathbf{b} \propto \mathbf{j} \propto \boldsymbol{\omega}$$

But even more alignment than just $\mathbf{u}$ and $\mathbf{b}$
 Searching to weaken the nonlinearities



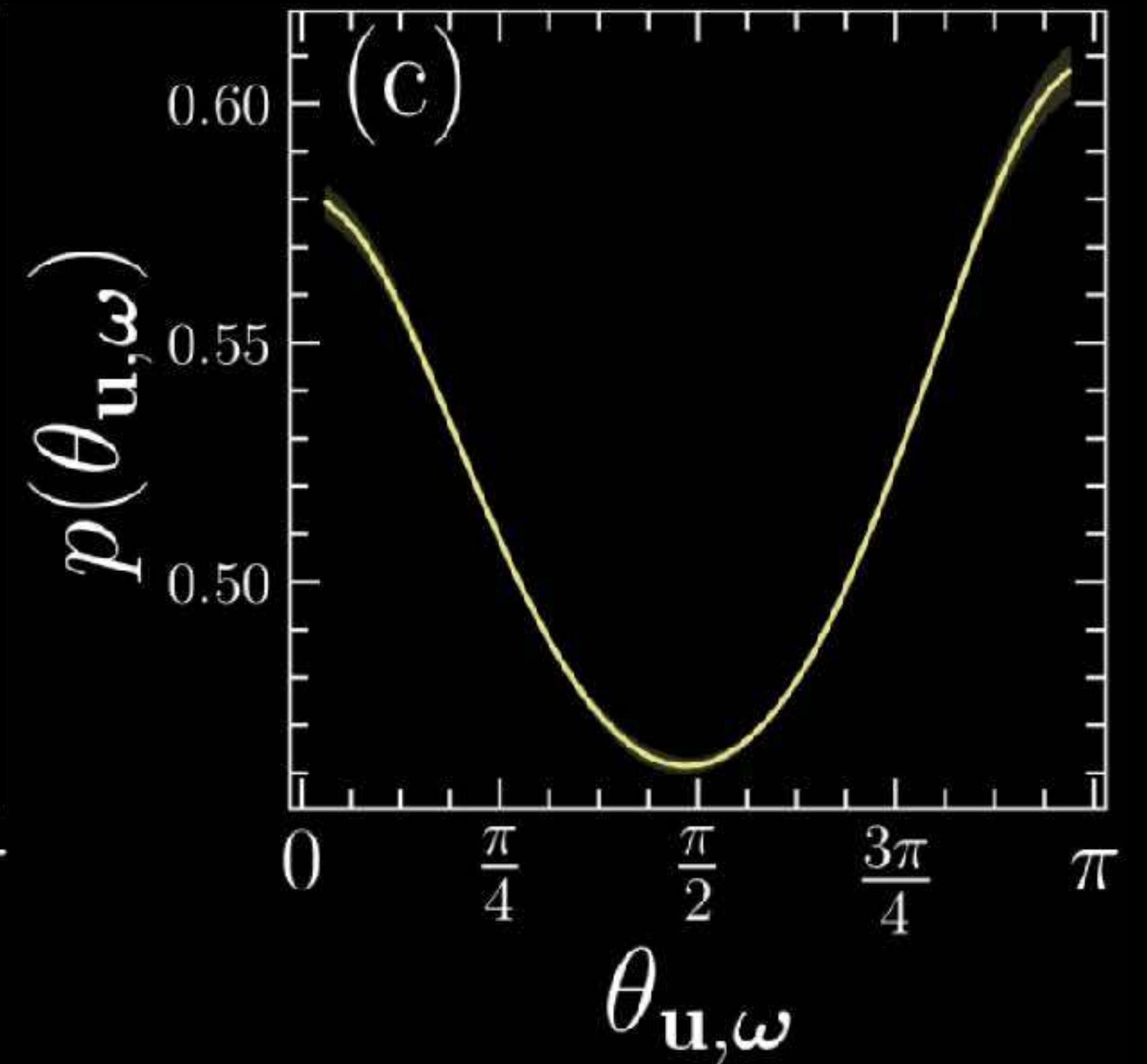
$$\nabla \times (\mathbf{u} \times \mathbf{b})$$

Induction



$$\mathbf{j} \times \mathbf{b}$$

Lorentz force



$$\nabla \cdot (\mathbf{u} \otimes \mathbf{u}) \sim \boldsymbol{\omega} \times \mathbf{u}$$

Reynolds nonlinearity

Magnetic Relaxation — main idea?

Searching to weaken the nonlinearities

Define constraint equation based on quadratic (ideal) MHD invariants

$$H_m = \langle \mathbf{a} \cdot \mathbf{b} \rangle$$

$$H_c = \langle \mathbf{u} \cdot \mathbf{b} \rangle$$

total energy

cross helicity

$$\mathcal{E} - \lambda_1 H_m - \lambda_2 H_c = \text{const.}$$

magnetic helicity

Use variational principle on magnetic energy eq., for perturber δ

$$\delta \left(\mathcal{E} - \lambda_1 H_m - \lambda_2 H_c \right) = 0,$$

Minimize to find minimum energy state.

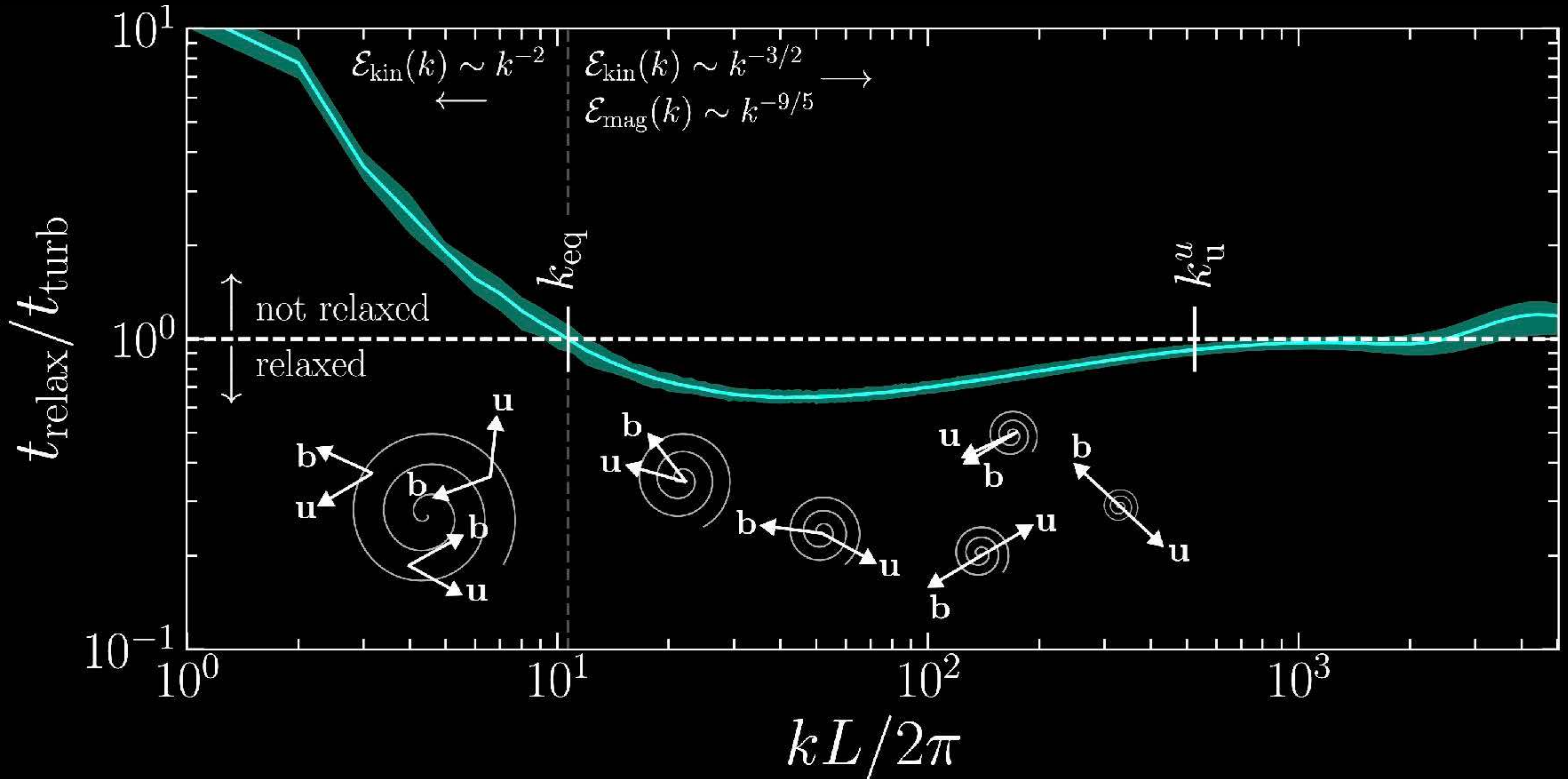
$$\boldsymbol{\omega} = \nabla \times \mathbf{u}$$

$$\mathbf{u} = \lambda_1 \mathbf{b}, \quad \mathbf{j} = 2\lambda_2 \mathbf{b} + \lambda_1 \boldsymbol{\omega}$$

Banerjee+(2023)

Competitive relaxation: turbulence versus relaxation

Can we relax faster than the turbulence can perturb us away from minimum energy?



On these scales...

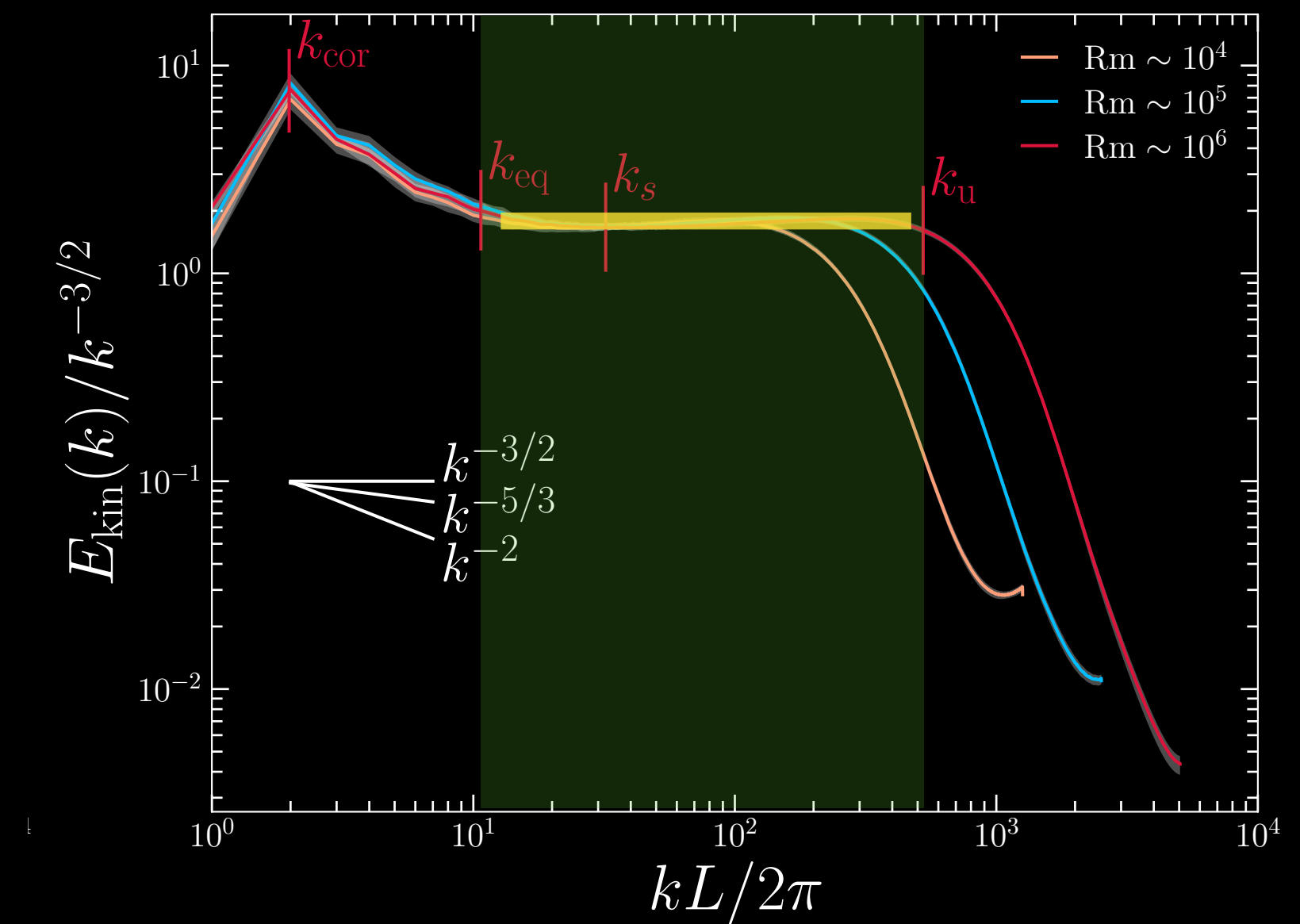
$$\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$\partial_t(\rho \mathbf{u}) + \nabla \cdot \left(\cancel{\rho \mathbf{u} \otimes \mathbf{u}} + p \mathbb{I} - \frac{1}{4\pi} \cancel{\mathbf{b} \otimes \mathbf{b}} \right) = \cancel{\rho \mathbf{f}} + \nabla \cdot \mathbb{D}_\nu(\rho \mathbf{u})$$

$$\partial_t \mathbf{b} + \nabla \cdot (\cancel{\mathbf{u} \otimes \mathbf{b}} - \mathbf{b} \otimes \mathbf{u}) = \nabla \cdot \mathbb{D}_\eta(\mathbf{b})$$

$$\nabla \cdot \mathbf{b} = 0$$

$$p = c_s^2 \rho + \frac{1}{8\pi} \cancel{\mathbf{b} \cdot \mathbf{b}}$$



The subsonic energy cascade

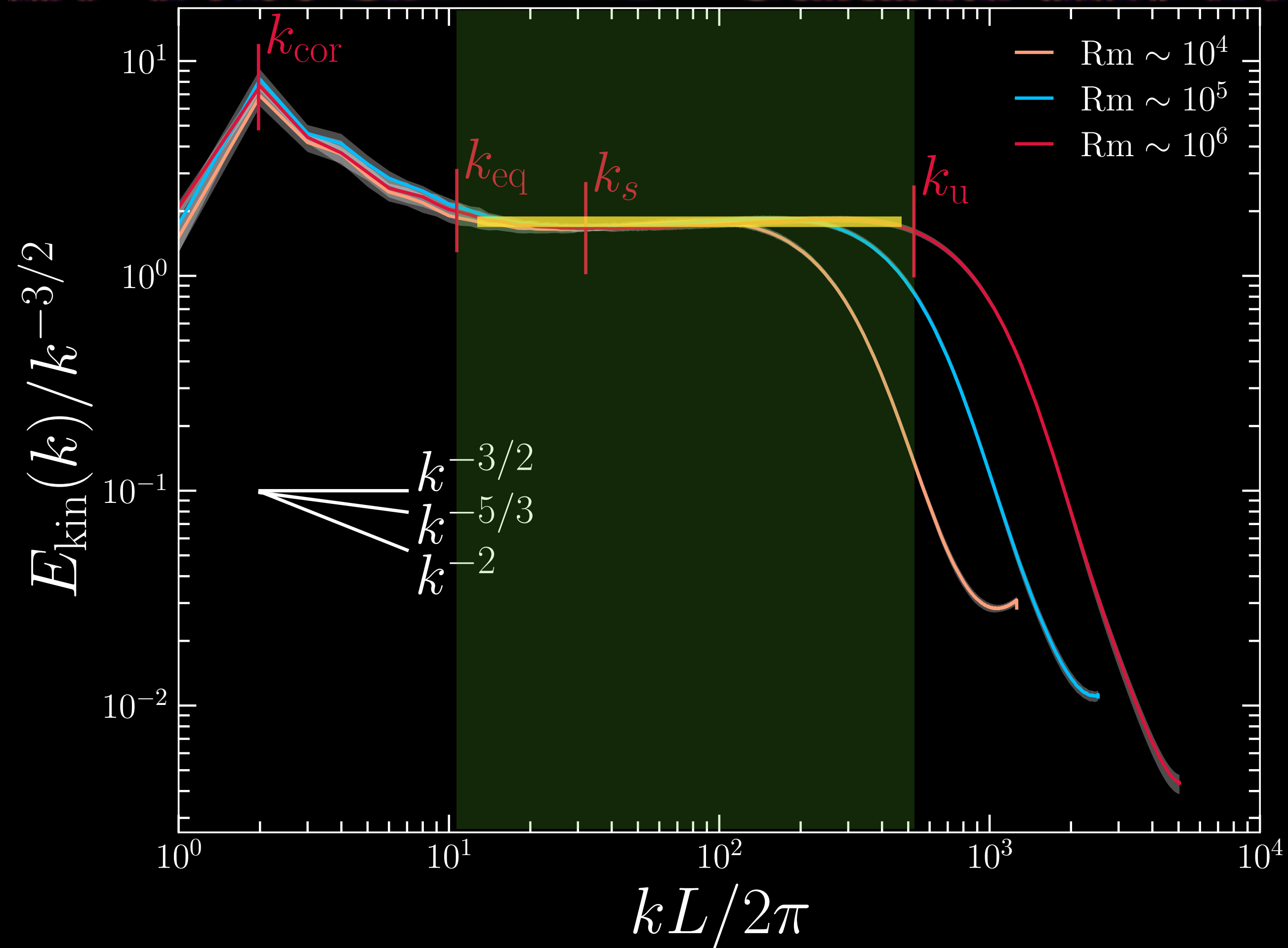
$$\mathcal{E}(k) \sim k^{-3/2}, \quad k > k_{\text{eq}}$$

IK-type turbulence (Kolmogorov + b flux)?

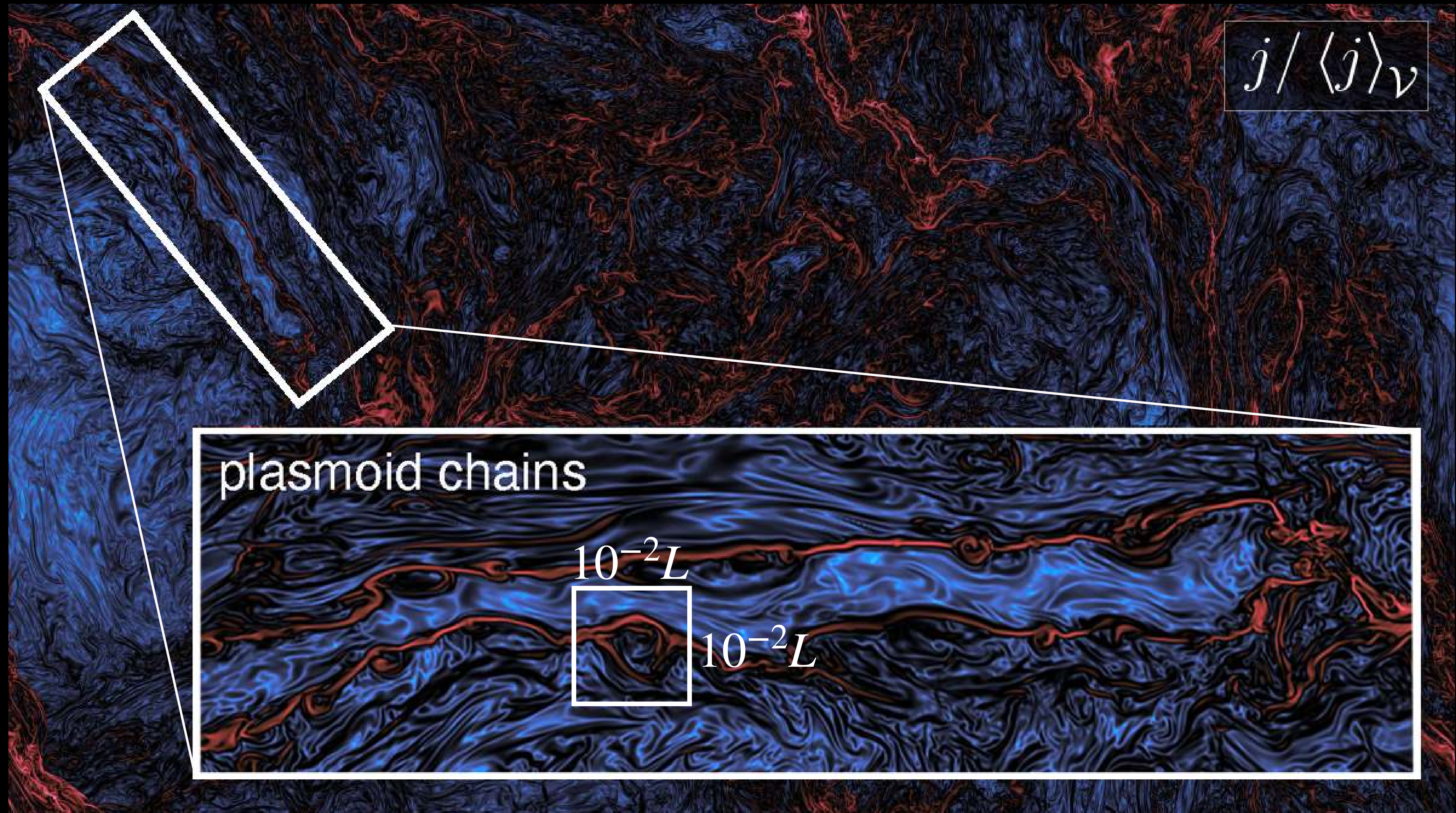
Dynamical alignment?

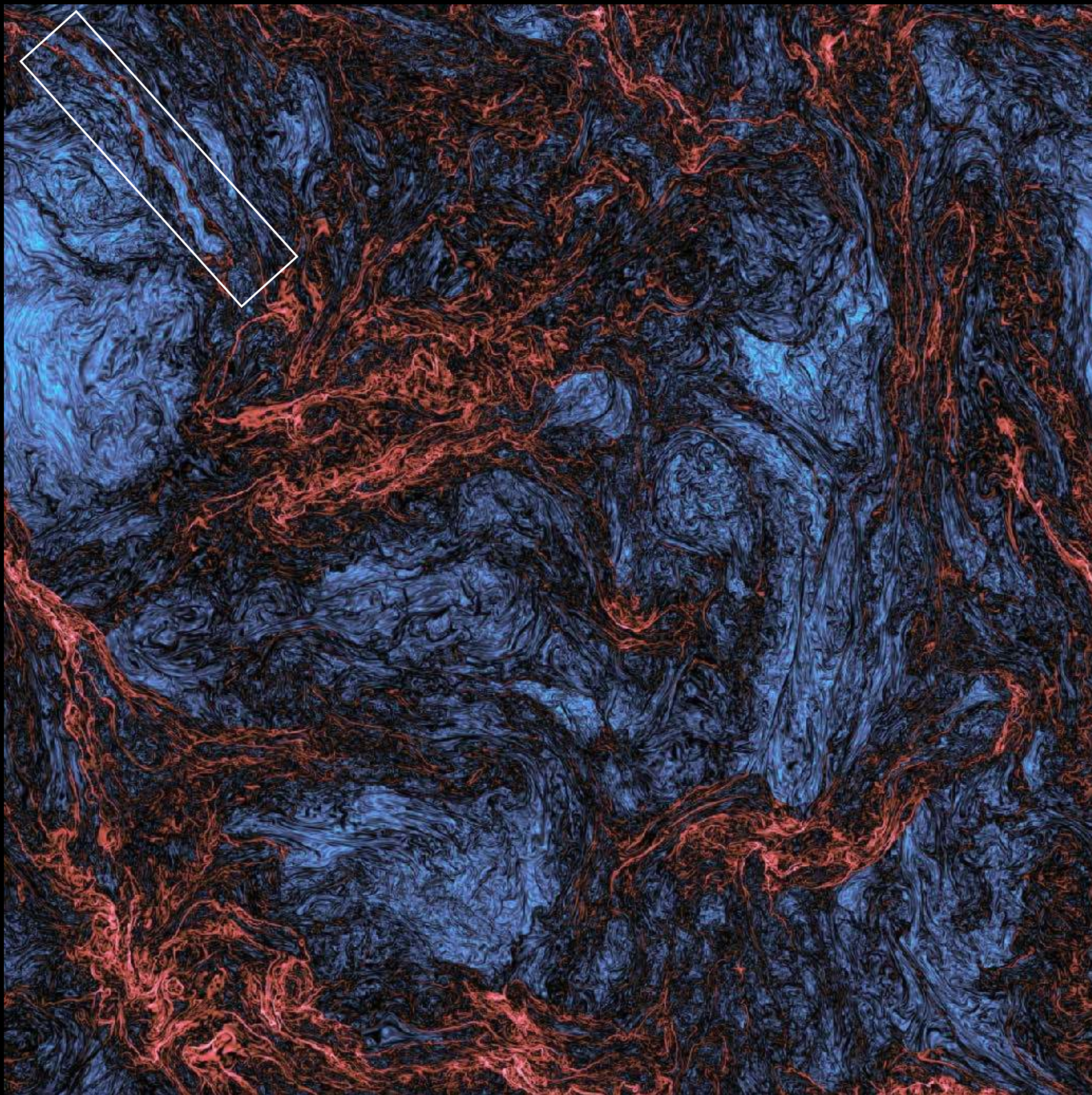
?

A mechanism for depleting nonlinearities
in the turbulence



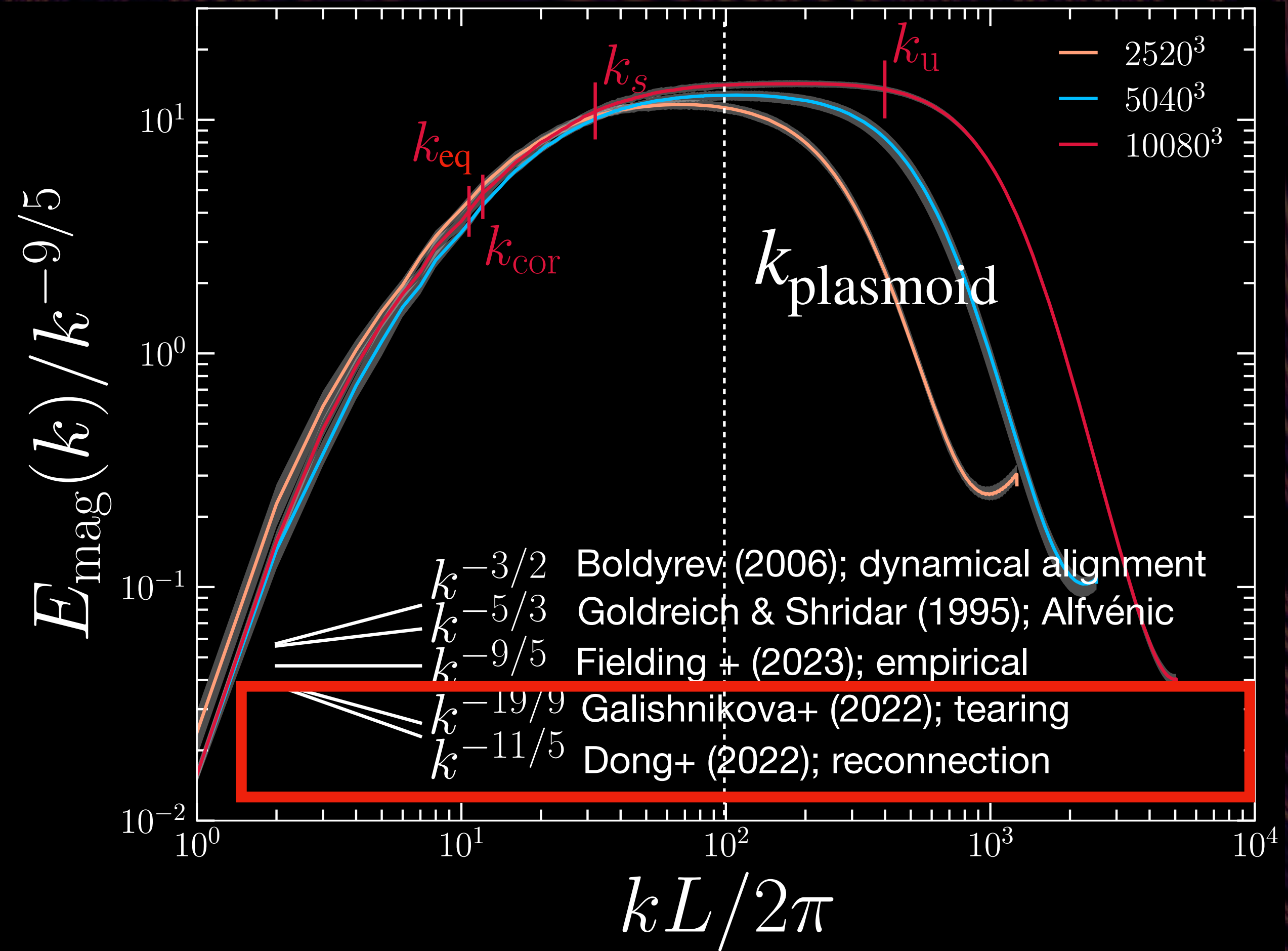
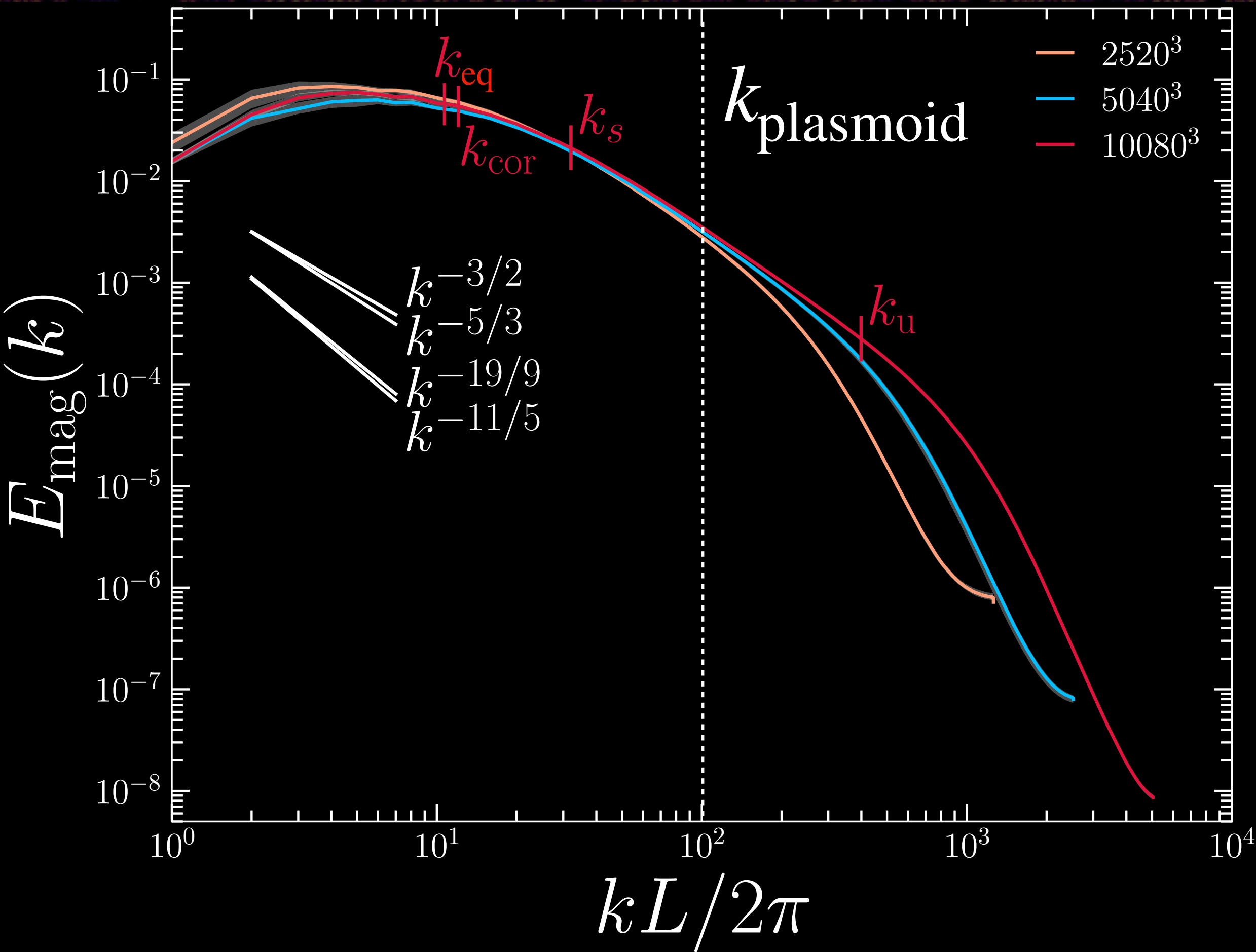
The magnetic energy cascade





Occupy a very small
volume filling-factor
in this regime

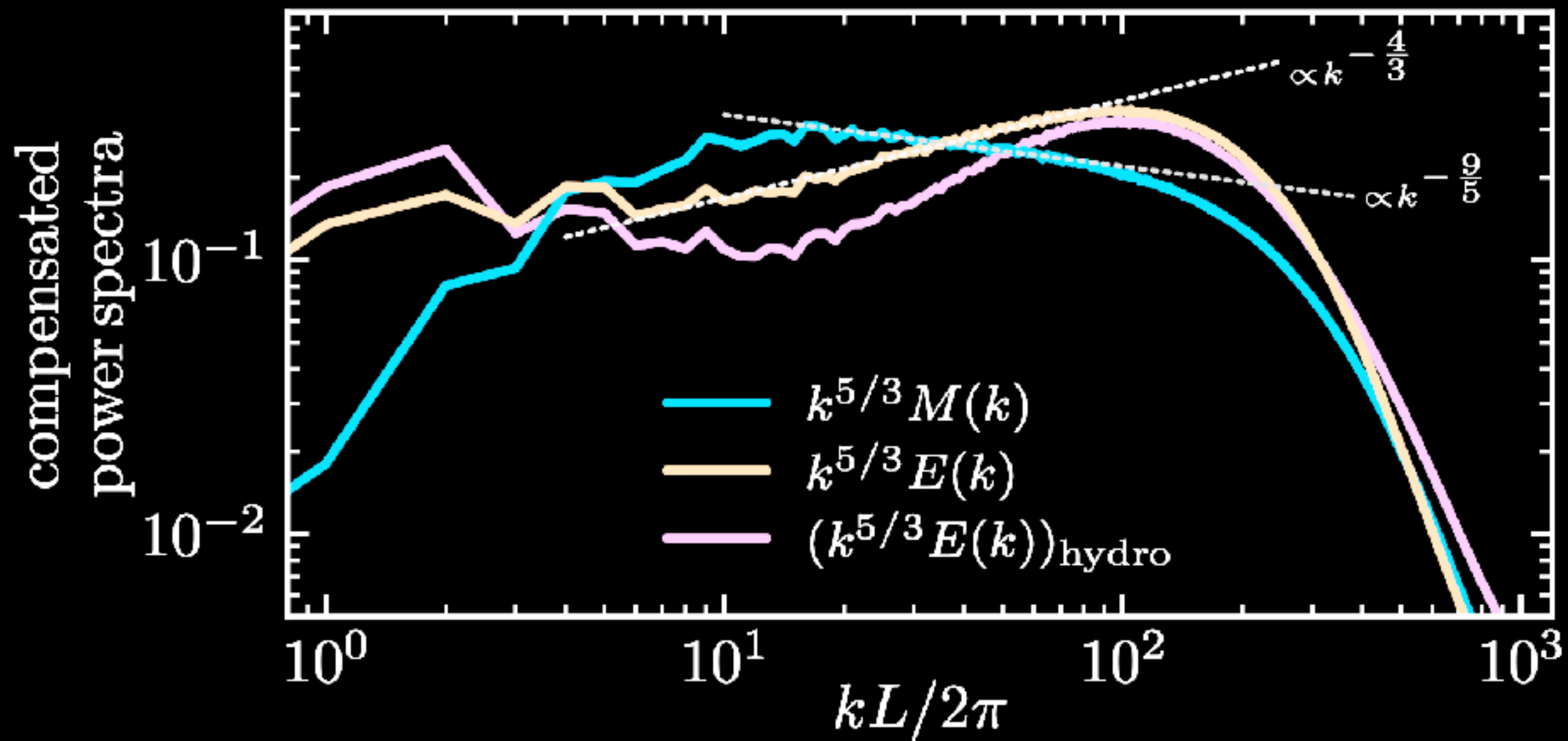
The magnetic energy cascade



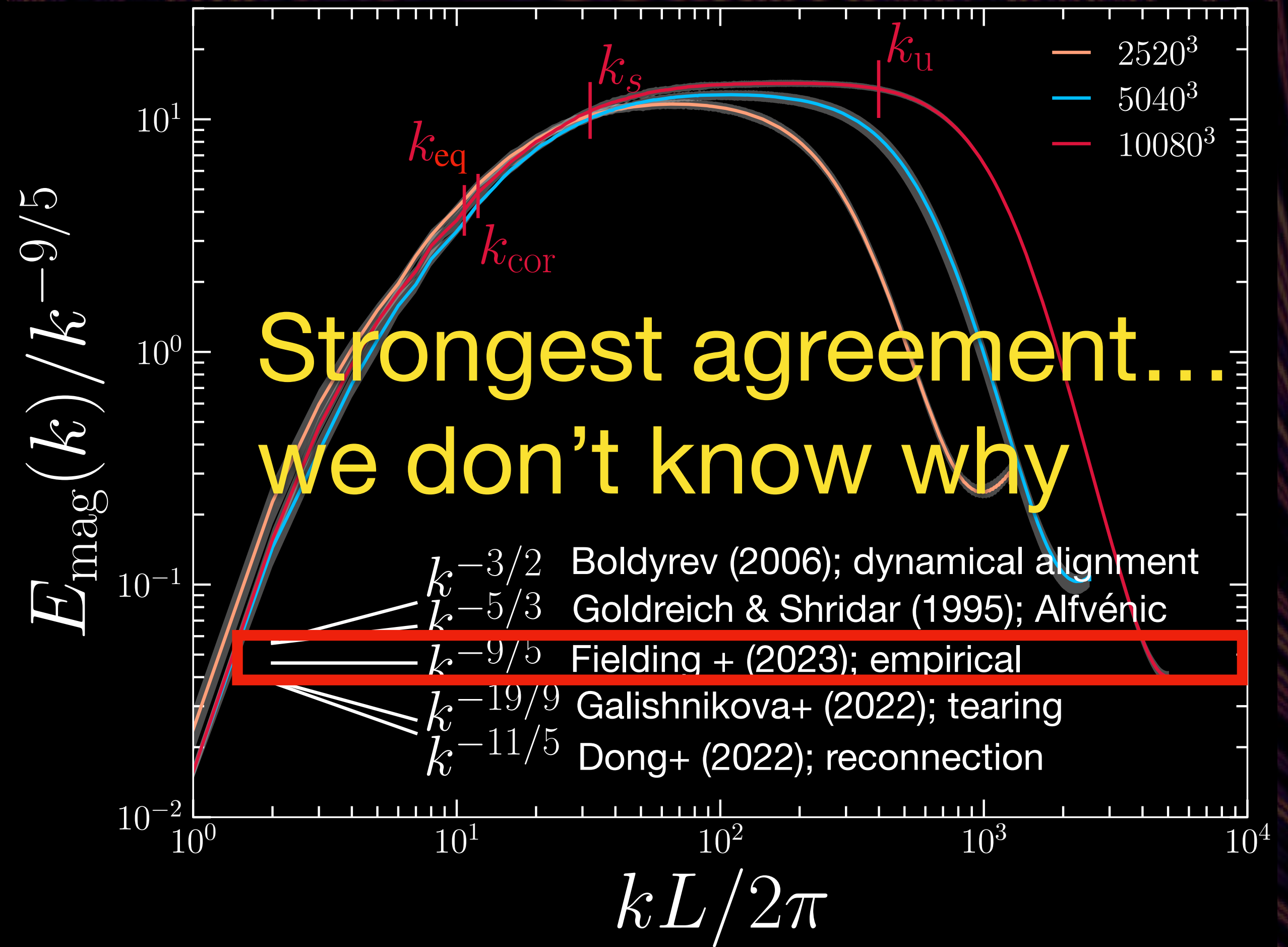
The magnetic energy cascade

Fielding+2024 (bistable medium)

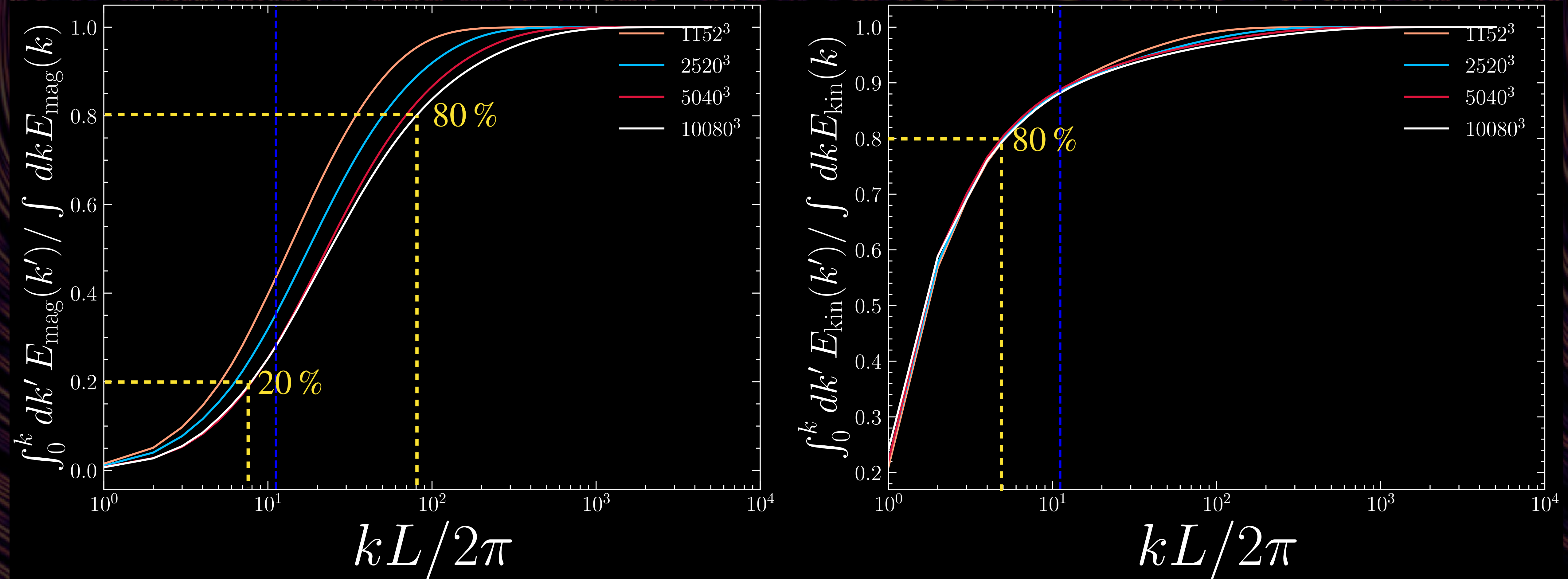
Turbulent Thermal Instability



Magnetic fluctuations don't seem to care about phases



The magnetic energy cascade: small scale structure



Magnetic field is intrinsically a small-scale field! Need a lot of resolution to resolve it properly (energetically)!

Next steps with observers



james.beattie@princeton.edu



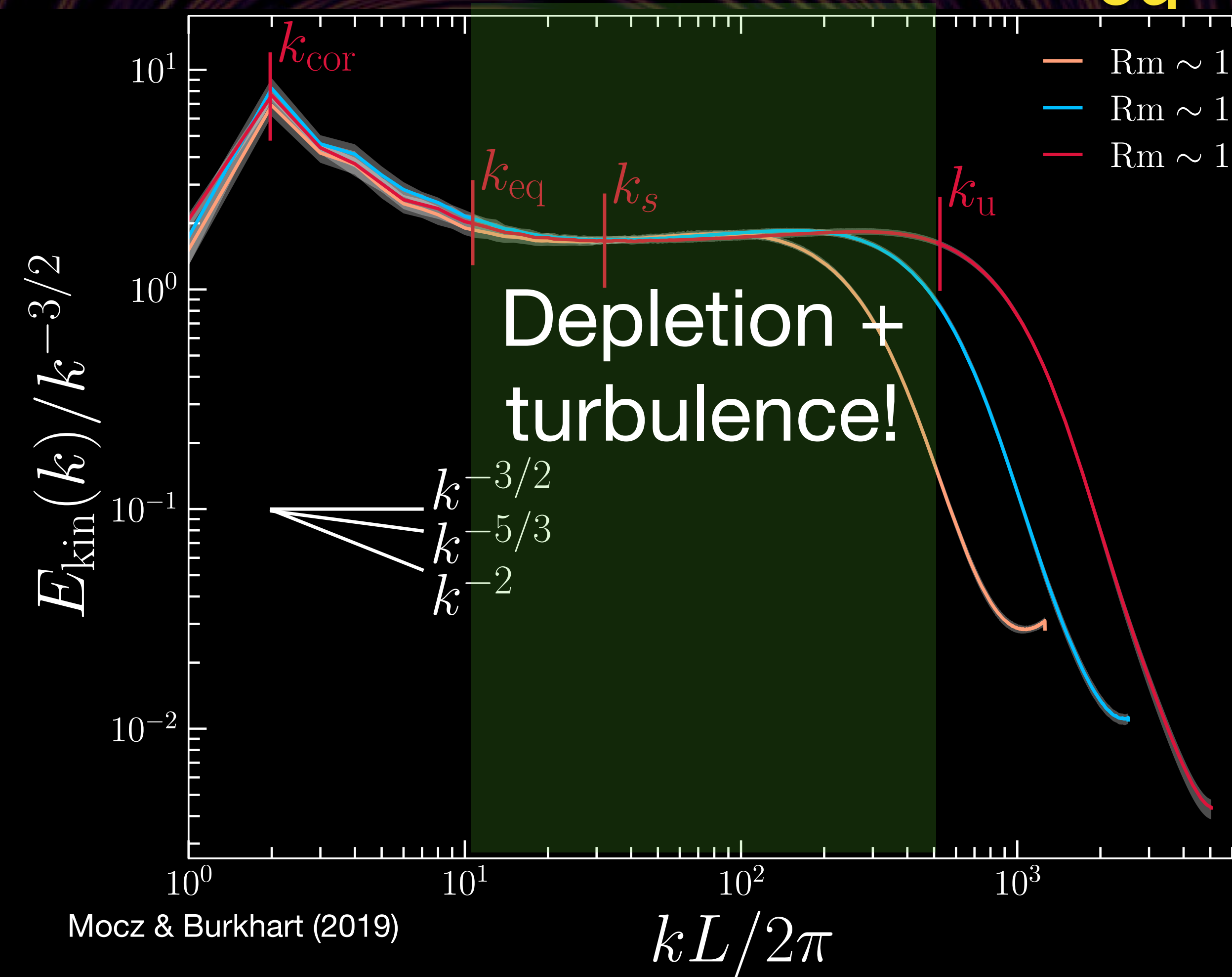
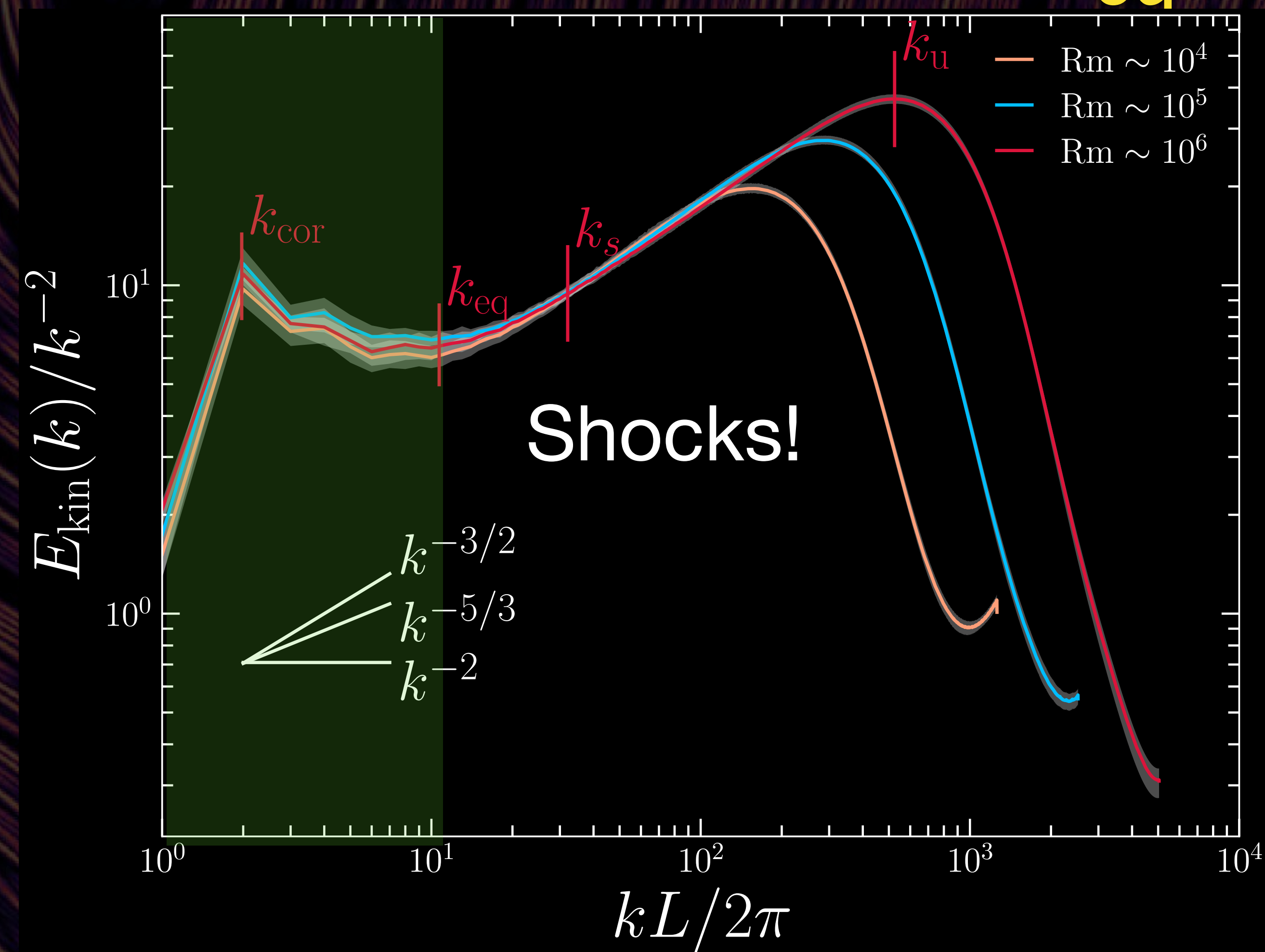
[@astro_magnetism](https://twitter.com/astro_magnetism)

- Post-process to better map to observables (tell me what's interesting, I can make this work at $\sim 10,000^3$ but will take some collaboration + time)
- I want observers to tell me what they can access about alignment — seems ubiquitous in a turbulent dynamo generated magnetic field (related to the saturation of the ISM dynamo), and is in tension with models that have a strong large scale field embedded in ICs (e.g., Lazarian's velocity gradient methods)

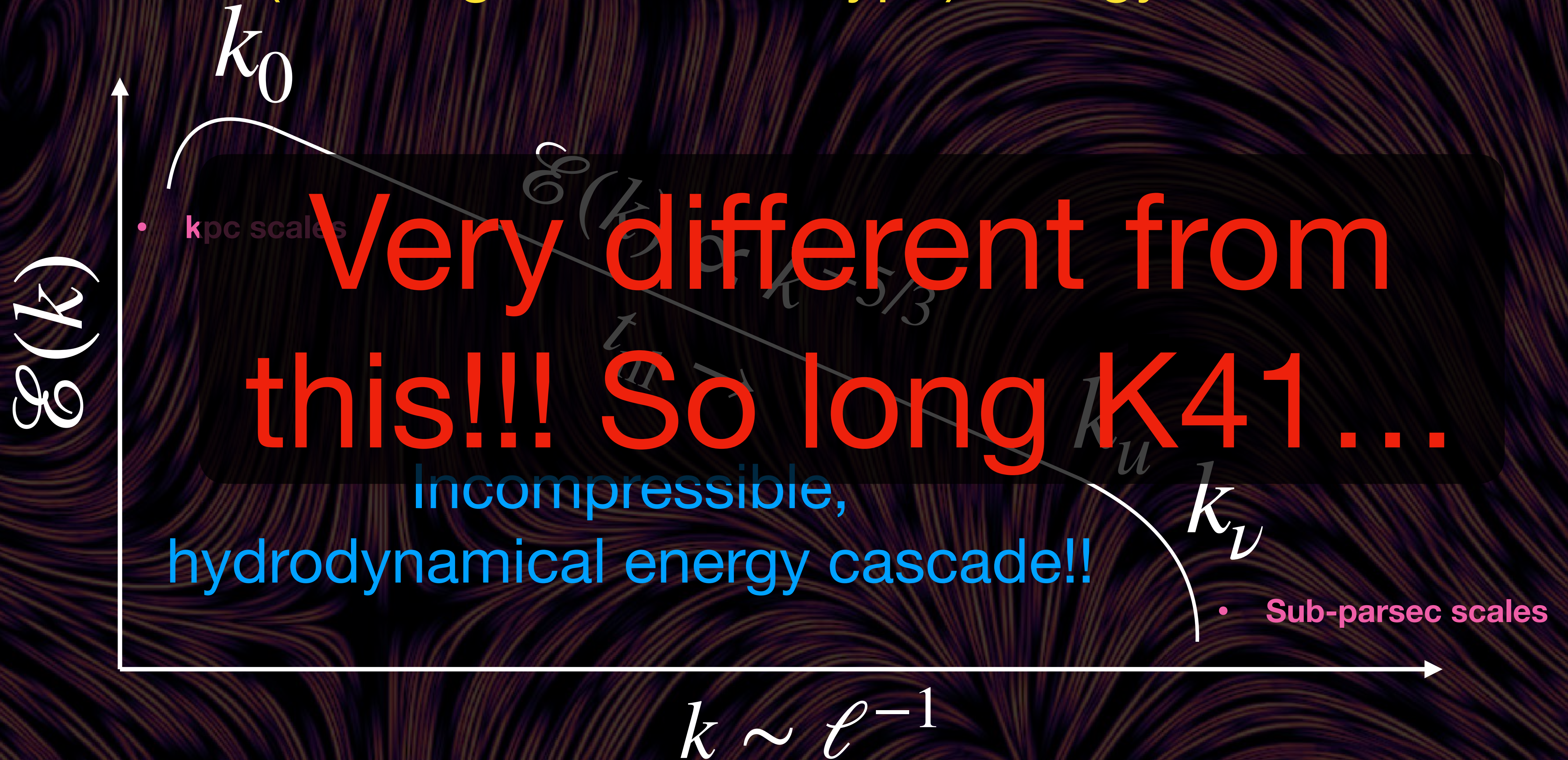
Final thoughts...

$$\mathcal{E}(k) \sim k^{-2}, k \leq k_{\text{eq}}$$

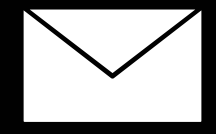
$$\mathcal{E}(k) \sim k^{-3/2}, k > k_{\text{eq}}$$



The (Kolmogorov, 1941 -type) energy cascade



Thanks, questions?



james.beattie@princeton.edu



[@astro_magnetism](https://twitter.com/astro_magnetism)

