The interstellar cascade II: supernova driven turbulence

a story of compressible and solenoidal modes

Château de Goutelas

James Beattie

Postdoc research associate / fellow
Princeton / CITA

In collaboration: Anne Noer Kolborg (UCSC)

A beautiful degeneracy… or universality?

Is The Starry Night Turbulent?

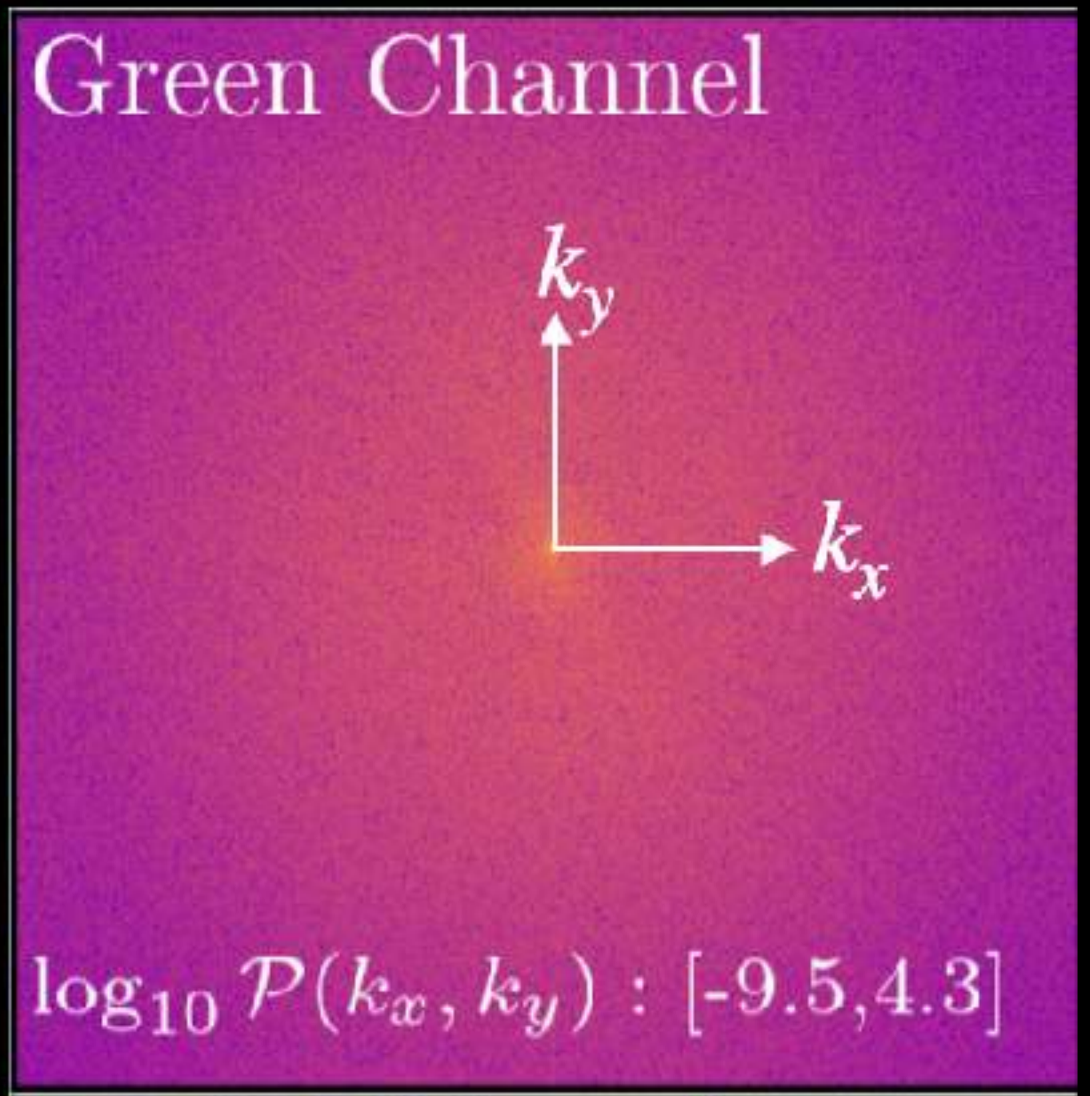
James R. Beattie[1] and Neco Kriel[2]

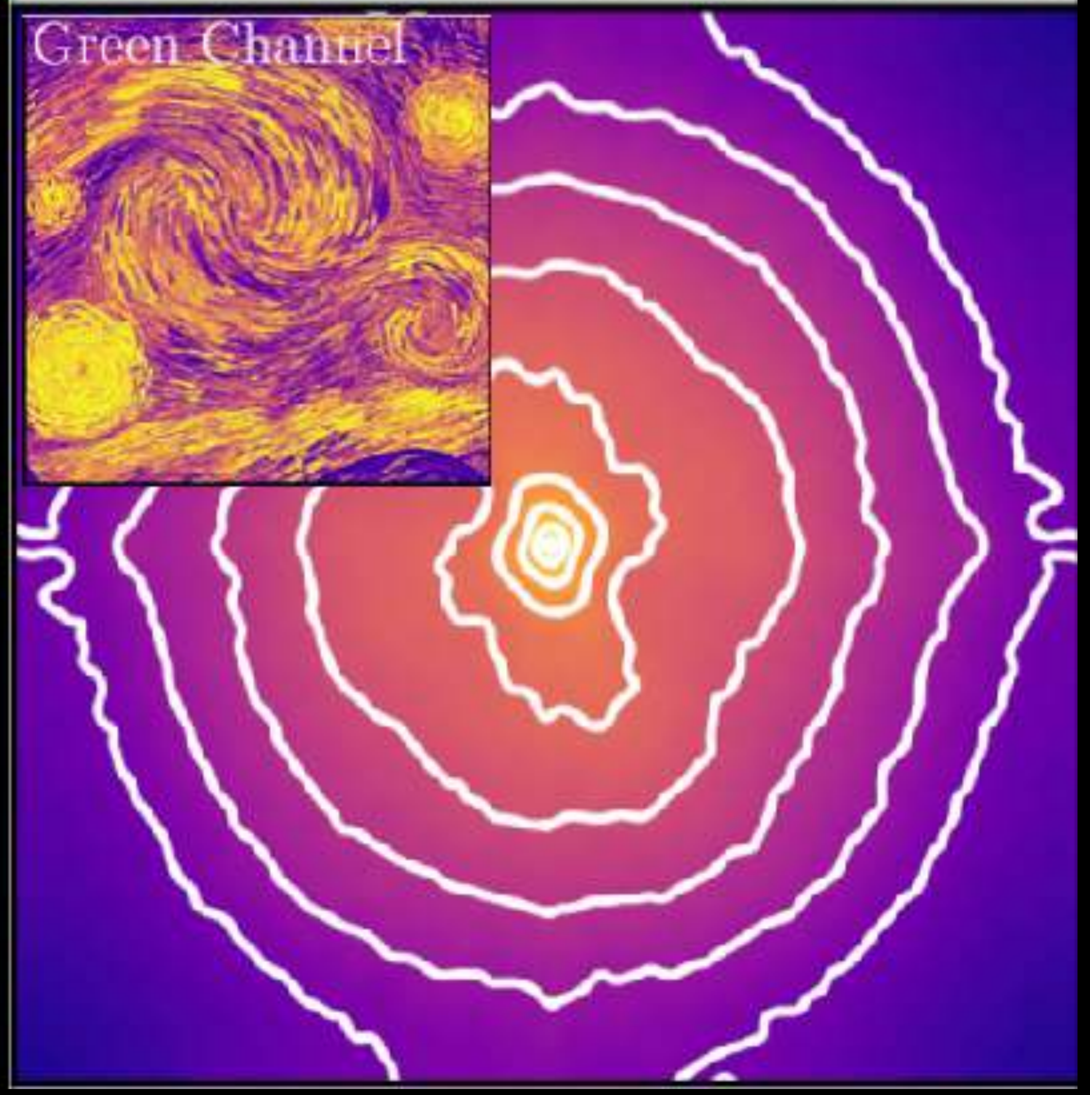
[1] *Research School of Astronomy and Astrophysics, Australian National University, Canberra, Australia*
[2] *Science and Engineering Faculty, Queensland University of Technology, Brisbane, Australia*

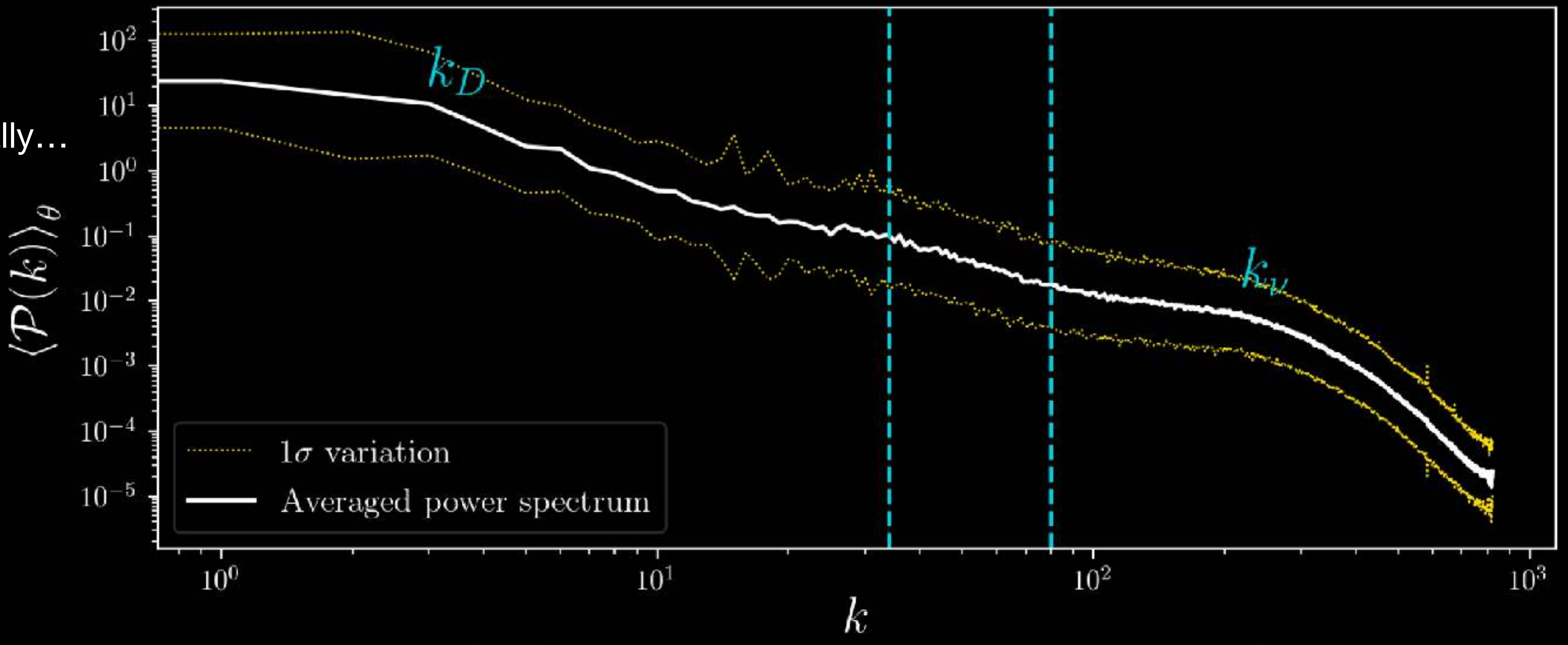
Vincent van Gogh's painting, The Starry Night, is an iconic piece of art and cultural history. The painting portrays a night sky full of stars, with eddies (spirals) both large and small. Kolmogorov (1941)'s description of subsonic, incompressible turbulence gives a model for turbulence that involves eddies interacting on many length scales, and so the question has been asked: is The Starry Night turbulent? To answer this question, we calculate the azimuthally averaged power spectrum of a square region (1165×1165 pixels) of night sky in The Starry Night. We find a power spectrum, $\mathcal{P}(k)$, where k is the wavevector, that shares the same features as supersonic turbulence. It has a power-law $\mathcal{P}(k) \propto k^{-2.1\pm0.3}$ in the scaling range, $34 \leq k \leq 80$. We identify a driving scale, $k_{\mathrm{D}} = 3$, dissipation scale, $k_\nu = 220$ and a bottleneck. This leads us to believe that van Gogh's depiction of the starry night closely resembles the turbulence found in real molecular clouds, the birthplace of stars in the Universe.

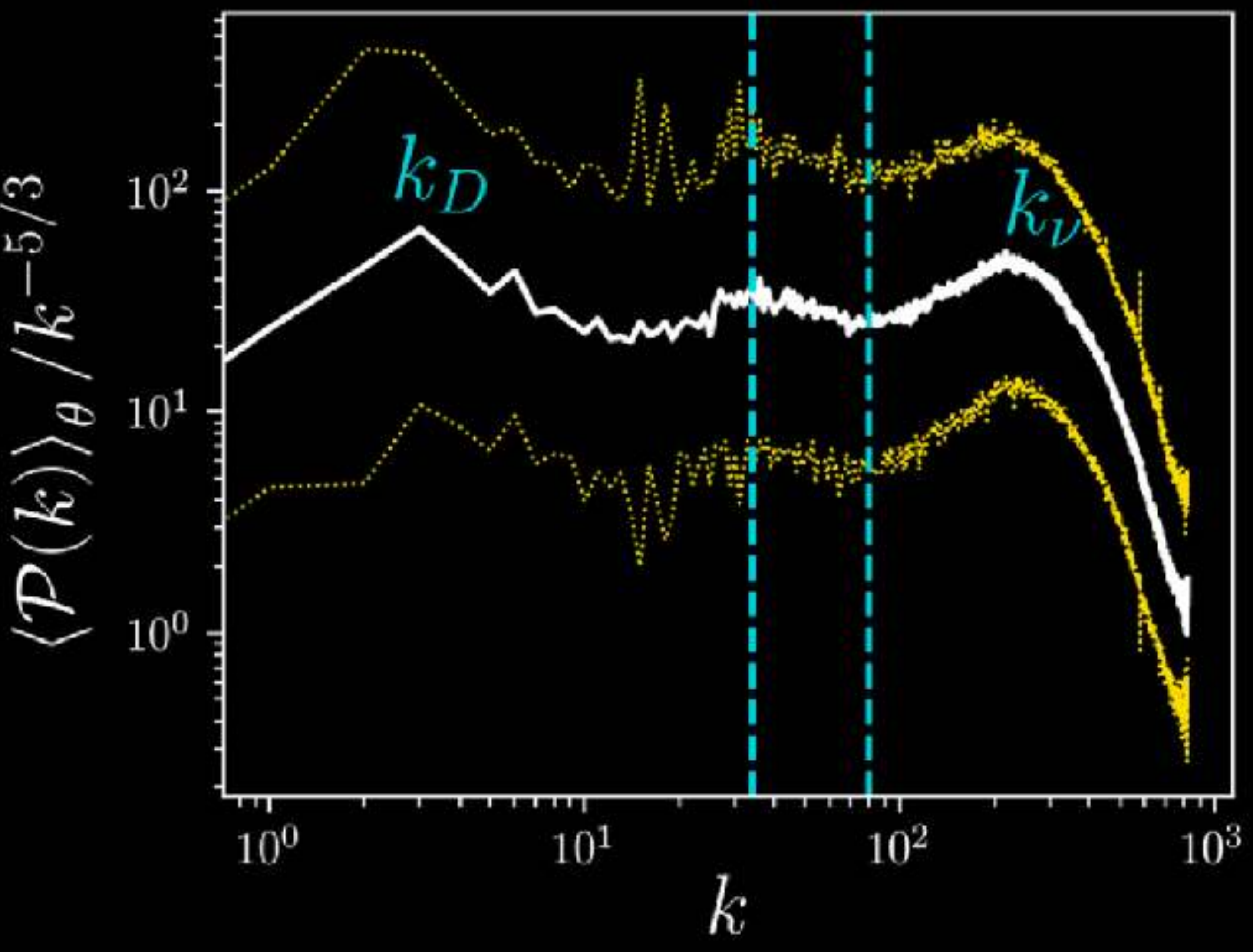
back in 2017…

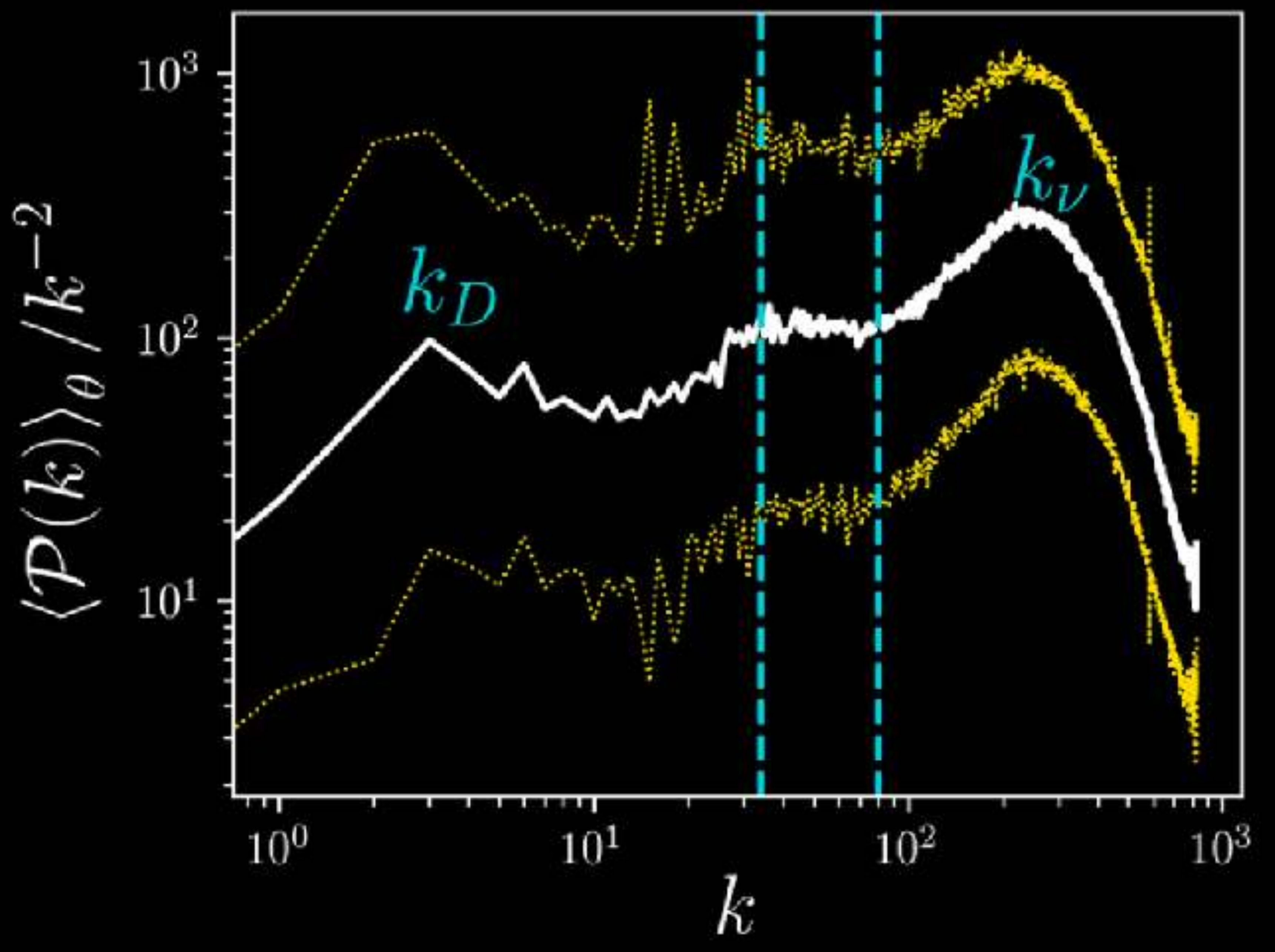
Van Gogh painted quite isotropically…

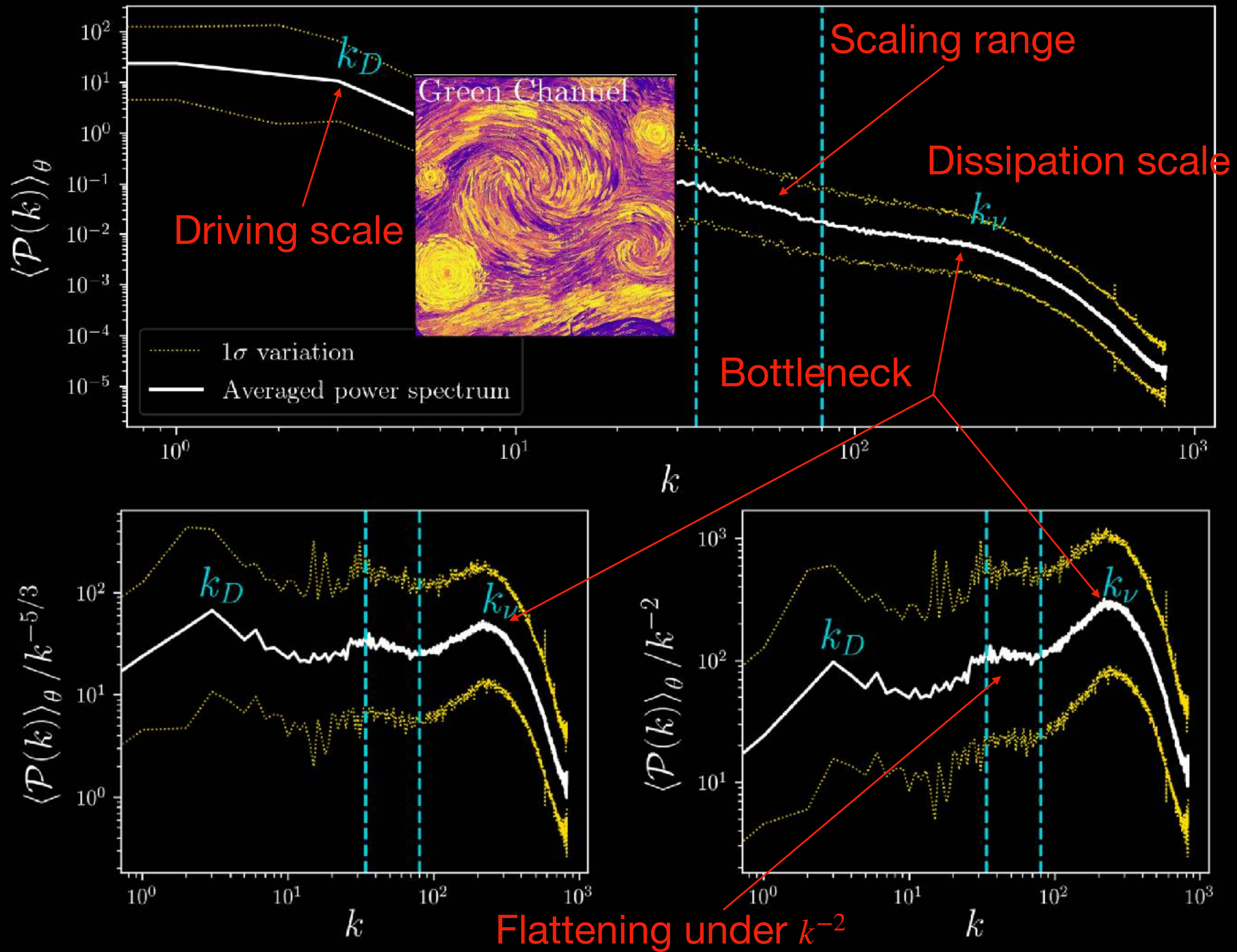












A beautiful degeneracy… or universality?

$$k_\nu \approx 220\,\mathrm{px}^{-1} \approx \frac{2\pi}{\ell_\nu}$$

$$\ell_\nu \approx (1-2)\,\mathrm{cm}$$

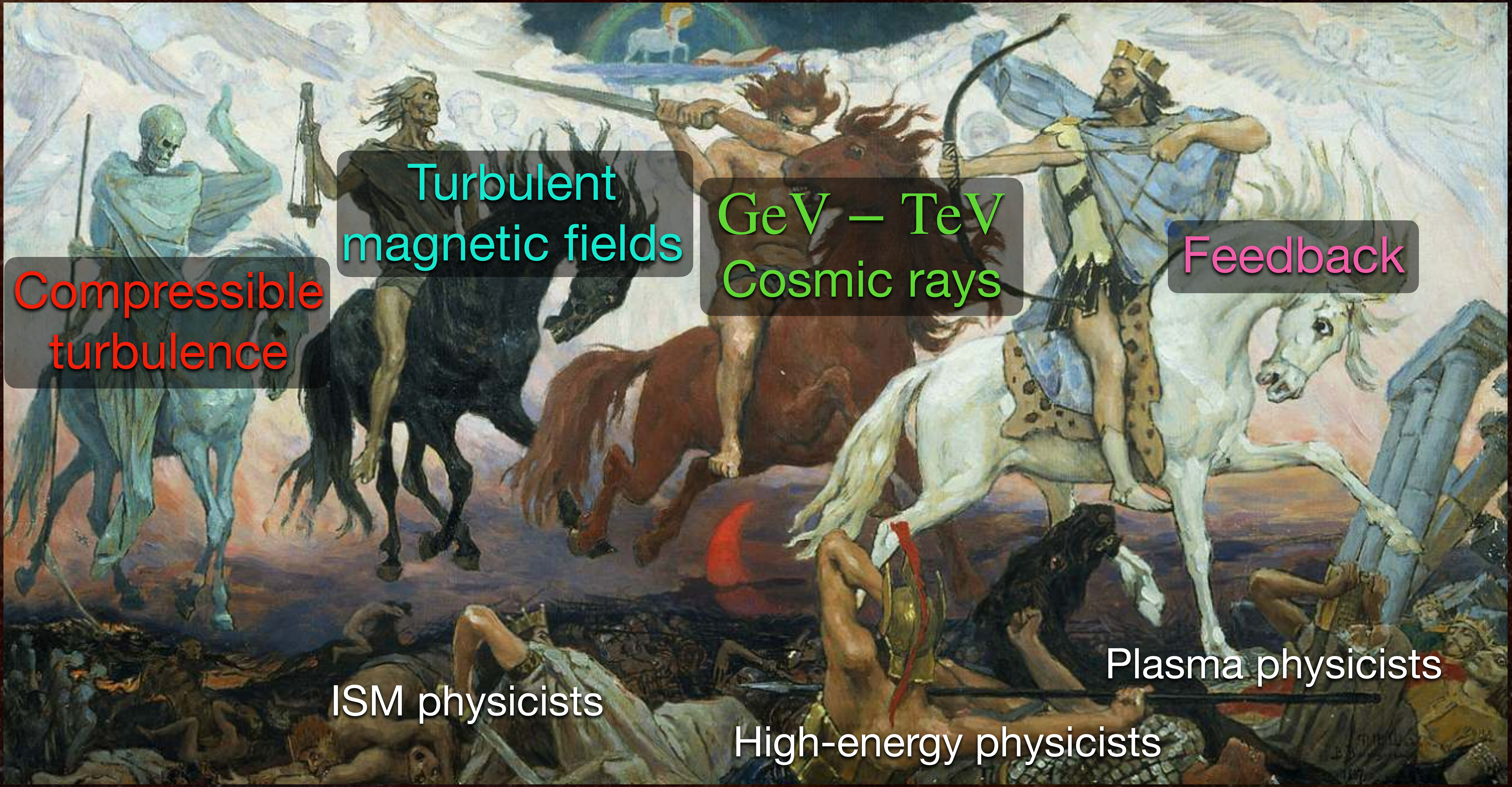
Objectives

1. Understand the nature of supernova-driven turbulence to directly see how much it resembles our simple Kolmogorov-style models *(philosophy: build the simplest models first, understand them in great detail)*.

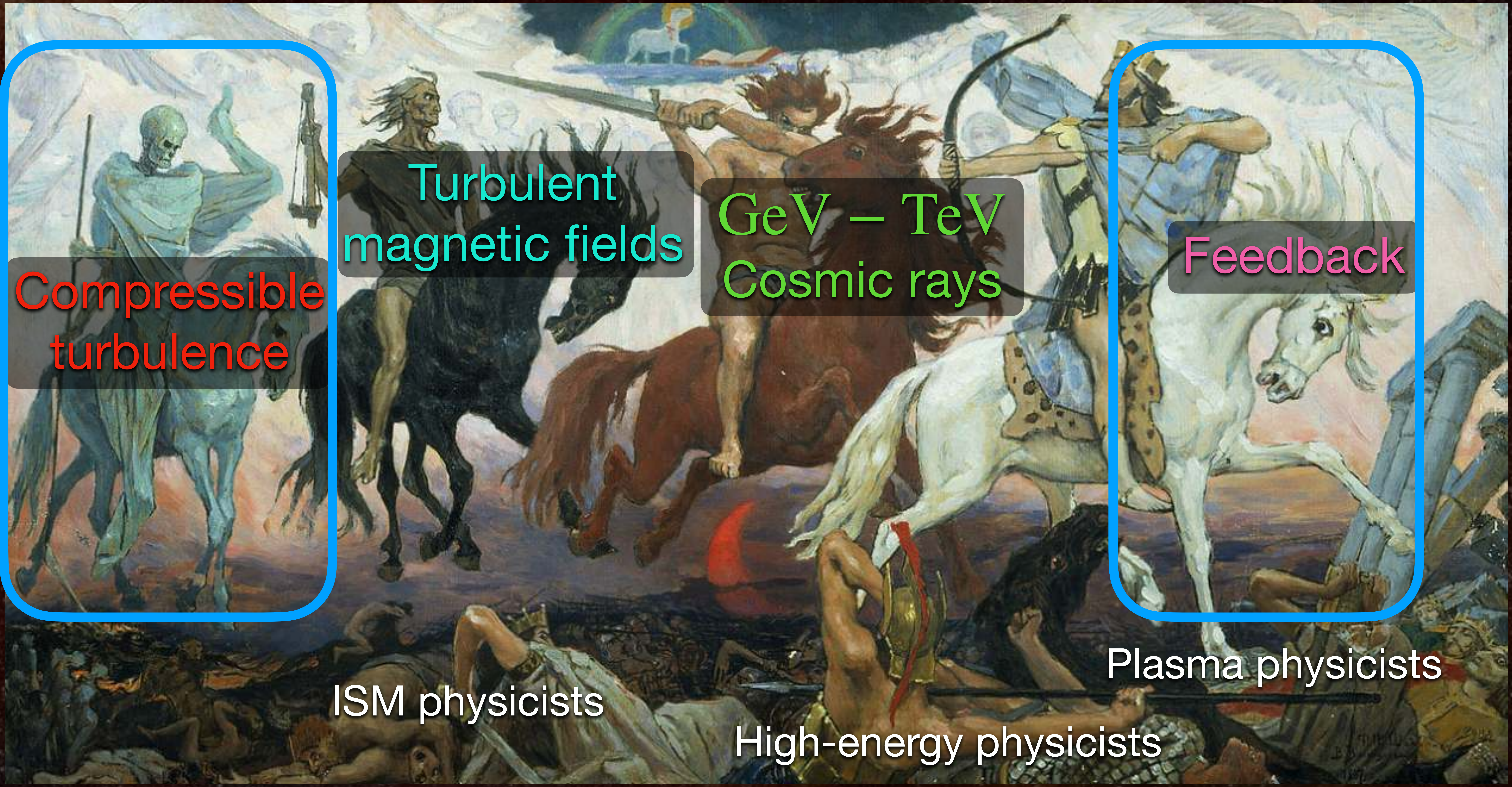
2. Introduce the idea of energy flux density statistics, defining some new opportunities for using these directly on observations

James Beattie. Magnetic reconnection in a turbulent fluid. Iterative LIC.

My four horsemen of ISM physics



My four horsemen of ISM physics



Supernova driven turbulence

To drive the interstellar cascade you only need a few percent of the energy from SNe to be into turbulence

Chamandy & Shukurov (2020)

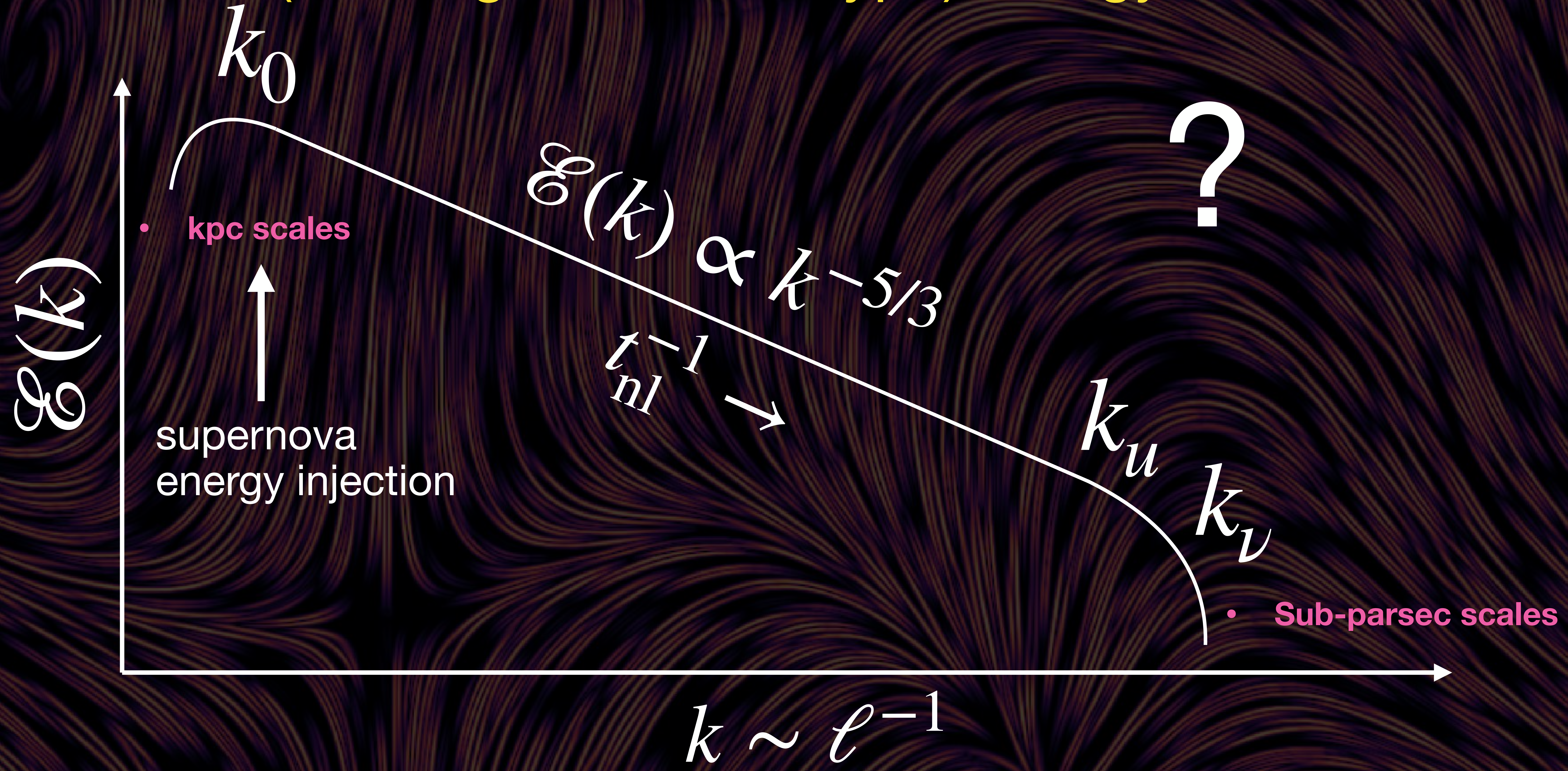
W3/W4/W5 MCs and star forming region complex.

Supernova driven turbulence

To drive the interstellar cascade you only need a few percent of the energy from SNe to be into turbulence

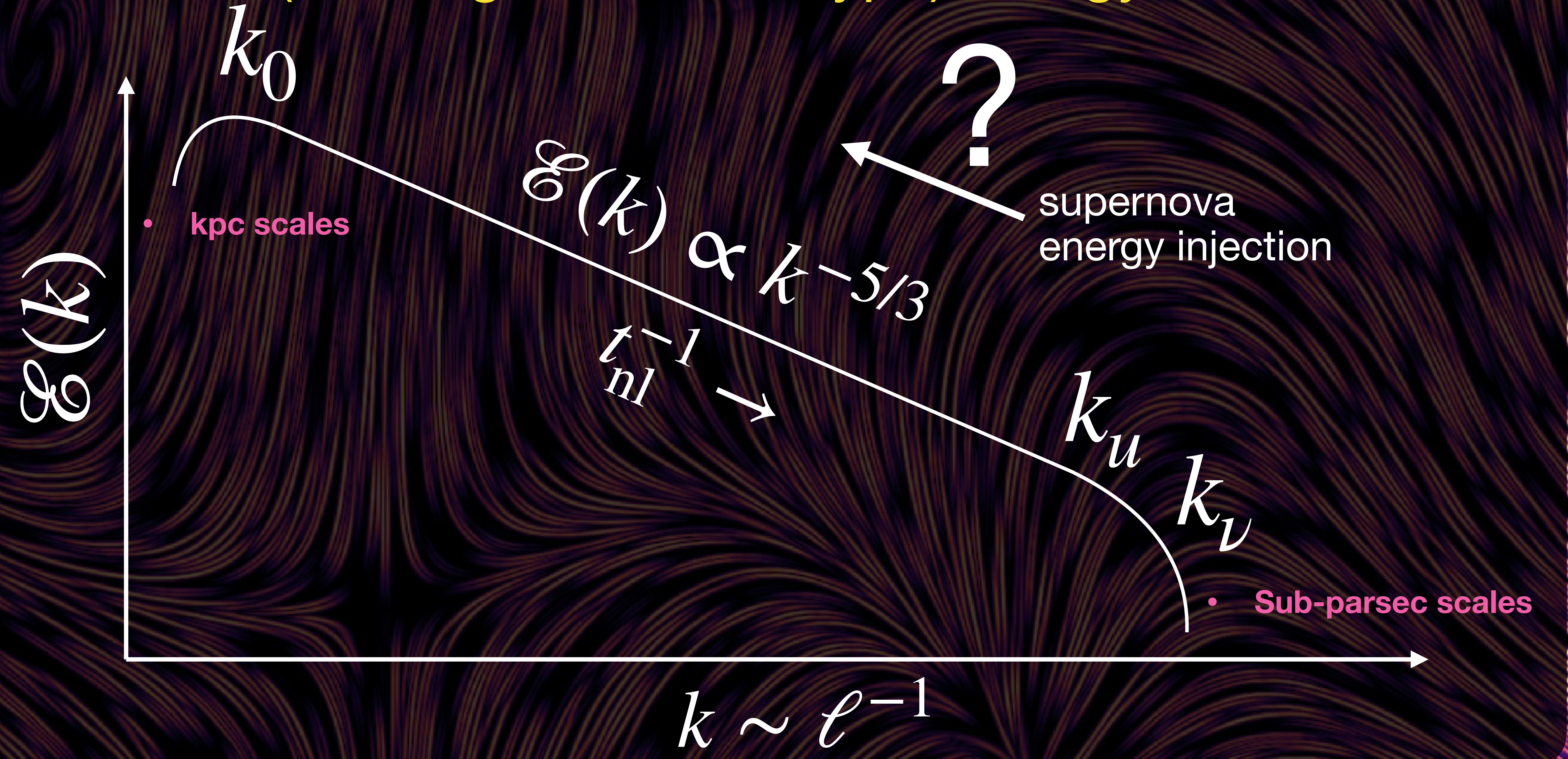
Chamandy & Shukurov (2020)

The (Kolmogorov, 1941-type) energy cascade



Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

The (Kolmogorov, 1941-type) energy cascade



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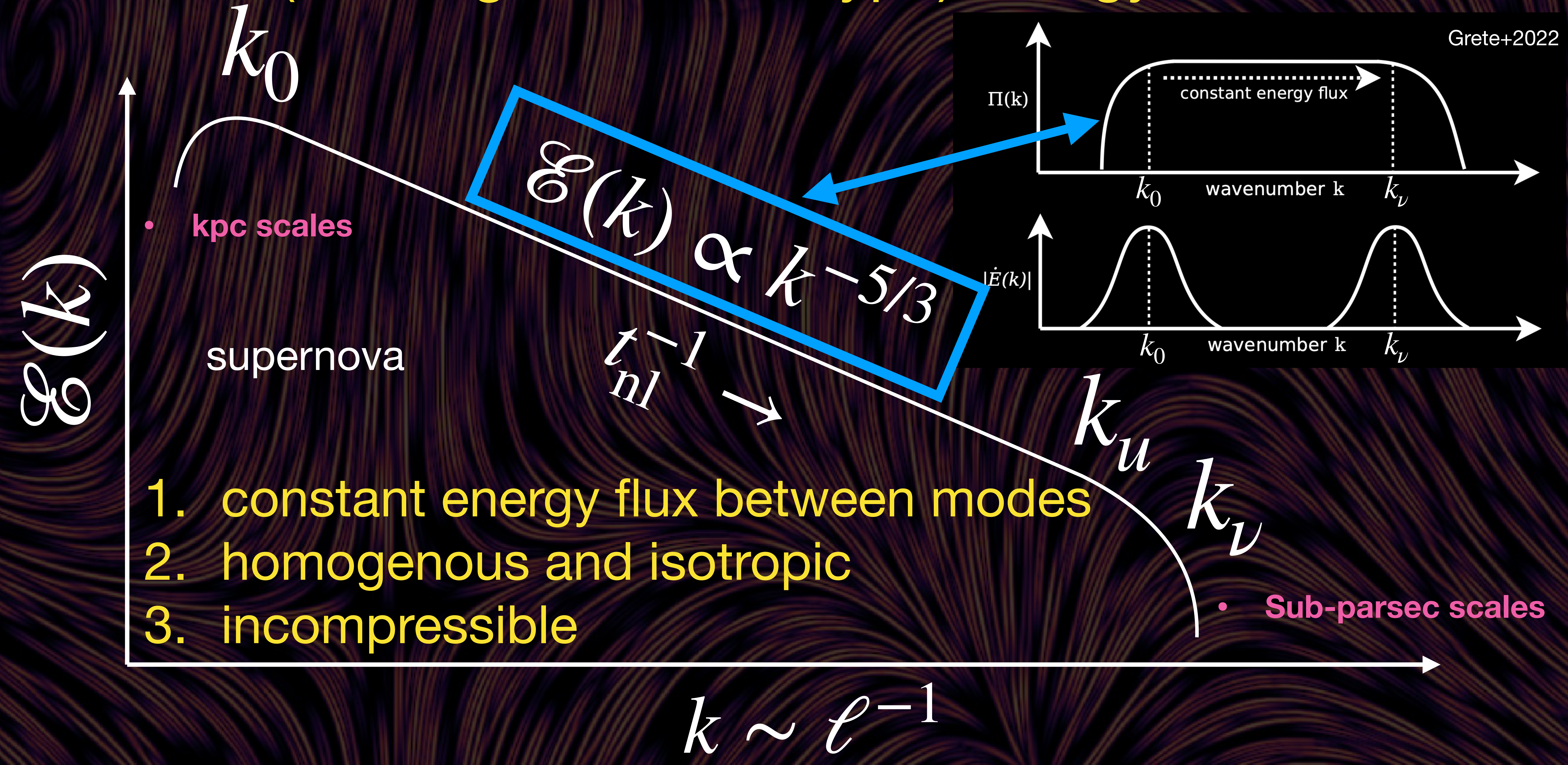
The (Kolmogorov, 1941-type) energy cascade

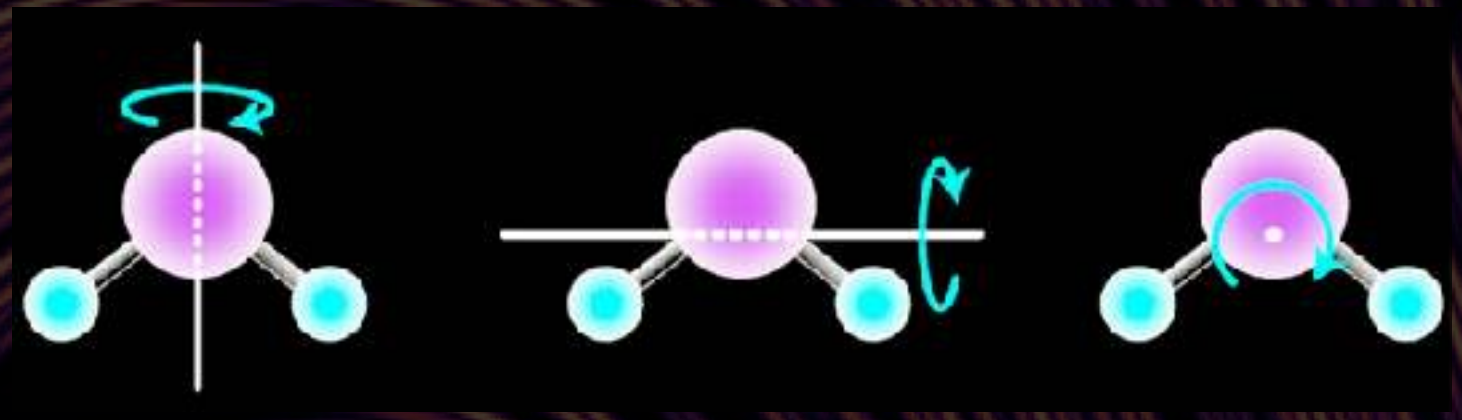


1. constant energy flux between modes
2. homogenous and isotropic
3. incompressible

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Energy flux density gives rise to Kolmogorov

$$\varepsilon \sim u^3/\ell \implies u \sim (\varepsilon\ell)^{1/3} \implies u^2(k) \sim k^{-5/3}$$



$$\varepsilon_{\mathrm{in}} = \varepsilon_{\mathrm{out}} = Q_{\mathrm{heating}}$$

$Q_{\mathrm{heating}} \rightarrow Q_{\mathrm{ions}},\ Q_{\mathrm{electrons}},\ Q_{\mathrm{neutrals}},\ Q_{\mathrm{molecules}},\ Q_{\ldots}$

$Q_{\mathrm{ions}},\ Q_{\mathrm{neutrals}},\ Q_{\mathrm{molecules}} \rightarrow Q_{\mathrm{dof}}$

For details on heating and partition between ions and electrons and dof, read -> Greg Howes' latest work. arxiv.org/abs/2402.12829

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

How to probe the energy flux density?

$$\overbrace{\partial_t \mathcal{E}_{\mathrm{kin}}}^{\text{kinetic energy density}} + \underbrace{\mathbf{u} \cdot \nabla \cdot \mathbb{F}_{\rho \mathbf{u}}}_{\substack{\text{energy flux density} \\ \text{from transport}}} = \frac{1}{\mathrm{Re}} \overbrace{\mathbf{u} \cdot \mathbb{D}_{\nu}(\mathbf{u})}^{\substack{\text{energy flux density} \\ \text{from viscosity}}}$$

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

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$$\overbrace{\partial_t \mathcal{E}_{\mathrm{kin}}}^{\text{kinetic energy density}} + \underbrace{\mathbf{u} \cdot \nabla \cdot \mathbb{F}_{\rho \mathbf{u}}}_{\text{energy flux density from transport}} = \frac{1}{\mathrm{Re}} \overbrace{\mathbf{u} \cdot \mathbb{D}_{\nu}(\mathbf{u})}^{\text{energy flux density from viscosity}}$$

$$\varepsilon \sim u^3/\ell$$

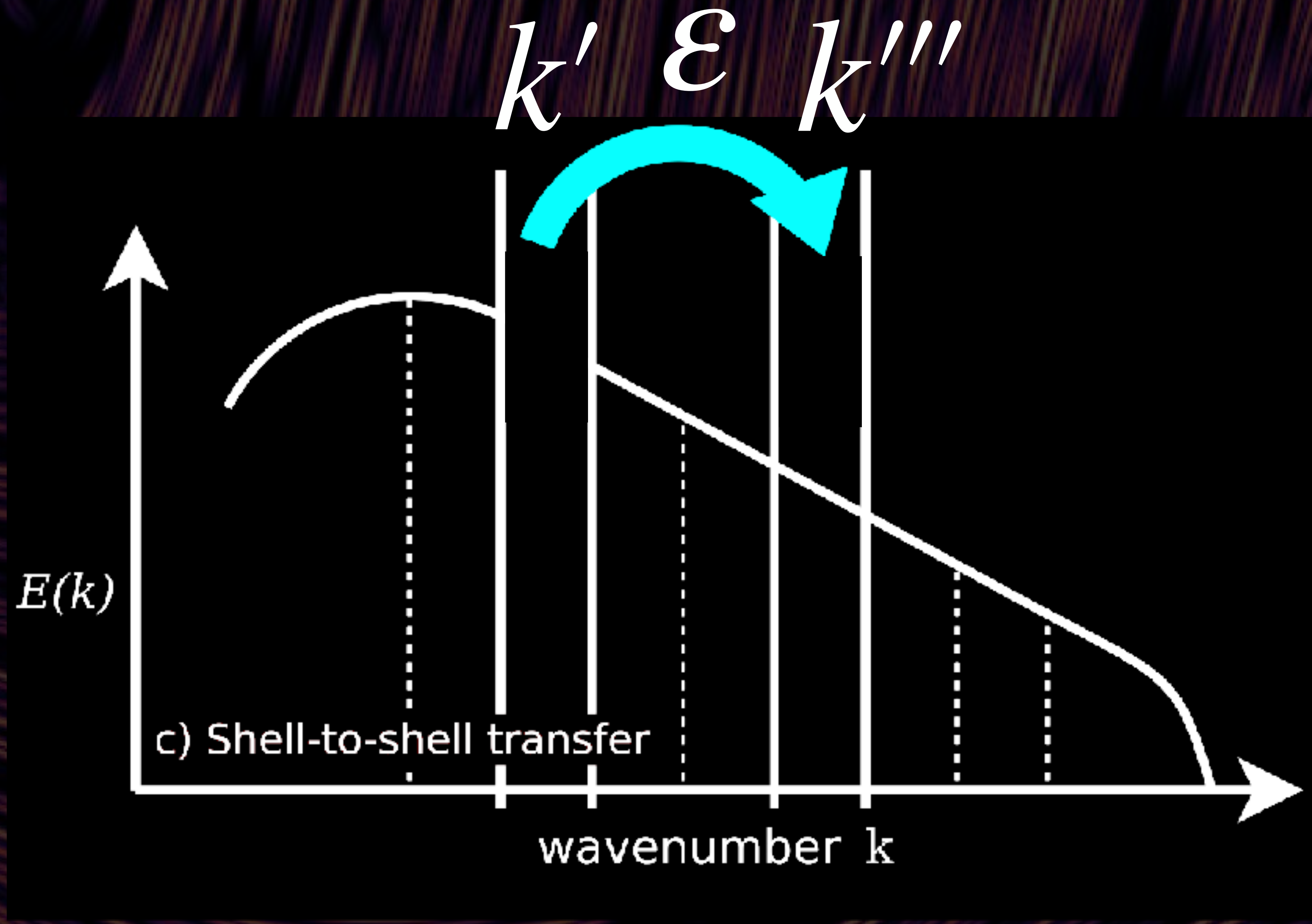
$$\mathbf{u} \cdot \nabla \cdot \mathbb{F}_{\rho \mathbf{u}} = \underbrace{\mathbf{u} \otimes \mathbf{u} : \nabla \otimes \mathbf{u}}_{\text{turbulence}} + \dots$$

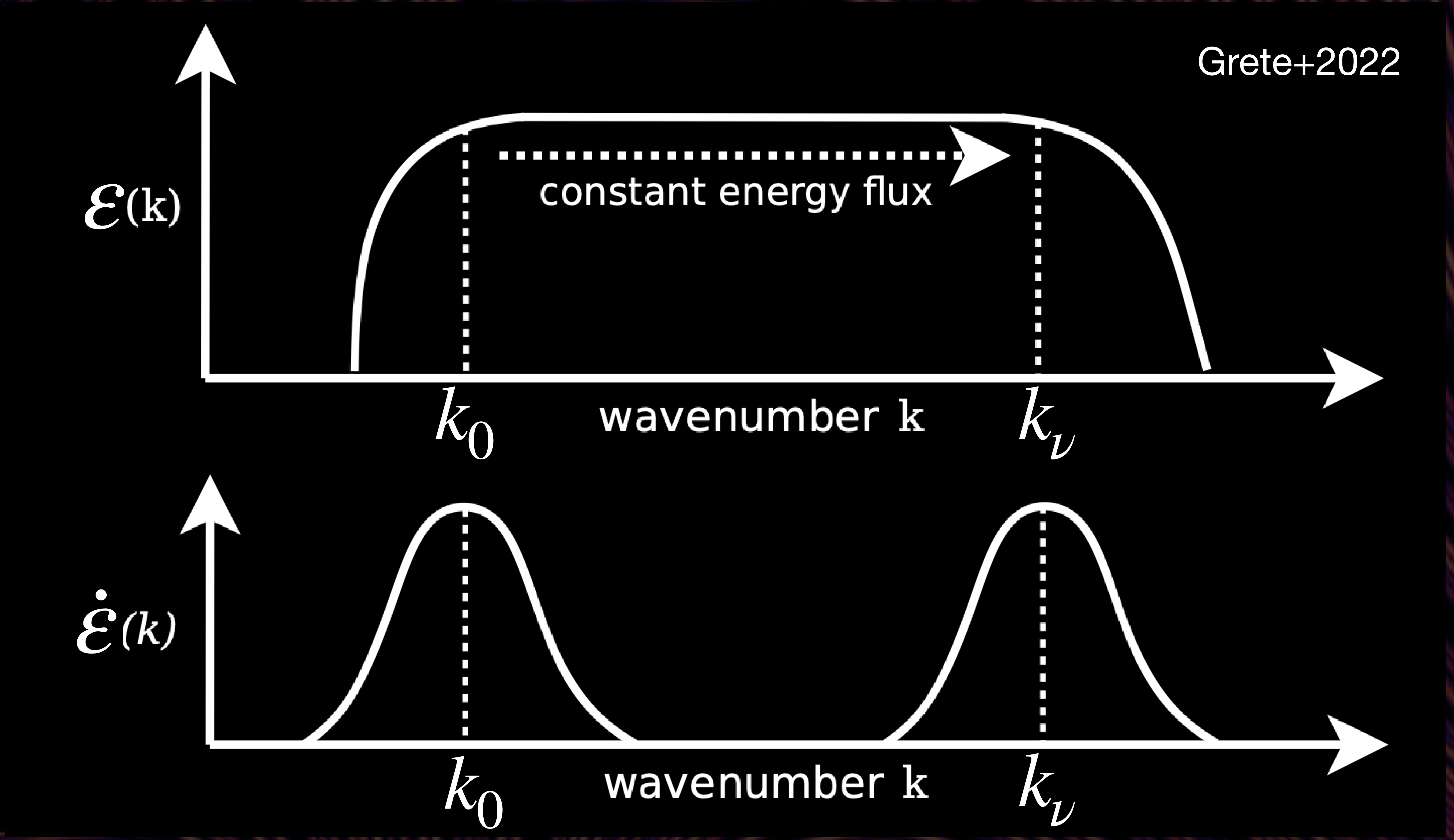
That nonlinearity!

$$\mathbf{u} \cdot \mathbf{u} \cdot \nabla \otimes \mathbf{u}$$

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

But to understand the turbulence we must understand the flux from mode to mode!





Grete+2022a,b,23

Mininni+2005

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

But to understand the turbulence we must understand the flux from mode to mode!

Momentum conservation:

doner receiver

$$\mathbf{k}' + \mathbf{k}'' + \mathbf{k}''' = 0$$

mediator

doner receiver

$$\mathbf{k}' \xrightarrow{\mathbf{k}''} \mathbf{k}''' = -\,\mathbf{k}''' \xrightarrow{\mathbf{k}''} \mathbf{k}'$$

mediator

$$\mathbf{u}' = \mathbf{u}(\mathbf{r}') = \int \delta^3(\mathbf{k} - \mathbf{k}')\mathbf{u}(\mathbf{k})\exp\left\{2\pi i \mathbf{k}\cdot\mathbf{r}\right\}$$

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

But to understand the turbulence we must understand the flux from mode to mode!

$$\mathbf{k}' \xrightarrow{\mathbf{k}''} \mathbf{k}'''$$

$$\frac{1}{2}\partial_t \overbrace{\rho \mathbf{u}' \cdot \mathbf{u}'''}^{\text{kinetic energy density}} + \mathbf{u}''' \cdot \nabla \cdot \underbrace{\mathbb{F}_{\rho\mathbf{u}}(\mathbf{u}'', \mathbf{u}')}_{\substack{\text{energy flux density} \\ \text{from transport between } \mathbf{u}' \text{ and } \mathbf{u}'''}} = 0$$

$$\mathbf{u}''' \cdot \nabla \cdot \mathbb{F}(\mathbf{u}'', \mathbf{u}')_{\rho\mathbf{u}} = \mathbf{u}''' \otimes \mathbf{u}'' : \nabla \otimes \mathbf{u}' + \ldots$$

$$\varepsilon \sim u^3/\ell$$

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

But to understand the turbulence we must understand the flux from mode to mode!

$$\mathbf{u}''' \cdot \nabla \cdot \mathbb{F}(\mathbf{u}'', \mathbf{u}')_{\rho\mathbf{u}} = \mathbf{u}''' \otimes \mathbf{u}'' : \nabla \otimes \mathbf{u}' + \dots$$

$$\mathbf{k}' \xrightarrow{\mathbf{k}''} \mathbf{k}'''$$

$$\mathscr{T}_{uu}(k', k''' \,|\, k'') = -\int dV\, \mathbf{u}''' \otimes \mathbf{u}'' : \nabla \otimes \mathbf{u}'$$

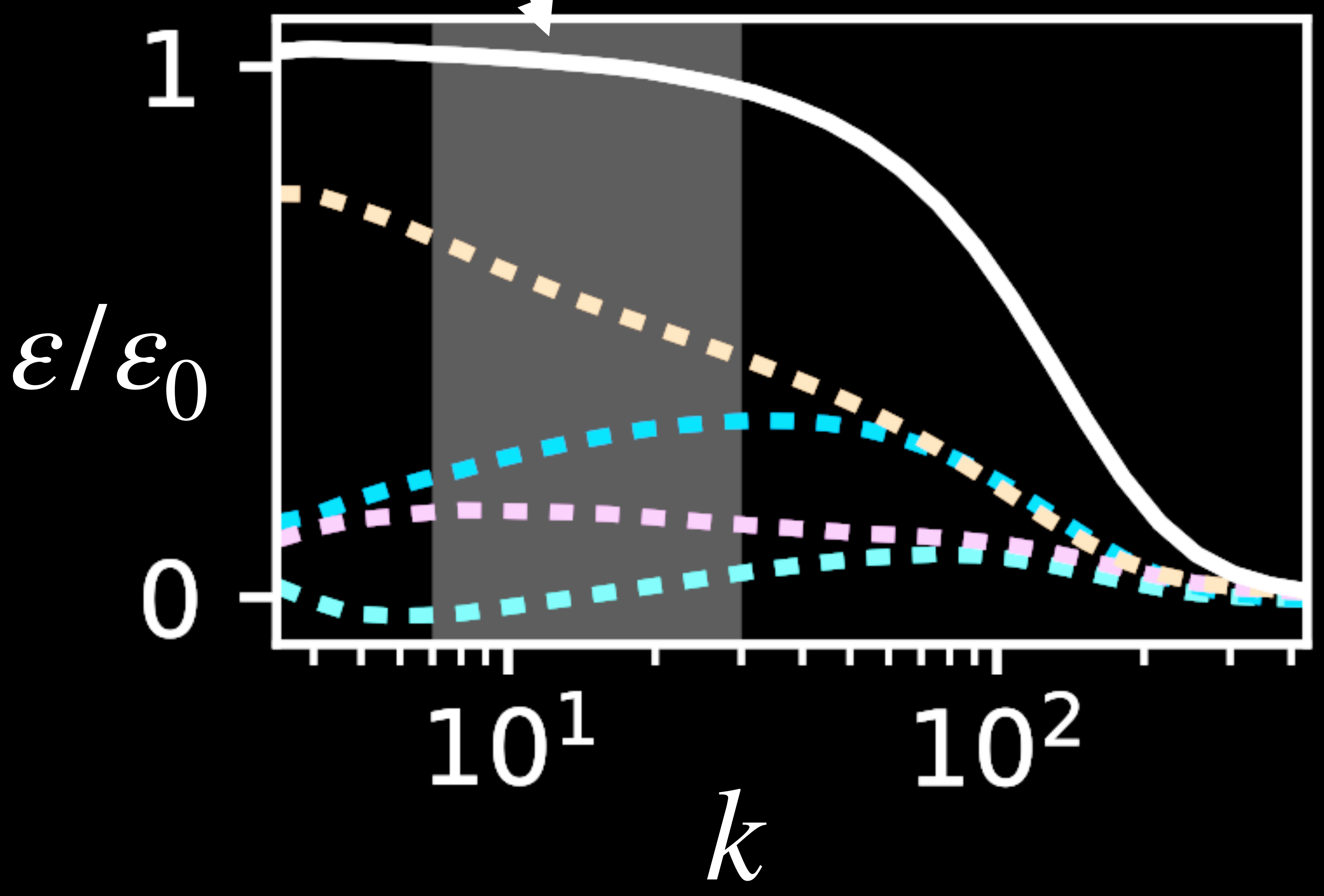
$$\varepsilon \sim u^3/\ell$$

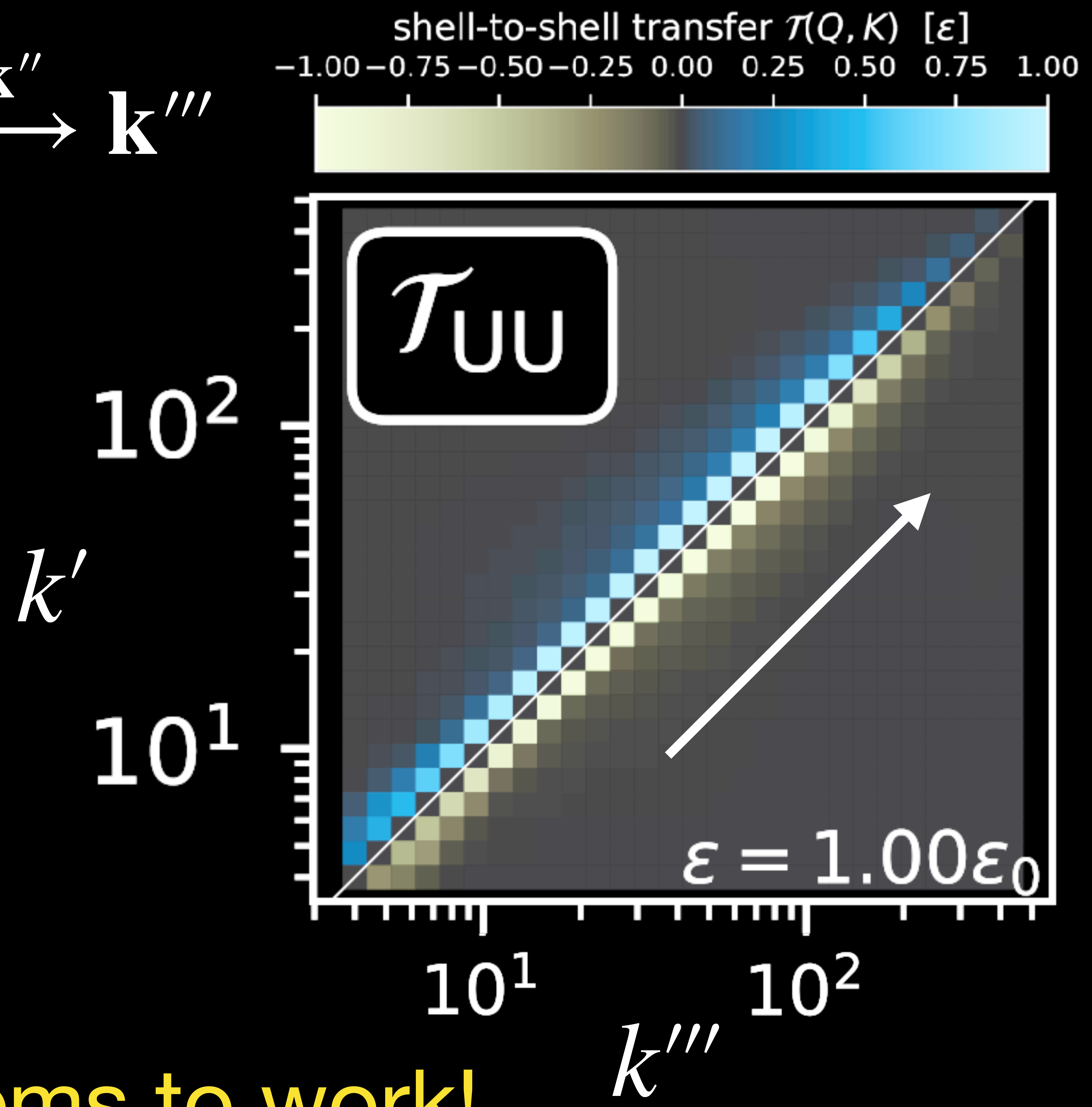
Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Compressible MHD turbulence in a box
Grete+2022

$$\mathbf{k}' \xrightarrow{\mathbf{k}''} \mathbf{k}'''$$

$$\varepsilon \sim u^3/\ell \approx \text{const.}$$

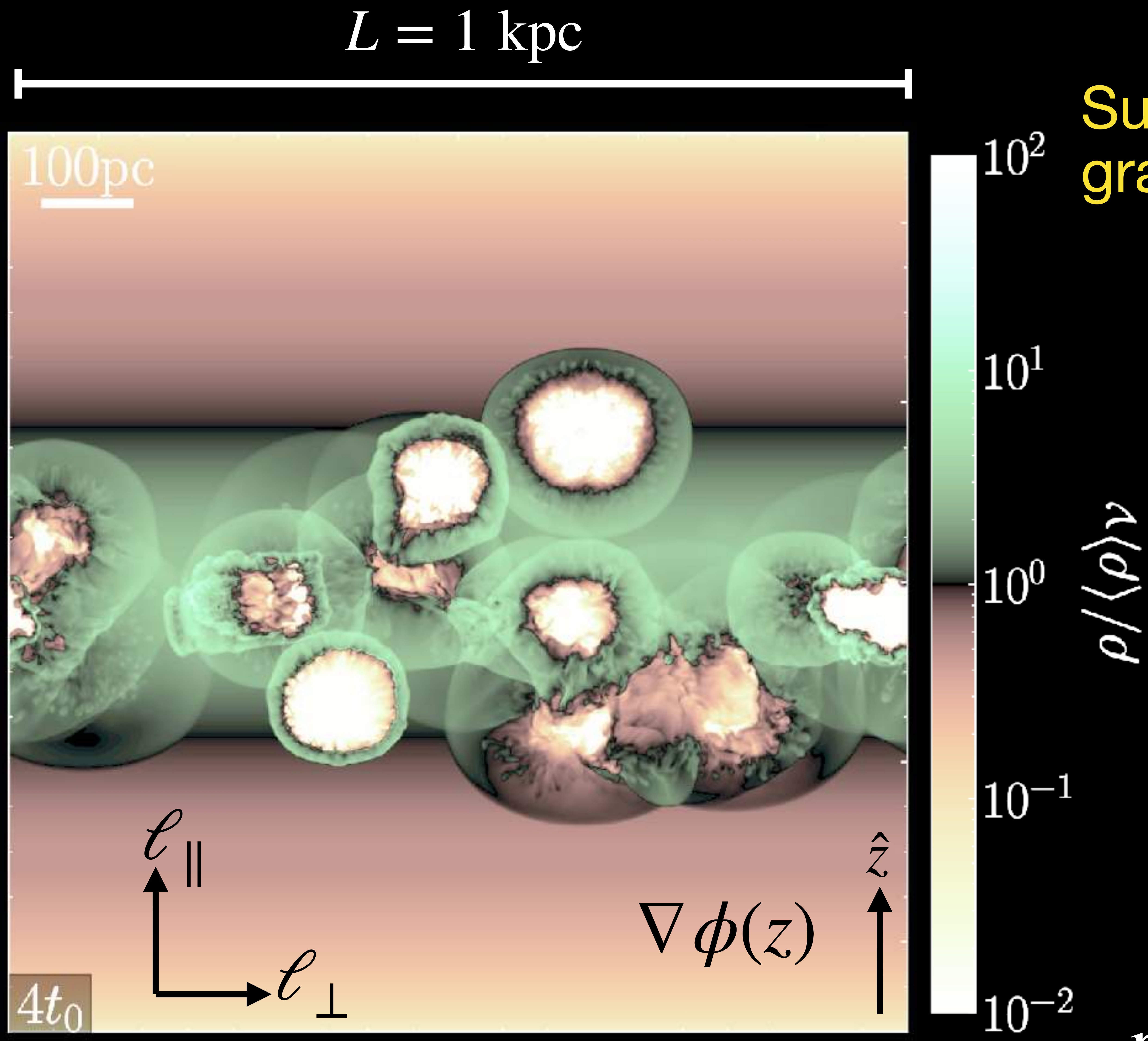




Kolmogorov assumption seems to work!

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Let's do this for more realistic ISM turbulence



Martizzi+2016

RAMSES (Teyssier 2002)

Supernova driven gravito-hydro dynamical model

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\boldsymbol{u}) = \dot{n}_{\rm SNe} M_{\rm ej}, \tag{1}$$

$$\frac{\partial \rho\boldsymbol{u}}{\partial t} + \nabla\cdot(\rho\boldsymbol{u}\otimes\boldsymbol{u} + P\mathbb{I}) = -\,\rho\nabla\phi + \dot{n}_{\rm SNe}\mathbf{p}_{\rm SNe}(Z, n_{\rm H}), \tag{2}$$

$$\frac{\partial \rho e}{\partial t} + \nabla\cdot[\rho\,(e+P)\,\boldsymbol{u}] = -\,n_{\rm H}^2\Lambda - \tag{3}$$

$$\rho\boldsymbol{u}\cdot\nabla\phi + \dot{n}_{\rm SNe}\left[E_{\rm th,SNe}(Z,n_{\rm H}) + \frac{\mathbf{p}_{\rm SNe}(Z,n_{\rm H})^2}{2(M_{\rm ej}+M_{\rm swept})}\right], \tag{4}$$

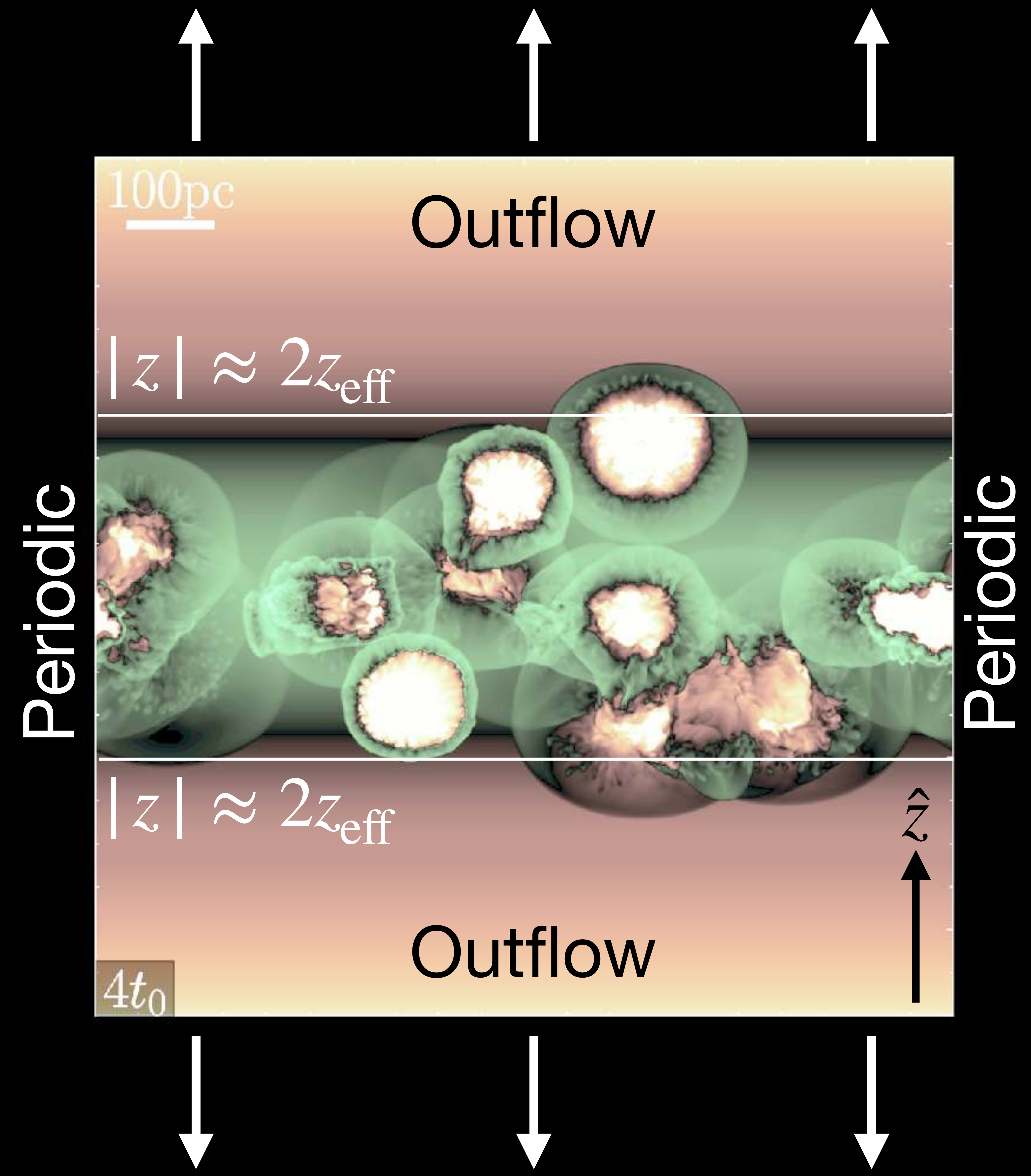
$$e = \epsilon + \frac{u^2}{2},\quad P = \frac{2}{3}\rho e, \tag{5}$$

$$N_{\rm grid}^3 = 1024^3 \implies \Delta x \sim 1\ \text{pc}$$

$$\text{numerical diss.} \implies \Delta x \sim 10\ \text{pc}$$

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Let's do this for more realistic ISM turbulence



Static gravitational potential

$$\phi(z) = \underbrace{2\pi G\Sigma_*\left(\sqrt{z^2 - z_0^2} - z_0\right)}_{\text{stratified disk}} + \underbrace{\frac{2\pi G\rho_{\rm halo}}{3}z^2}_{\text{spherical halo}}$$

Supernova driving prescription

$$\dot{n}_{\rm SNe} = \frac{\dot{\Sigma}_*}{2z_{\rm eff}100M_\odot}$$

1 SNe per $100M_\odot$ of SF

$$\dot{\Sigma}_* \propto \Sigma^{1.4}$$

KS relation

$$p(z) = \begin{cases} \frac{1}{4z_{\rm eff}}, & |z| \leq 2z_{\rm eff} \\ 0, & |z| > 2z_{\rm eff} \end{cases}$$

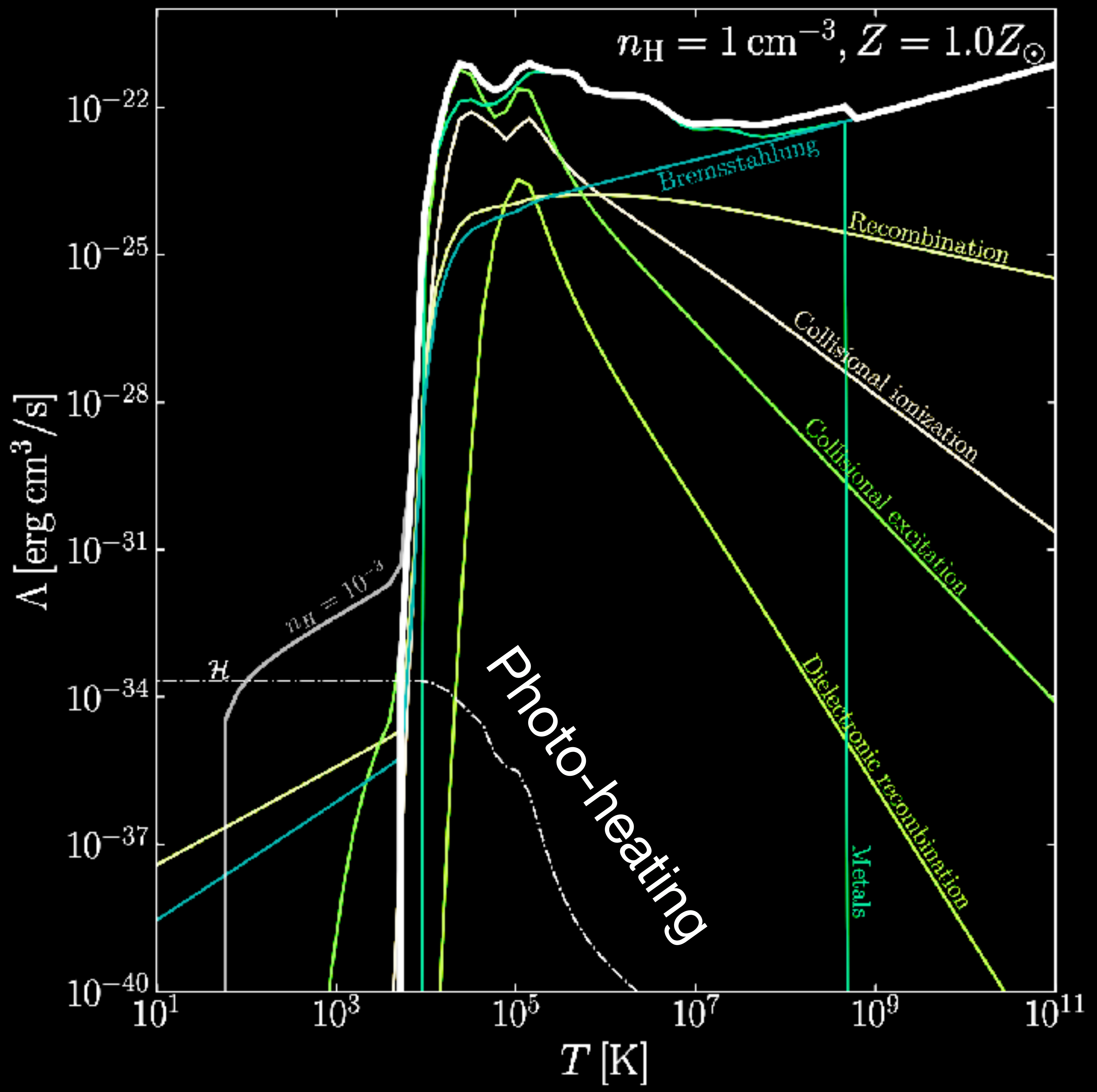
Inject many passive scalar metals into medium

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Let's do this for more realistic ISM turbulence

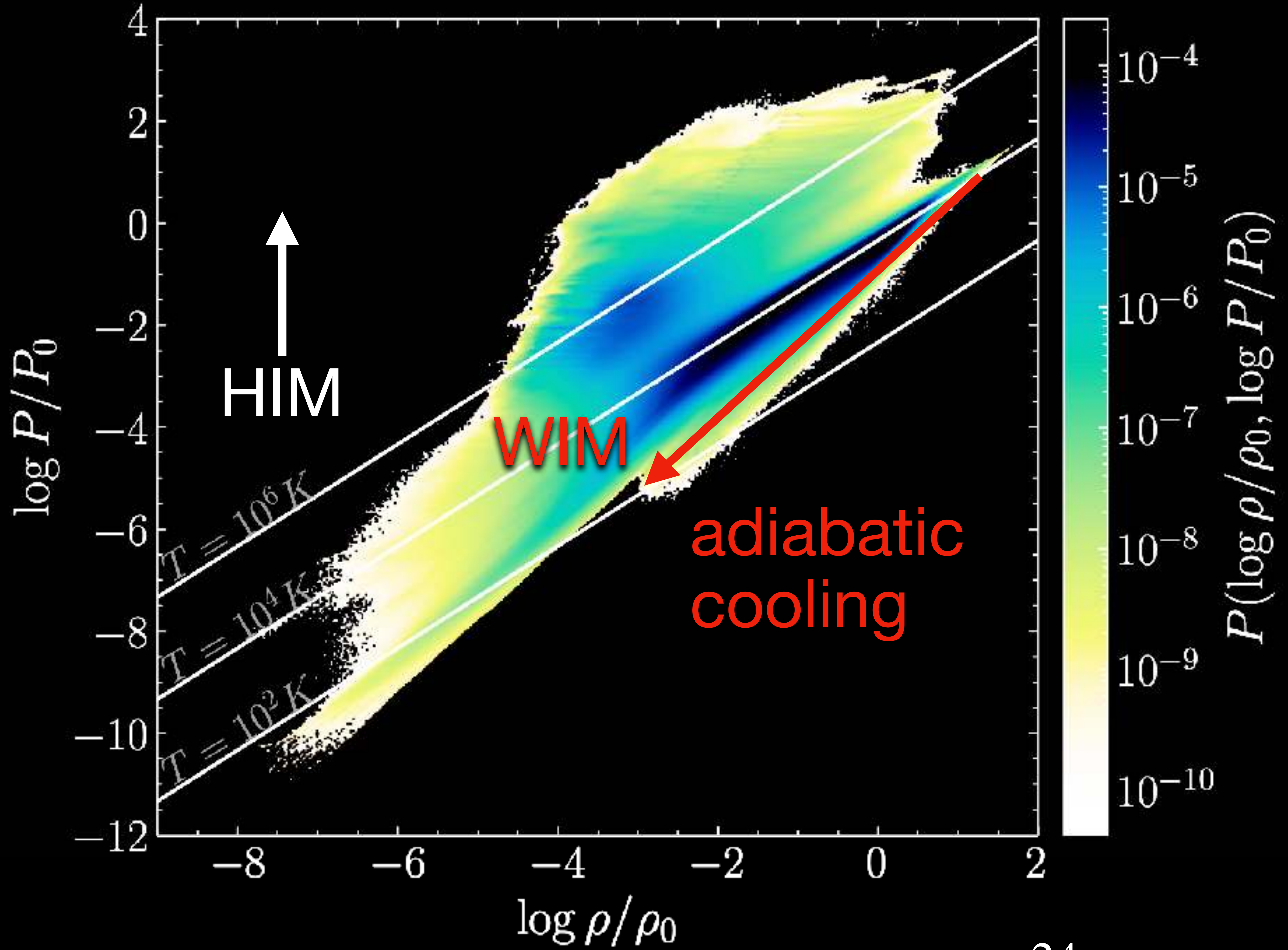
Cooling function

Theuns+(1998)
Sutherland & Dopita(1993)



HI, HII, HeI, HeII, HeIII and free electrons

Phase space



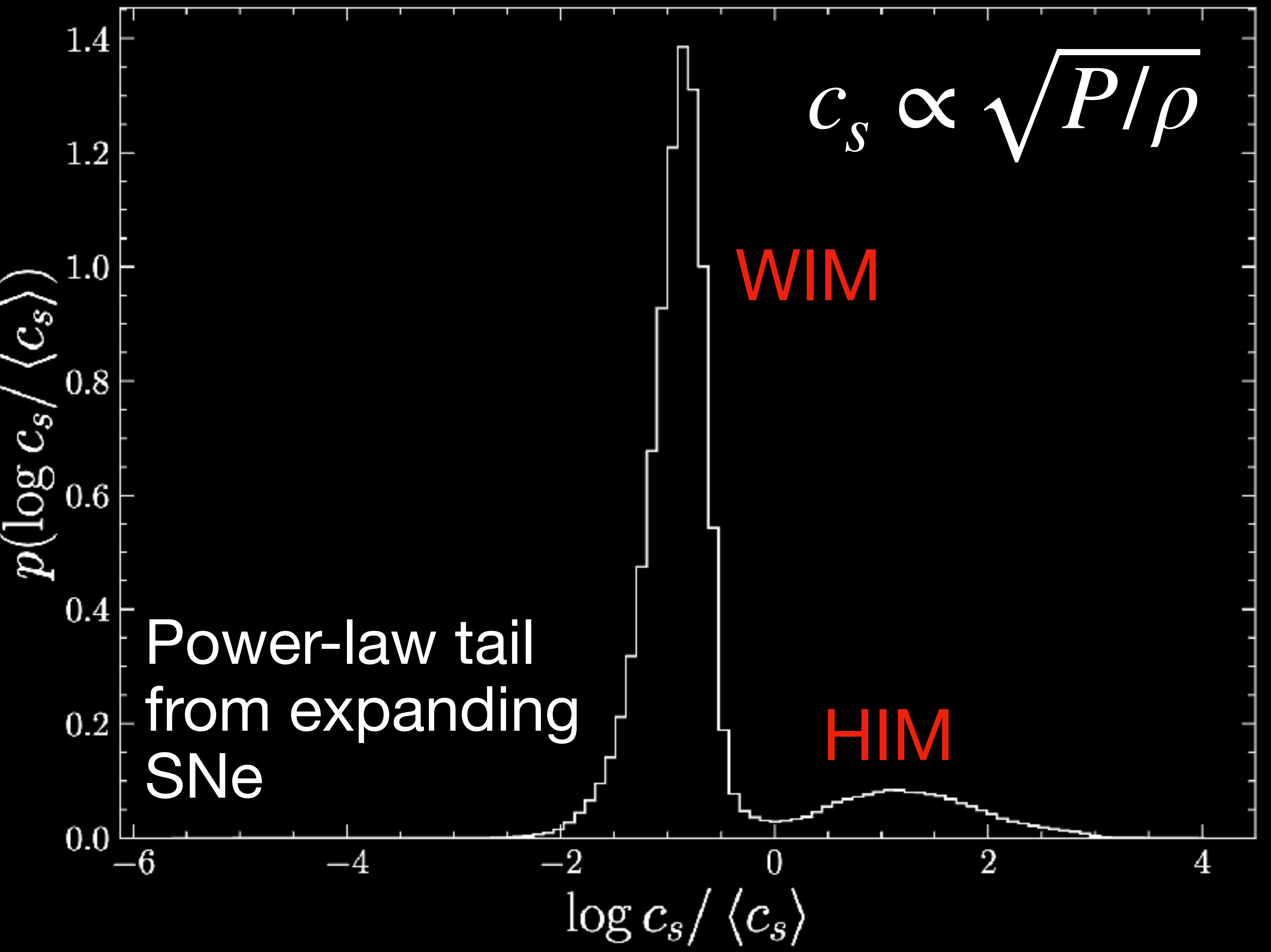
$$\rho_0 = 2.1 \times 10^{-24}\ \mathrm{g\ cm^{-3}}$$
$$P_0 = 2.2 \times 10^{-12}\ \mathrm{Ba}$$

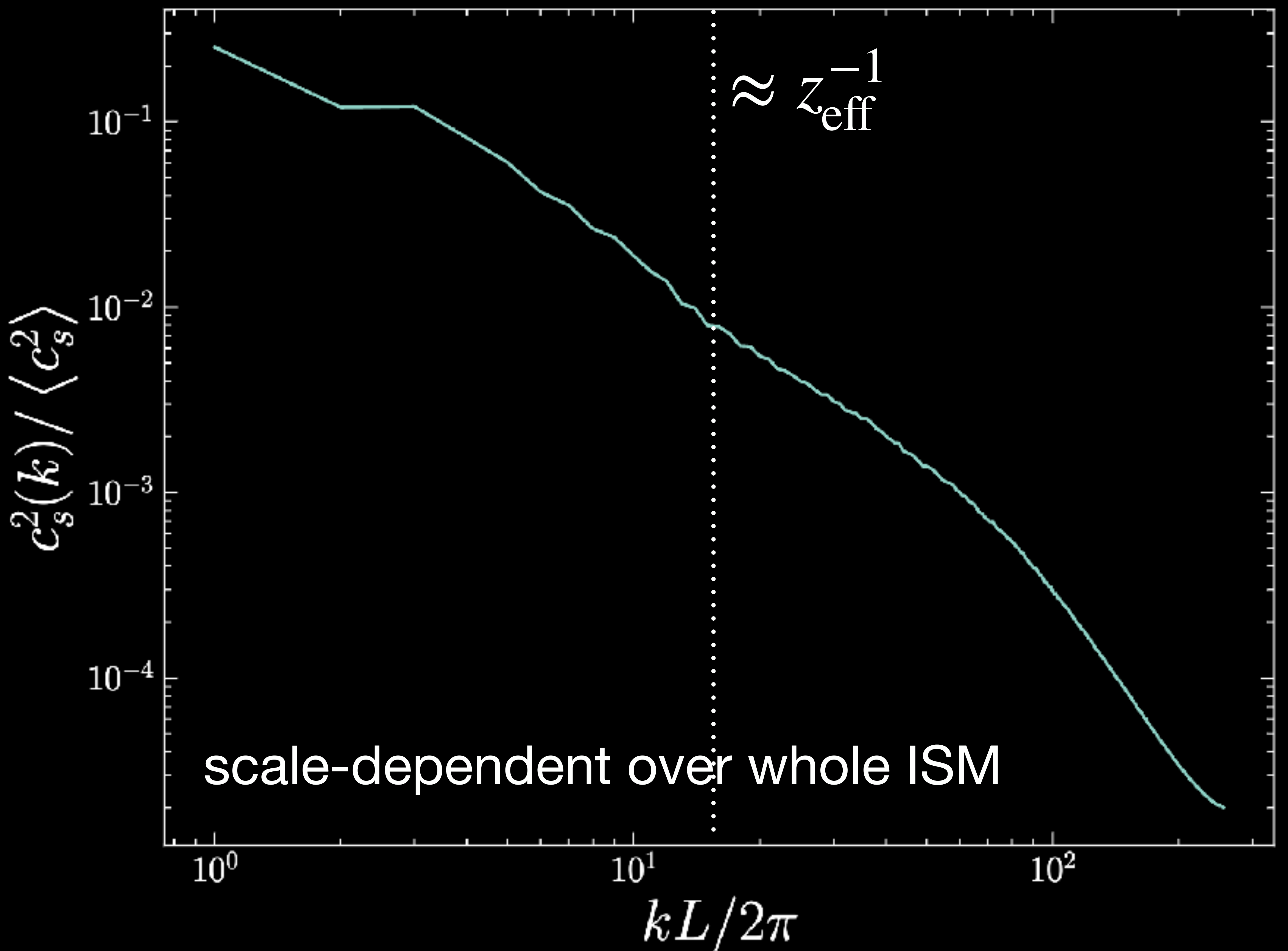
Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Let's do this for more realistic ISM turbulence

Sound speed statistics

$N_{\rm grid}^3 = 512^3$



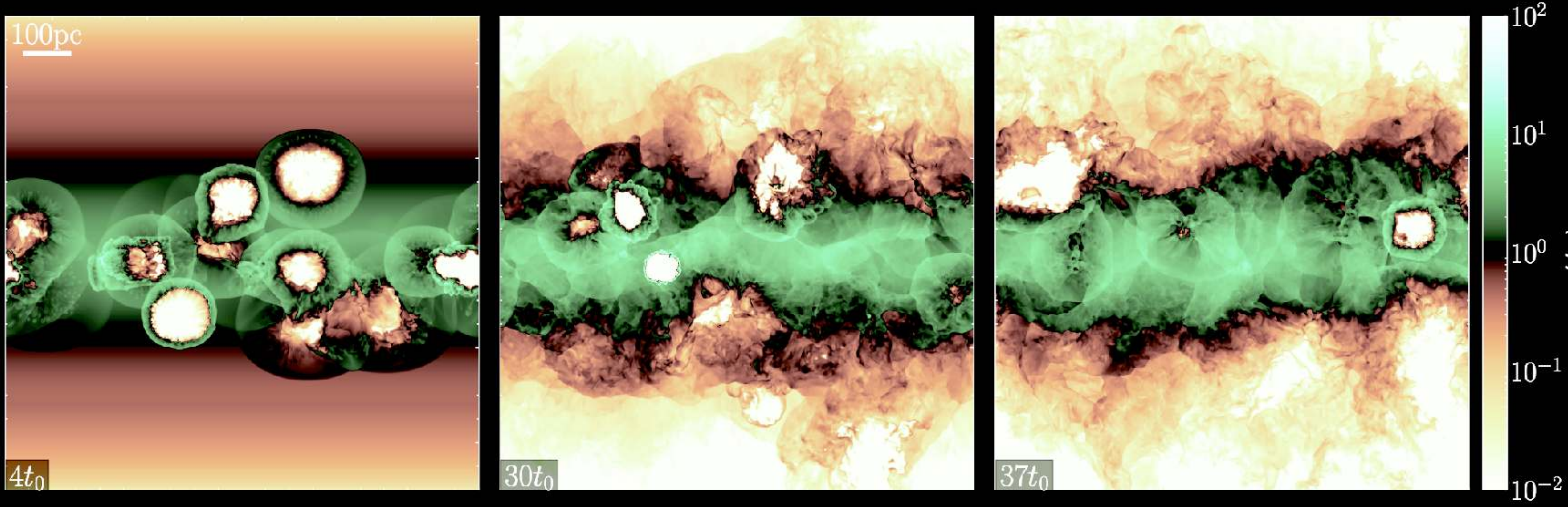


Embracing the idea of a turbulent ISM is to embrace that every quantity is scale-dependent!

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Let’s do this for more realistic ISM turbulence

The journey towards stationarity: the mass density

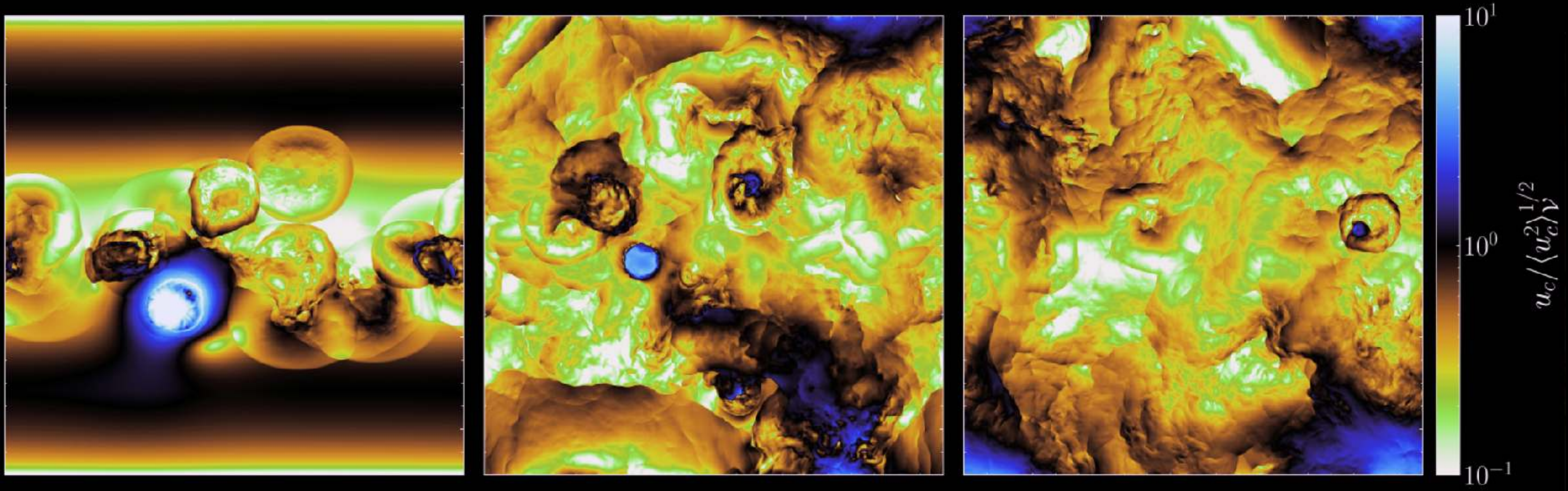


time →

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Let’s do this for more realistic ISM turbulence

The journey towards stationarity: compressible modes



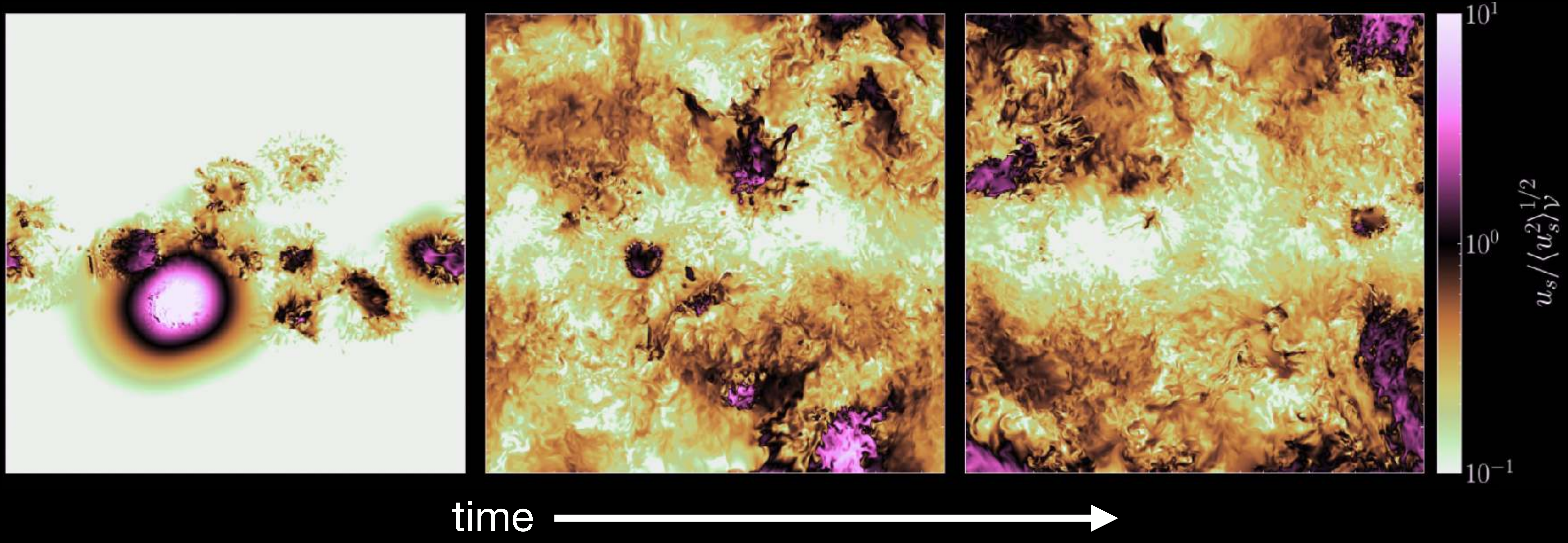
time →

the velocity modes don’t seem to care about the phases…

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Let's do this for more realistic ISM turbulence

The journey towards stationarity: solenoidal modes

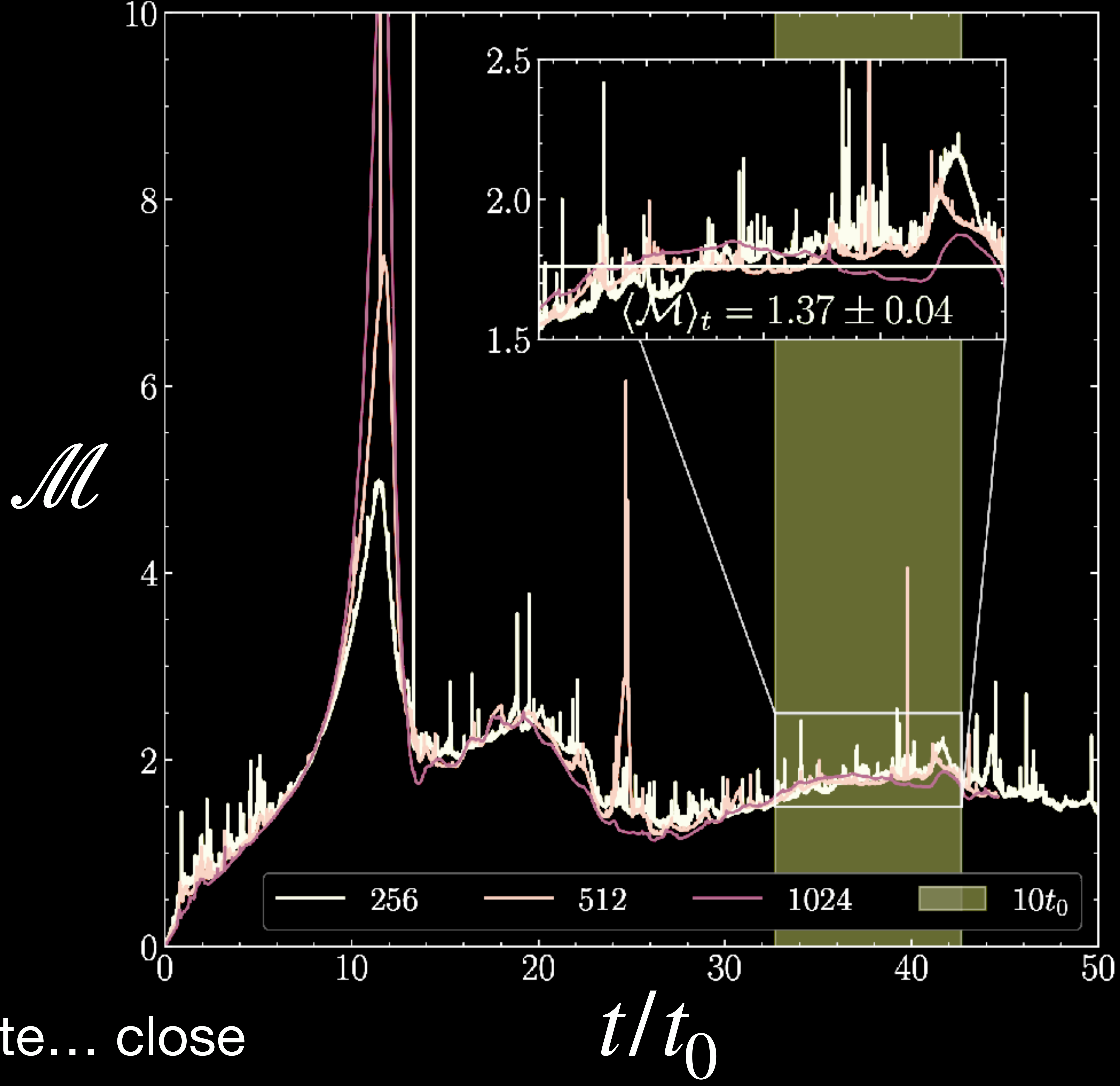


Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Stationarity and Mach number

$$\mathscr{M} = \left\langle \left(\frac{u}{c_s} \right)^2 \right\rangle^{1/2}$$

$$t_0 \sim \frac{z_{\text{eff}}}{\langle u^2 \rangle^{1/2}} \sim 80 \text{ Myr}$$



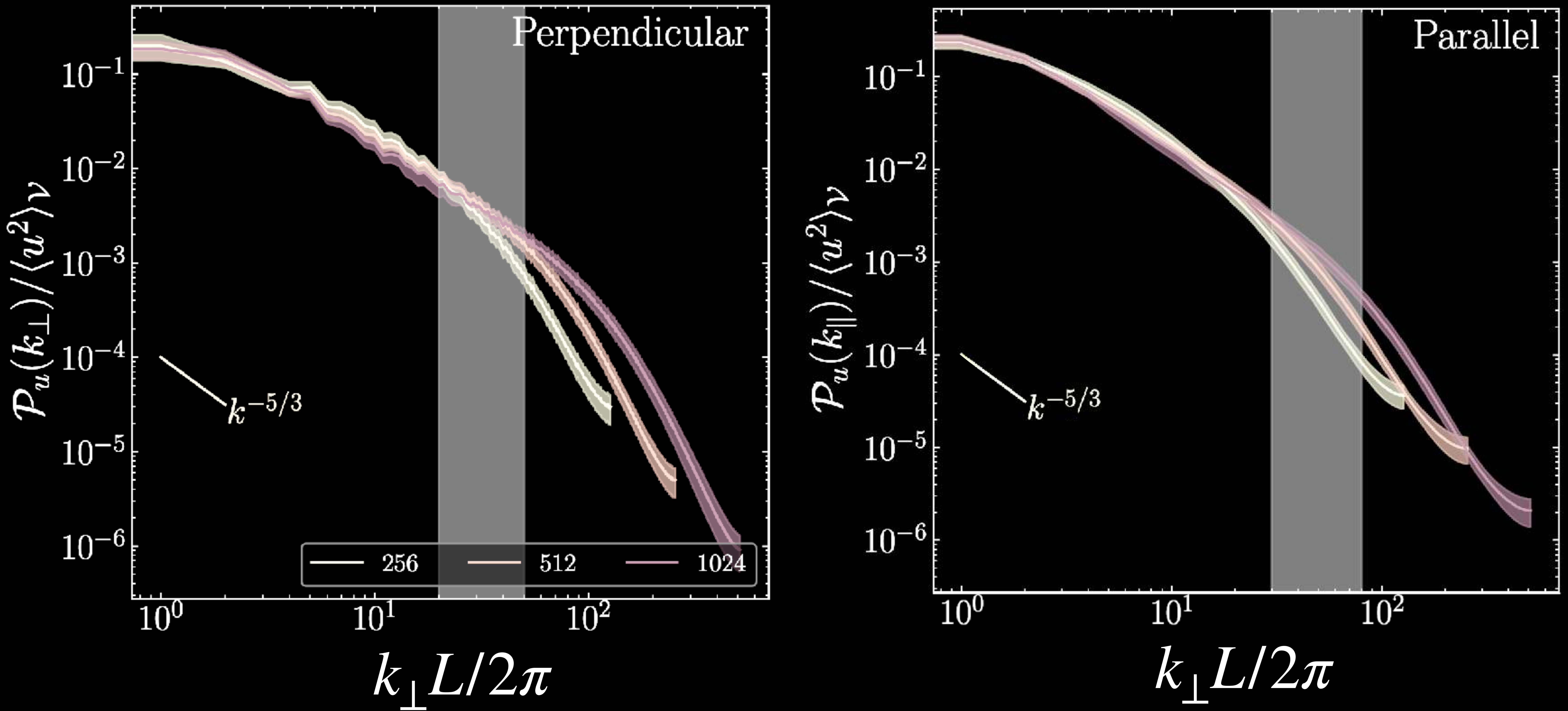
Reaches a close to stationary state… close

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Velocity Spectra

Fairly isotropic up and across $\nabla\phi$

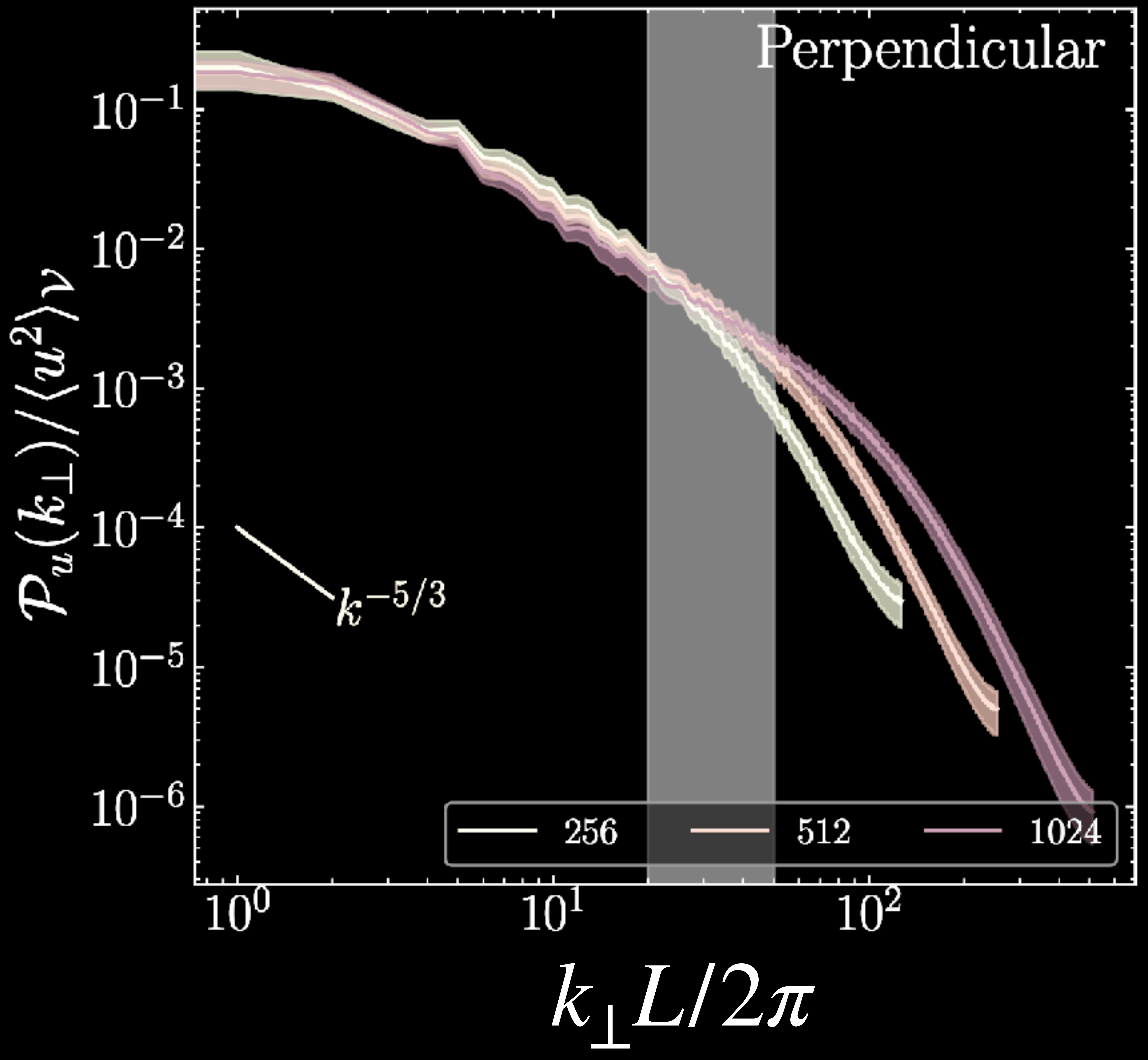
Roughly Kolmogorov in total energy

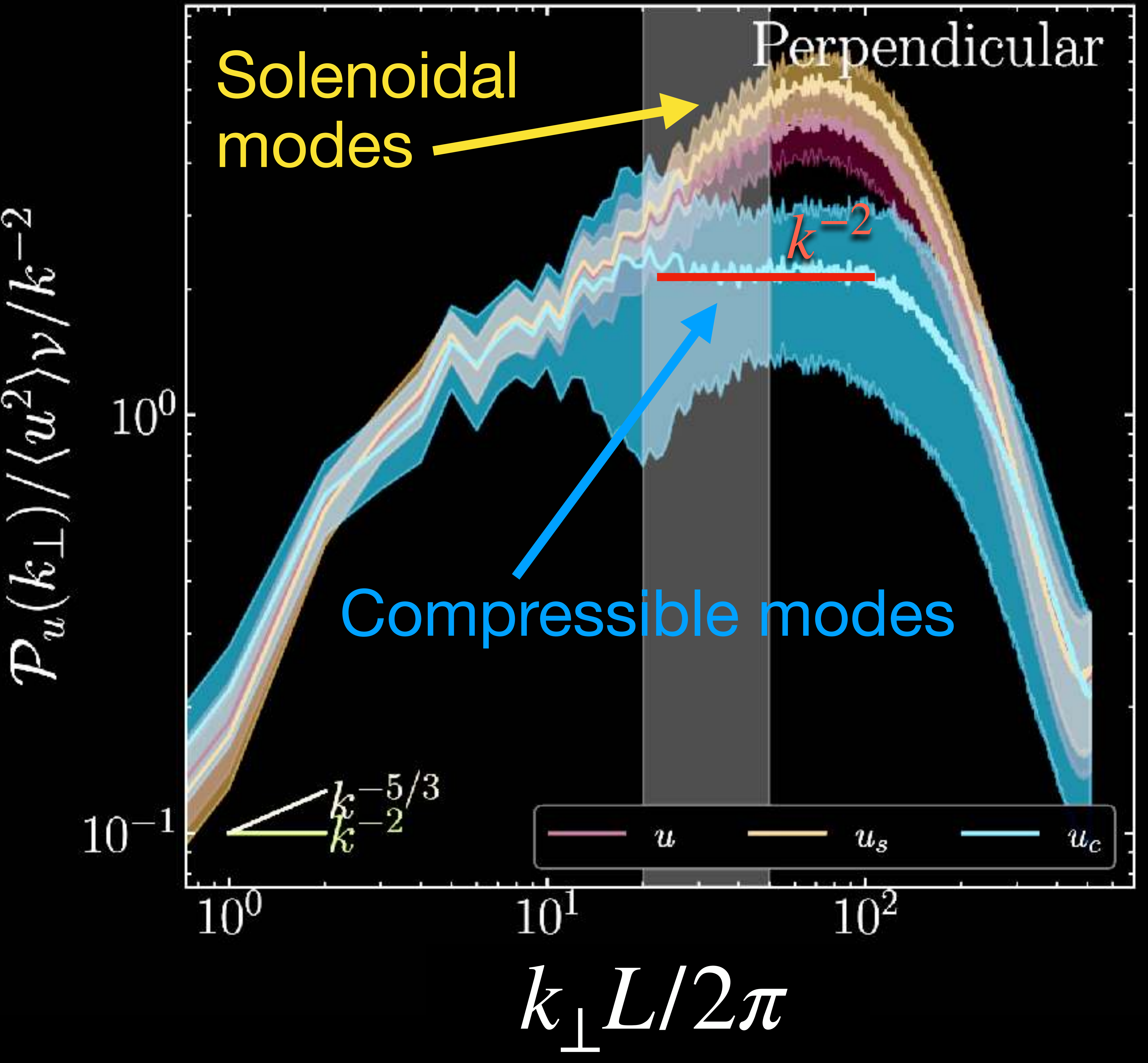


Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Velocity Spectra

Compressible modes and solenoidal modes live very different lives!





Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Compressible modes and solenoidal modes live very different lives!
We should treat them differently!

$$\mathbf{k}' \xrightarrow{\mathbf{k}''} \mathbf{k}'''$$

$$\mathscr{T}^{c}_{cc}(k', k''') = -\int dV\, \mathbf{u}_c''' \otimes \mathbf{u}_c'' : \nabla \otimes \mathbf{u}_c'$$

cascade transfers

$$\mathscr{T}^{s}_{ss}(k', k''') = -\int dV\, \mathbf{u}_s''' \otimes \mathbf{u}_s'' : \nabla \otimes \mathbf{u}_s'$$

$$\vdots$$

$$\mathscr{T}^{s}_{cs}(k', k''') = -\int dV\, \mathbf{u}_s''' \otimes \mathbf{u}_s'' : \nabla \otimes \mathbf{u}_c'$$

interaction transfers

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Compressible modes and solenoidal modes live very different lives!
We should treat them differently!

$$\mathbf{k}' \xrightarrow{\mathbf{k}''} \mathbf{k}'''$$

$$\boxed{\begin{aligned}\mathcal{T}^{c}_{cc}(k', k''') &= -\int dV\, \mathbf{u}_c''' \otimes \mathbf{u}_c'' : \nabla \otimes \mathbf{u}_c' \\ \mathcal{T}^{s}_{ss}(k', k''') &= -\int dV\, \mathbf{u}_s''' \otimes \mathbf{u}_s'' : \nabla \otimes \mathbf{u}_s'\end{aligned}}$$

cascade transfers

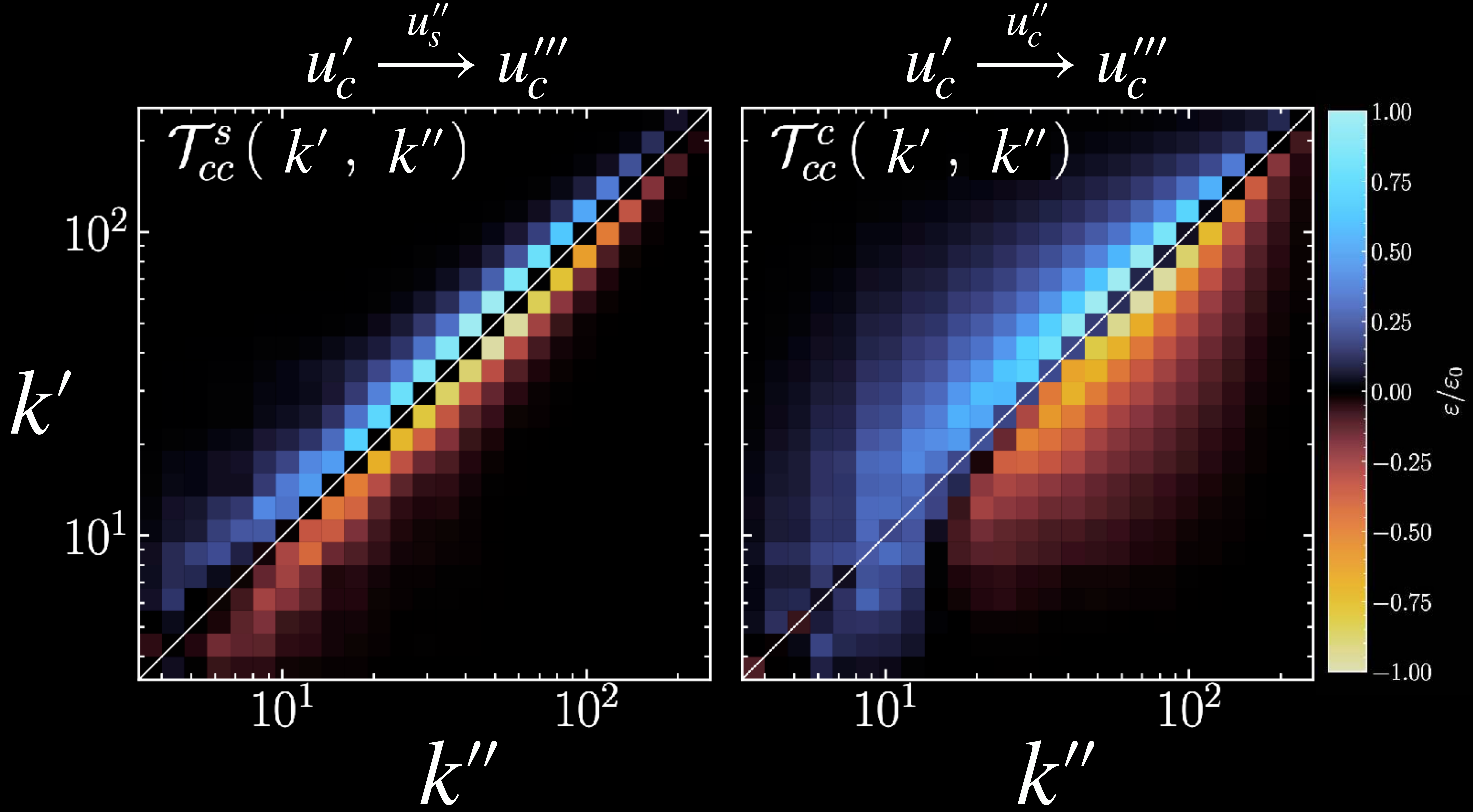
$$\vdots$$

$$\mathcal{T}^{s}_{cs}(k', k''') = -\int dV\, \mathbf{u}_s''' \otimes \mathbf{u}_s'' : \nabla \otimes \mathbf{u}_c'$$

interaction transfers

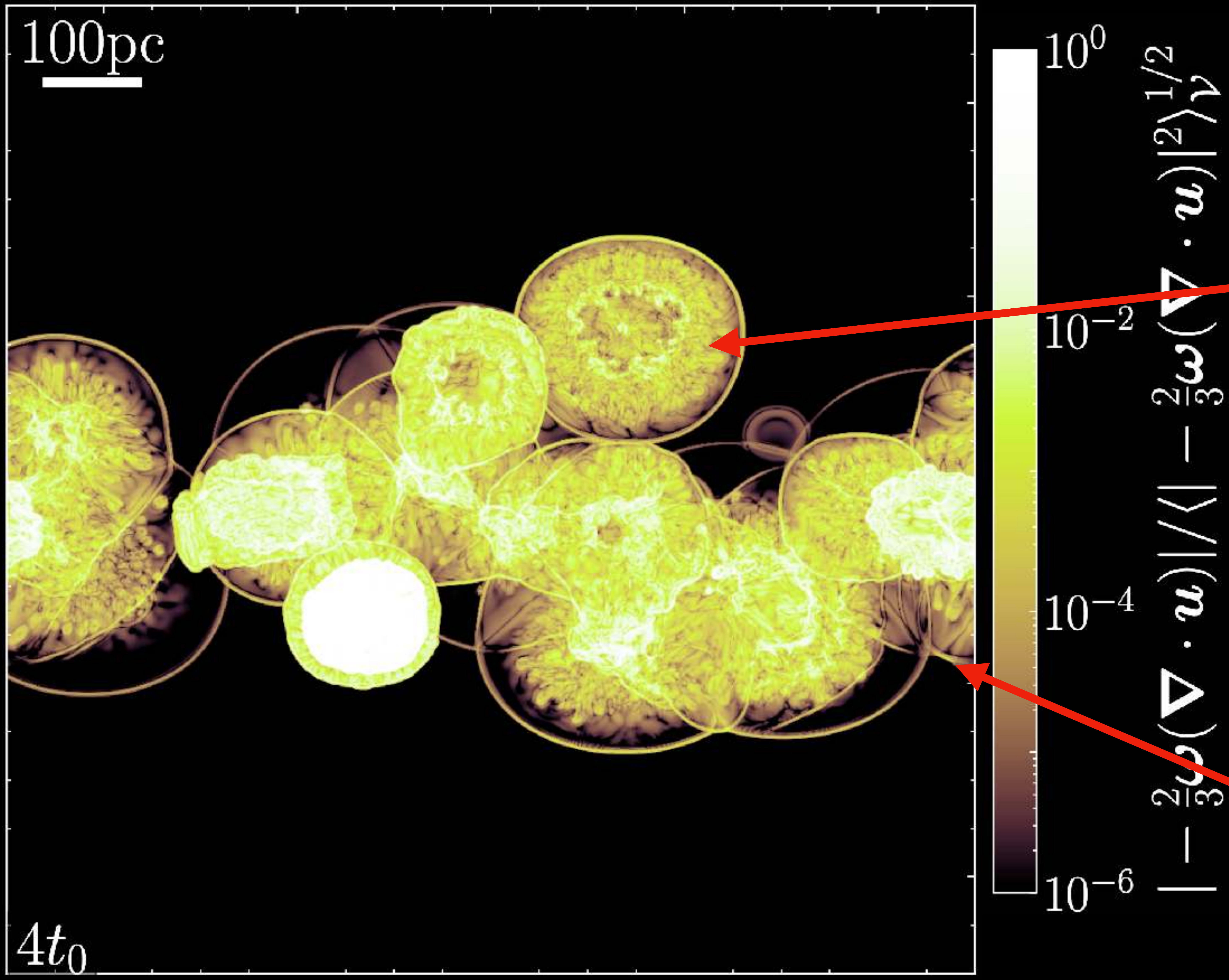
Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Cascade transfers: compressible modes



Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Cascade transfers: compressible modes



$$u_c' \xrightarrow{u_s''} u_c'''$$

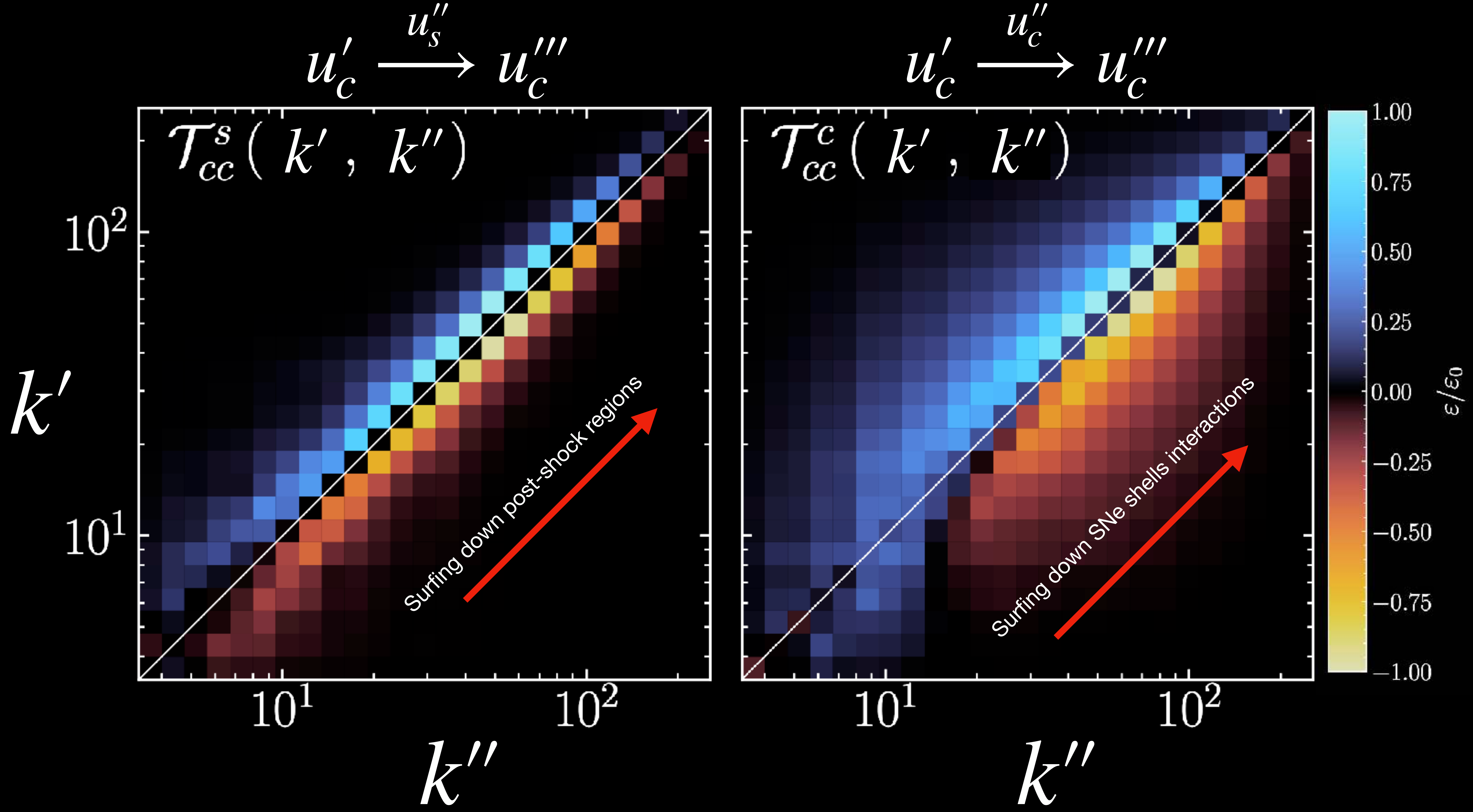
Compressible modes surfing solenoidal modes down post-shock regions

$$u_c' \xrightarrow{u_c''} u_c'''$$

Compressible modes forming other compressible modes through SNe shells

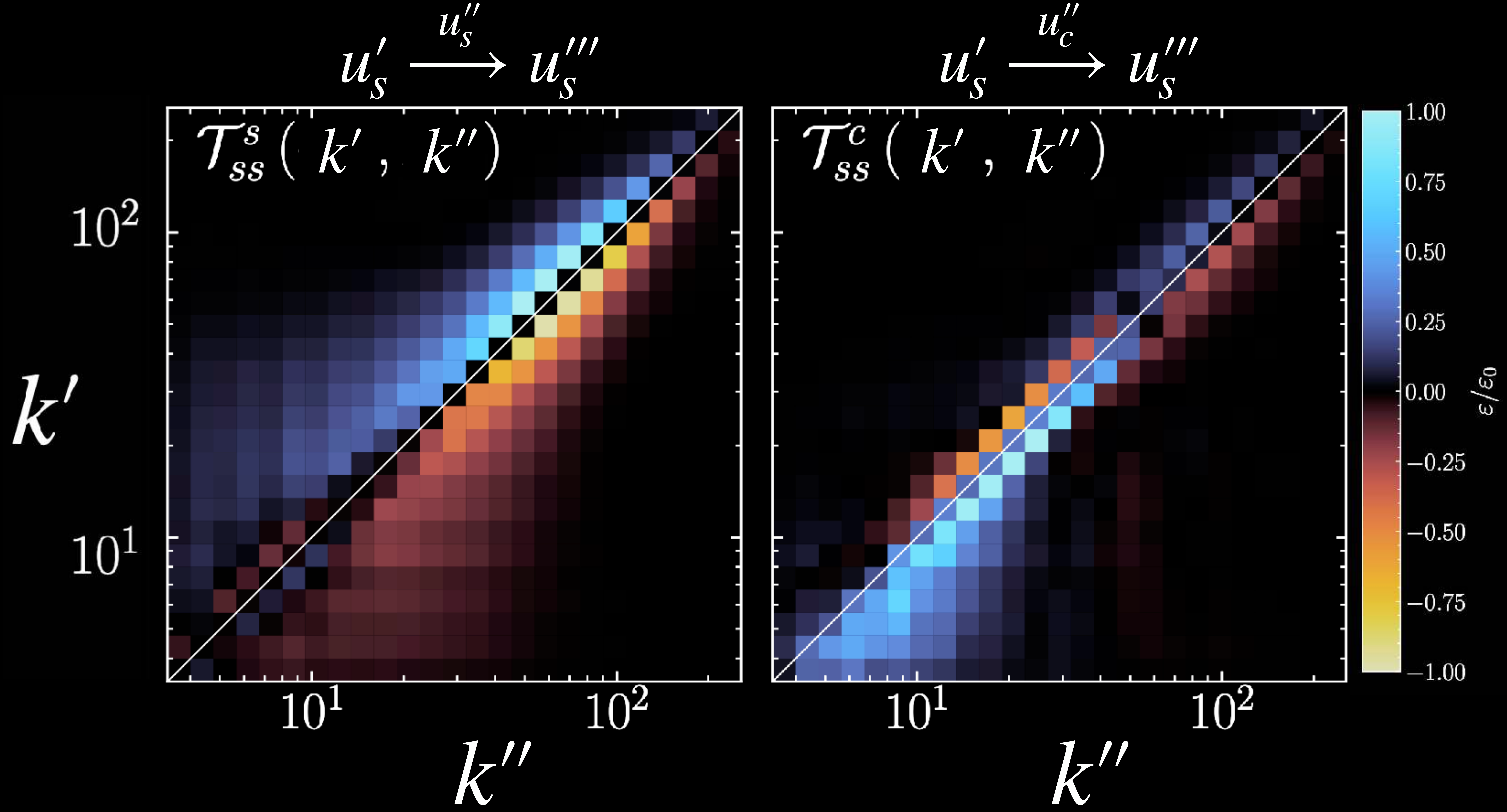
Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Cascade transfers: compressible modes



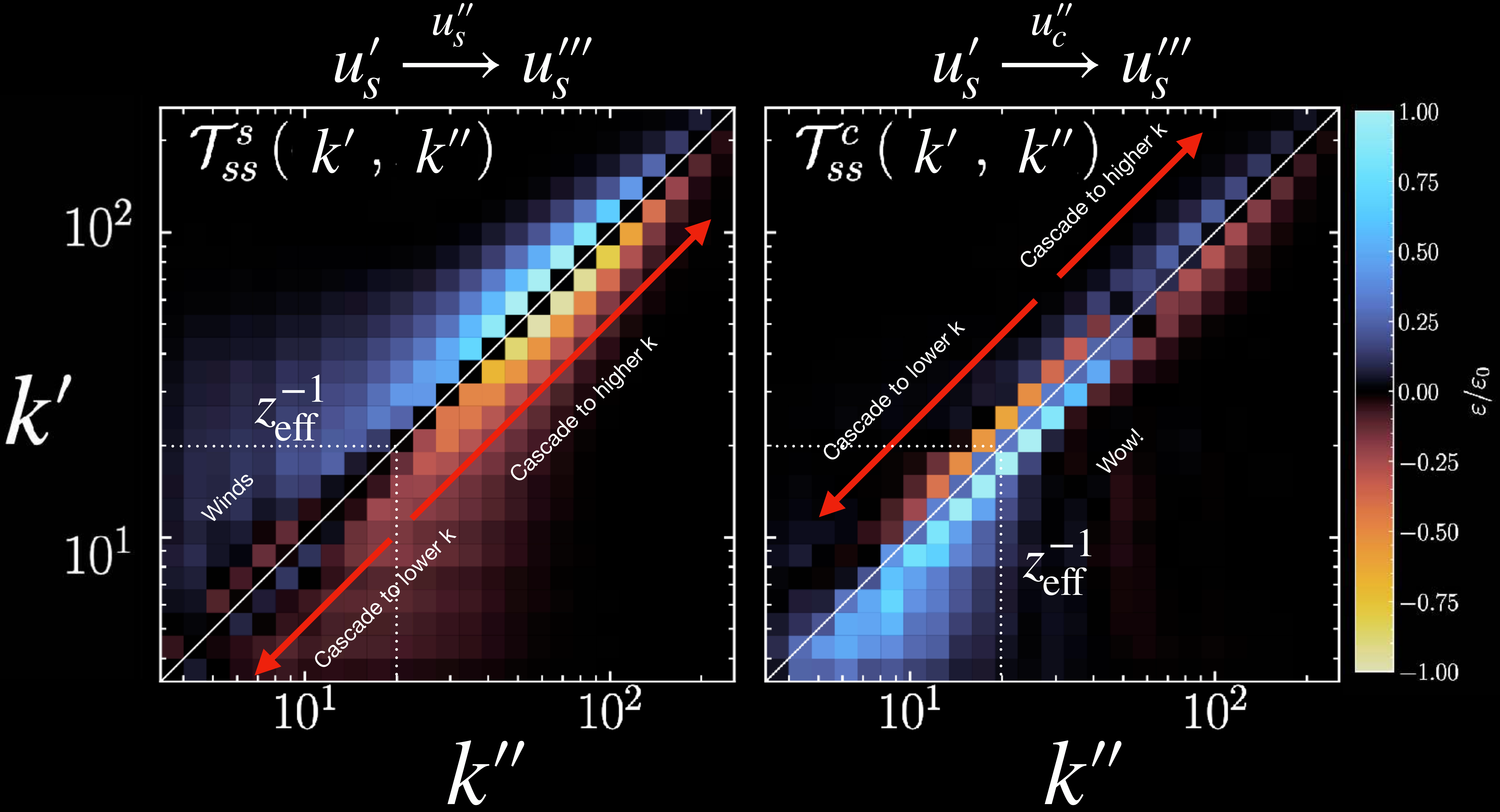
Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Cascade transfers: solenoidal modes



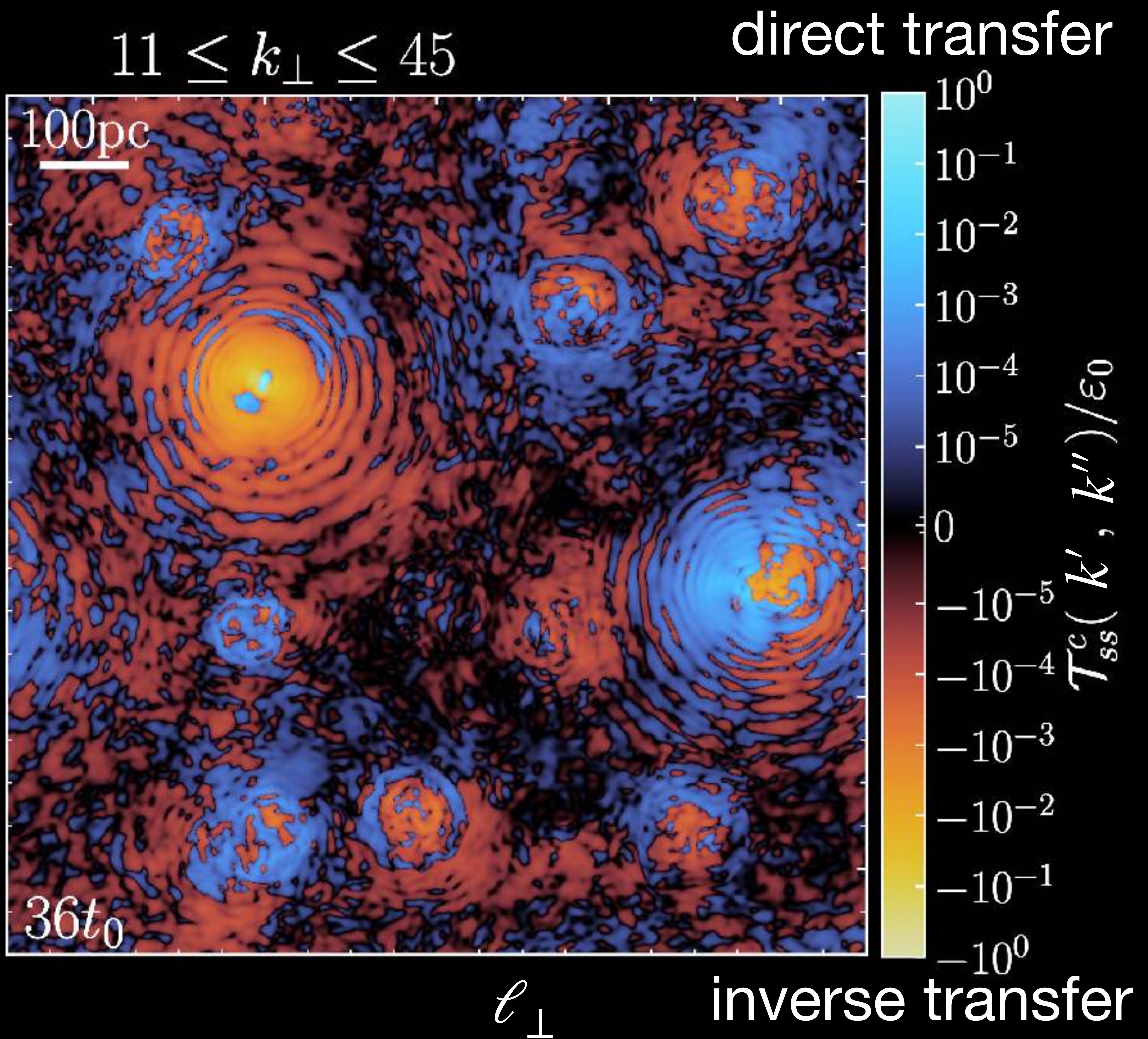
Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

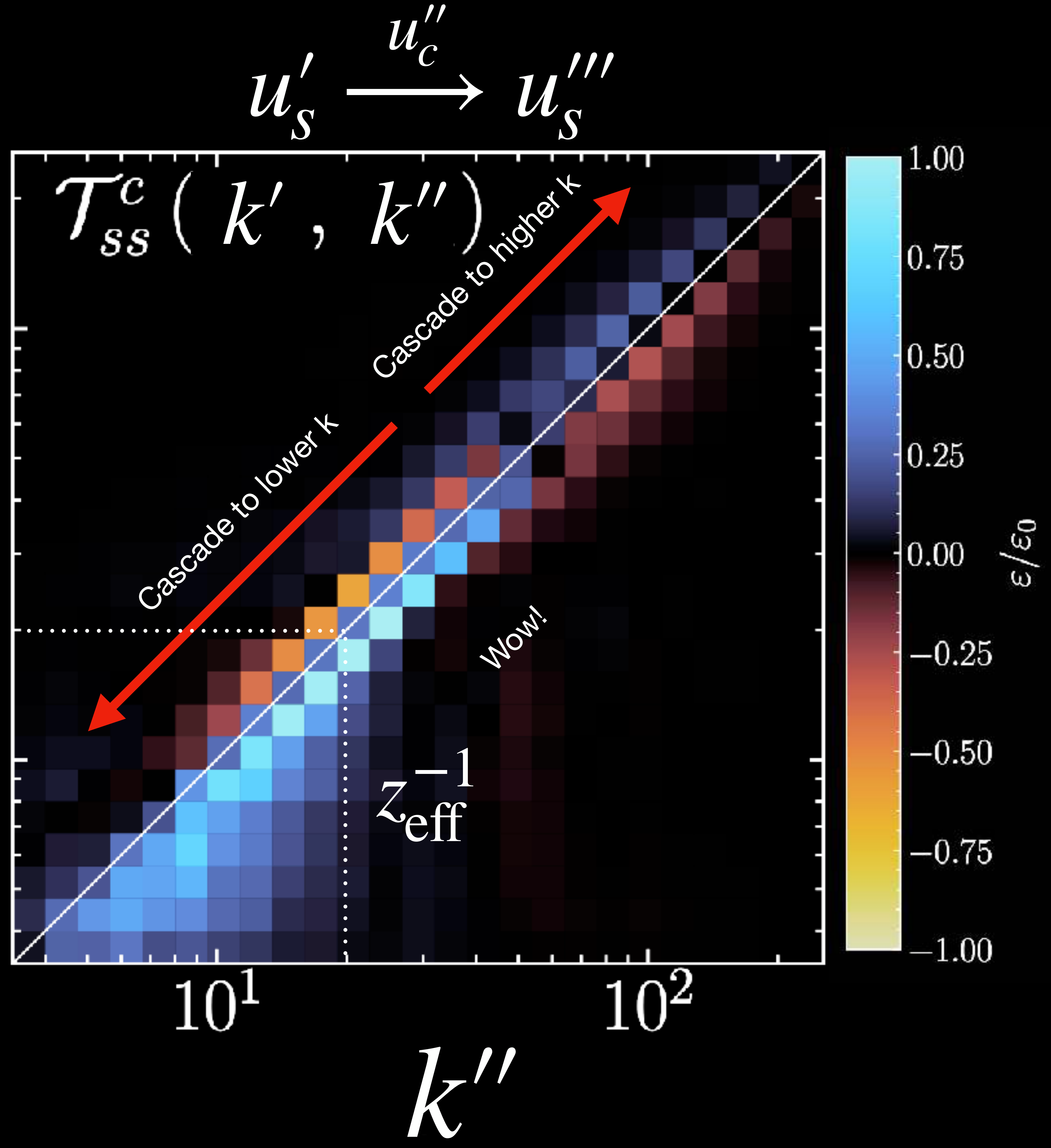
Cascade transfers: solenoidal modes



Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

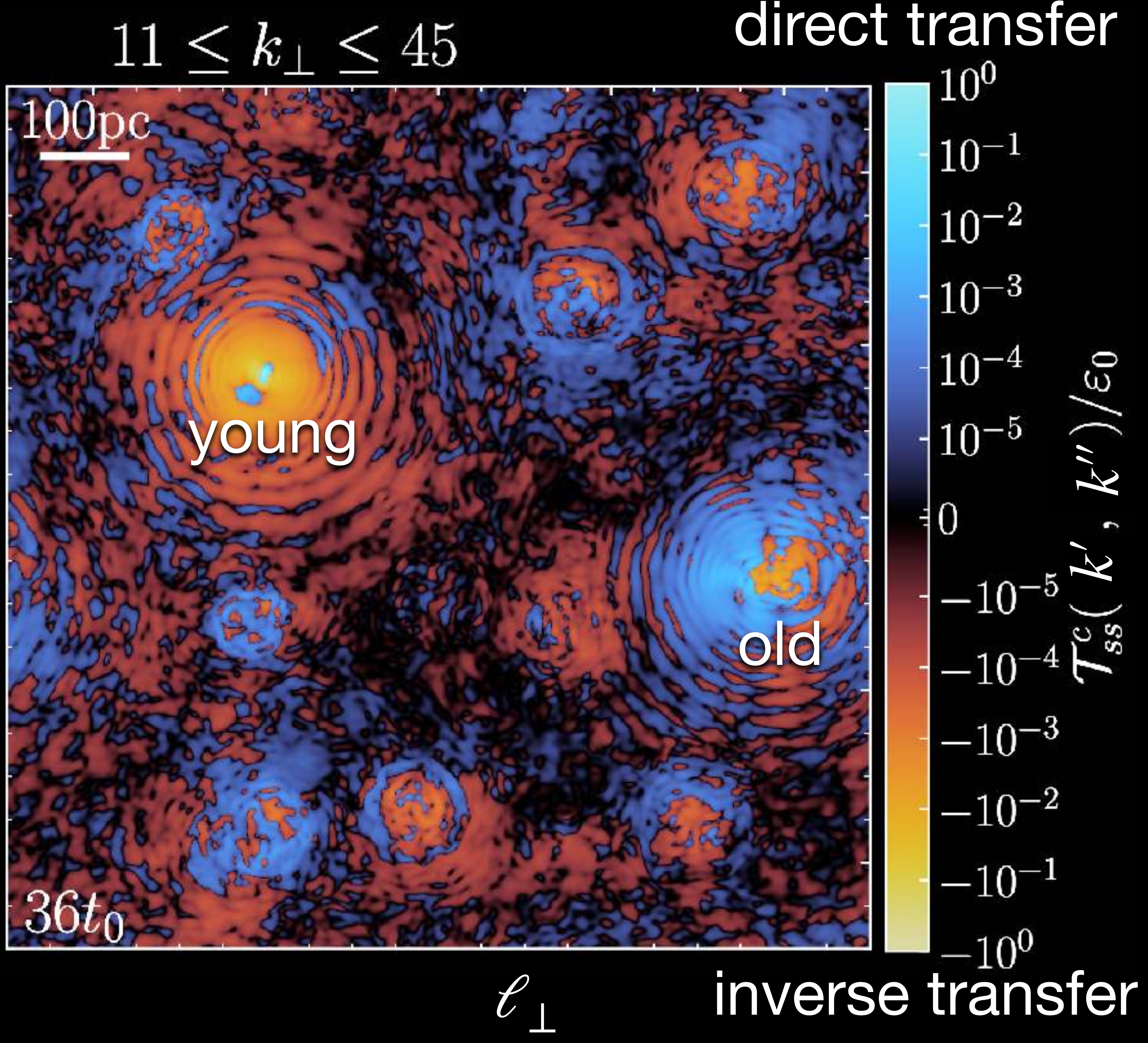
Cascade transfers: solenoidal modes

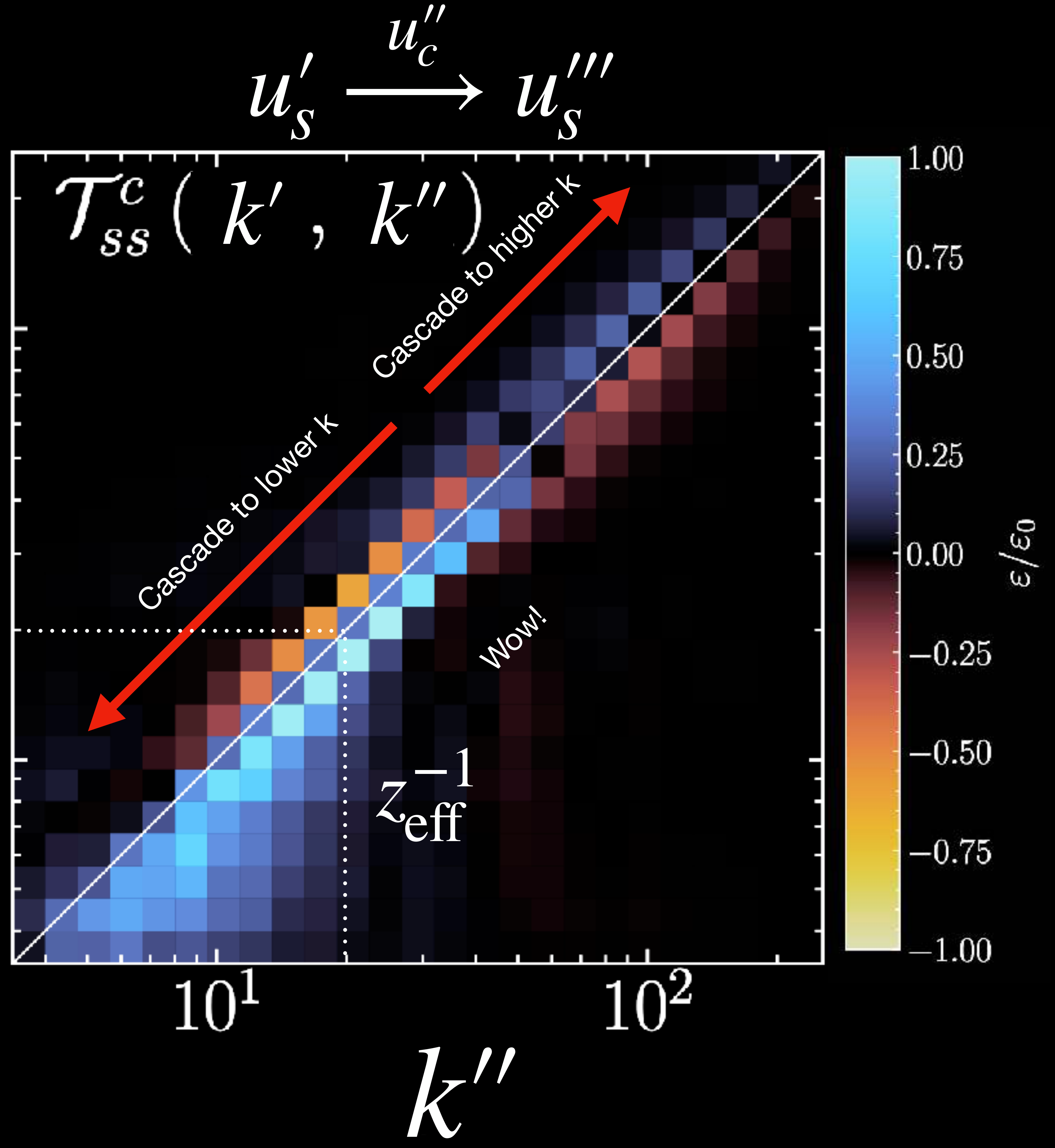




Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

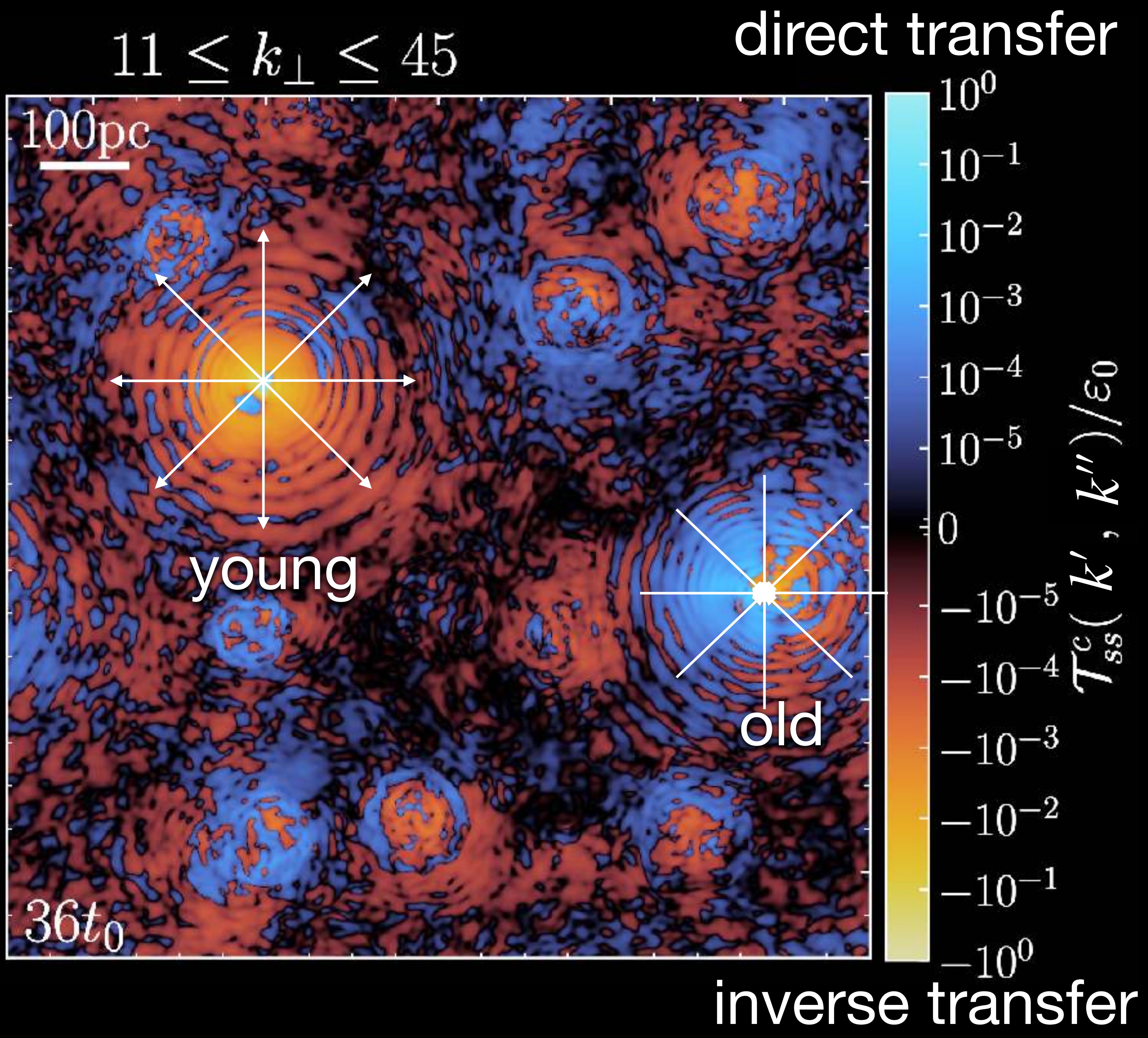
Cascade transfers: solenoidal modes — surfing on supernova shells



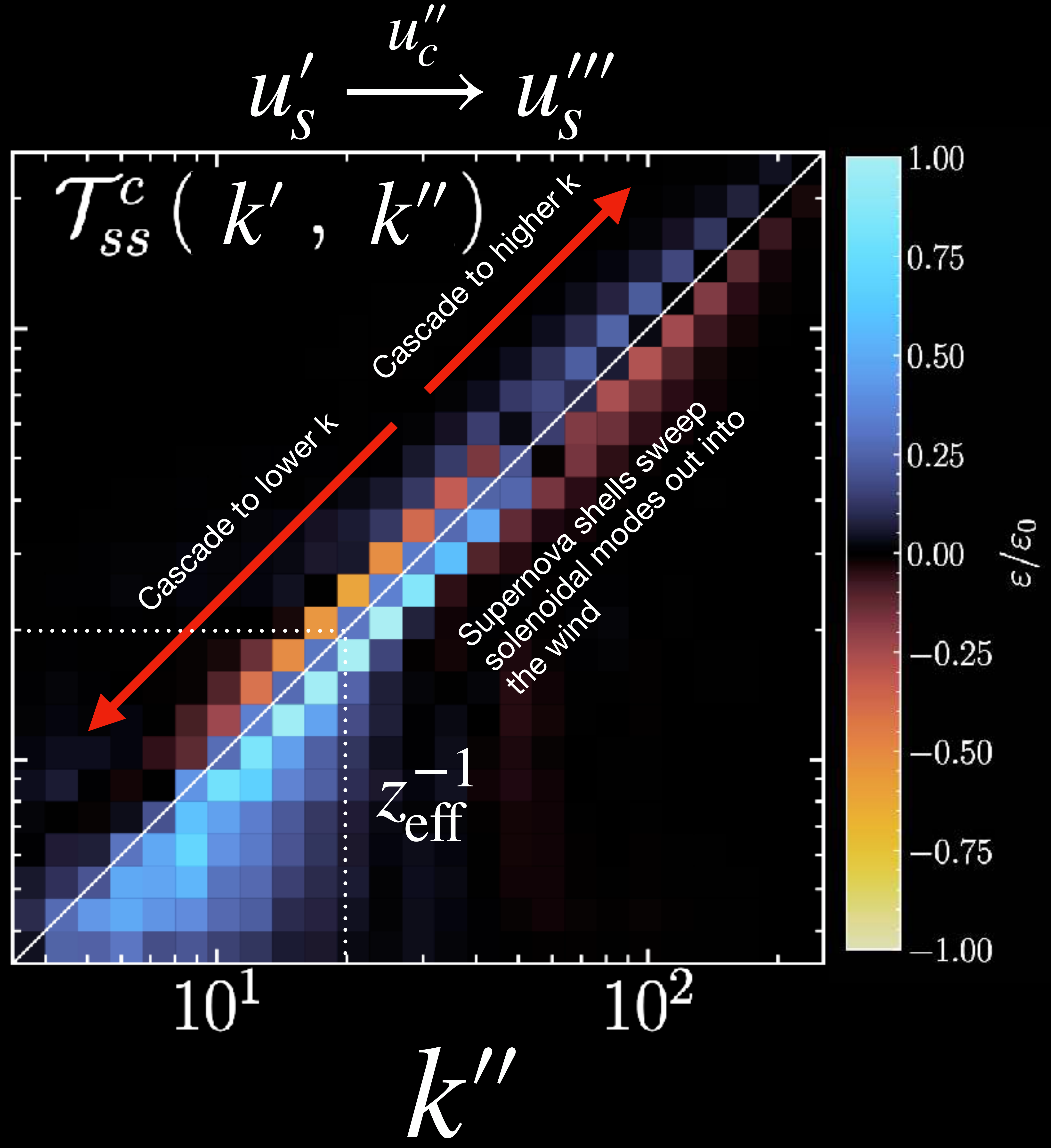


Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Cascade transfers: solenoidal modes — surfing on supernova shells



slice in the plane of the disk



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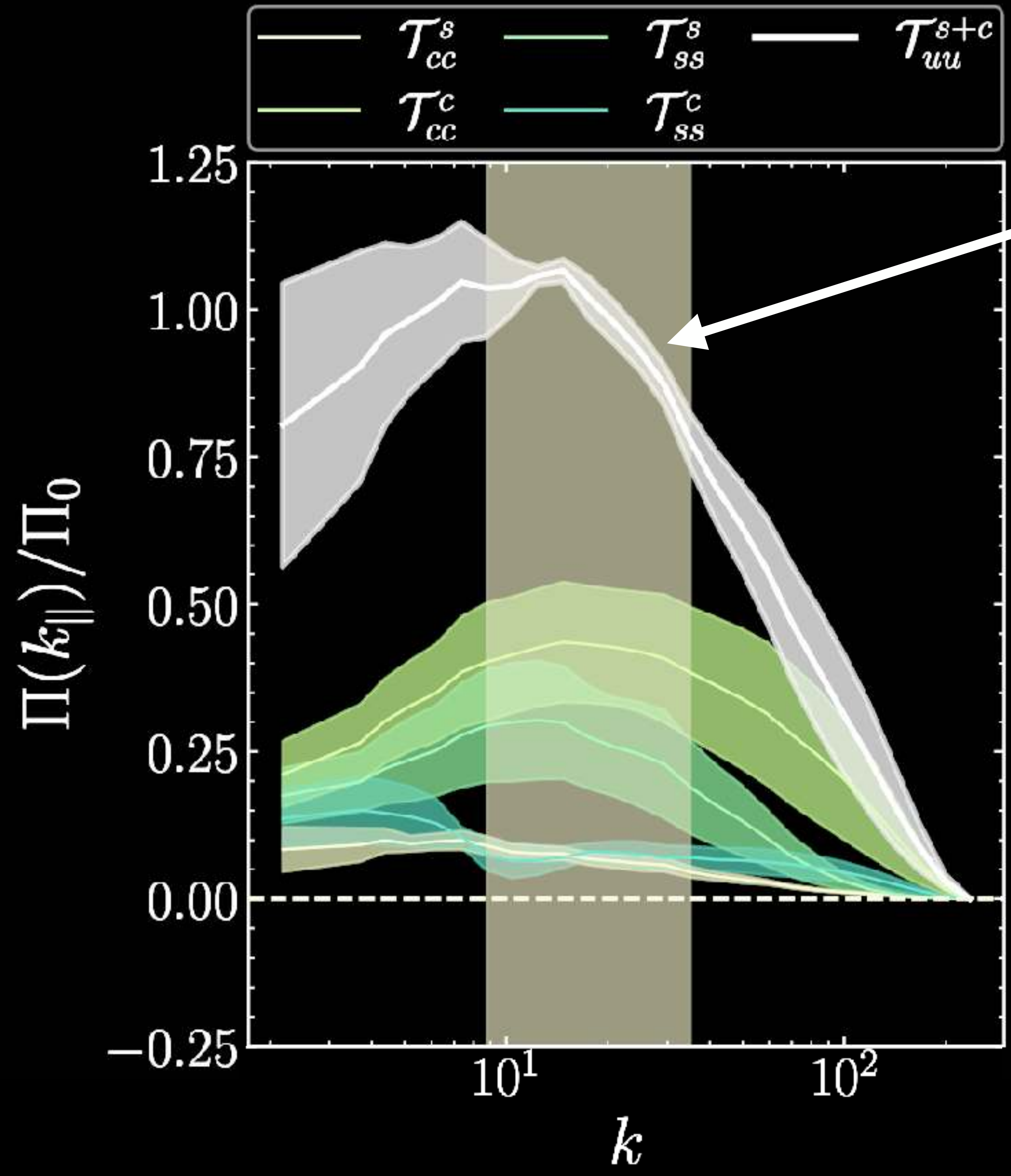
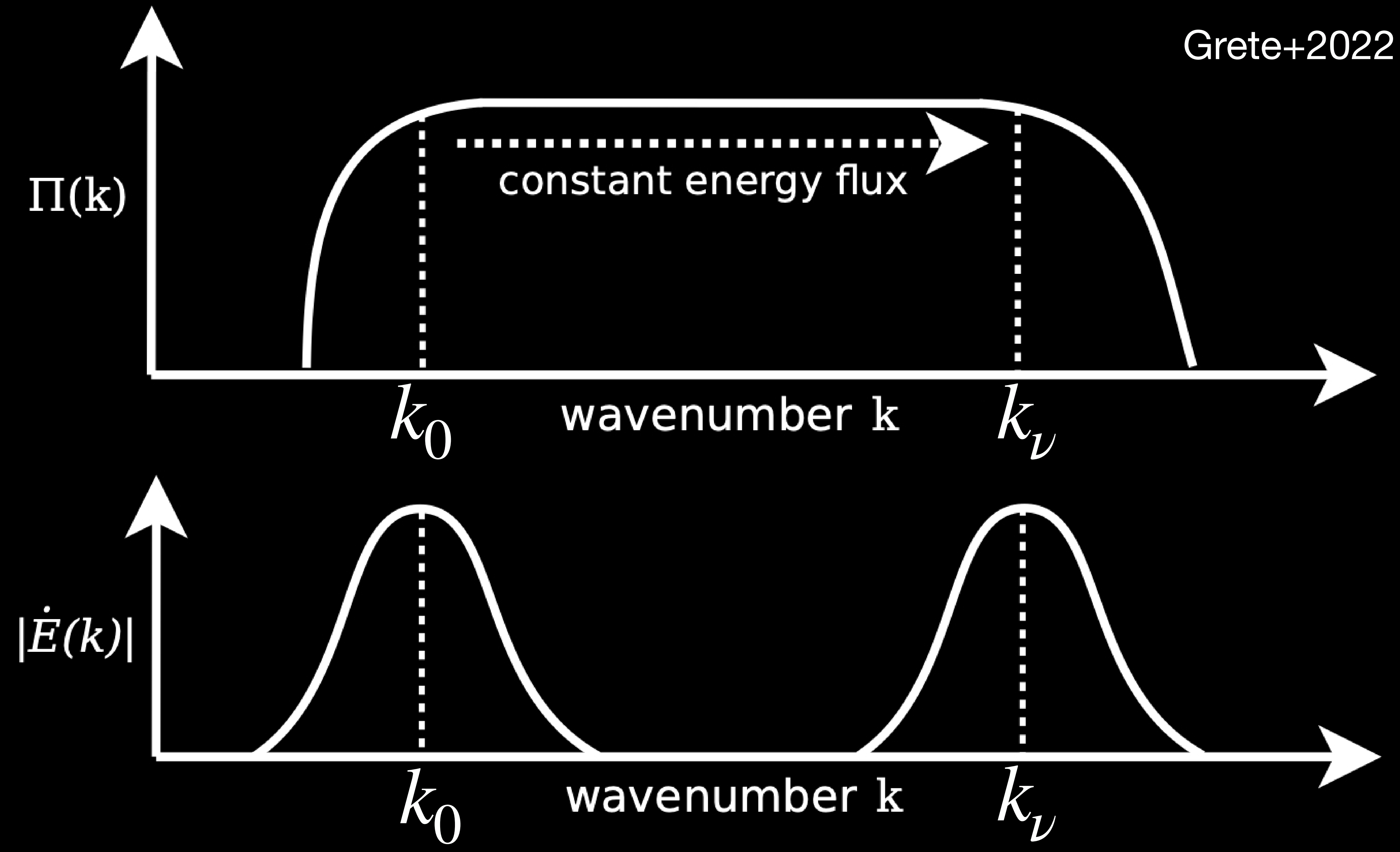


Not very constant at all!



Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Next steps with observers

james.beattie@princeton. @astro_magnetism

- Mostly the density is accessed via observation… any hope to measure energy flux density? Yes!

- With sufficiently resolved column density (Francois, M-A ;)), one should be able to compute:

compression mediated cascade

$$\mathscr{T}_{\rho\rho}(k', k''' \,|\, k'') = -\int dV\, \rho'''\mathbf{u}'' \cdot \nabla \otimes \rho' - \int dV\, \rho'''\rho'\nabla \cdot \mathbf{u}''$$

advection mediated cascade

assuming isotropy. Never been done before…

Carina Nebula, JWST

Next steps with observers

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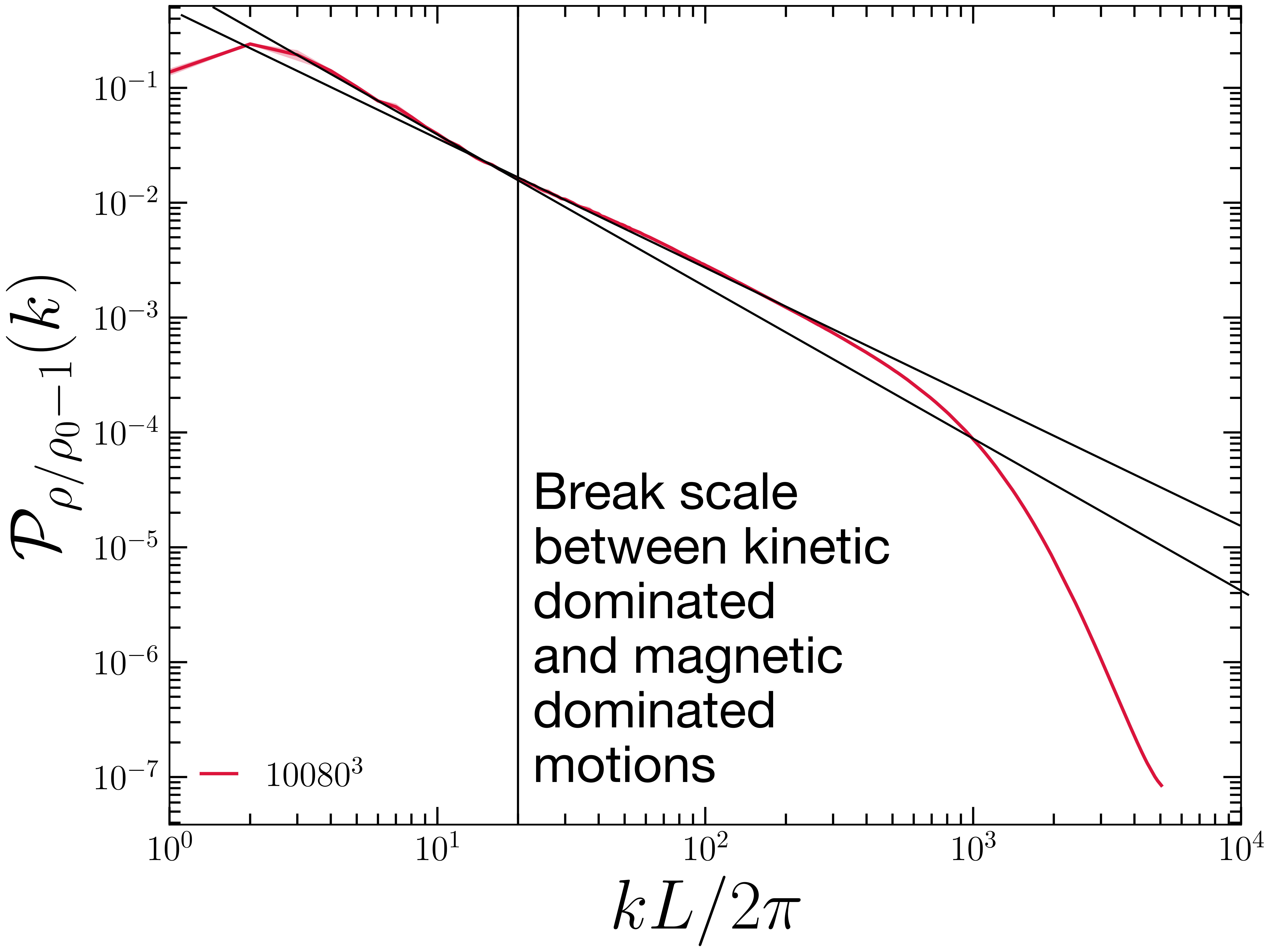
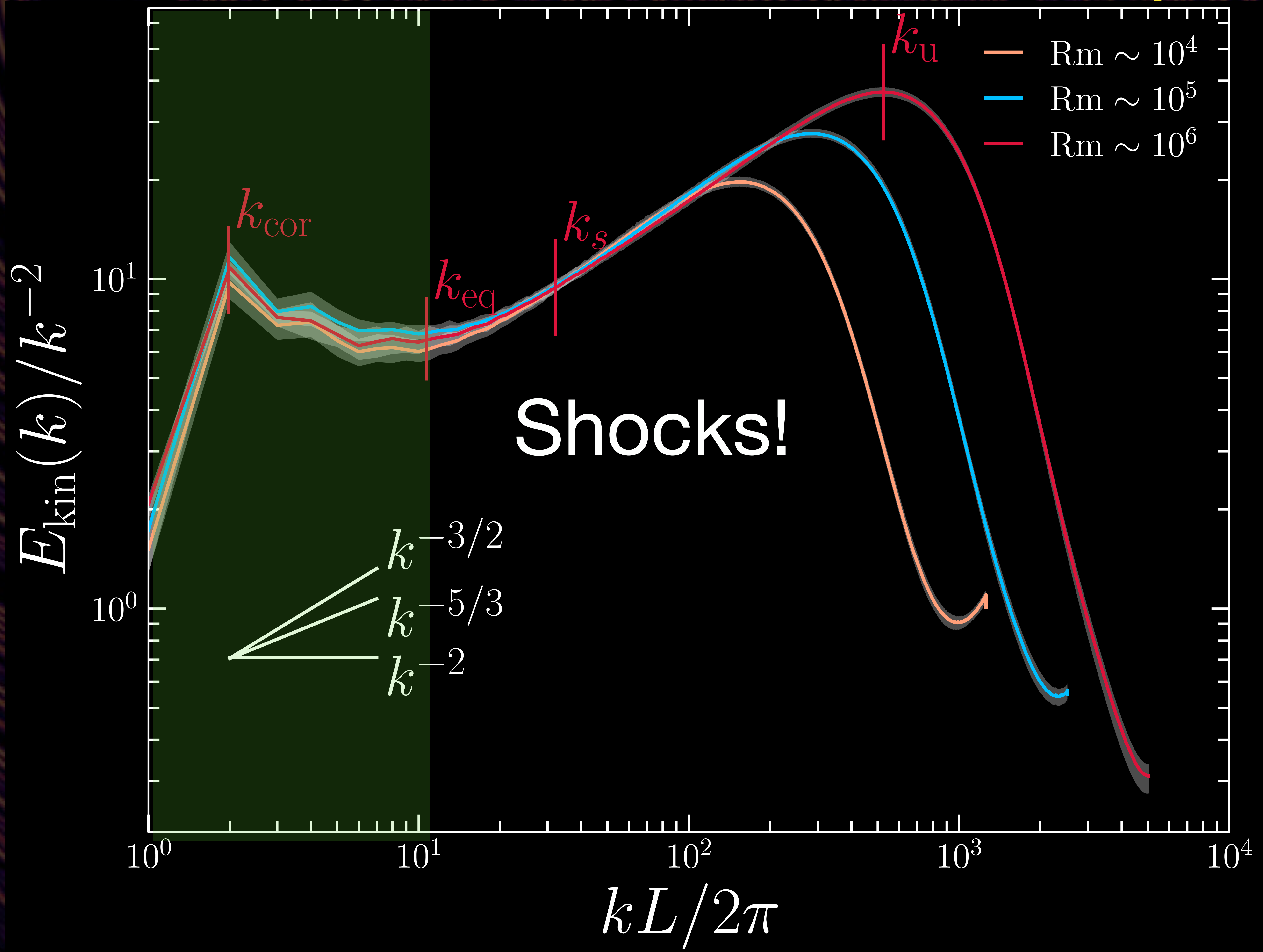
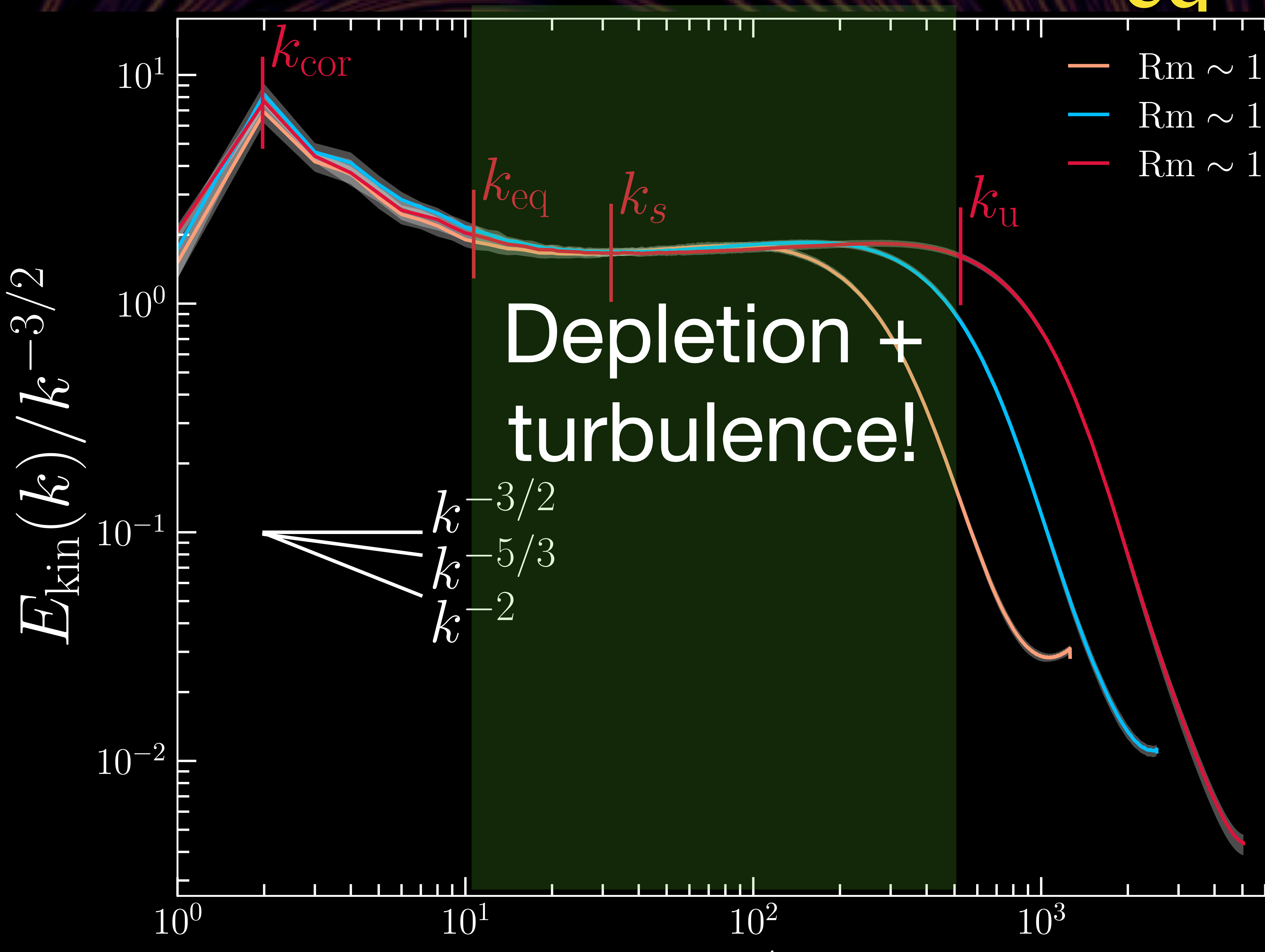


Carina Nebula, JWST

Final thoughts (from Tuesday)...

$\mathcal{E}(k) \sim k^{-2},\ k \leq k_{\mathrm{eq}}$ $\qquad$ $\mathcal{E}(k) \sim k^{-3/2},\ k > k_{\mathrm{eq}}$

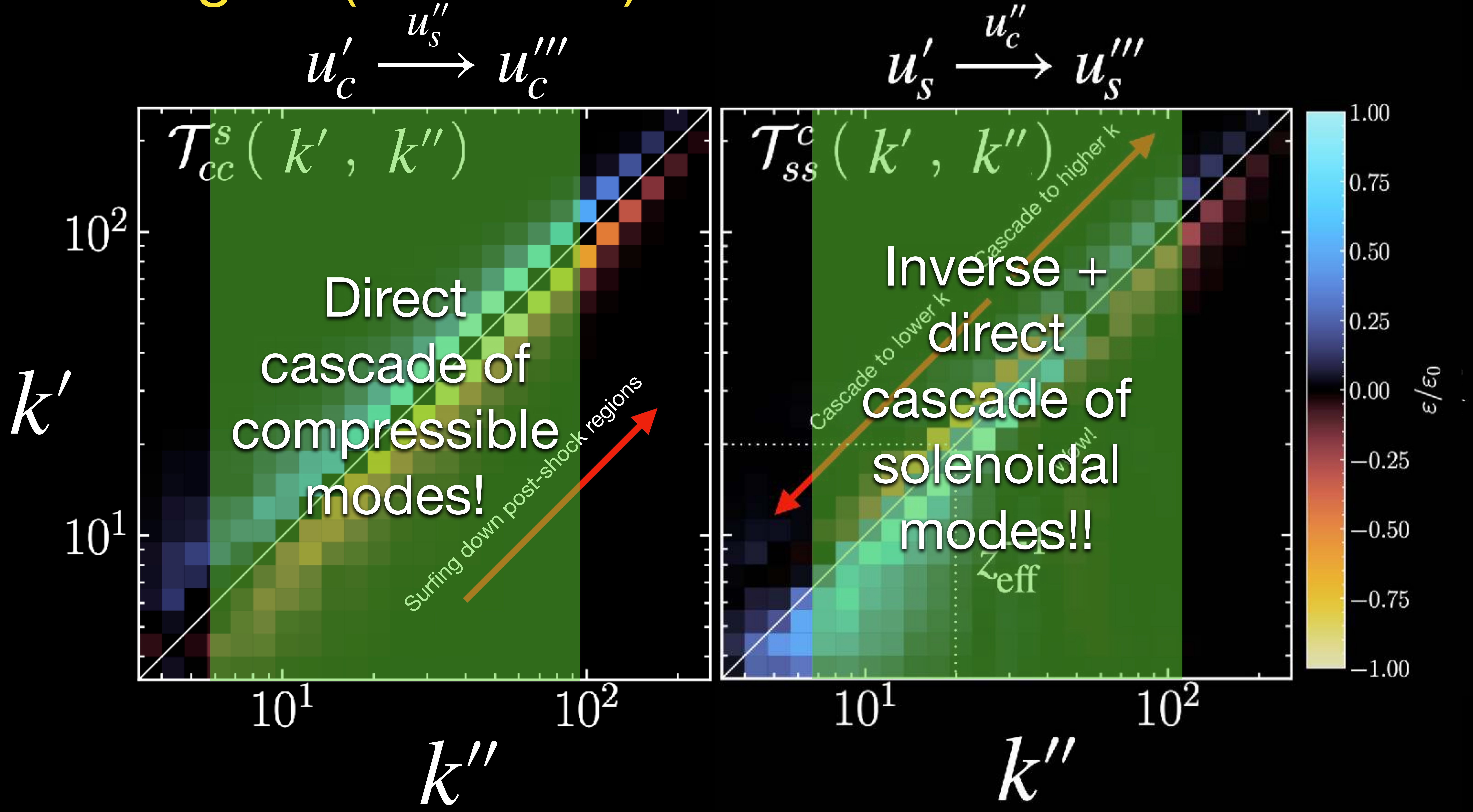




Mocz & Burkhart (2019)

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Final thoughts (from now)...



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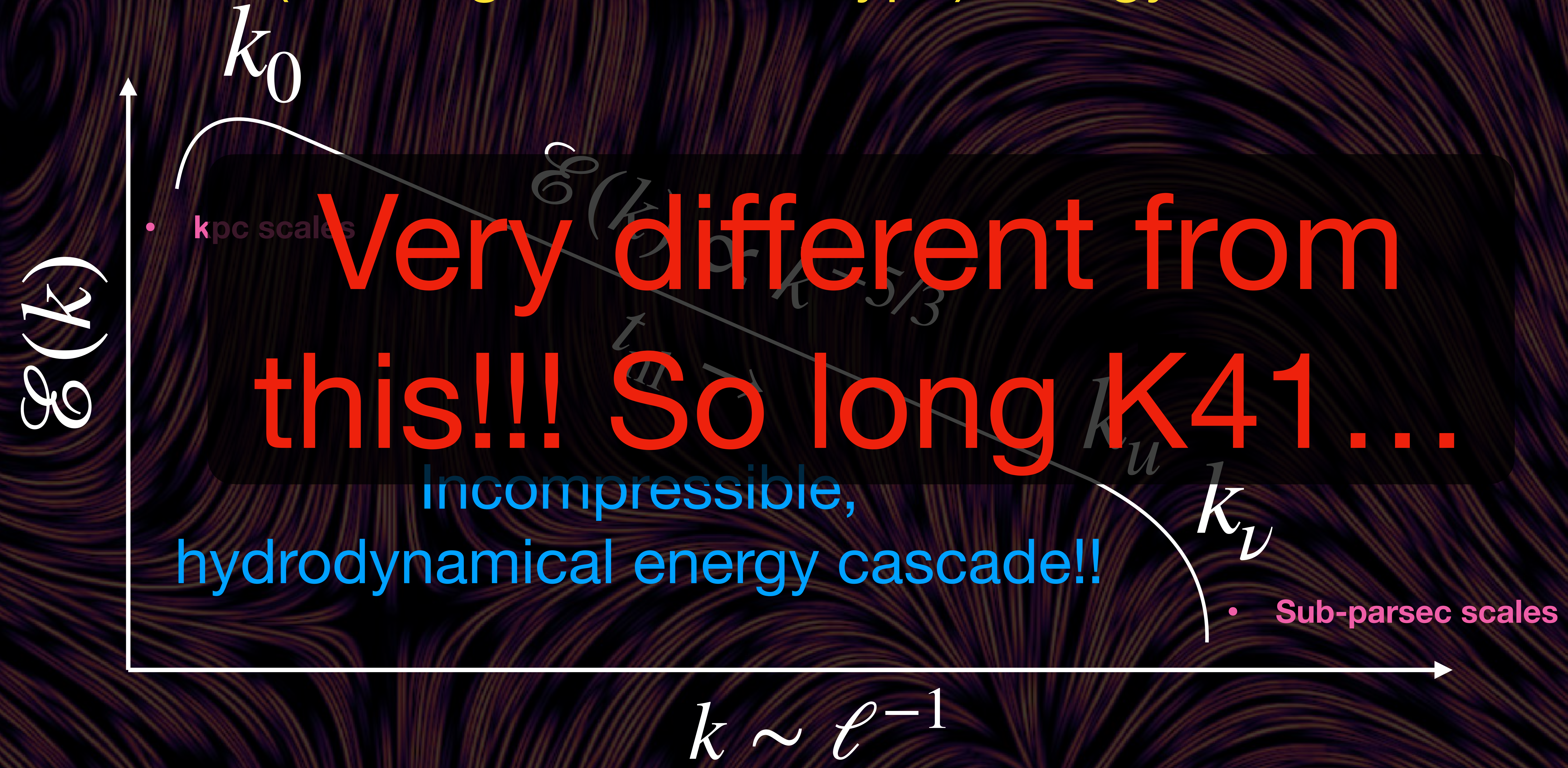
The (Kolmogorov, 1941 -type) energy cascade



Very different from this!!! So long K41…

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Thanks, questions?

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