From fast growth to saturation of the intracluster medium dynamo

Galaxy Clusters & Radio Relics II | Center for Astrophysics | Harvard & Smithsonian

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Objectives

1. Discuss and confirm some theoretical models in the fast growth phases of the small scale dynamo in the context of ICM.

2. Show some new ideas about dynamo saturation that do not rely on resistivity or effective resistivity.

James Beattie. Magnetic reconnection in a turbulent fluid. Iterative LIC.

Basic plasma properties of ICM

Based on St-Onge+(2020) & Kunz, Jones & Zhuravleva (2022), +

- Weakly collisional, $\nu_{\rm i} t_0 \sim 10^2$ $(\Delta p \neq 0)$ — Schekochihin+(2005); Kulsrud & Zweibel (2008)
- Hot plasma, $T \sim 10^8$ K, $u_{\rm therm,i} \sim 10^3$ km s^{-1}, $t_{\rm cluster} \sim$ Gyr — Subramanian+(2008)
- Turbulence stirred by thermal instabilities, AGN winds, shock-vorticity interactions, $\ell_0 \sim 100$ kpc, $u_0 \sim 200$ km s^{-1}, $t_0 \sim 10^2$ Myr — Hitomi Collaboration (2016); Zhuravleva+(2018); Simionescu+(2019)
- Subsonic $u_0/u_{\rm therm,i} = \mathcal{M} \sim 0.1$ (quasi incompressible)
- $\mathrm{Re}_{||} \sim |\nabla \cdot (u_0 \otimes u_0)|/|\nabla \cdot \Pi_{||}| \sim 100$, $k_\nu^{-1} \sim 3$ kpc, $t_\nu \sim$ 10 Myr — St-Onge+(2020); John ZuHone (slack)
- $B \sim \mathcal{O}(\mu\mathrm{G})$, $\beta \sim 100$, $\ell_{\rm cor} \sim 10$ kpc — Carilli & Taylor (2002); Govoni+(2017)
- Very conductive $\mathrm{Pm} \sim 10^{29} \left(\frac{T}{10^8\ \mathrm{K}}\right)^4 \left(\frac{n}{10^{-3}\ \mathrm{cm}^{-3}}\right)^{-1}$, $k_\eta^{-1} \sim 10^4$ km — Schekochihin & Cowley (2006)

Botteon+(2022) the merging cluster, Abell 2255

Inevitability of the turbulent dynamo

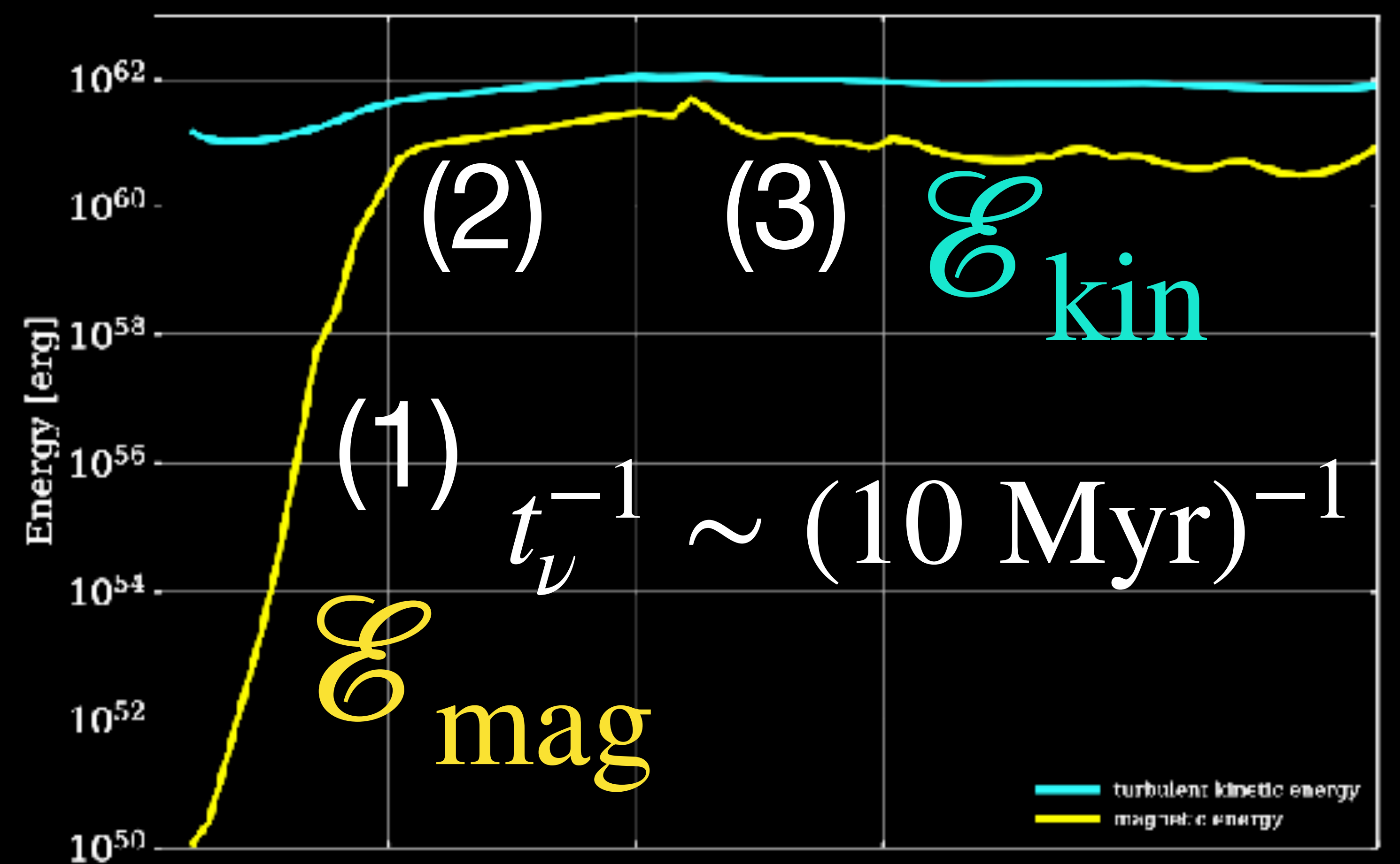
$B \sim \mathcal{O}(\mu\mathrm{G})$!

Very simple ingredients:

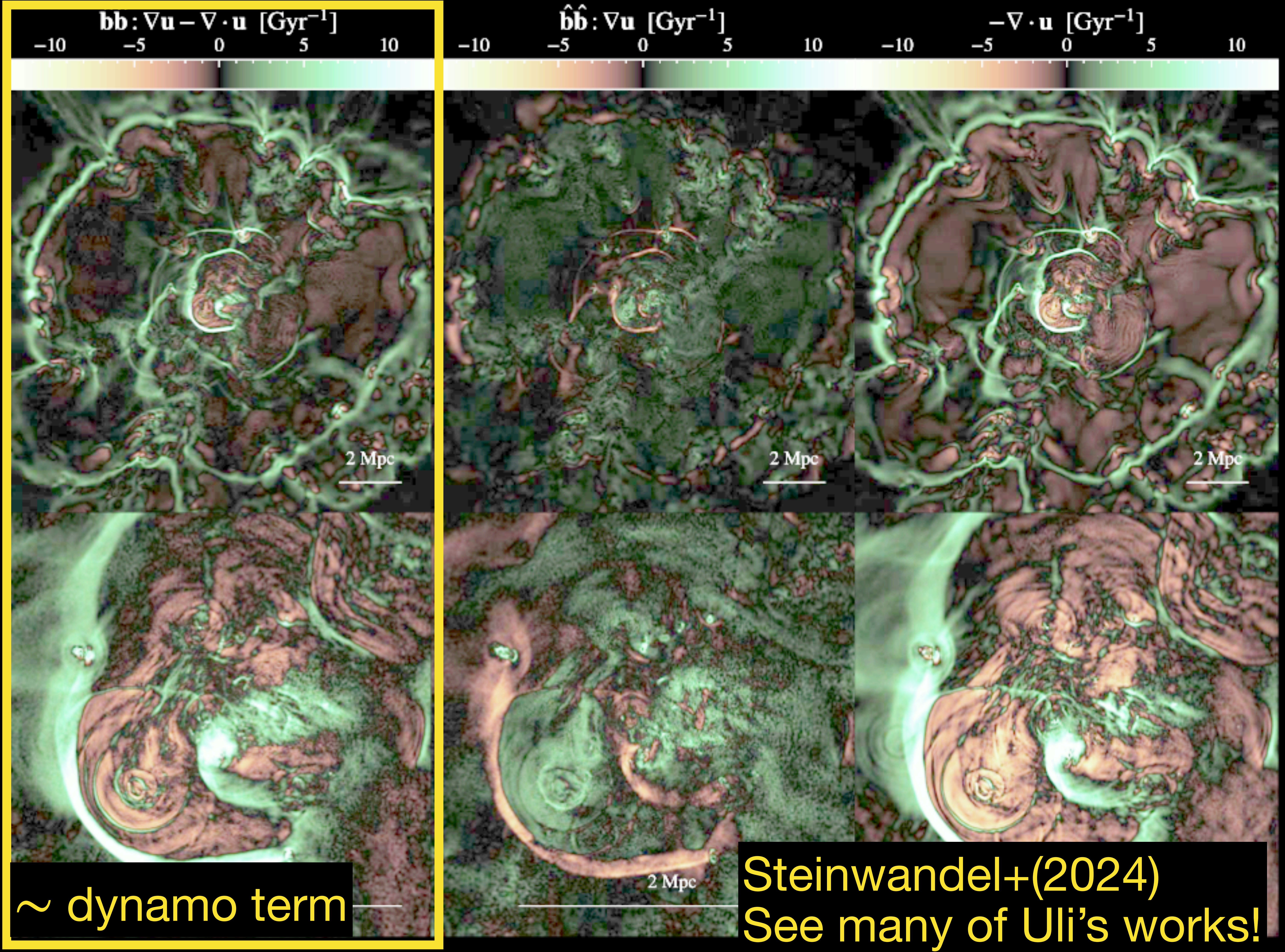
seed field + stochastic $\nabla \mathbf{u}$

any global geometry,
extremely universal!

(1) kinematic
(2) nonlinear
(3) saturation



time ⟶



Steinwandel+(2024)
See many of Uli's works!

Inevitability of the turbulent dynamo

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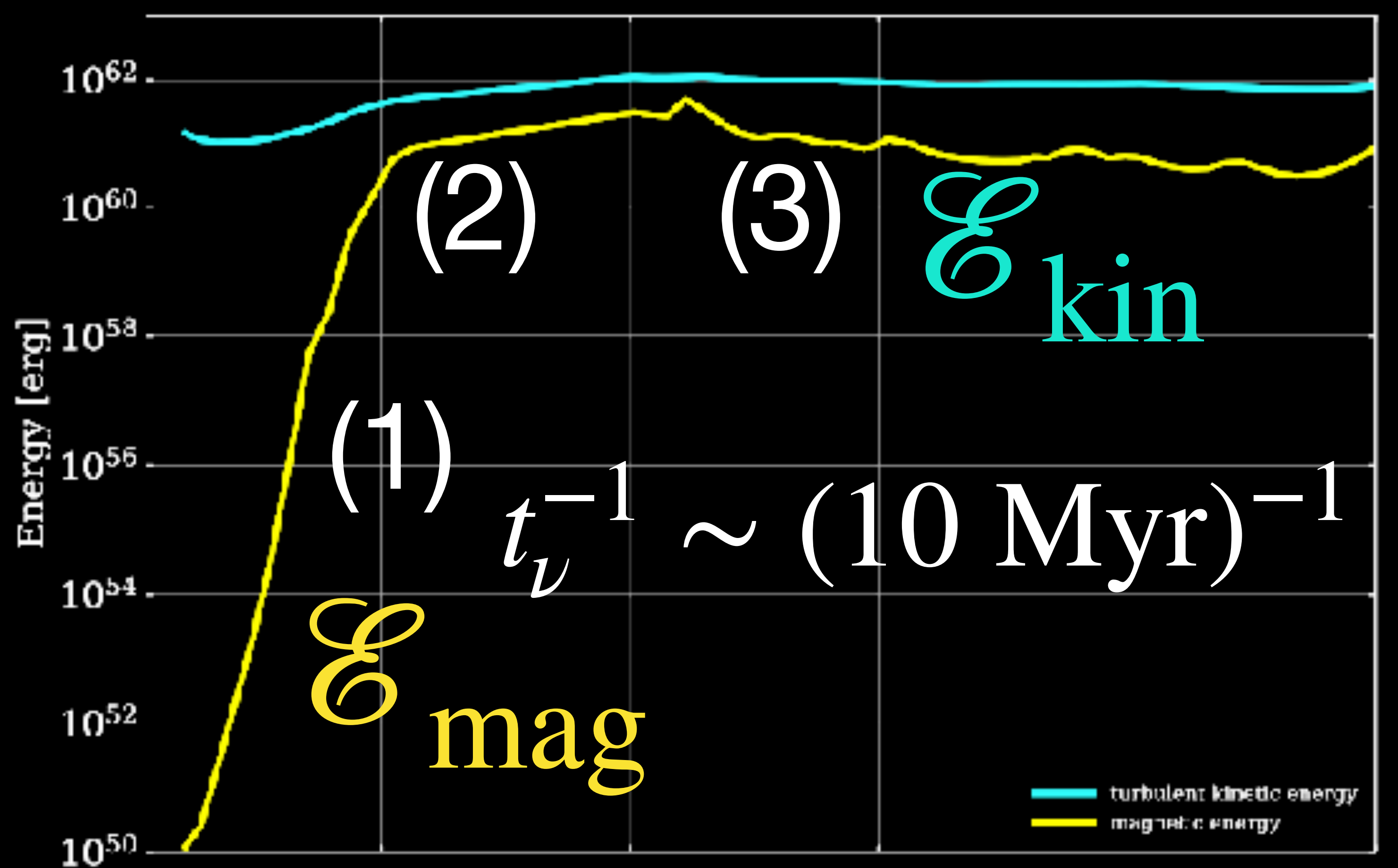

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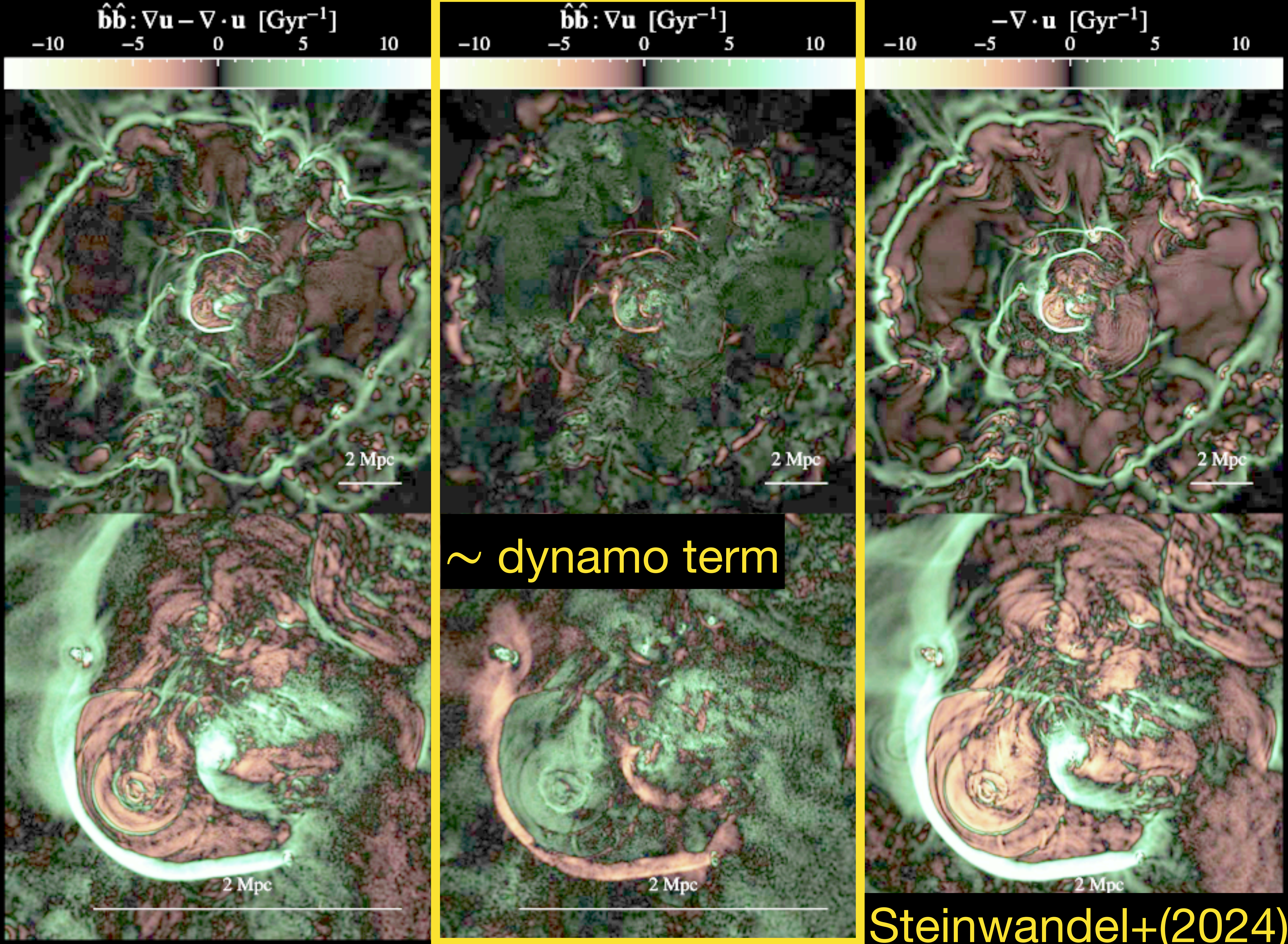


Steinwandel+(2024)

Simulations in this talk

- Highly-modified version of finite volume code *FLASH*, second-order in space approximate Riemann (PPM) solver with framework outlined in Bouchut+(2010), tested in *FLASH* in Waagen+ (2011), ~ 200 simulations: $72^3 - 10{,}080^3$

- Compressible non-helical, isothermal visco/resistive MHD turbulence driven with finite correlation time (OU process; Federrath+(2022)) on $L/2$.

- No net magnetic flux. Pure turbulent magnetic field.

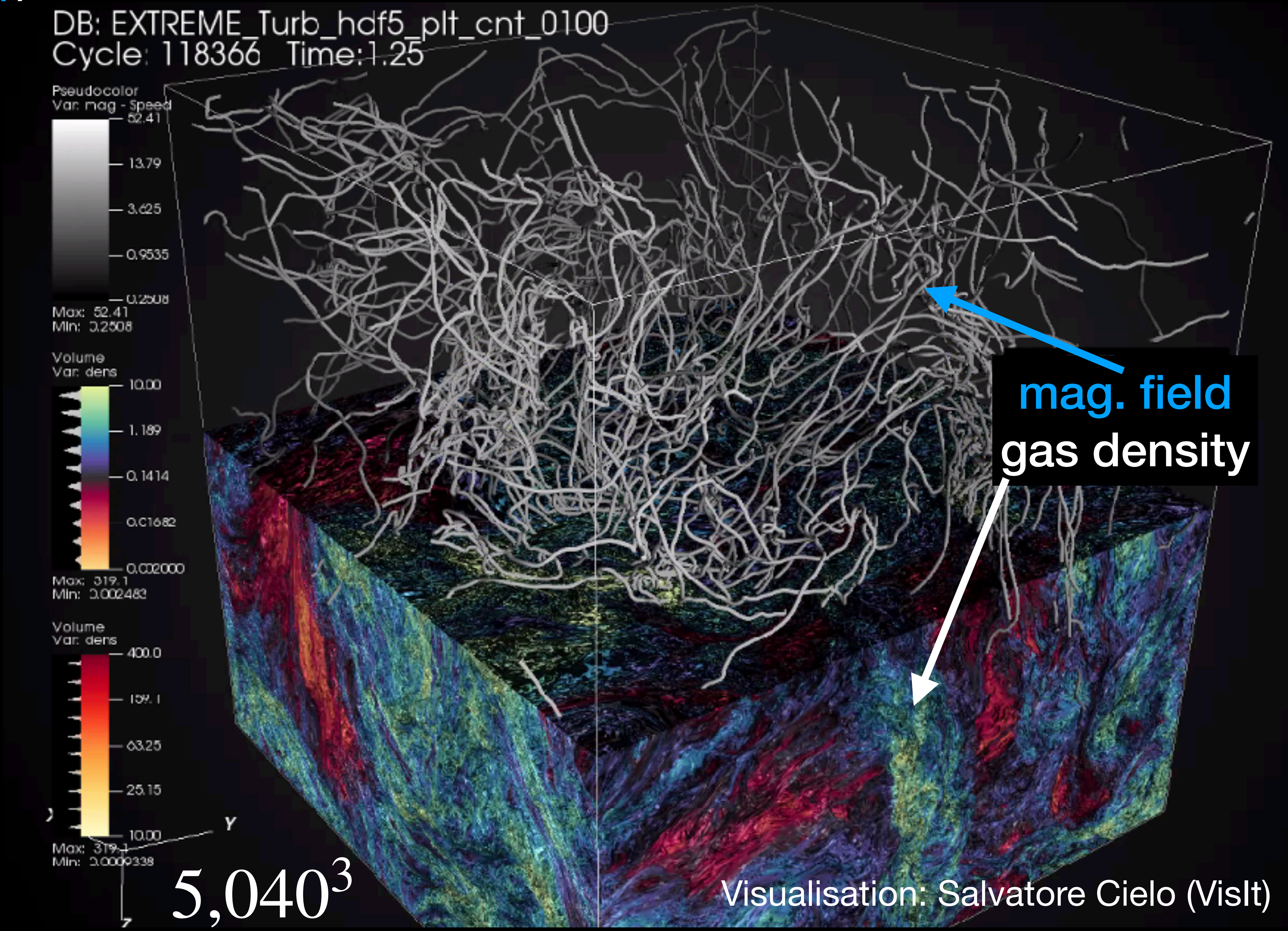
$$\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$d_t(\rho \mathbf{u}) + \nabla \cdot \mathbb{F}_{\rho u} = \frac{1}{\mathrm{Re}} \nabla \cdot \sigma_{\mathrm{viscous}} + \rho \mathbf{f}$$

$$\partial_t \mathbf{b} + \nabla \cdot \mathbb{F}_b = \frac{1}{\mathrm{Rm}} \nabla^2 \mathbf{b}$$

$$\nabla \cdot \mathbf{b} = 0 \quad p = c_s \rho$$

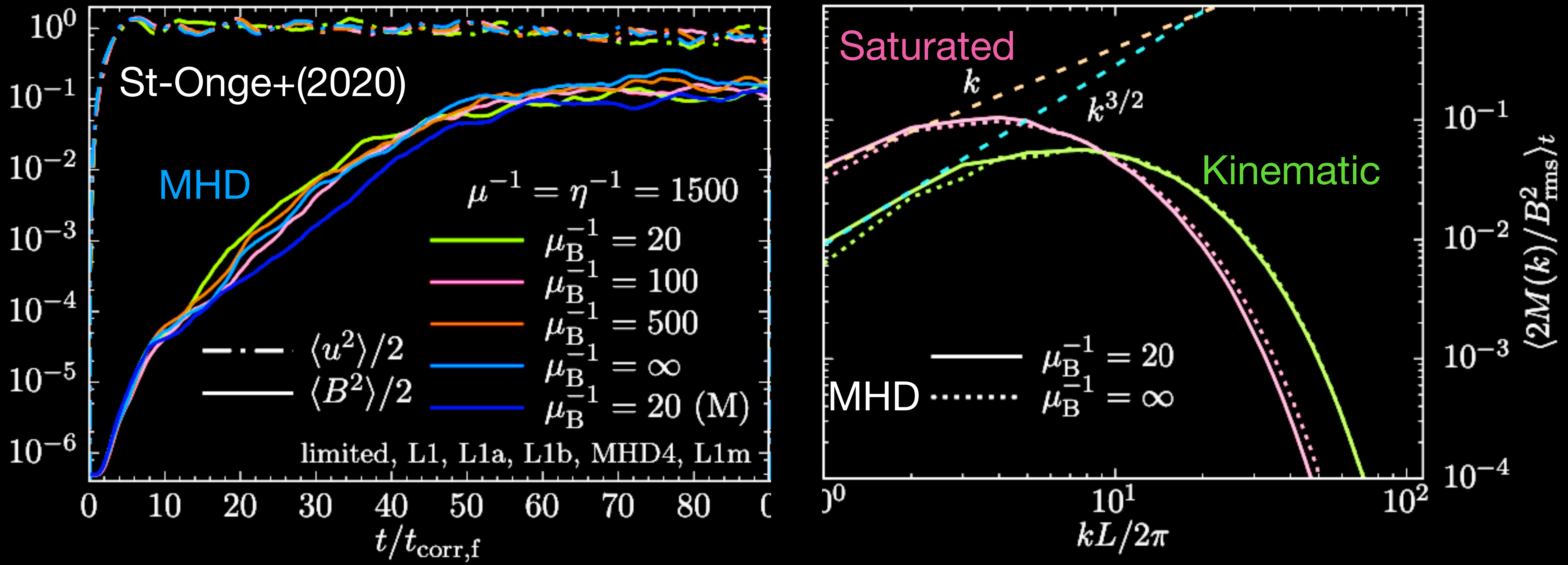
$$\mathrm{Pm} = \frac{\mathrm{Rm}}{\mathrm{Re}}$$



Inevitability of the turbulent dynamo

Somewhat* universal over collisionality (or at least pressure anisotropy)

Weakly collisional Braginksii MHD



(added anisotropic viscous Braginskii stress term into MHD)

$\nabla \cdot (\hat{\mathbf{b}} \otimes \hat{\mathbf{b}}(\hat{\mathbf{b}} \otimes \hat{\mathbf{b}} : \nabla \otimes \mathbf{u}))$ *Snoopy*

The turbulent dynamo story

The turbulent dynamo story

$$\text{Pm} = \frac{\nu}{\eta} \gg 1$$

$$k_\eta \gg k_\nu$$

Do we need to worry about the seed field in turbulent dynamos?

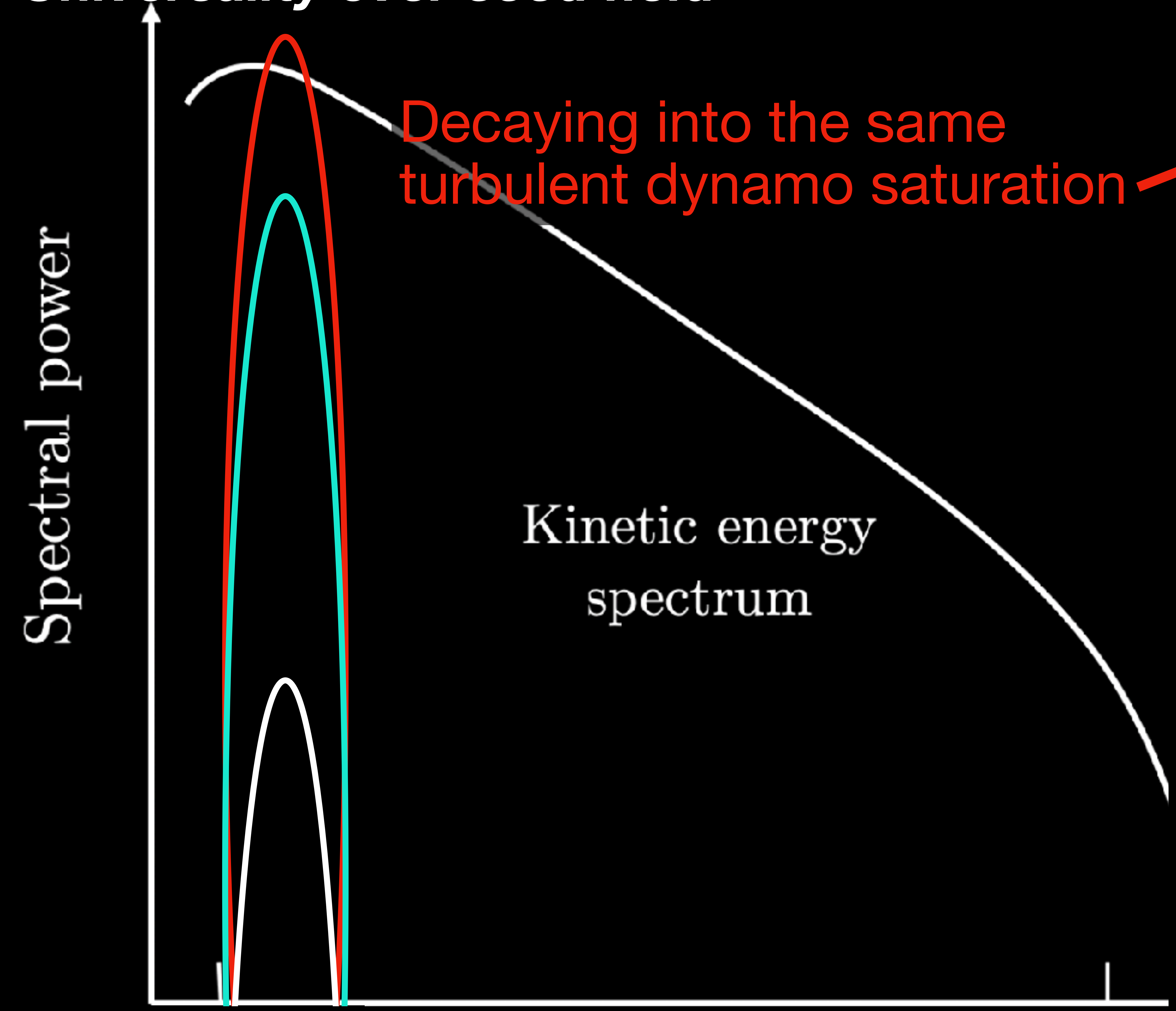
i.e., does the initial state influence the final state?

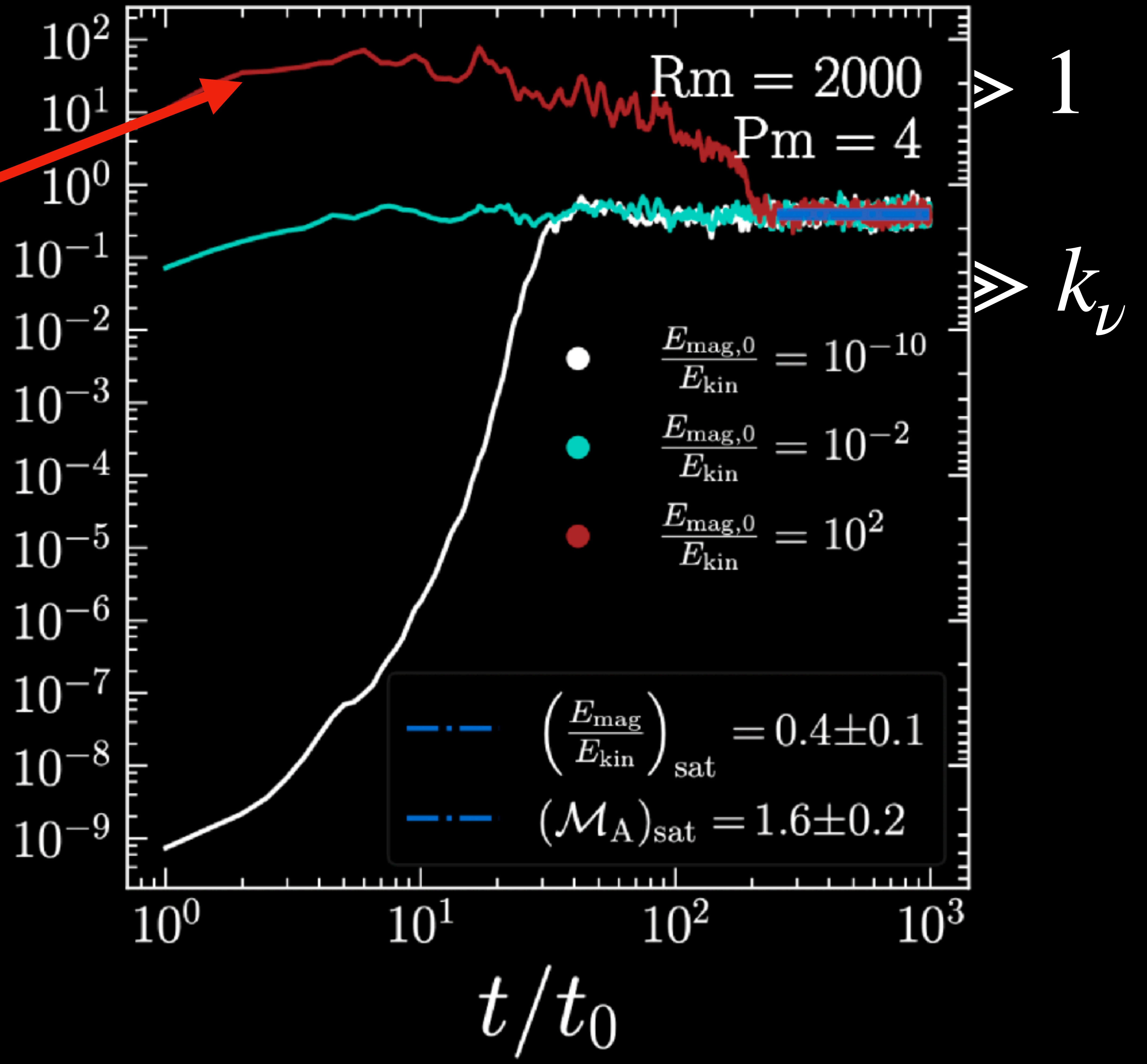
Inevitability of the turbulent dynamo

Universality over seed field

Inevitability of the turbulent dynamo

Universality over seed field

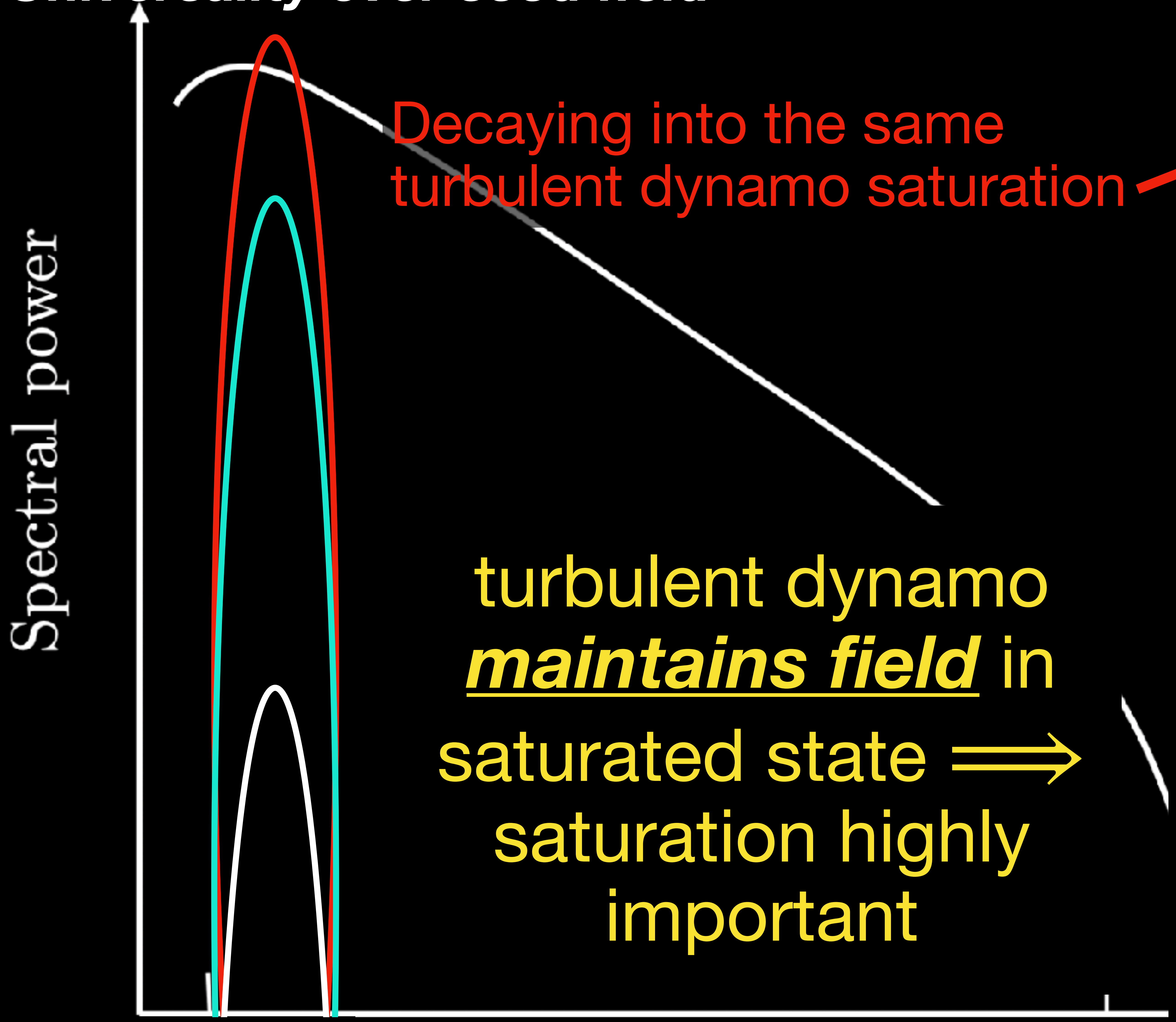




Beattie+ (2023). *Growth or Decay I: Universality of the turbulent dynamo saturation*

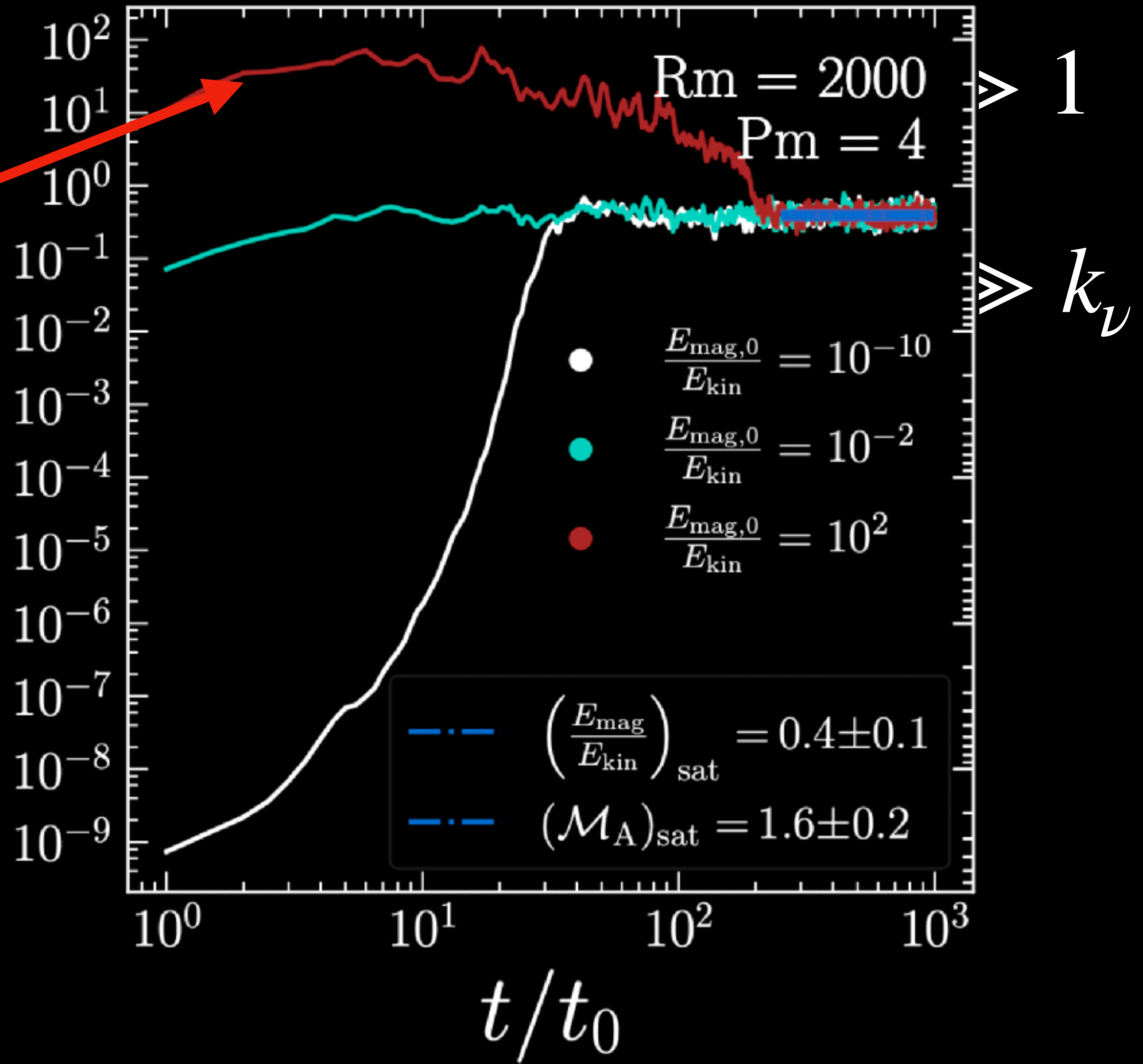
Inevitability of the turbulent dynamo

Universality over seed field



Decaying into the same turbulent dynamo saturation

turbulent dynamo ***maintains field*** in saturated state $\Longrightarrow$ saturation highly important



Beattie+ (2023). *Growth or Decay I: Universality of the turbulent dynamo saturation*

The turbulent dynamo story

First growth stage for Biermann seed: diffusion-free regime

$$\mathrm{d}_t\langle\mathcal{E}_{\mathrm{mag}}\rangle = 2\gamma\langle\mathcal{E}_{\mathrm{mag}}\rangle, \quad \gamma = \hat{\mathbf{b}}\otimes\hat{\mathbf{b}} : \nabla\mathbf{u}$$

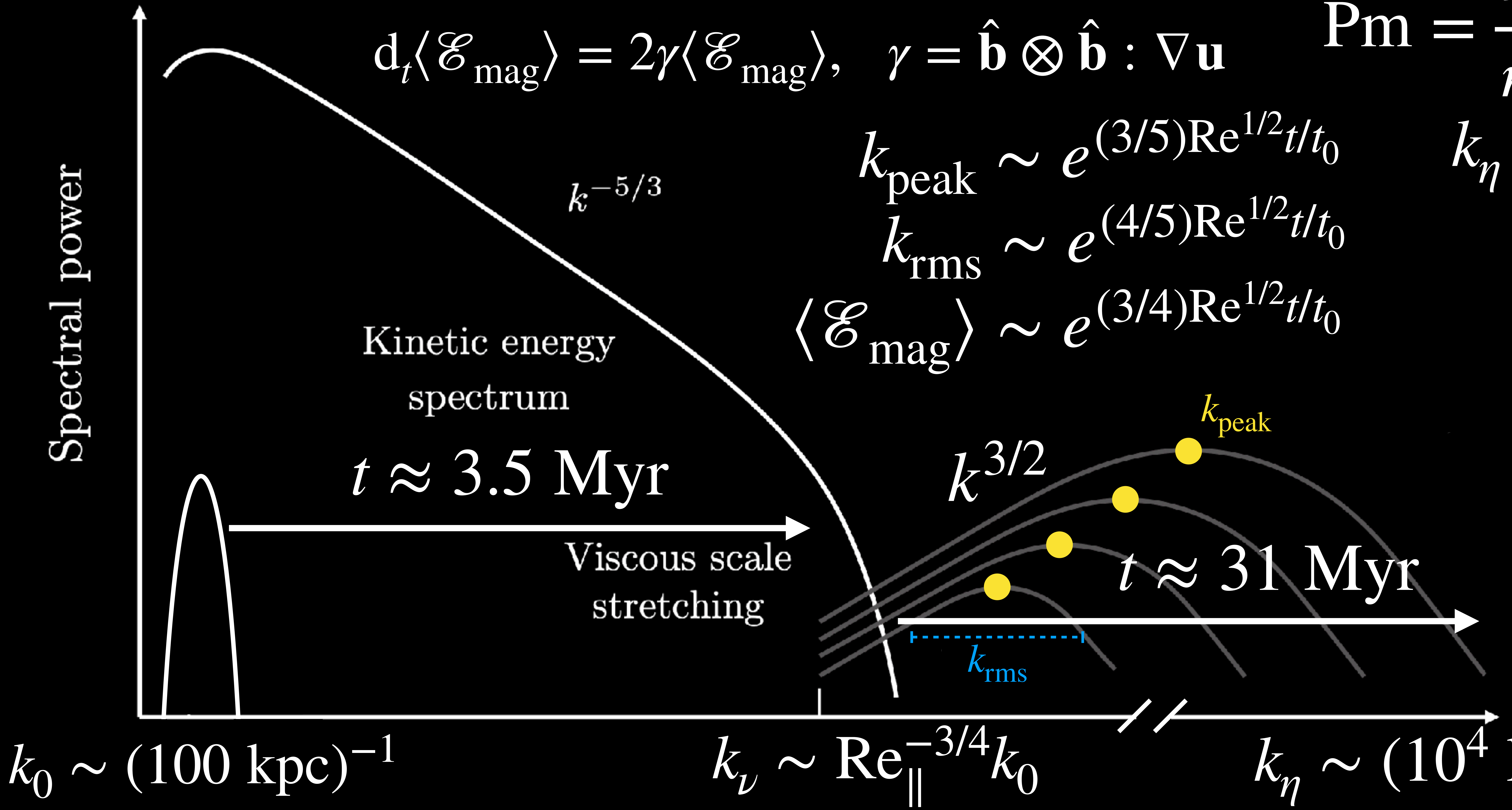
$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

$$k_\eta \gg k_\nu$$

$$k_{\mathrm{peak}} \sim e^{(3/5)\mathrm{Re}^{1/2}t/t_0}$$

$$k_{\mathrm{rms}} \sim e^{(4/5)\mathrm{Re}^{1/2}t/t_0}$$

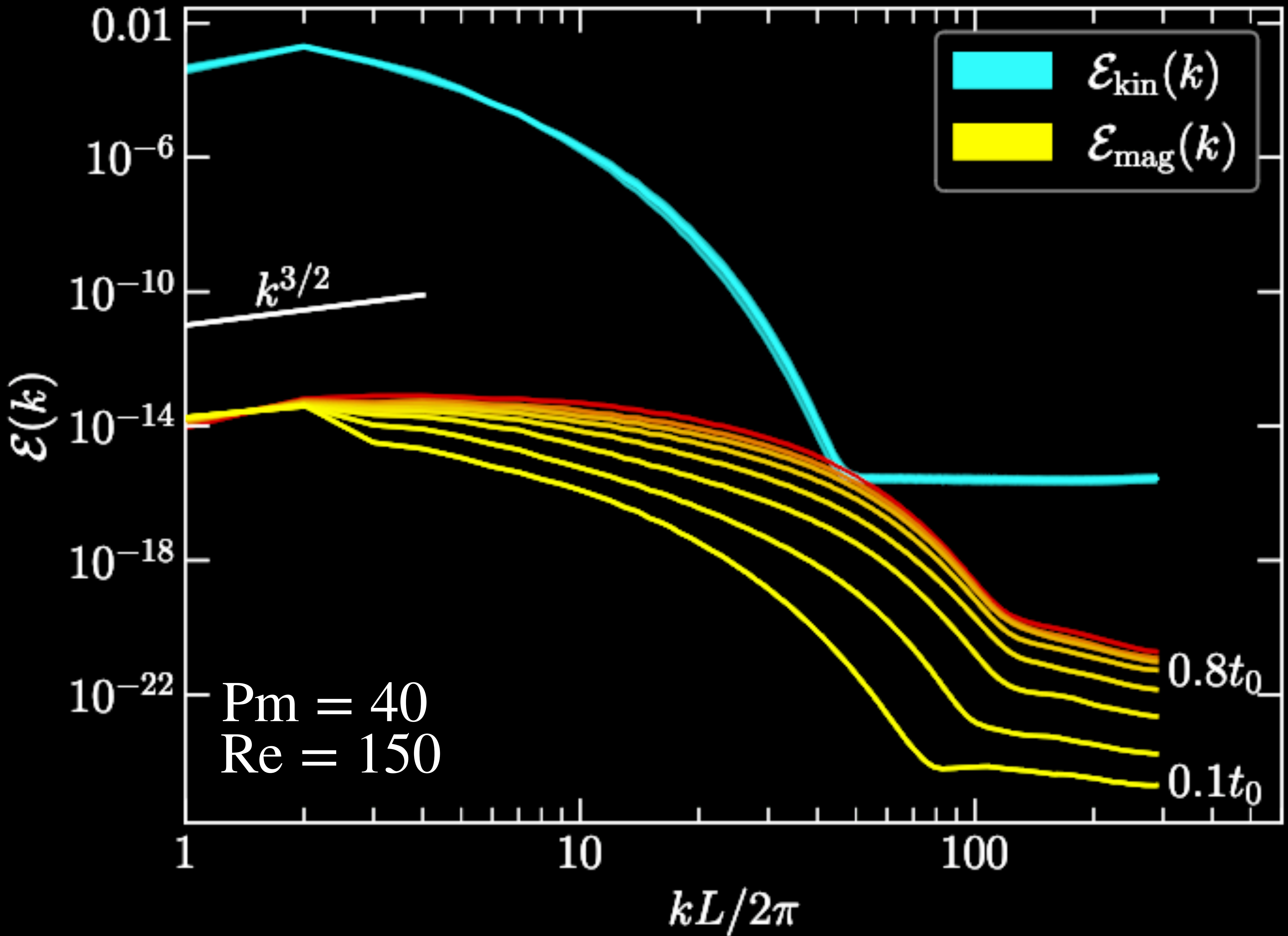
$$\langle\mathcal{E}_{\mathrm{mag}}\rangle \sim e^{(3/4)\mathrm{Re}^{1/2}t/t_0}$$



Schekochihin+(2002)

The turbulent dynamo story

First growth stage for Biermann seed: diffusion-free regime



Shashvat Varma
Grad. student (UofT)

$$k_{\mathrm{peak}} \sim e^{(3/5)\mathrm{Re}^{1/2} t/t_0}$$

$$k_{\mathrm{rms}} \sim e^{(4/5)\mathrm{Re}^{1/2} t/t_0}$$

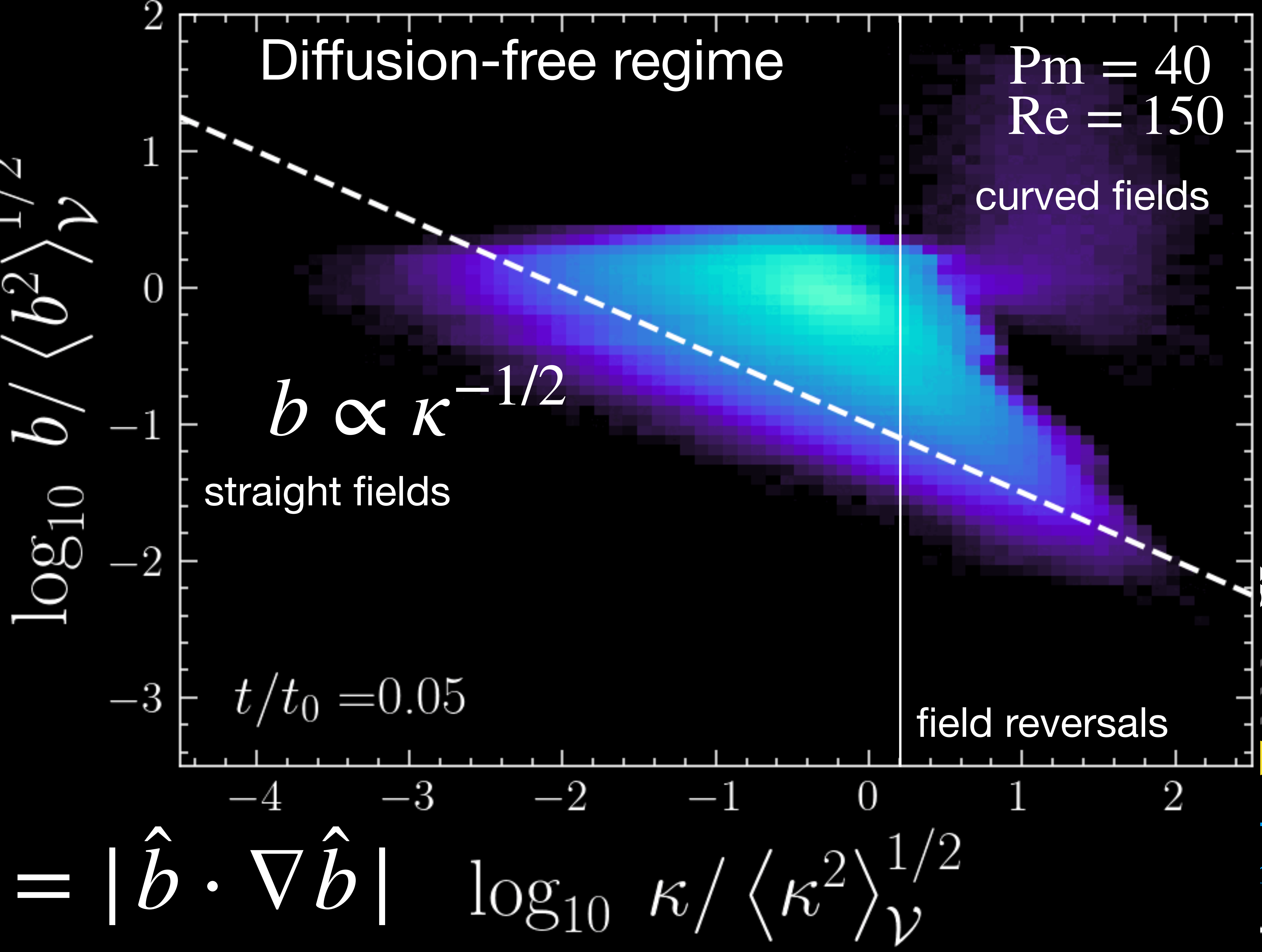
$$\langle \mathcal{E}_{\mathrm{mag}} \rangle \sim e^{(3/4)\mathrm{Re}^{1/2} t/t_0}$$

Varma, **Beattie**, Kriel, Ripperda (*in prep.*)

Schekochihin+ (2002)

Inevitability of the turbulent dynamo

First growth stage for Biermann seed: diffusion-free regime



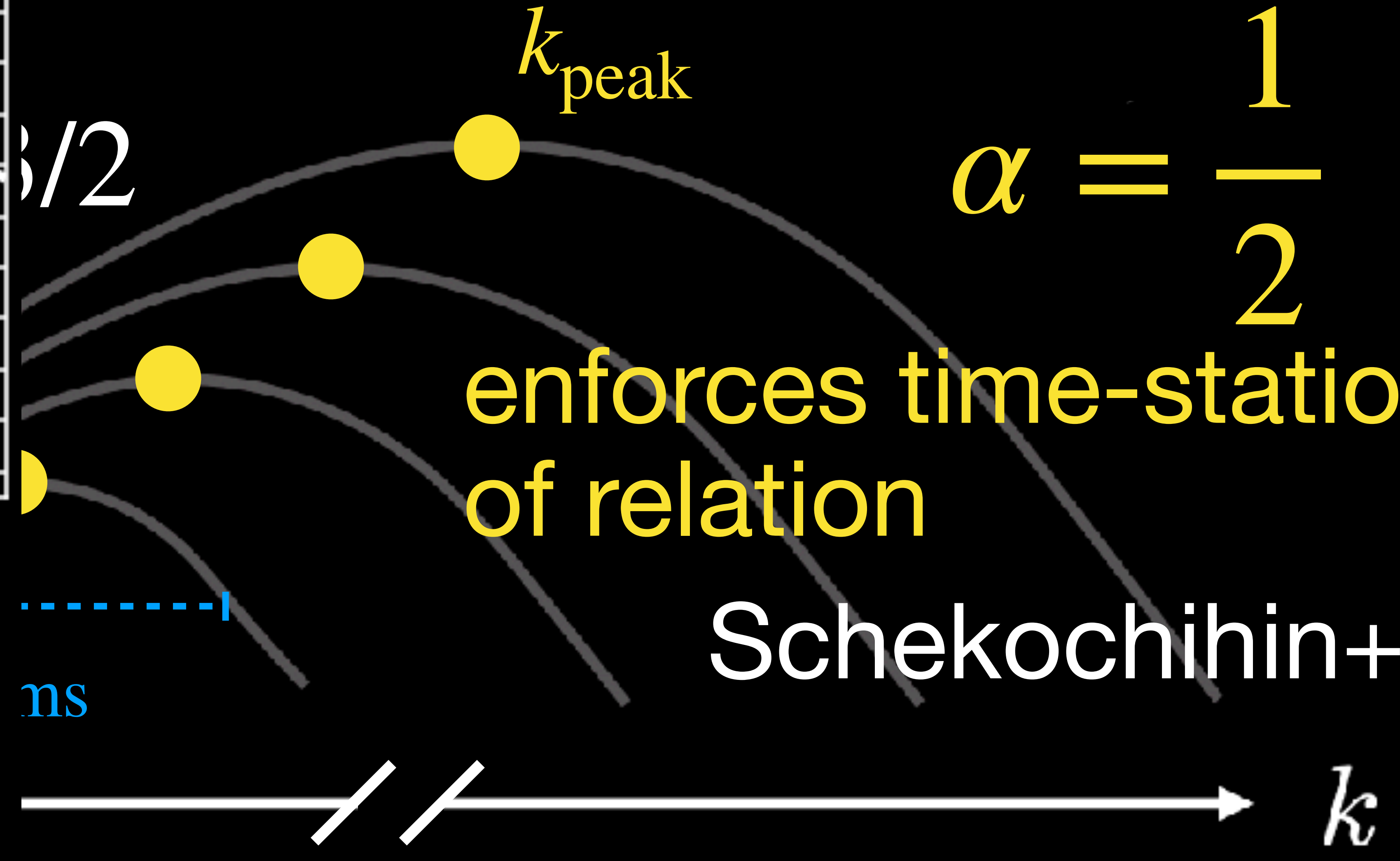
$\kappa = |\hat{b} \cdot \nabla \hat{b}|$

Varma, **Beattie**, Kriel, Ripperda (*in prep.*)

Establishing the curved field

Shashvat Varma
Grad. student (UofT)

$$d_t(b\kappa^{\alpha}) = \left(\frac{1}{2} - \alpha\right) \hat{\mathbf{b}} \otimes \hat{\mathbf{b}} : \nabla \mathbf{u} + \alpha \hat{\mathbf{n}} \otimes \hat{\mathbf{n}} : \nabla \mathbf{u}$$



$\alpha = \frac{1}{2}$

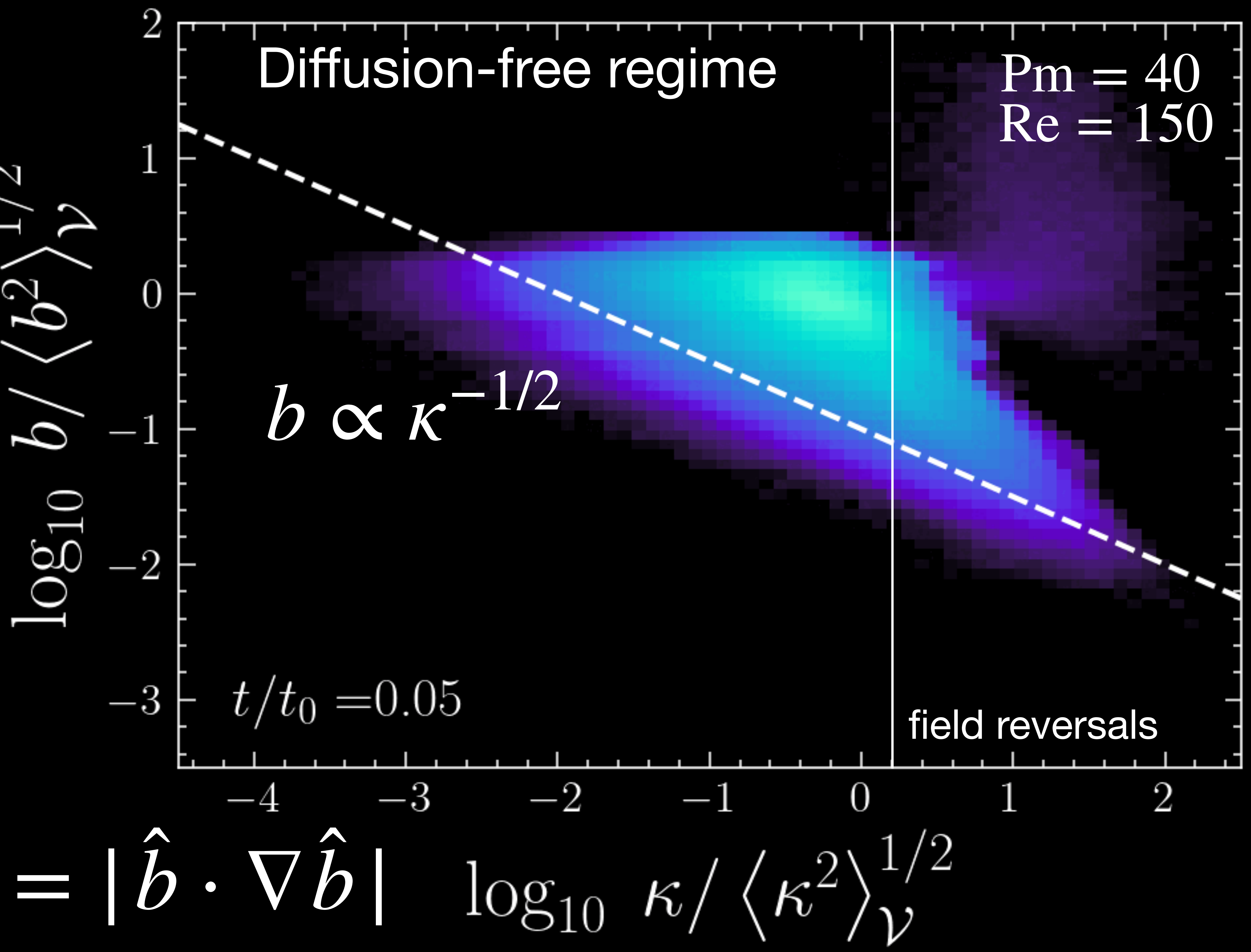
enforces time-stationarity of relation

Schekochihin+ (2004)


$10{,}080^3$, Rm ~ 10^6, Pm ~ 1

Inevitability of the turbulent dynamo

First growth stage for Biermann seed: diffusion-free regime



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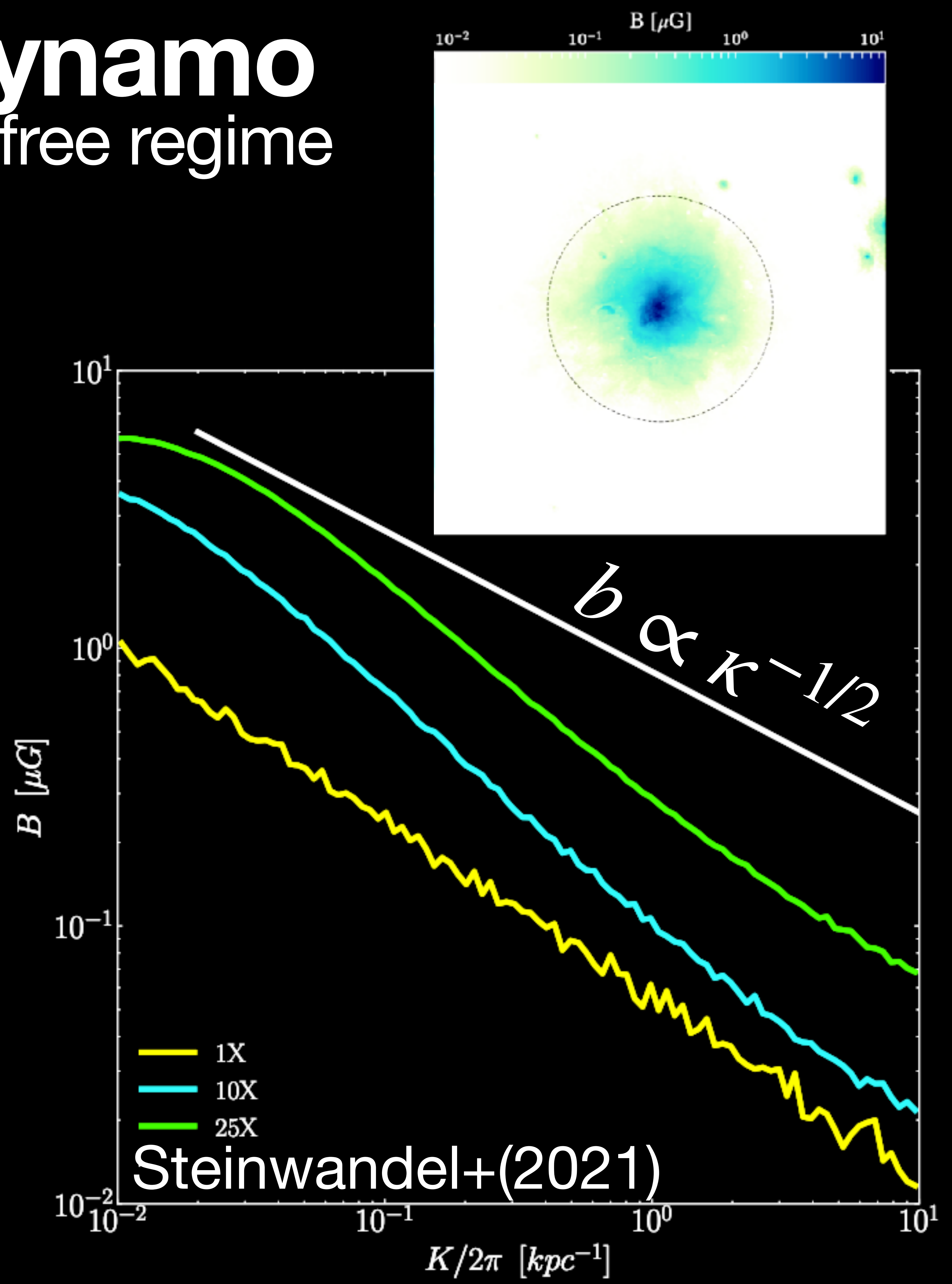


The turbulent dynamo story

First growth stage for ~ Weibel seed: kinematic regime

$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

$$k_\eta \gg k_\nu$$

stretching at the viscous scale

$$\hat{\mathbf{b}} \otimes \hat{\mathbf{b}} : \nabla \mathbf{u} \sim \frac{u_\nu}{\ell_\nu} \sim \frac{\eta}{\ell_\eta^2}$$

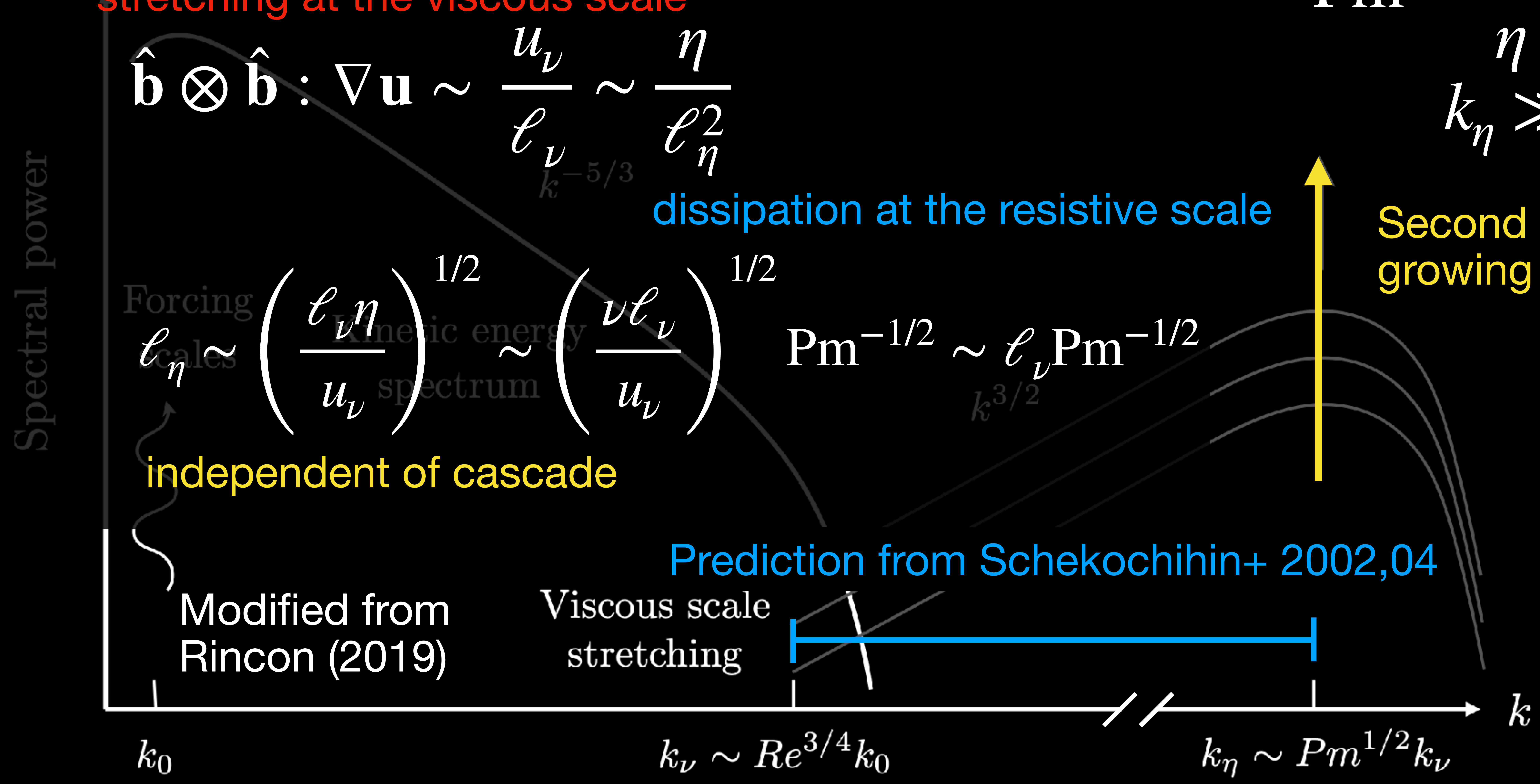
dissipation at the resistive scale

Second fastest growing stage

$$\ell_\eta \sim \left(\frac{\ell_\nu \eta}{u_\nu}\right)^{1/2} \sim \left(\frac{\nu \ell_\nu}{u_\nu}\right)^{1/2} \mathrm{Pm}^{-1/2} \sim \ell_\nu \mathrm{Pm}^{-1/2}$$

independent of cascade

Prediction from Schekochihin+ 2002,04



Modified from Rincon (2019)

The turbulent dynamo story

First growth stage for ~ Weibel seed: kinematic regime

Neco Kriel
Grad. Student (ANU)

Derived from $k^{-5/3}$ velocity spectrum

$$k_\nu \sim \mathrm{Re}^{3/4} k_0$$

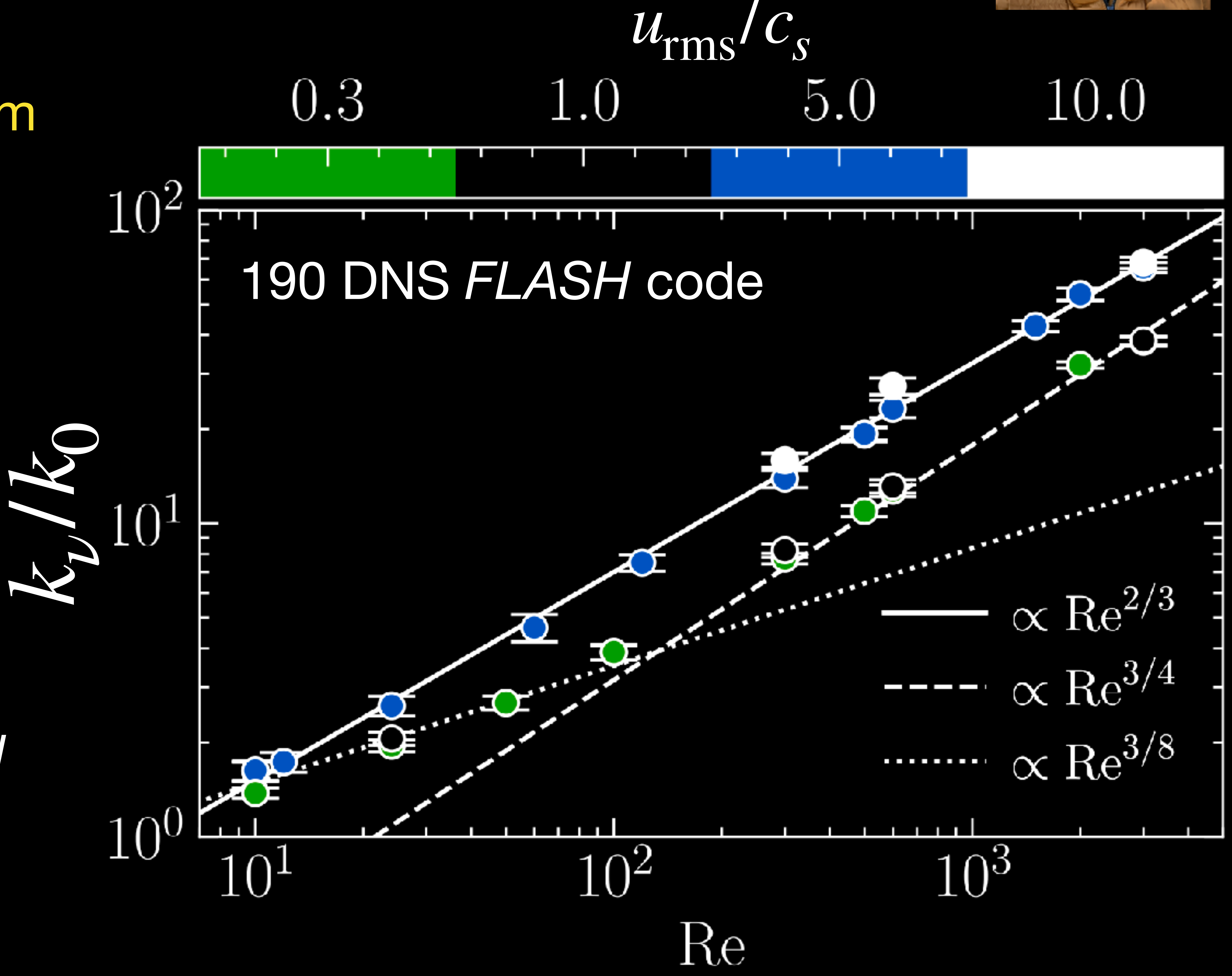
Kolmogorov

Derived from k^{-2} velocity spectrum

$$k_\nu \sim \mathrm{Re}^{2/3} k_0$$

Schober+(2015)

Kriel, **Beattie**+ (2024). *Fundamental scales II: the kinematic stage of the supersonic dynamo*



The turbulent dynamo story

First growth stage for ~ Weibel seed: kinematic regime

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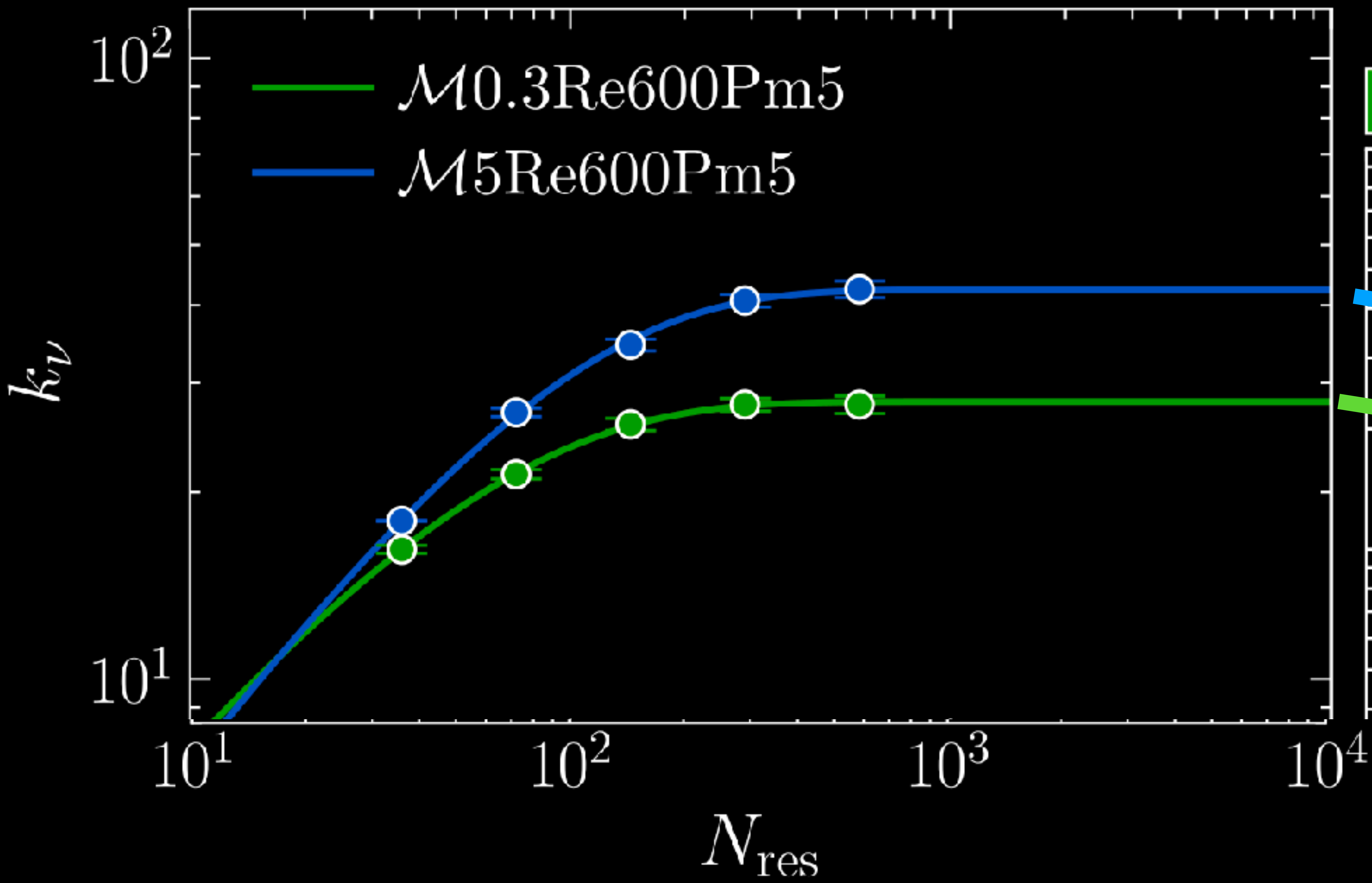

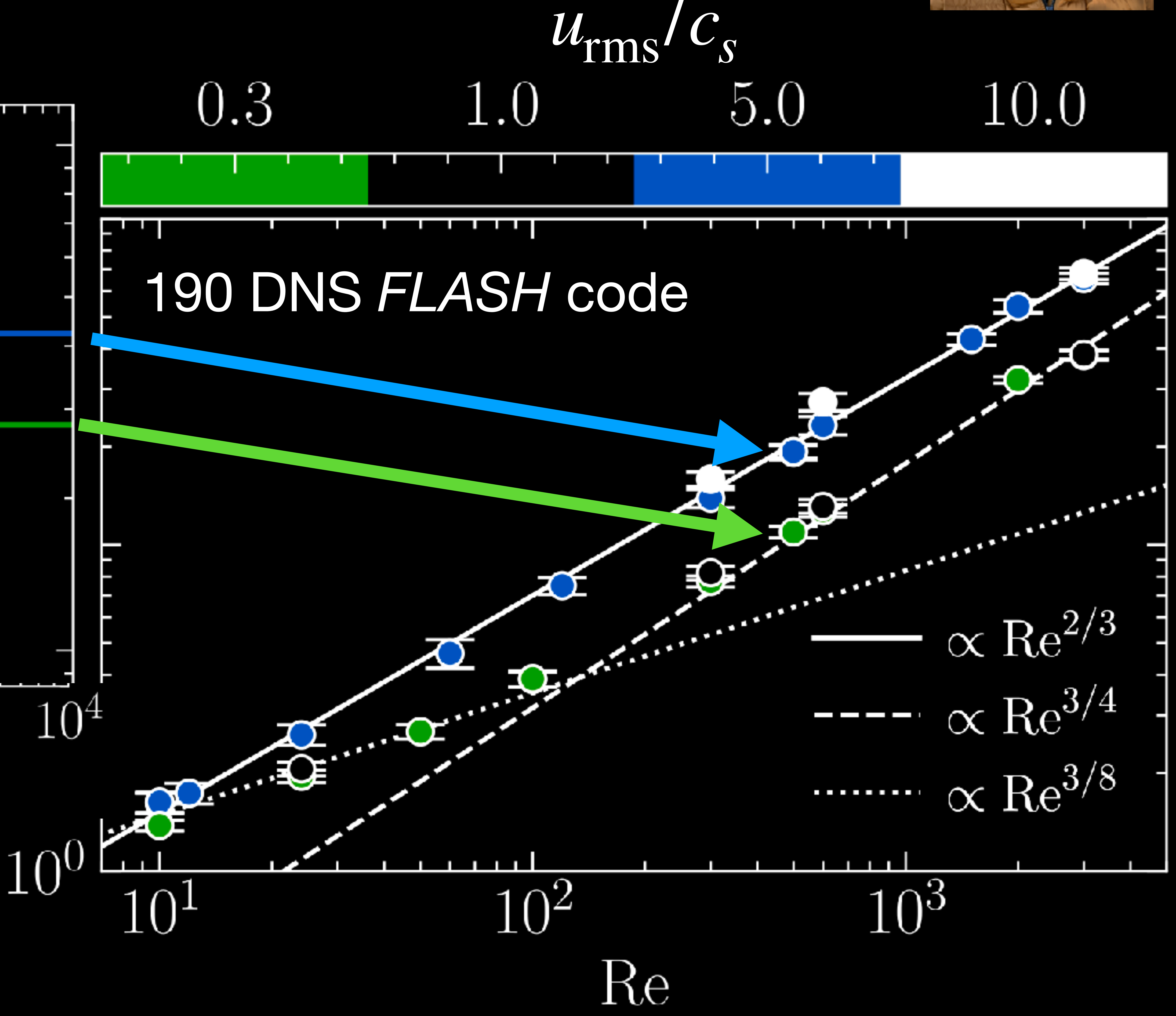

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The turbulent dynamo story

Neco Kriel
Grad. Student (ANU)

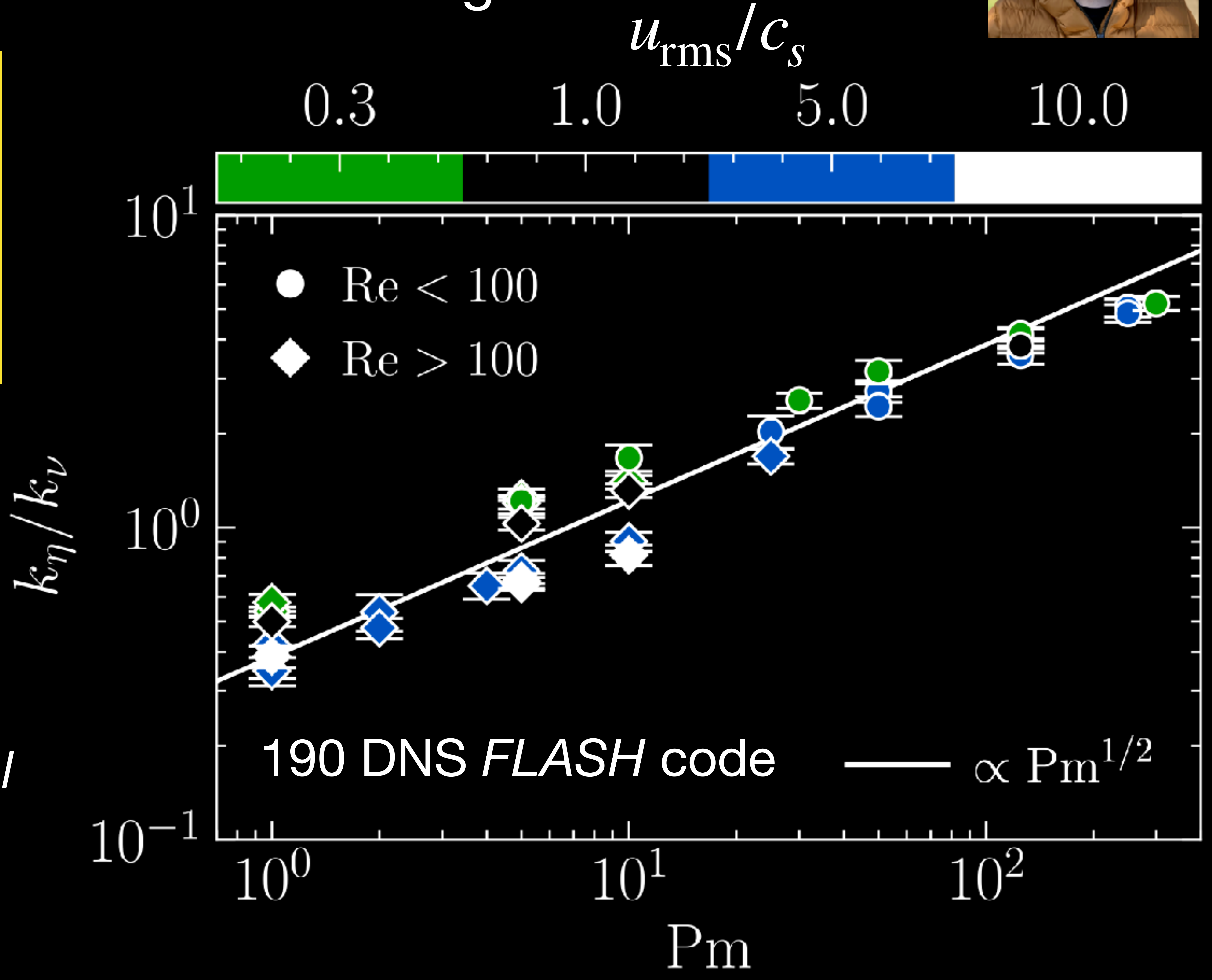
First growth stage for ~ Weibel seed: kinematic regime

stretching at the viscous scale

$$\frac{u_\nu}{\ell_\nu} \sim \frac{\eta}{\ell_\eta^2}$$

dissipation at the resistive scale

1. universal of cascade
2. implies the viscous scale eddies fuel the kinematic dynamo



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The turbulent dynamo story

Linear growth and backreaction

$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

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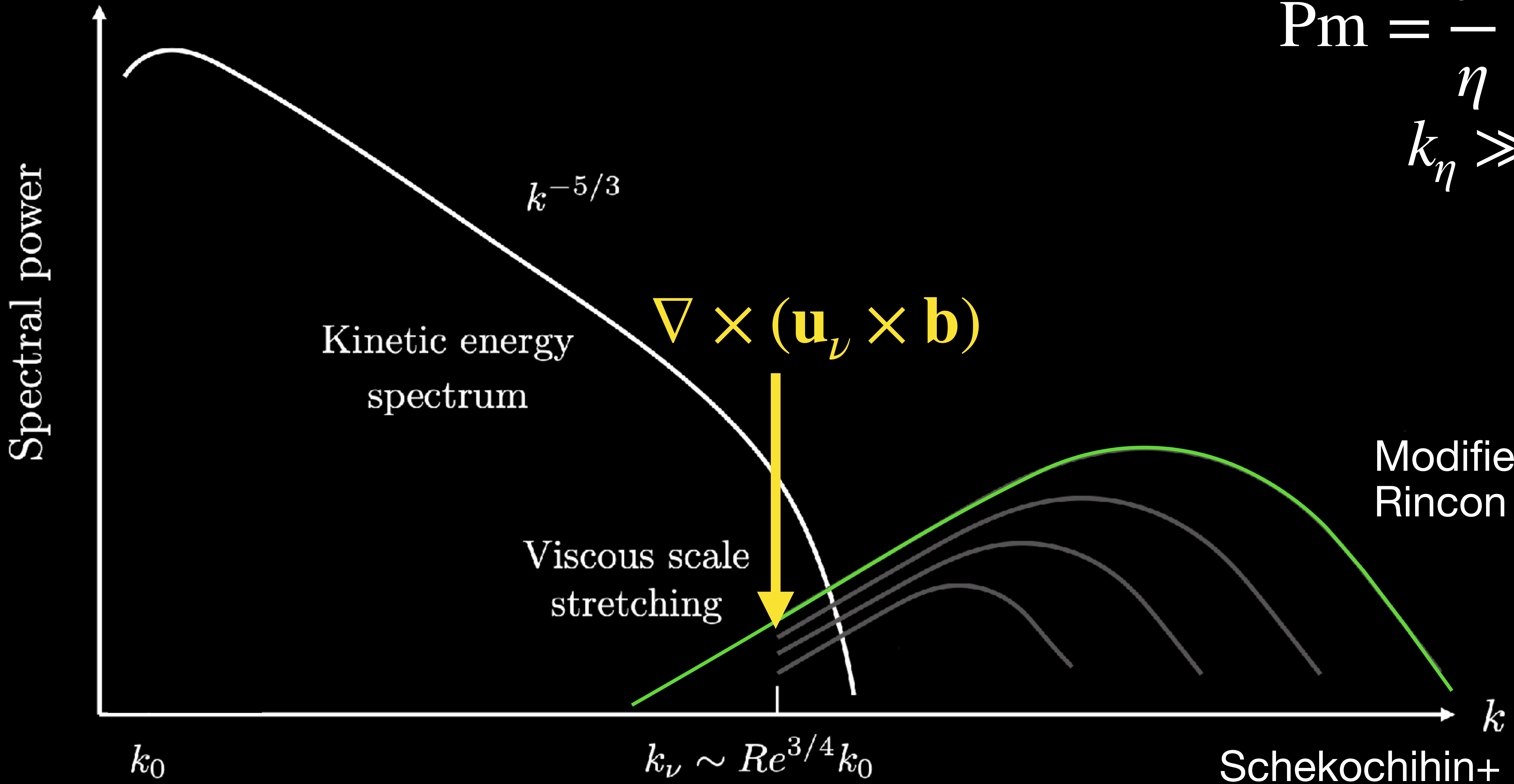
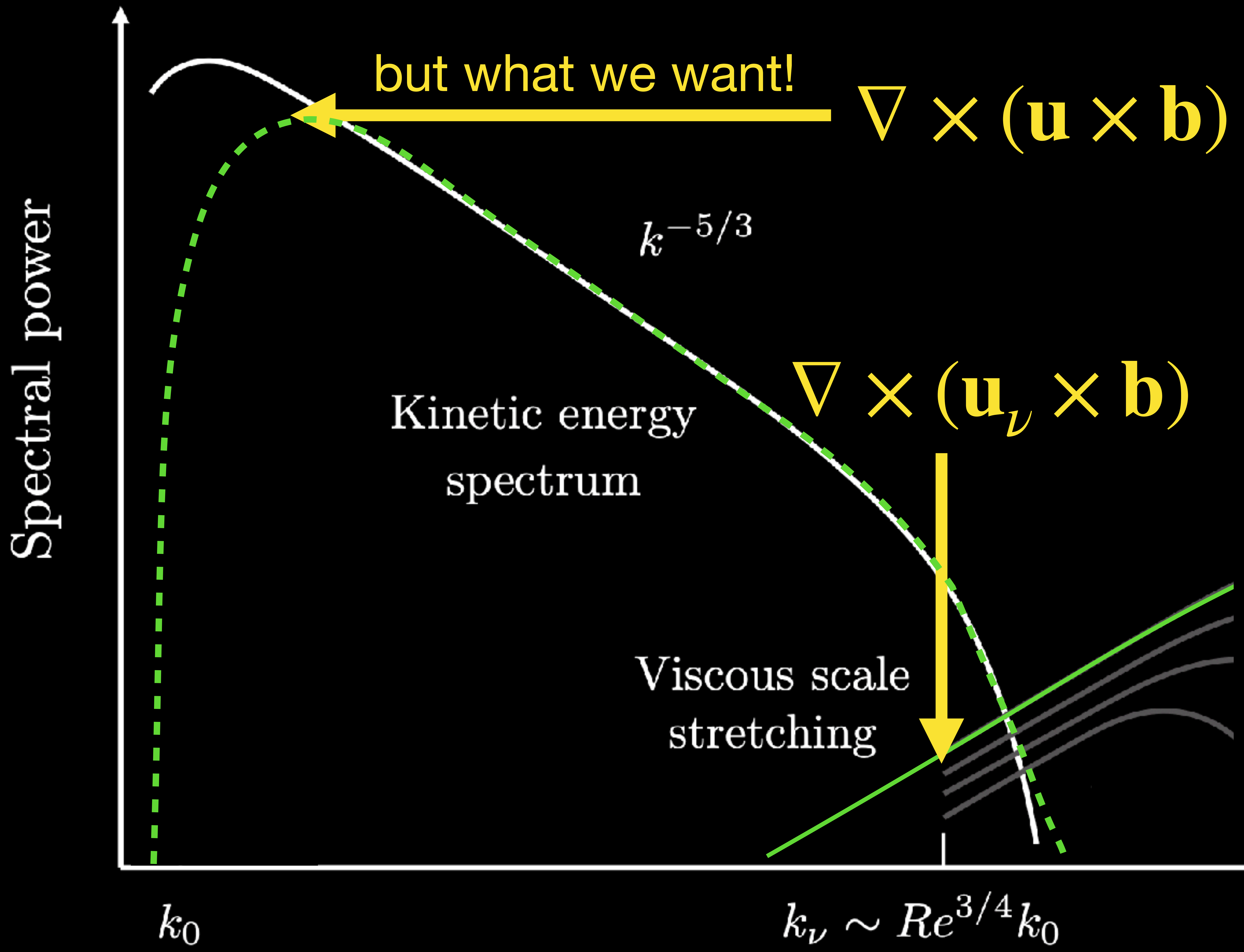


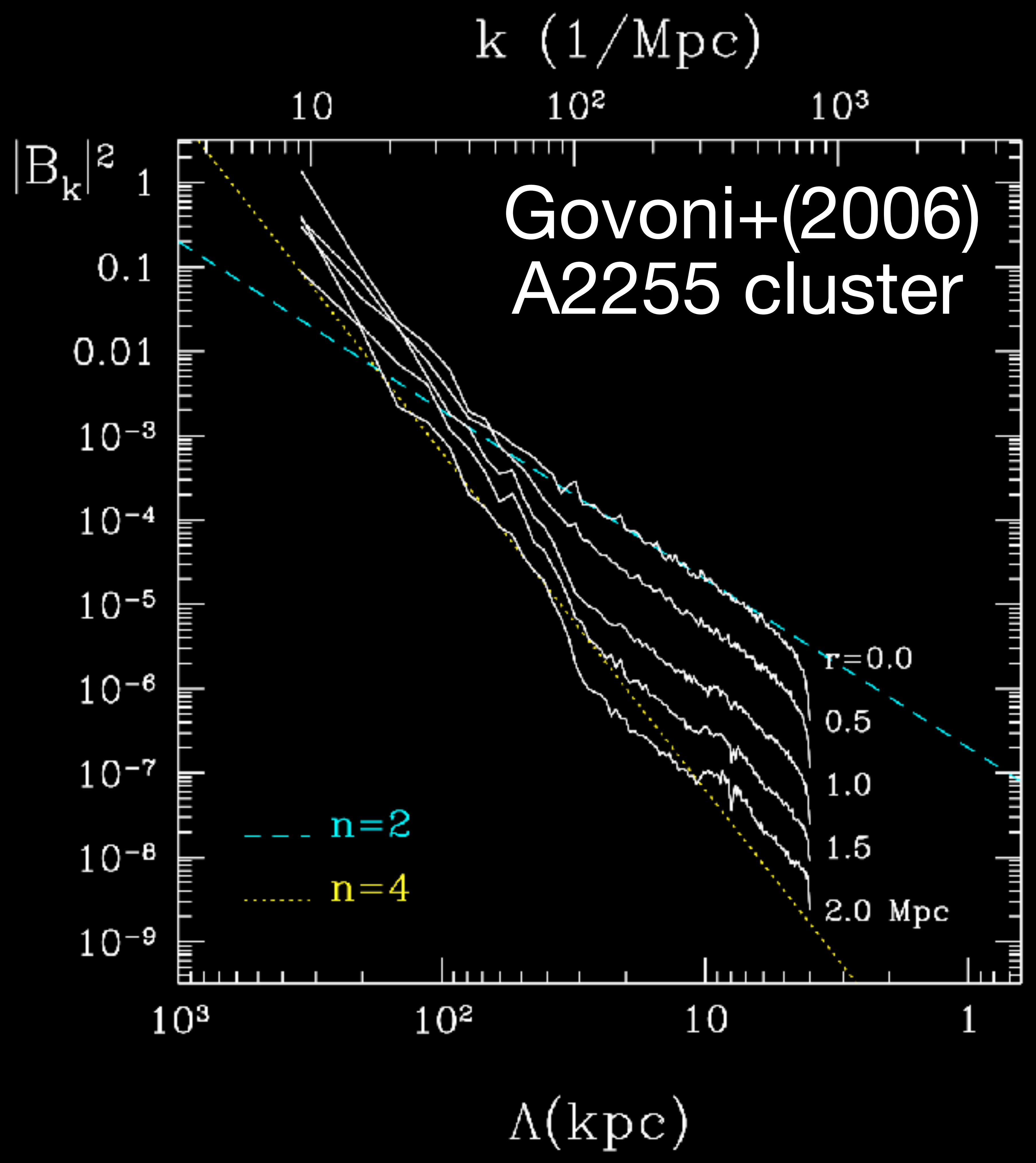
Modified from Rincon (2019)

Schekochihin+ (2002)

The turbulent dynamo story

Linear growth and backreaction





Schekochihin+ (2002)

The turbulent dynamo story
Linear growth and backreaction

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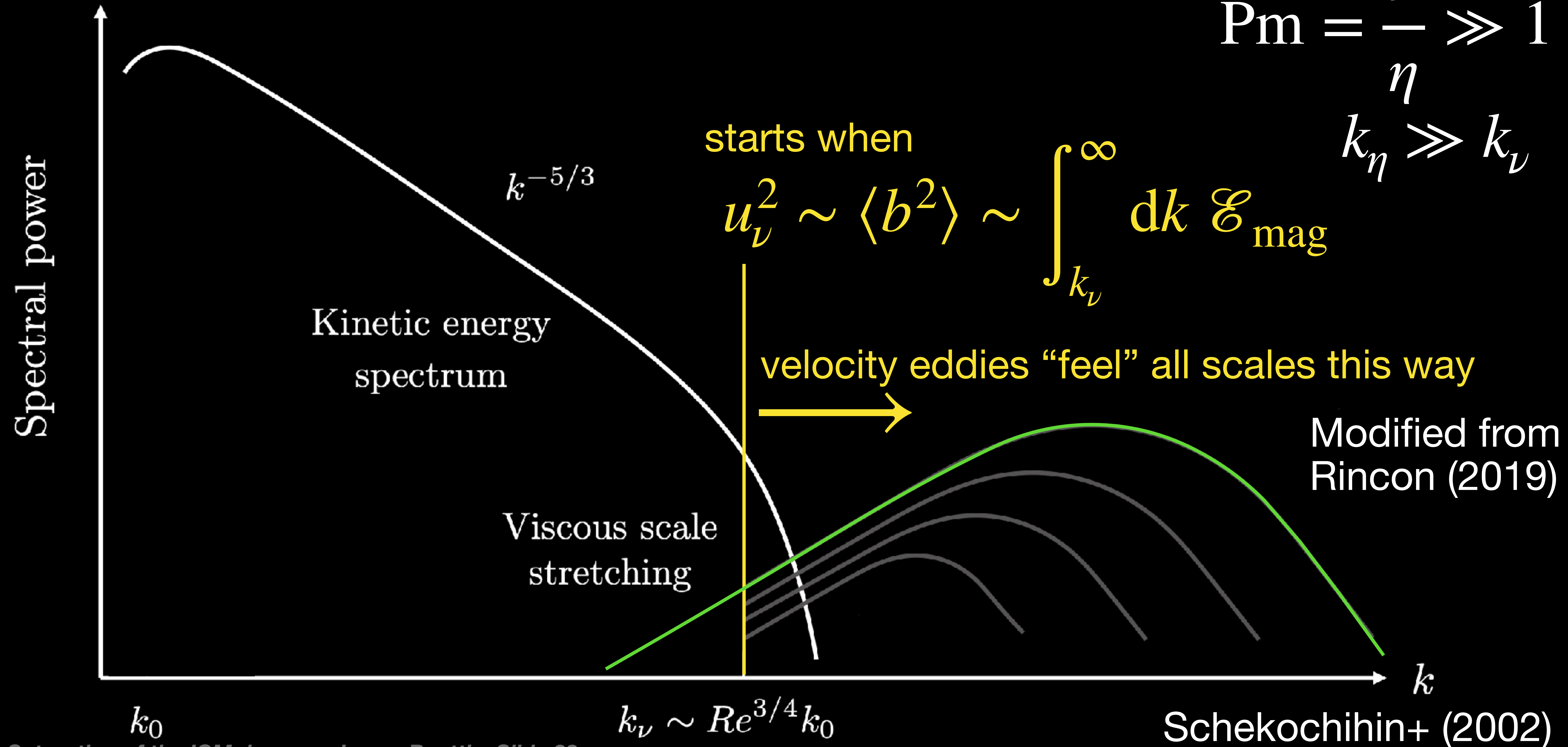

The turbulent dynamo story

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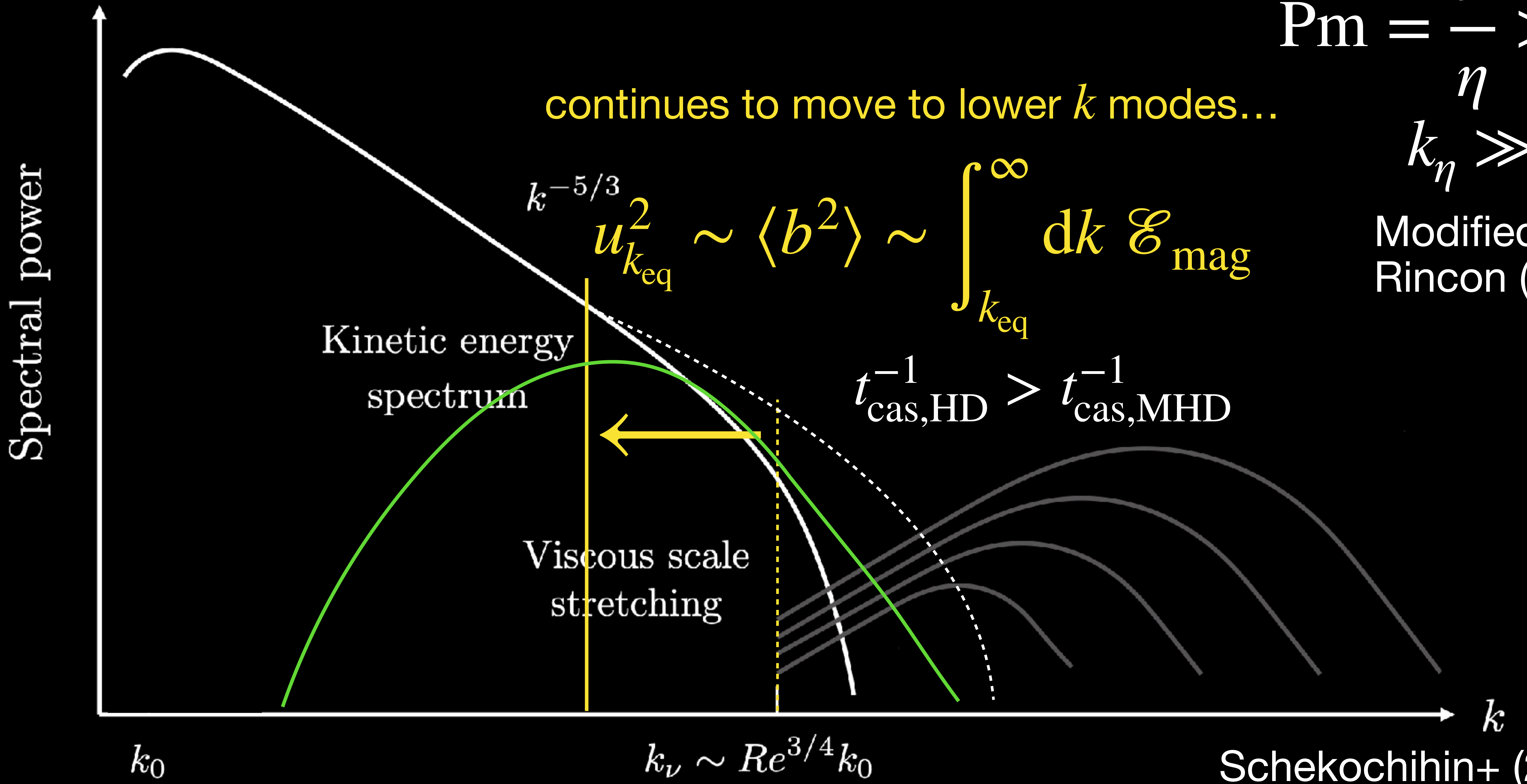
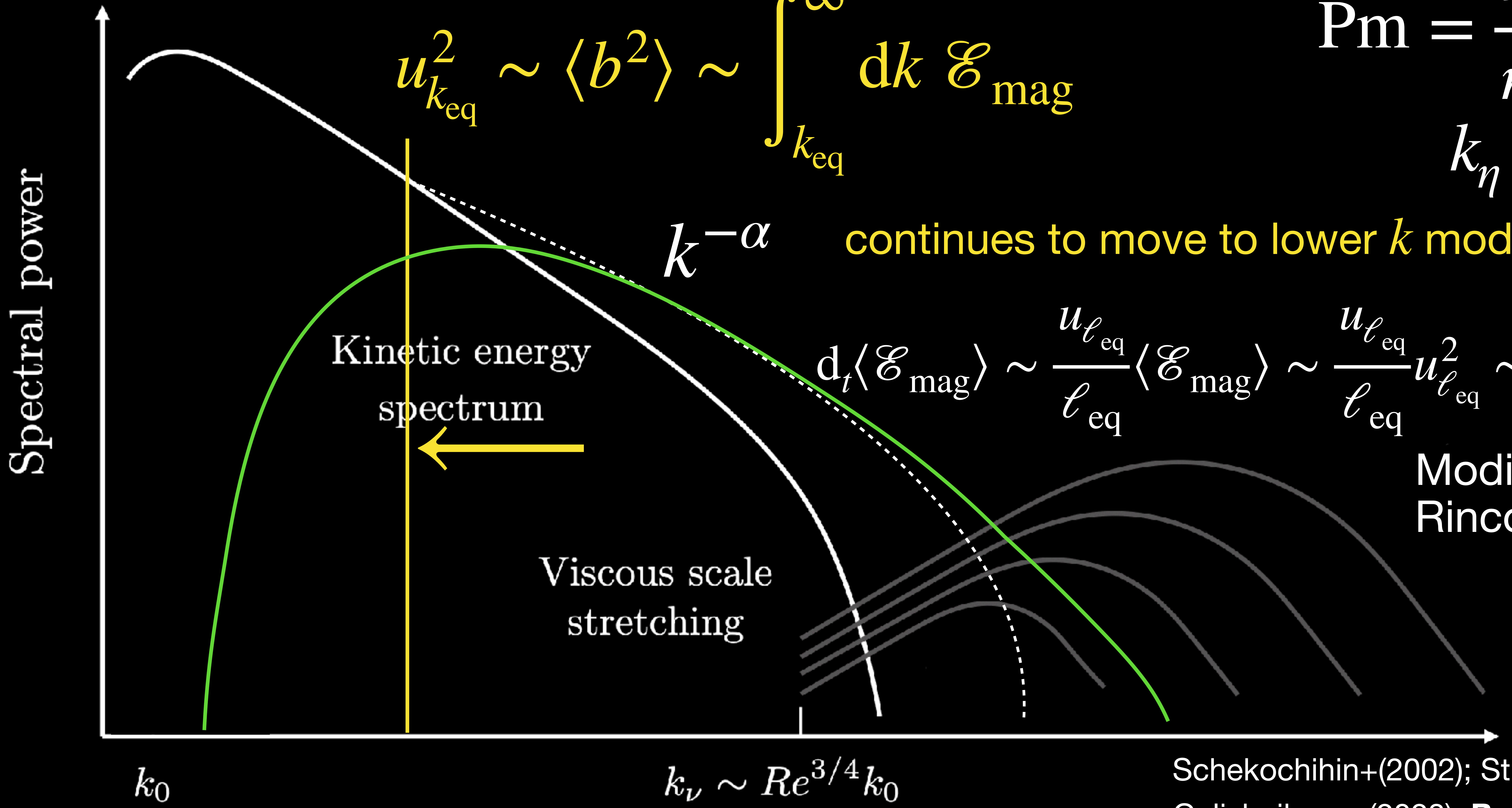


Schekochihin+ (2002)

The turbulent dynamo story

Linear growth and backreaction



$$u_{k_{\rm eq}}^2 \sim \langle b^2 \rangle \sim \int_{k_{\rm eq}}^{\infty} \mathrm{d}k \, \mathscr{E}_{\rm mag}$$

$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

$$k_\eta \gg k_\nu$$

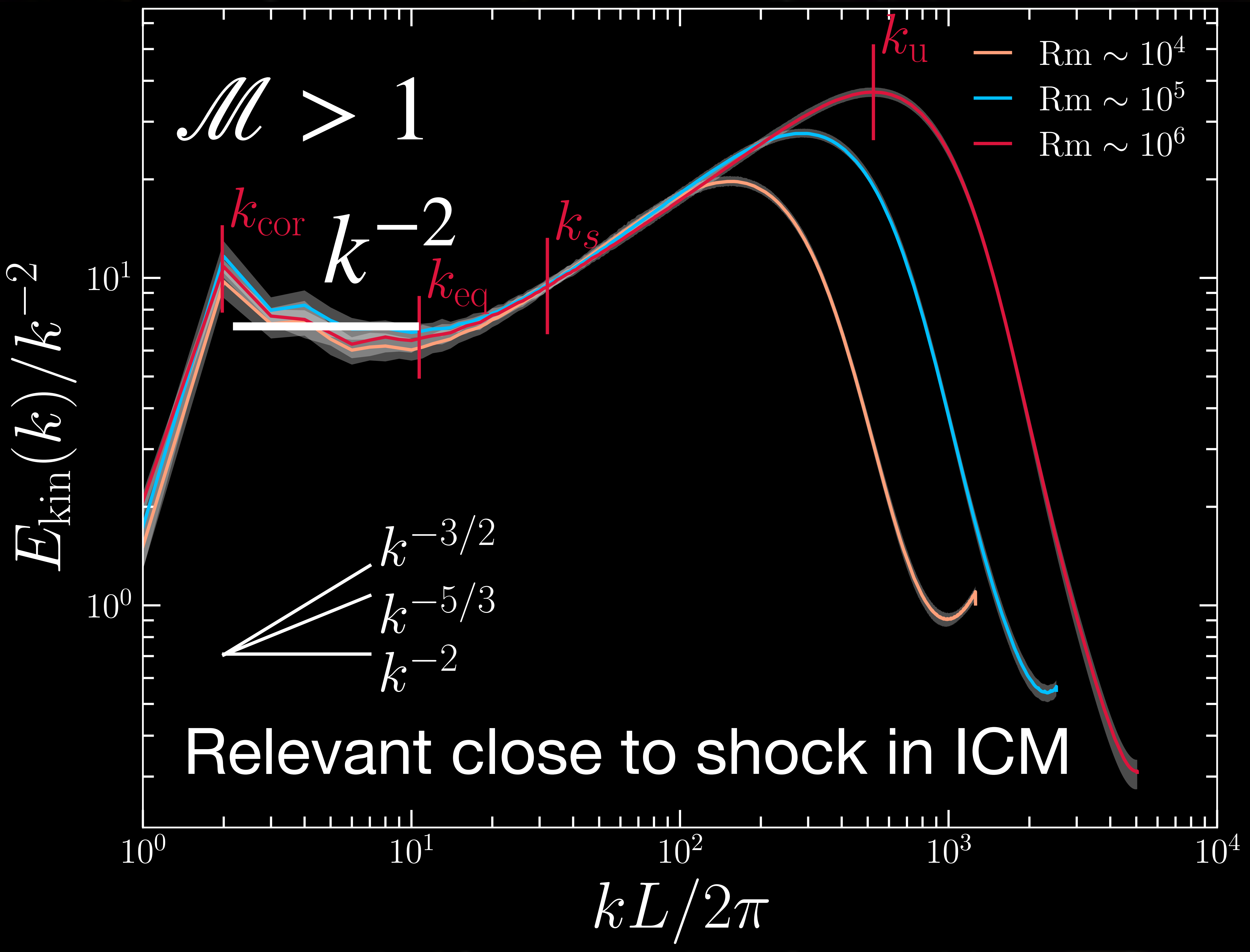
continues to move to lower k modes…

$$\mathrm{d}_t \langle \mathscr{E}_{\rm mag} \rangle \sim \frac{u_{\ell_{\rm eq}}}{\ell_{\rm eq}} \langle \mathscr{E}_{\rm mag} \rangle \sim \frac{u_{\ell_{\rm eq}}}{\ell_{\rm eq}} u_{\ell_{\rm eq}}^2 \sim \varepsilon_{\rm eq} \sim \varepsilon,$$

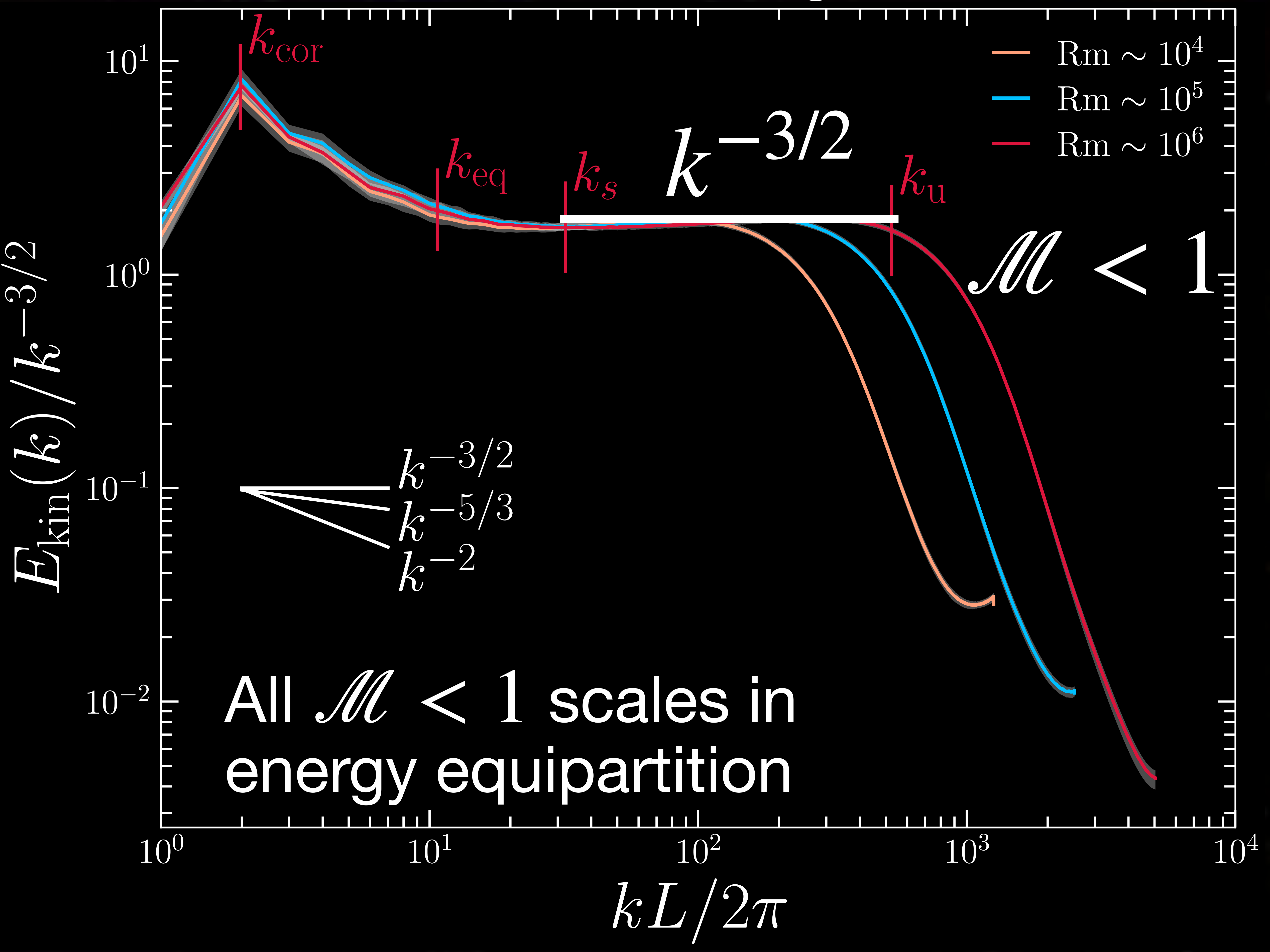
Modified from Rincon (2019)

Schekochihin+(2002); St-Onge+(2020)
Galishnikova+(2023); **Beattie**+(2024)

Kinetic cascade in saturated dynamo



Relevant to more general ICM

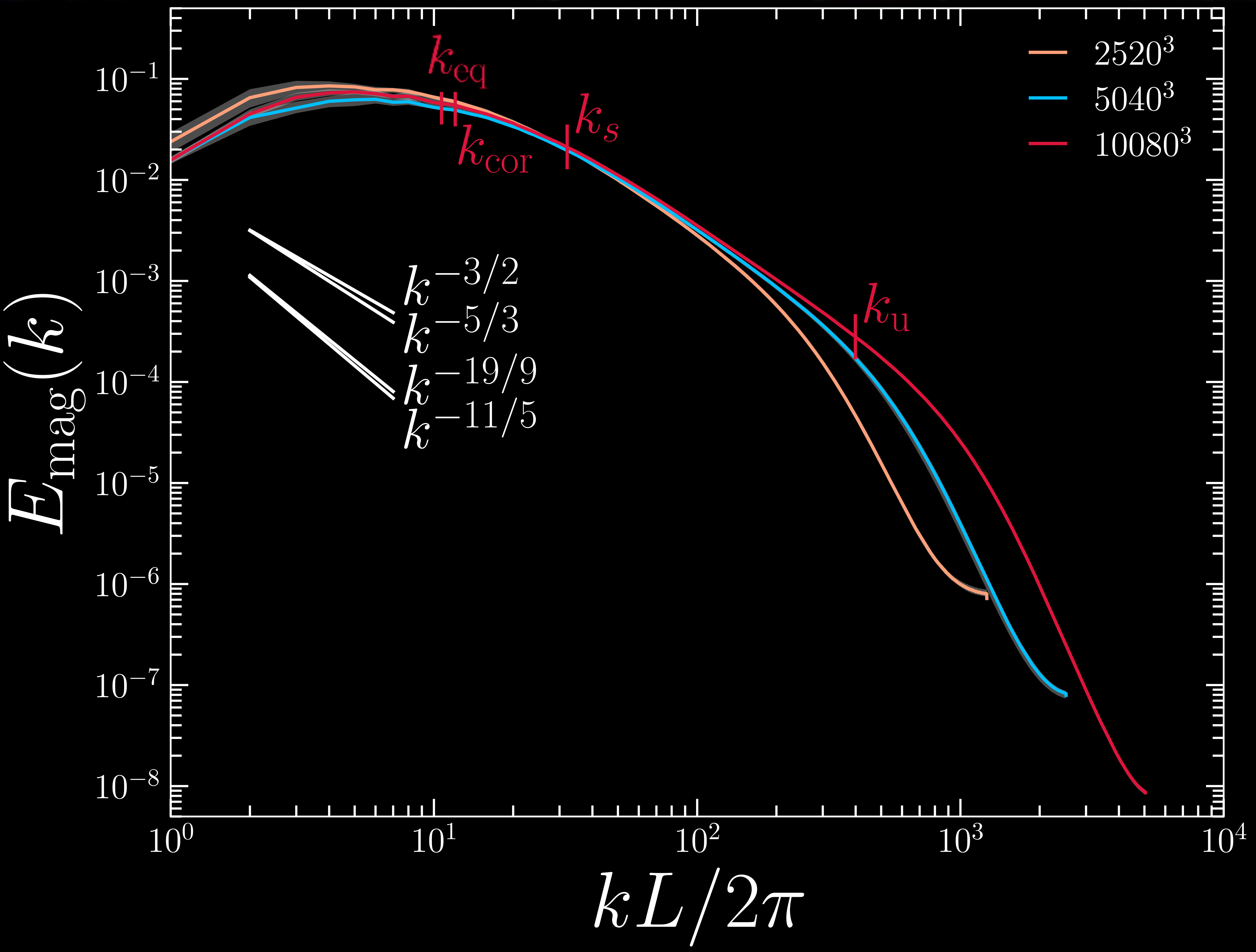


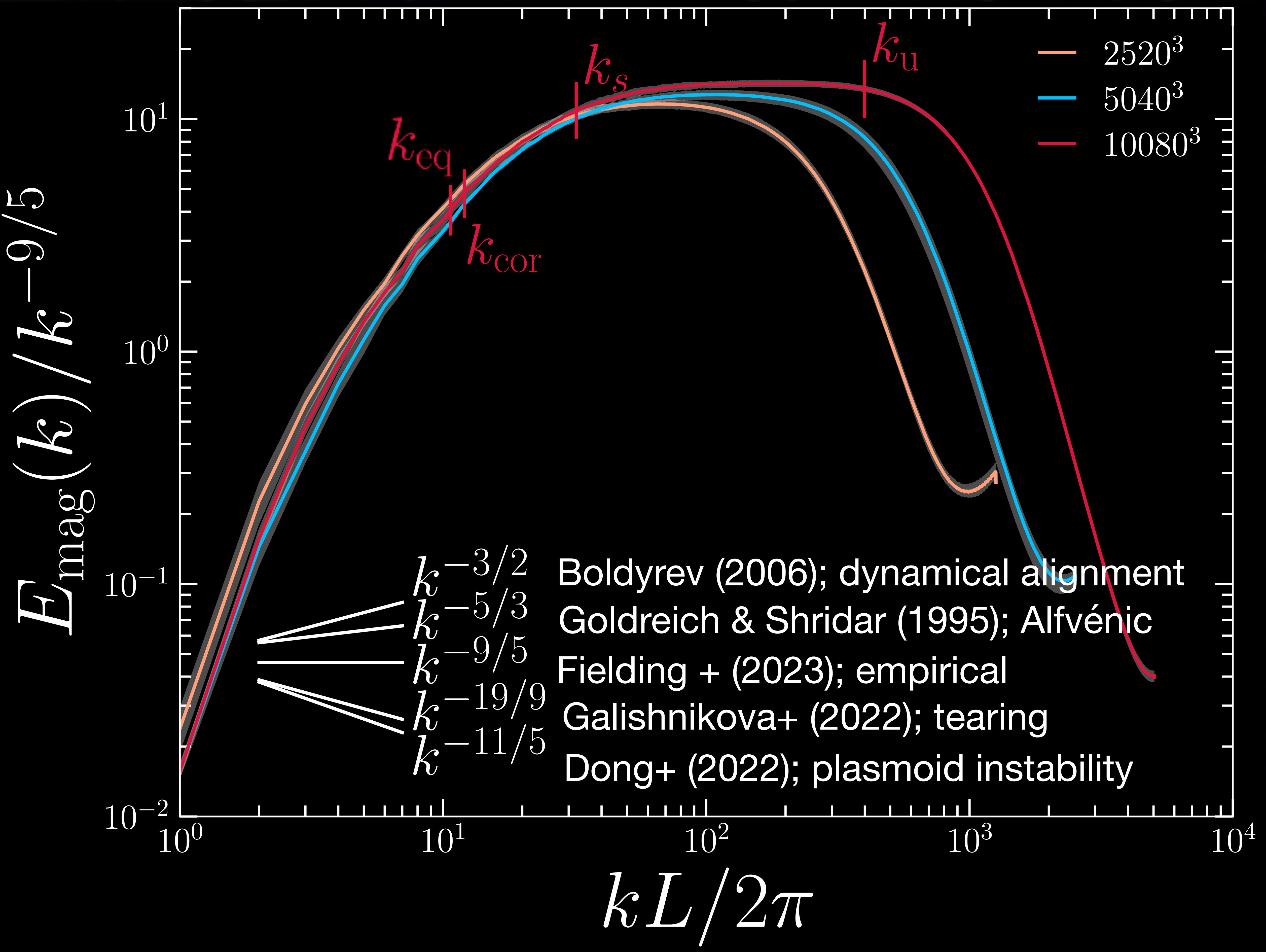
***Beattie**+(2024). Magnetized compressible turbulence with a fluctuation dynamo and Reynolds numbers over a million*

- **HIGH-RES: $10{,}080^3$ (80.0Mcore-h, 148,240cores)**
- **3.45PB of data products**
- **Factor of 4 higher linear grid resolution than IllustrisTNG**

Botteon+(2022) the merging cluster, Abell 2255

Magnetic cascade in saturated dynamo





Beattie+(2024). *Magnetized compressible turbulence with a fluctuation dynamo and Reynolds numbers over a million* arXiv:2405.16626

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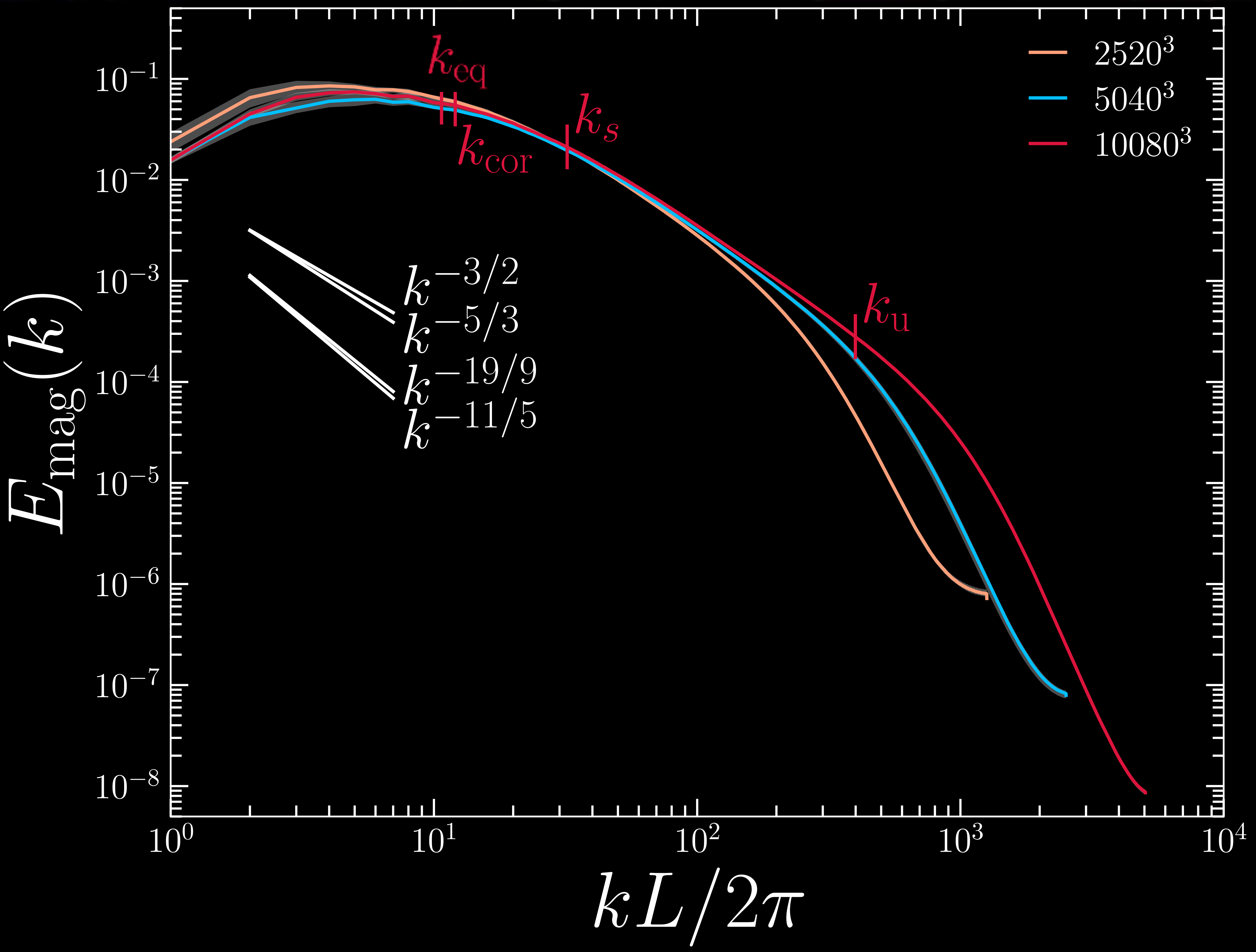
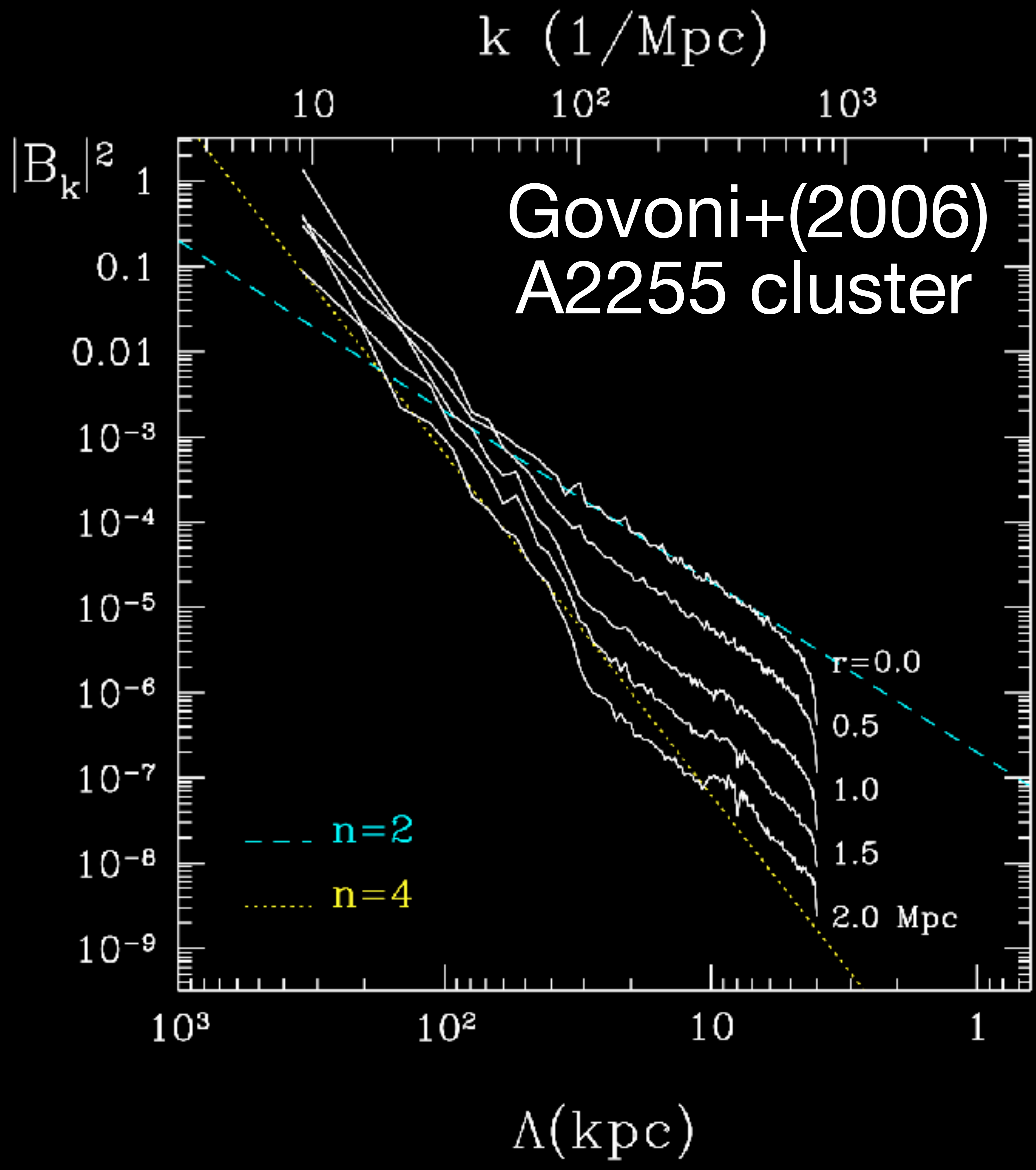
Magnetic cascade in saturated dynamo





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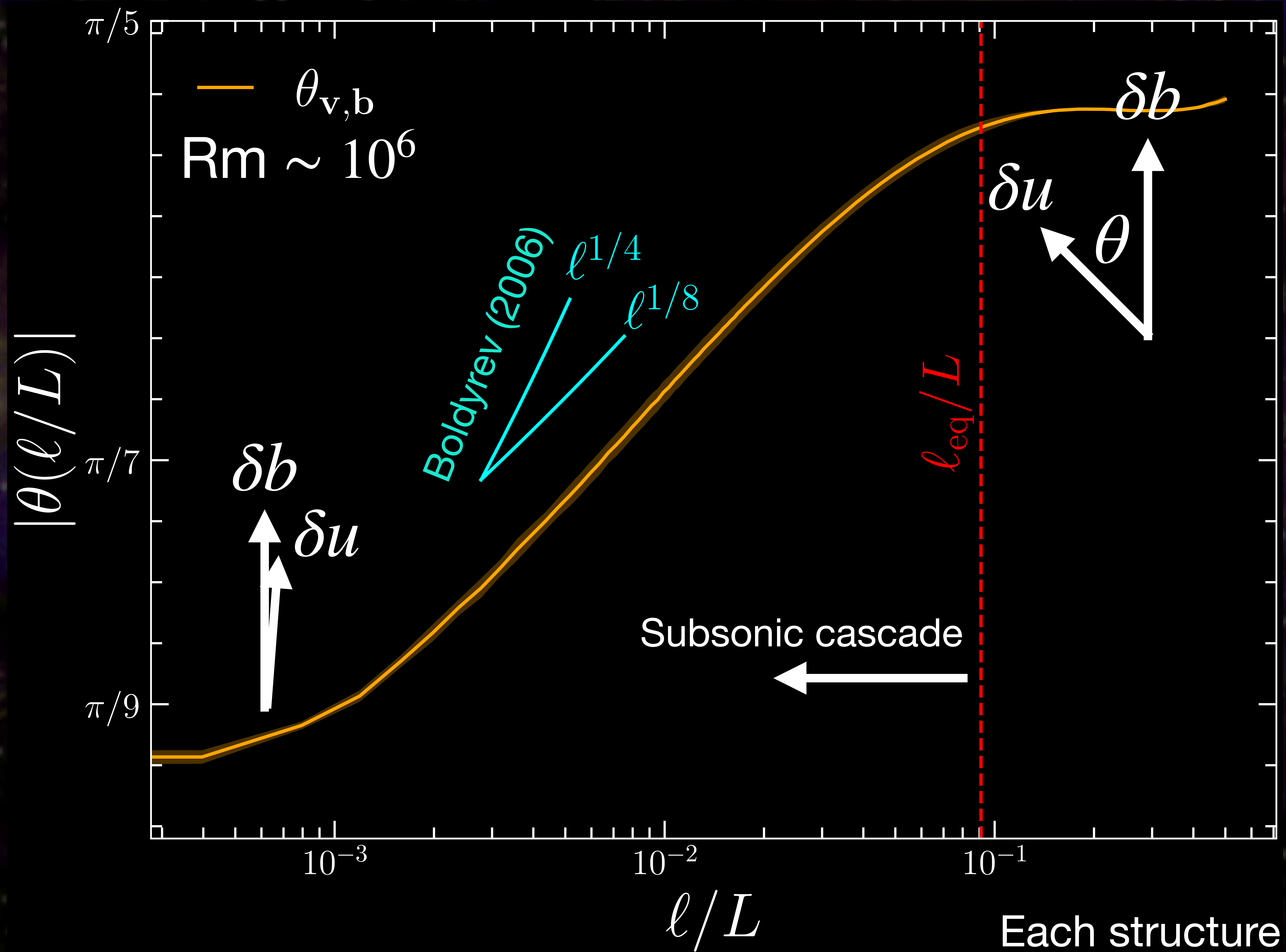
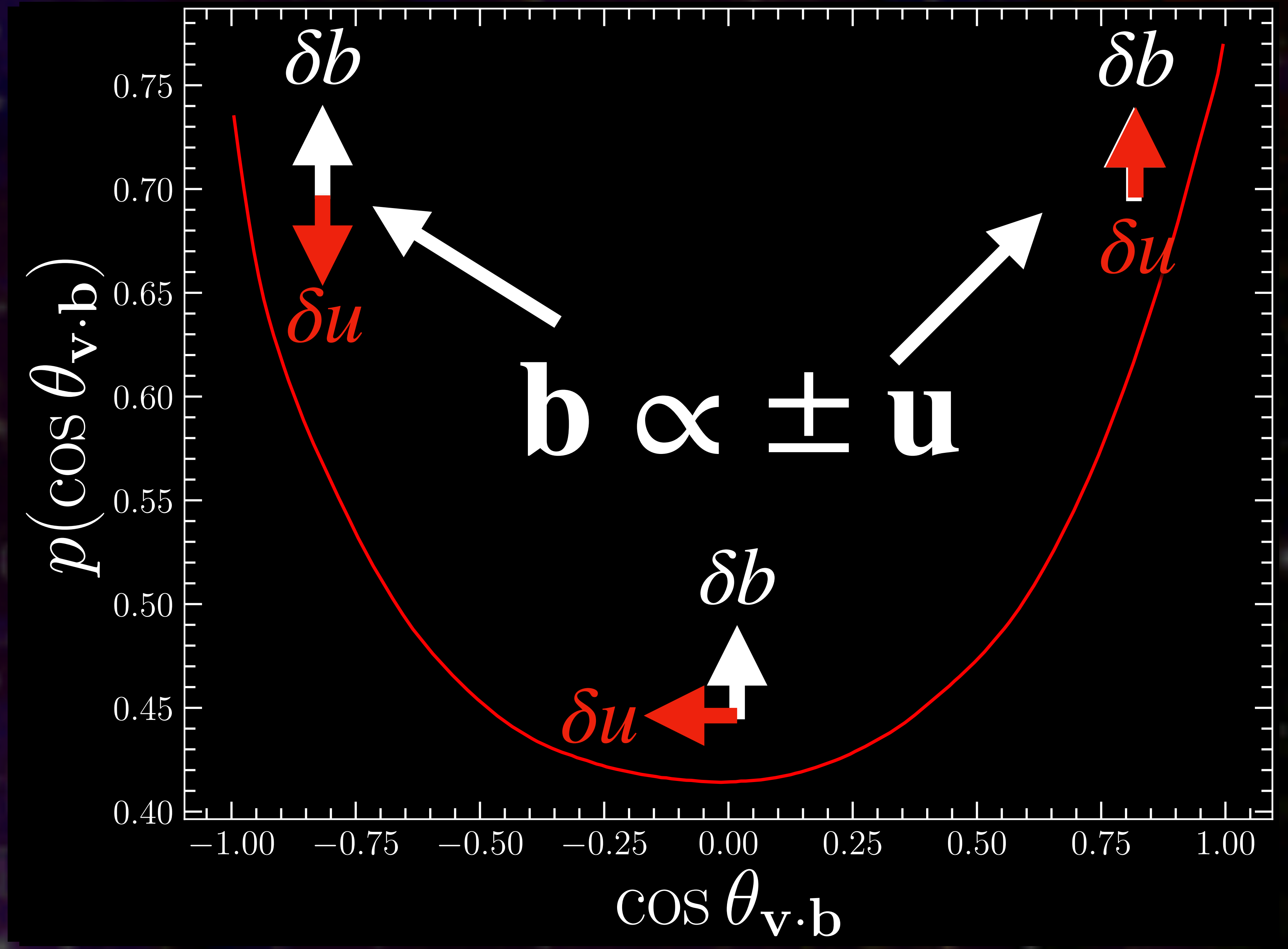
Botteon+(2022) the merging cluster, Abell 2255

Alignment in saturated dynamo

Beattie & Bhattacharjee *(in prep.). Scale dependent alignment in compressible magnetohydrodynamic turbulence*





We find

$$\theta(\ell) \sim \ell^{1/8}$$

scale-dependent alignment, but not Boldyrev (2006).

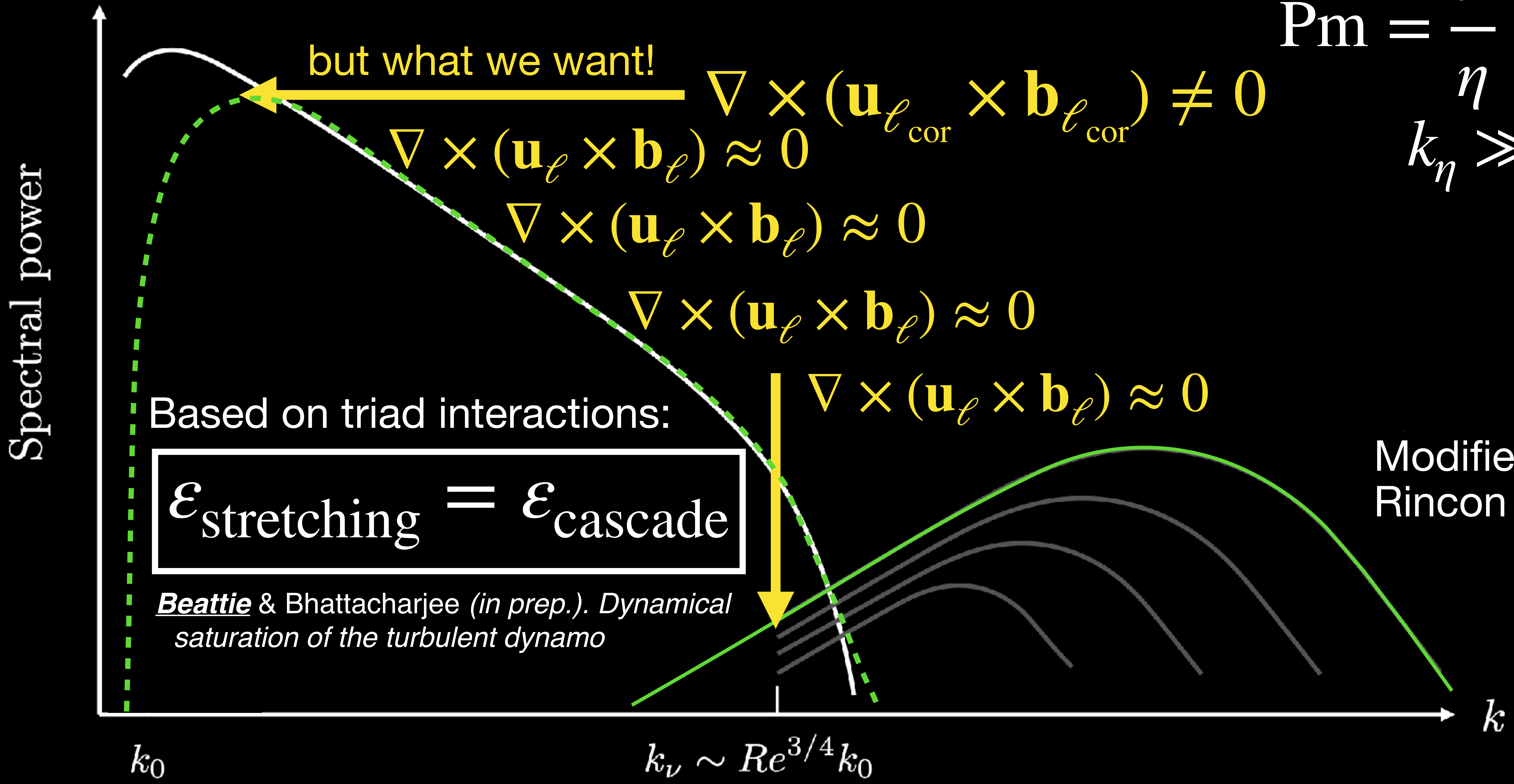
Each structure function costs 100,000 core hours!

The turbulent dynamo story

Saturation through alignment

$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

$$k_\eta \gg k_\nu$$



Modified from Rincon (2019)

Conclusions

1. Incompressible fast dynamo theories work well. Even capture some key features of the compressible fast dynamo (not all, e.g., growth rate is suppressed).

2. Scale-dependent alignment can turn off induction on small-scales, restricting magnetic flux generation to the largest scales, turning the Kazanstev spectrum into a more classical turbulent spectrum. Nothing needed other than transport.

Global simulators — help me test my model on realistic ICM!!!

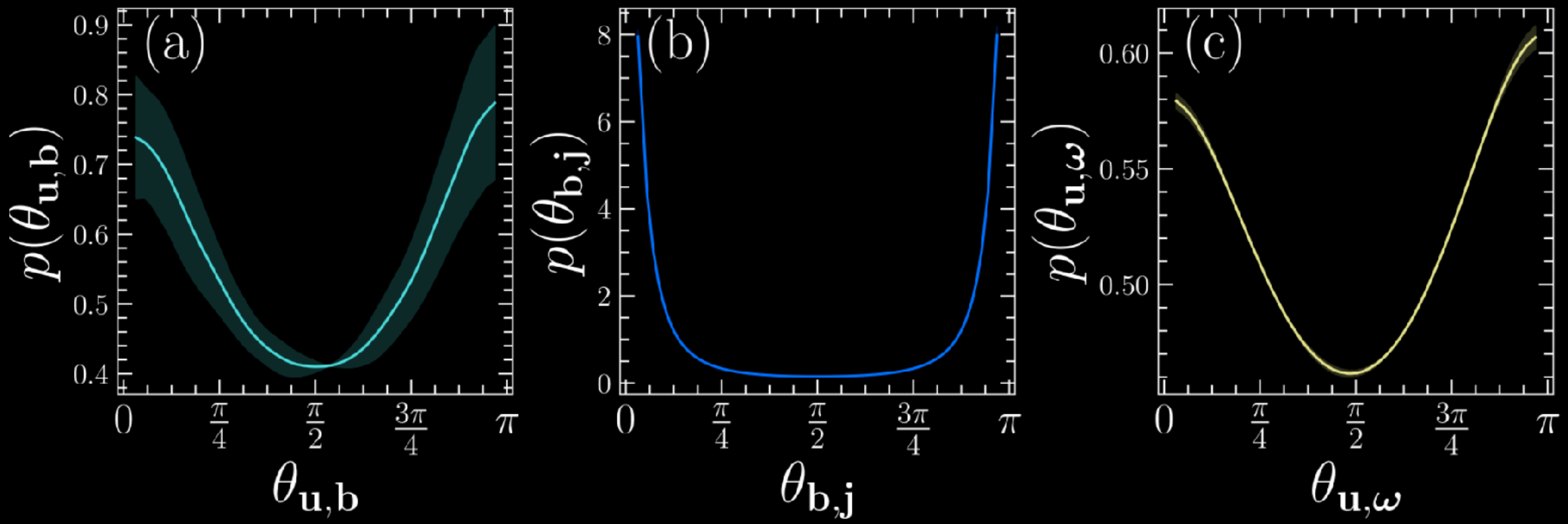
James Beattie. Magnetic reconnection in a turbulent fluid. Iterative LIC.

Extra slides

Botteon+(2022) the merging cluster, Abell 2255

But even more alignment than just u and b

Searching to weaken the nonlinearities



$$\nabla \times (\mathbf{u} \times \mathbf{b})$$

Induction

$$\mathbf{j} \times \mathbf{b}$$

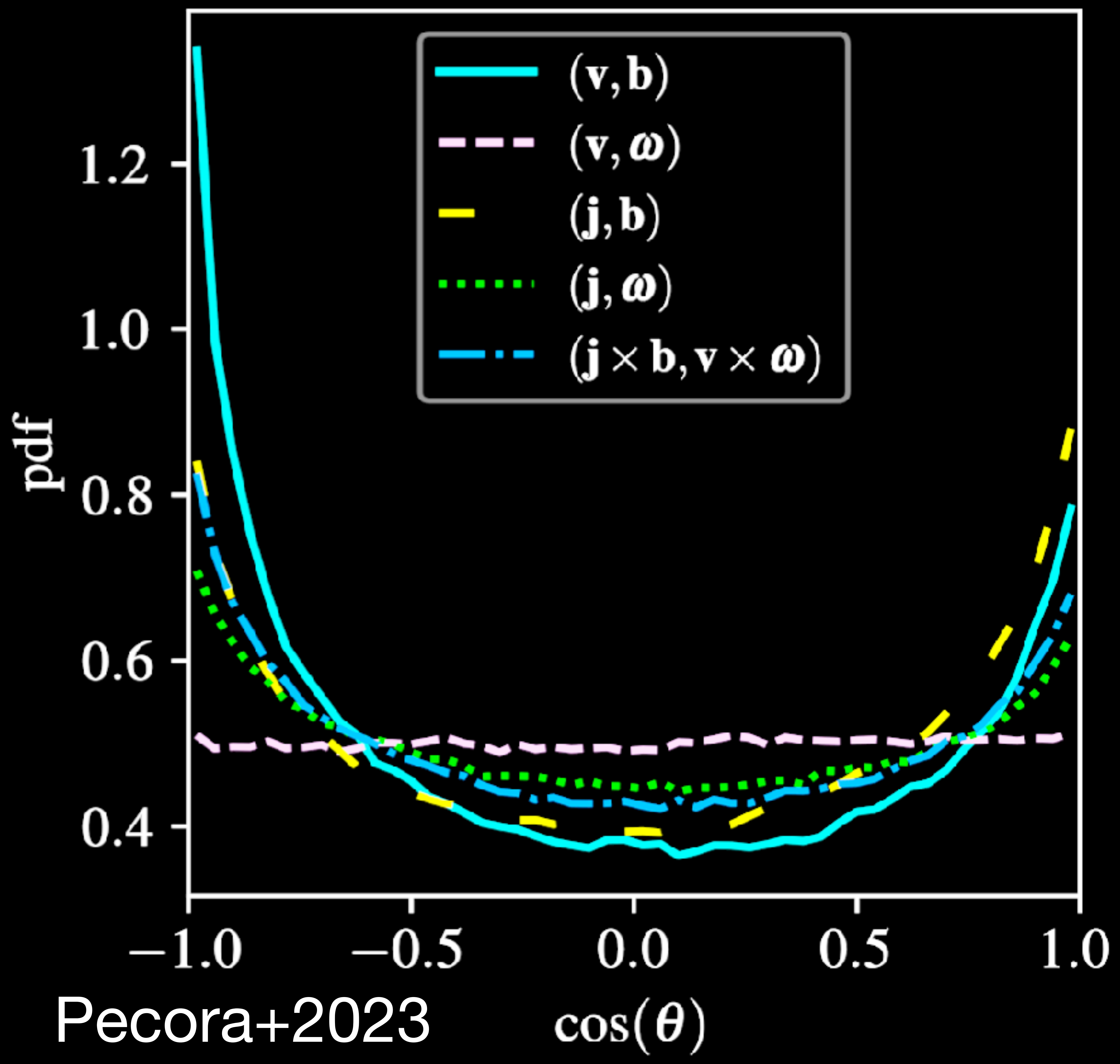
Lorentz force

$$\nabla \cdot (\mathbf{u} \otimes \mathbf{u}) \sim \boldsymbol{\omega} \times \mathbf{u}$$

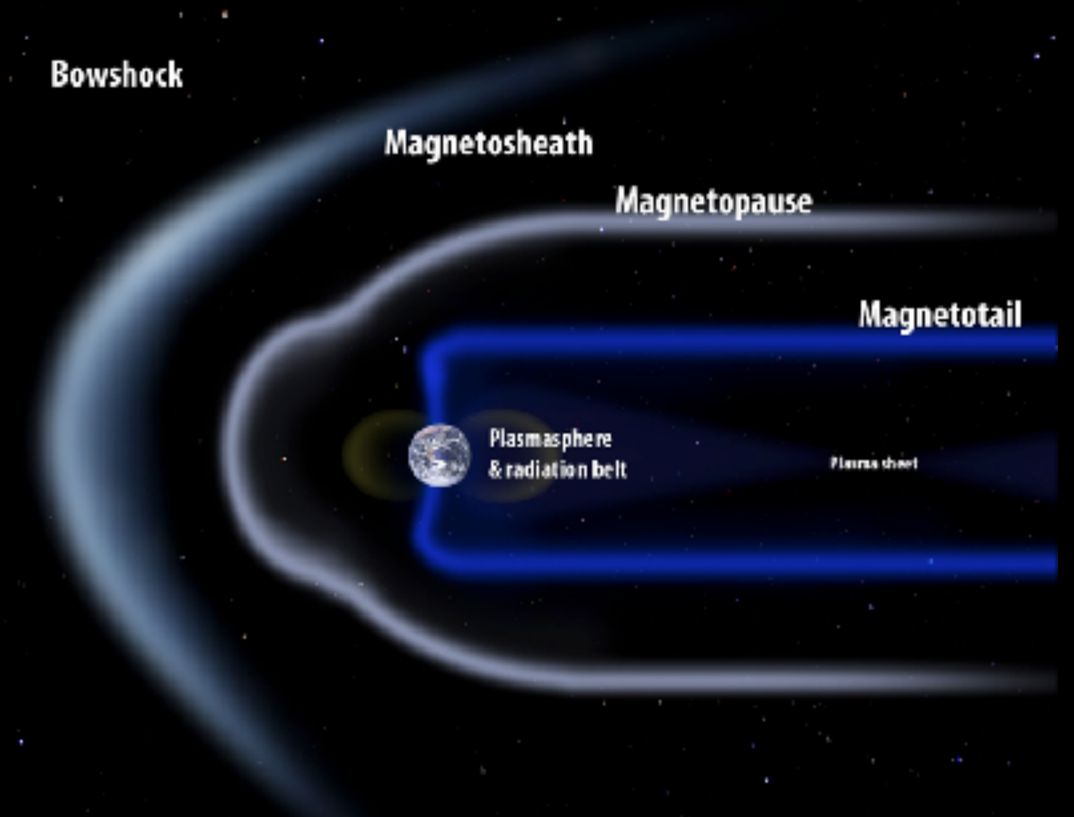
Reynolds nonlinearity

But even more alignment than just u and b

Searching to weaken the nonlinearities



Pecora+2023



Highly-aligned states in magneotsheath turbulence!

Magnetic Relaxation — main idea?
Searching to weaken the nonlinearities

$$H_m = \langle \mathbf{a} \cdot \mathbf{b} \rangle$$

$$H_c = \langle \mathbf{u} \cdot \mathbf{b} \rangle$$

Define constraint equation based on quadratic (ideal) MHD rugged invariants

total volume-integral energy

cross helicity

$$\mathscr{E} - \lambda_1 H_m - \lambda_2 H_c = \text{const}\,.$$

magnetic helicity

Use variational principle on magnetic energy eq., for perturber δ

$$\delta \iiint dV \, \left(\mathscr{E} - \lambda_1 H_m - \lambda_2 H_c \right) = 0,$$

Minimize to find a global minimum magnetic energy state.

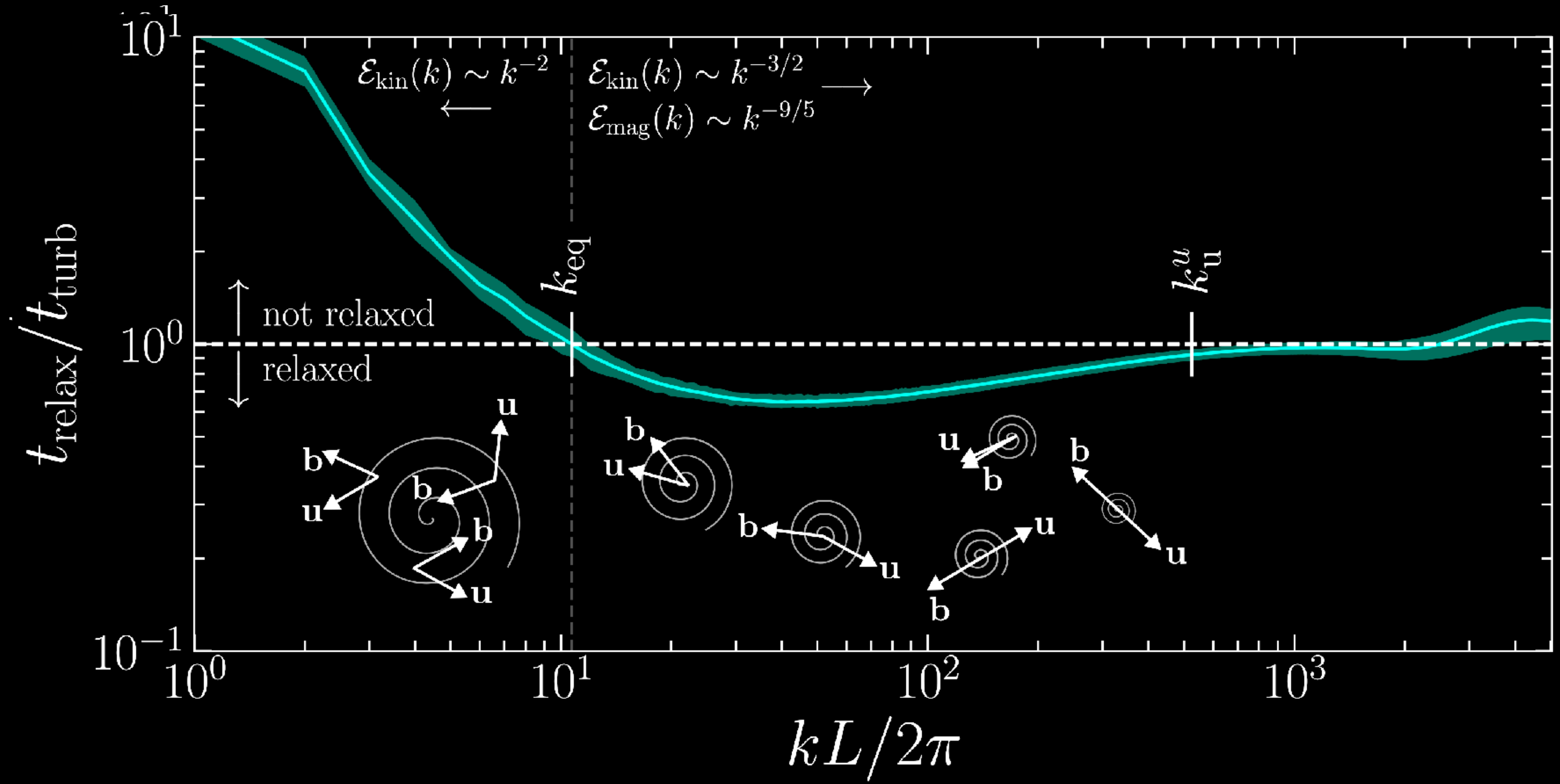
$$\mathbf{u} = \lambda_2 \mathbf{b} = \frac{\lambda_2 (1 - \lambda_1)^2}{\lambda_2} \mathbf{j} = \frac{(1 - \lambda_1)^2}{\lambda_2} \boldsymbol{\omega}$$

Banerjee+(2023)

Pecora+2023

Competitive relaxation: turbulence versus relaxation

Can we relax faster than the turbulence can perturb us away from minimum energy?



Inevitability of the turbulent dynamo

Saturation through alignment

$$\mathbf{k}' \xrightarrow{\mathbf{k}''} \mathbf{k}'''$$

Alignment implies a perfect balance between dynamo and cascade energy fluxes

$$\underbrace{\mathbf{b}''' \otimes \mathbf{b}'' : \nabla \otimes \mathbf{u}'}_{\text{kinetic to magnetic energy transfer}} = \overbrace{\mathbf{b}''' \otimes \mathbf{u}'' : \nabla \otimes \mathbf{b}' + \frac{1}{2}\mathbf{b}' \otimes \mathbf{b}''' : (\nabla \cdot \mathbf{u}'')\mathbb{I}}^{\text{magnetic cascade terms}}$$