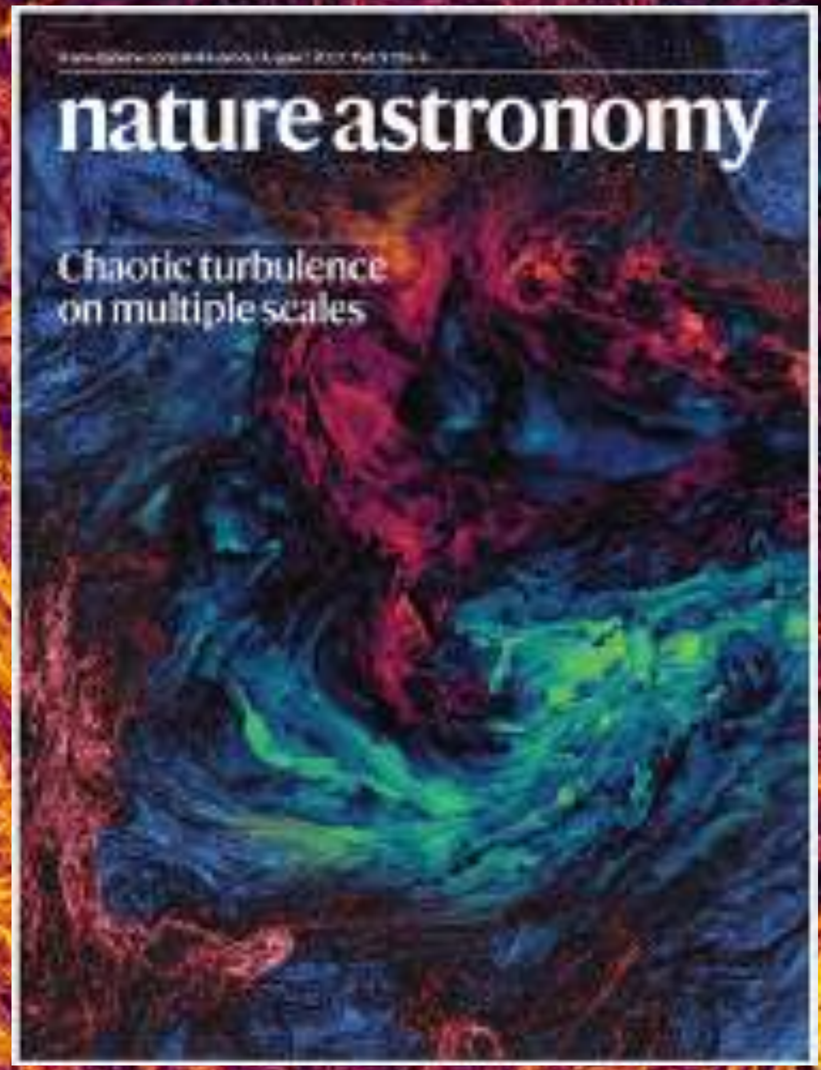

__Beattie__, Federrath, Klessen, Cielo, Bhattacharjee, 2025 (*Nature Astronomy*). The spectrum of magnetized turbulence in the interstellar medium

Why interstellar turbulence is non-Kolmogorov and how structures fit into turbulence phenomenology

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Scintillation 2025, Montreal

James Beattie
Post doc. Research Associate / Fellow
Princeton / Canadian Institute for Theoretical Astrophysics

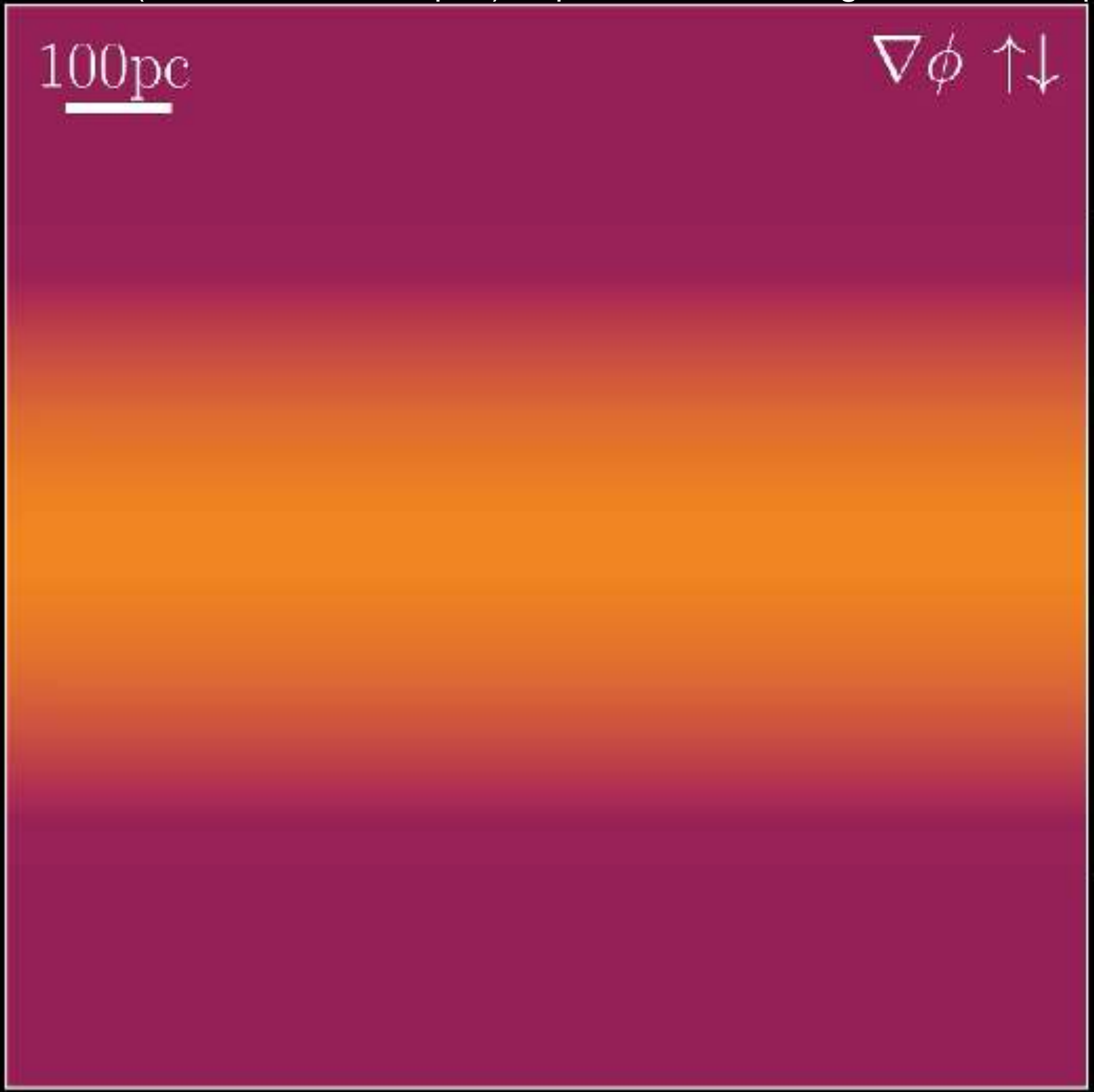
In collaboration: Amitava Bhattacharjee (Princeton), Christoph Federrath (ANU), Anna Tsai (UofT), Bart Ripperda (UofT), Reed Essick (UofT)

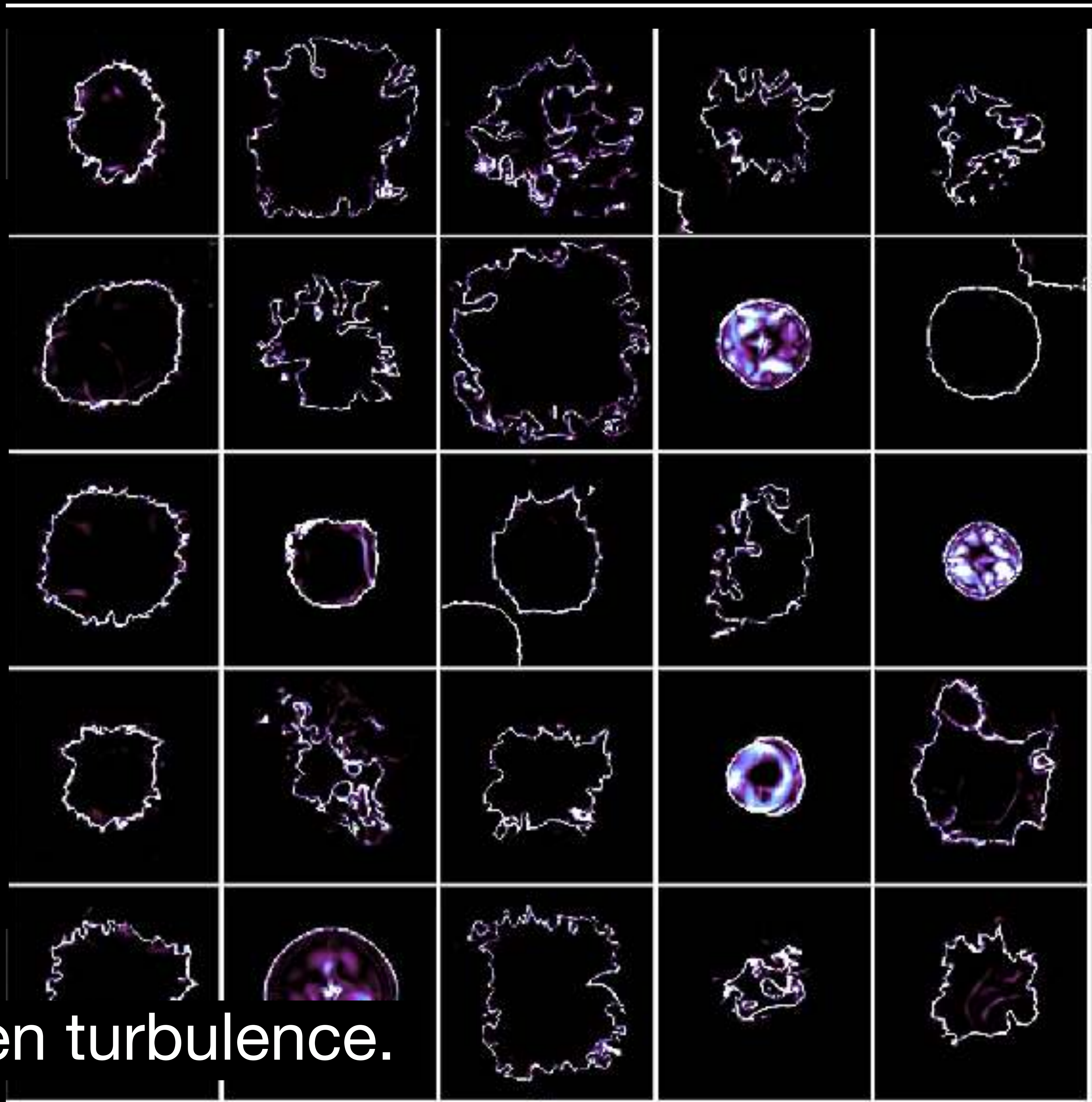
__Beattie__ + (2025; ApJ) So long Kolmogorov: the forward and backwards cascades in a supernovae-driven, multiphase ISM
Connor, **__Beattie__** + (2025, ApJ) Cascading from the winds to the disk: universality of supernovae-driven turbulence in different galactic ISMs
__Beattie__ (2025; submitted ApJL) Supernovae drive large-scale incompressible turbulence from small-scale instabilities
__Beattie__ *& Bhattacharjee (submitted PRL, 2025*). Scale-dependent alignment in compressible magnetohydrodynamic turbulence

I am a professional screen generator (unbeknownst to me until recently)

Beattie + (2025; ApJ) So long Kolmogorov: the forward and backwards cascades in a supernovae-driven, multiphase ISM

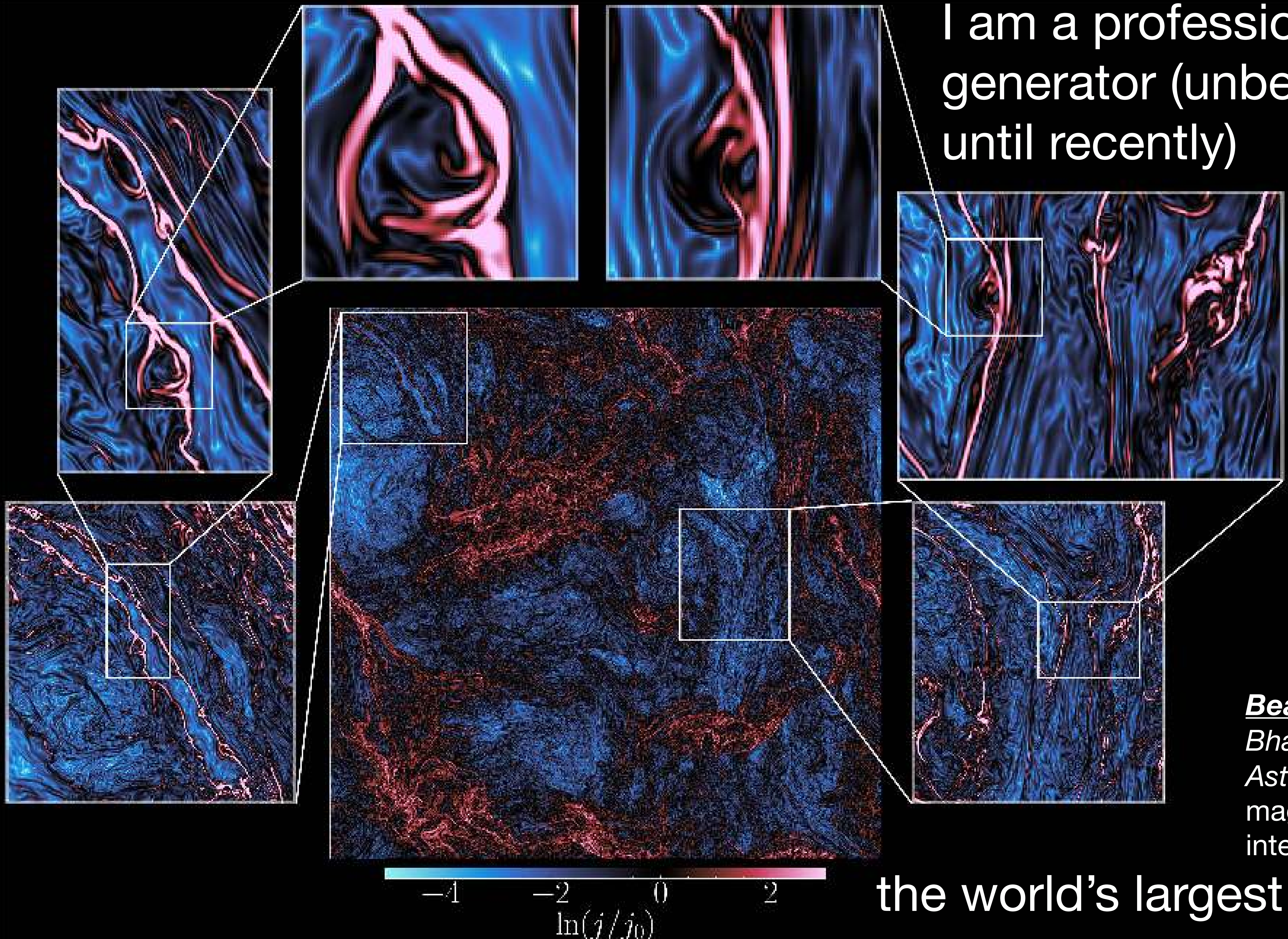
Beattie (2025; submitted ApJL) Supernovae drive large-scale incompressible turbulence from small-scale instabilities

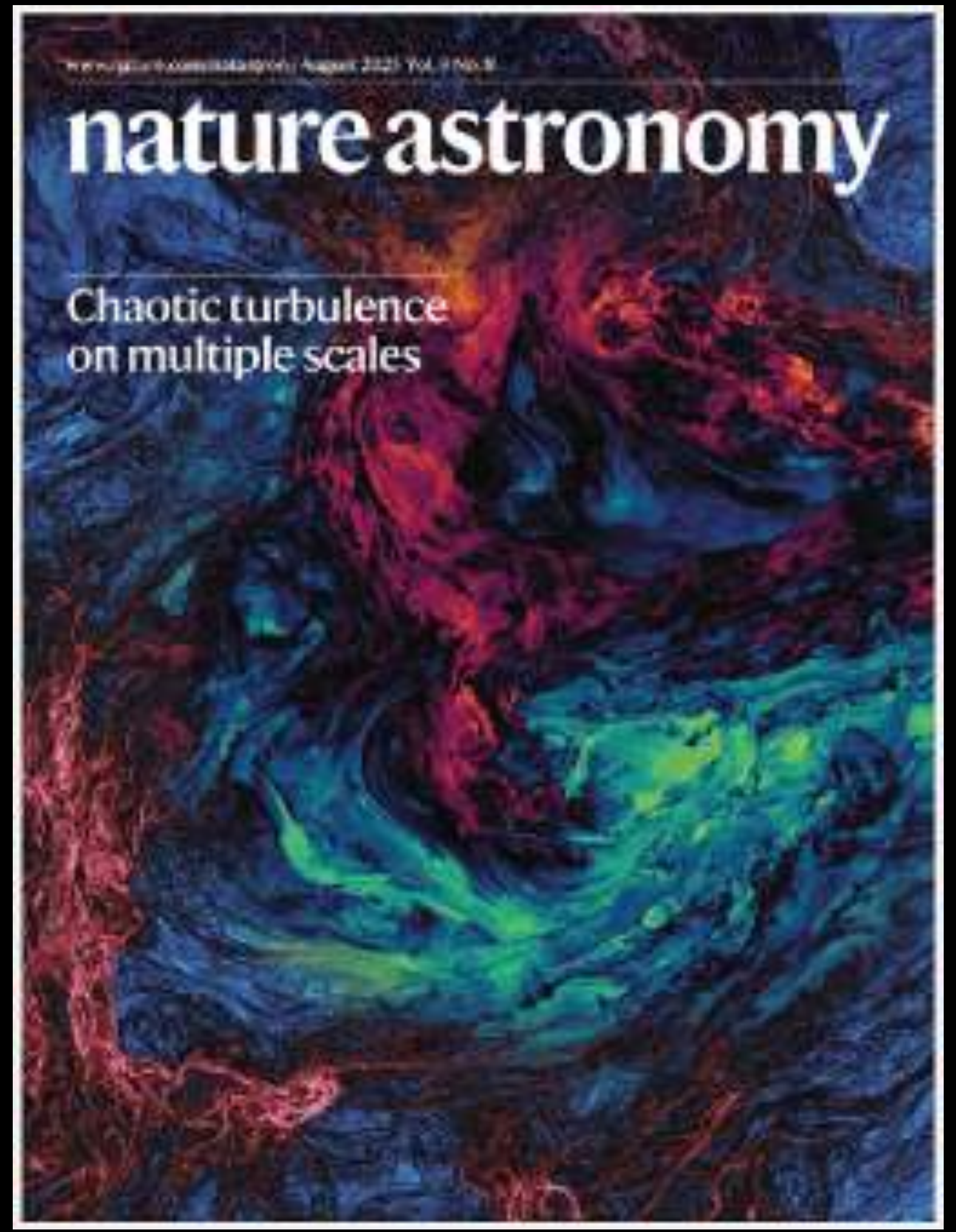




Galaxy cut-out simulations of SNe-driven turbulence.

I am a professional screen generator (unbeknownst to me until recently)





__Beattie__, *Federrath, Klessen, Cielo, Bhattacharjee*, 2025 (*Nature Astronomy*). The spectrum of magnetized turbulence in the interstellar medium

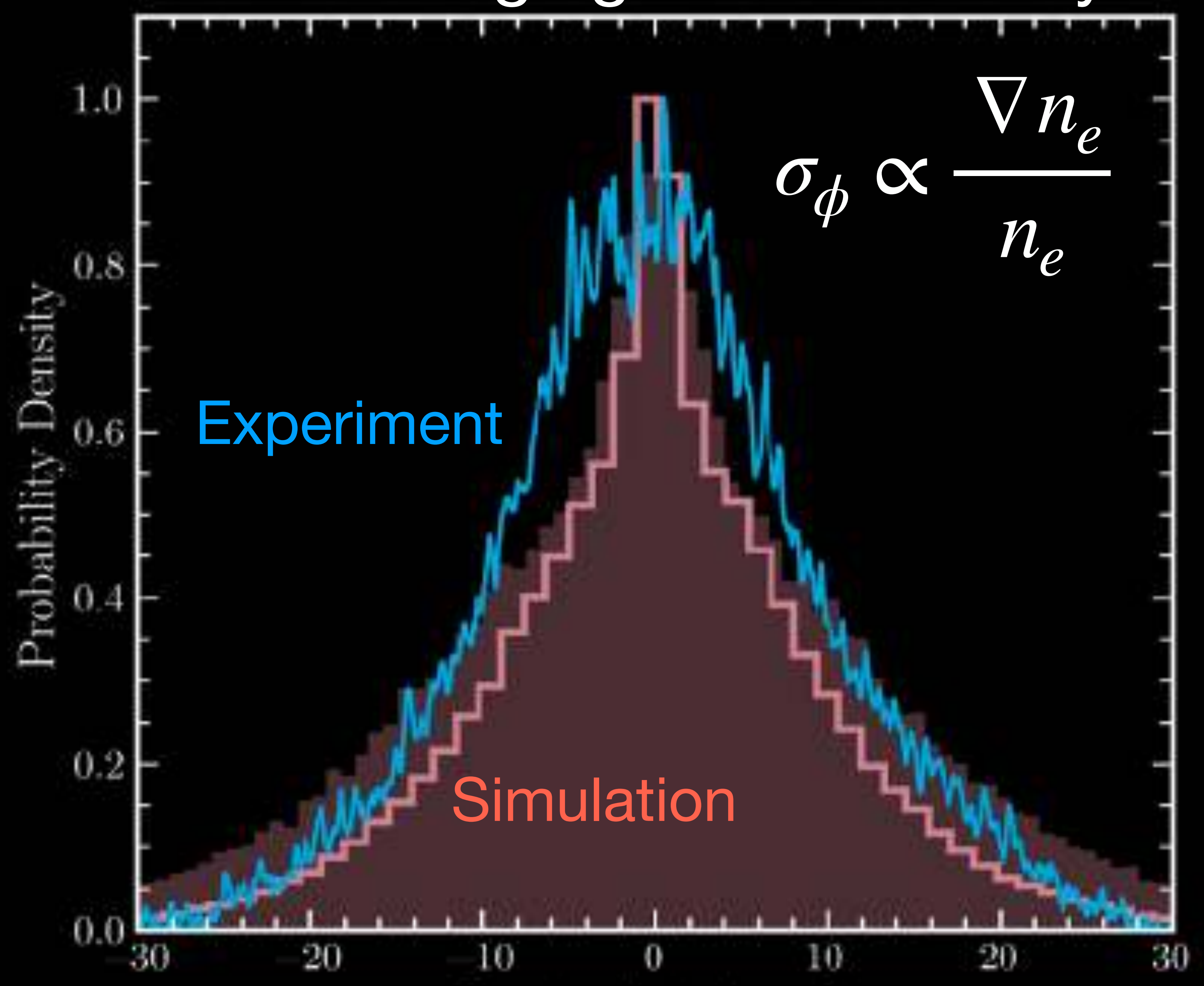
the world’s largest turbulence boxes

I am a professional screen generator (unbeknownst to me until recently)

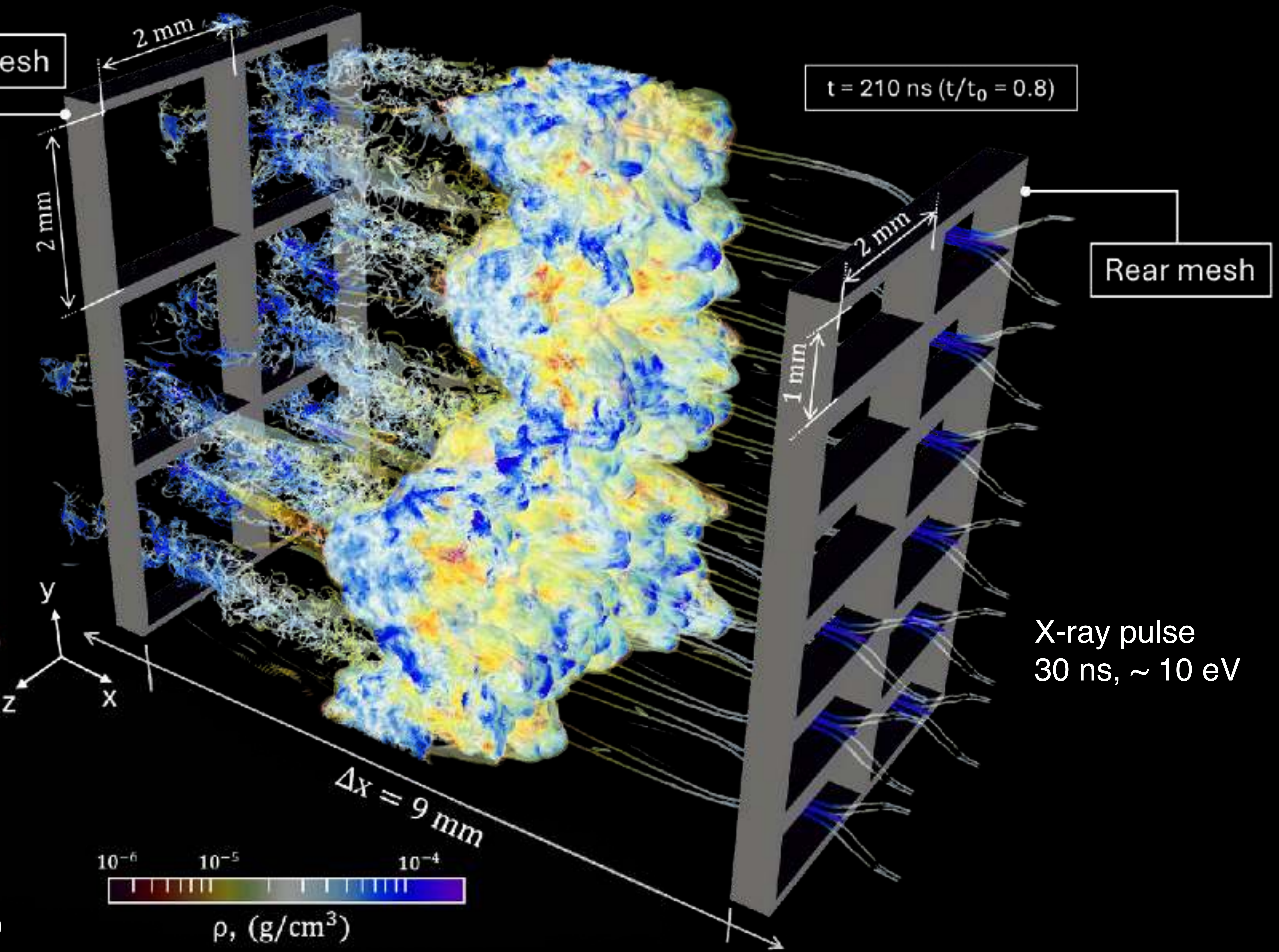
dynamo in KHI (for BNS mergers)

I am a professional screen generator (unbeknownst to me until recently)

Laser Imaging Refractometry



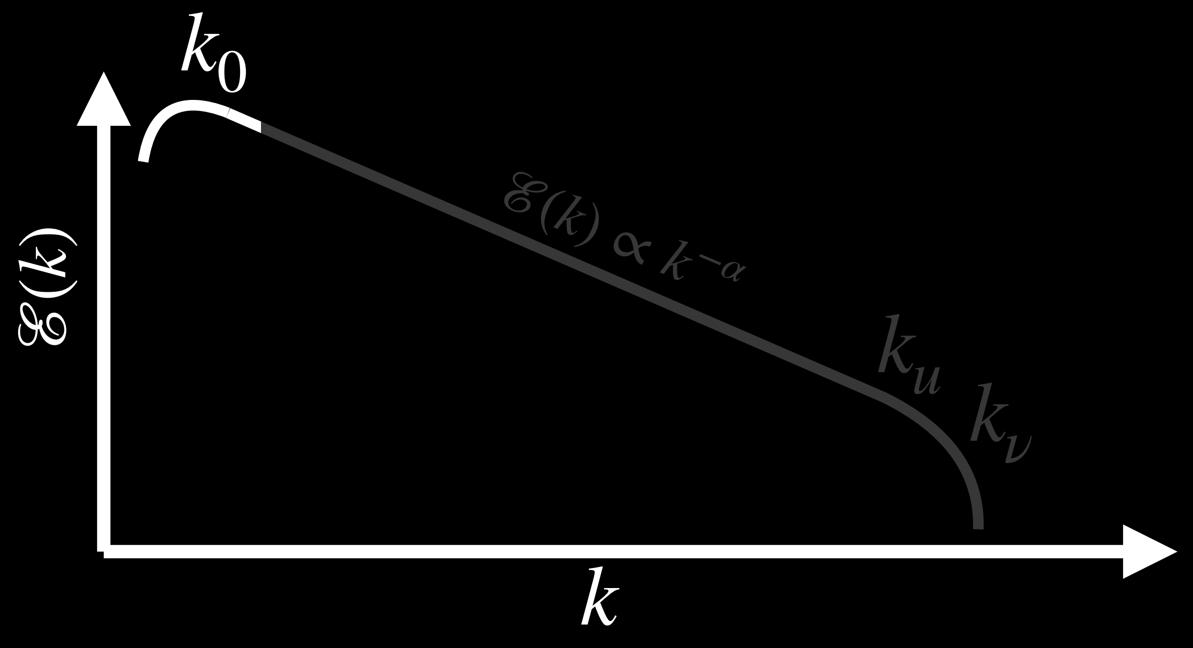
ϕ deflection angle



X-ray pulse
30 ns, ~ 10 eV

Merlini, **Beattie**, Valenzuela-Villaseca (in. prep)

Turbulence
Cascade



WIM: $\mathrm{Re} \sim 10^7$ $\lambda_{\mathrm{mfp}} \sim 5\ \mathrm{R}_\oplus$

WNM: $\mathrm{Re} \sim 10^7$ $\lambda_{\mathrm{mfp}} \sim 1.3\ \mathrm{AU}$

CNM: $\mathrm{Re} \sim 10^{10}$ $\lambda_{\mathrm{mfp}} \sim 0.3\ \mathrm{pc}$

Ferrière, 2020; Plasma Physics and Controlled Fusion

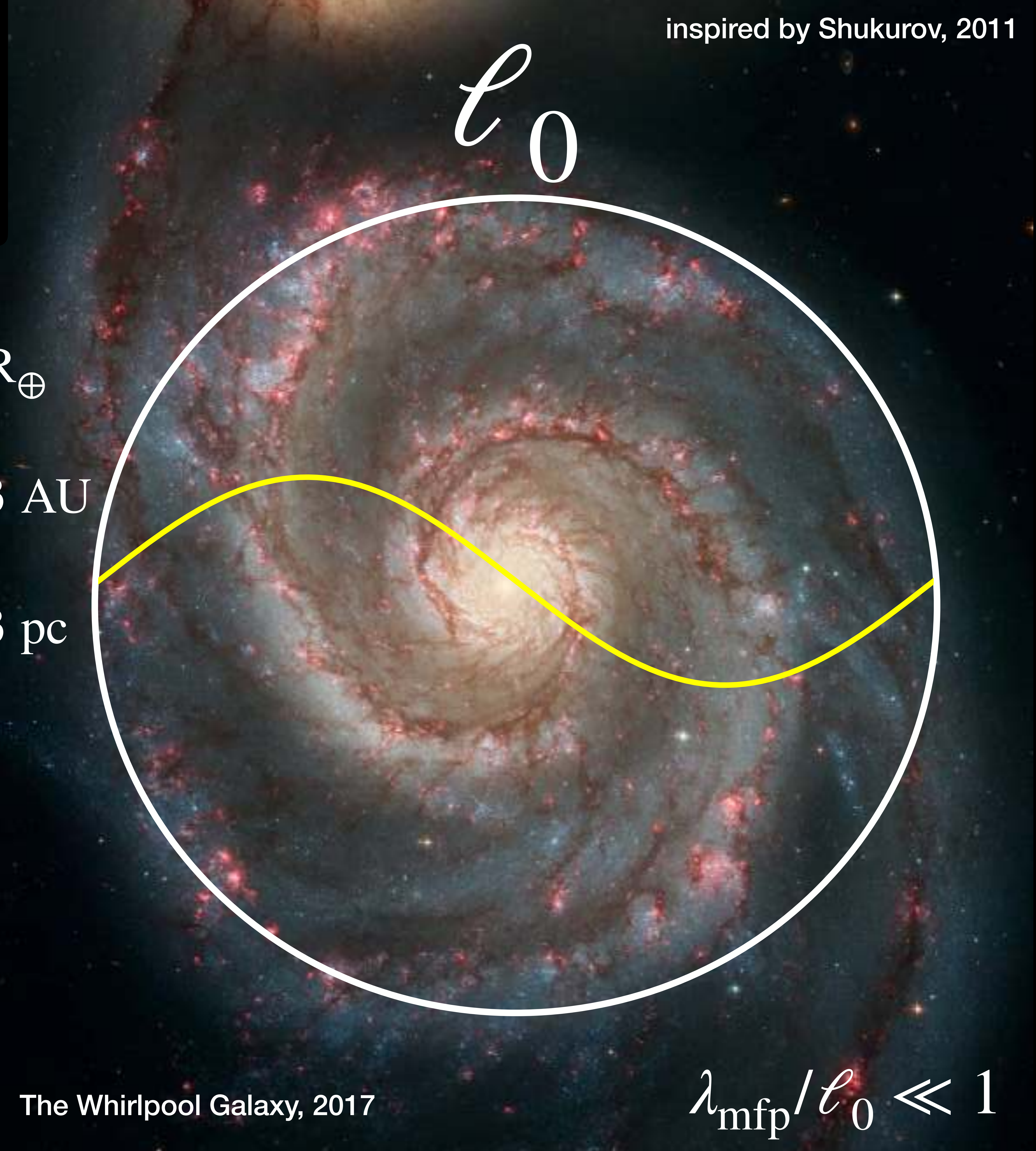
The quadratic nonlinear term

$$|\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})|$$

dominates on large scales, ℓ

Credit: NASA, ESA, S. Beckwith (STScI) and the Hubble Heritage Team (STScI/AURA)

inspired by Shukurov, 2011

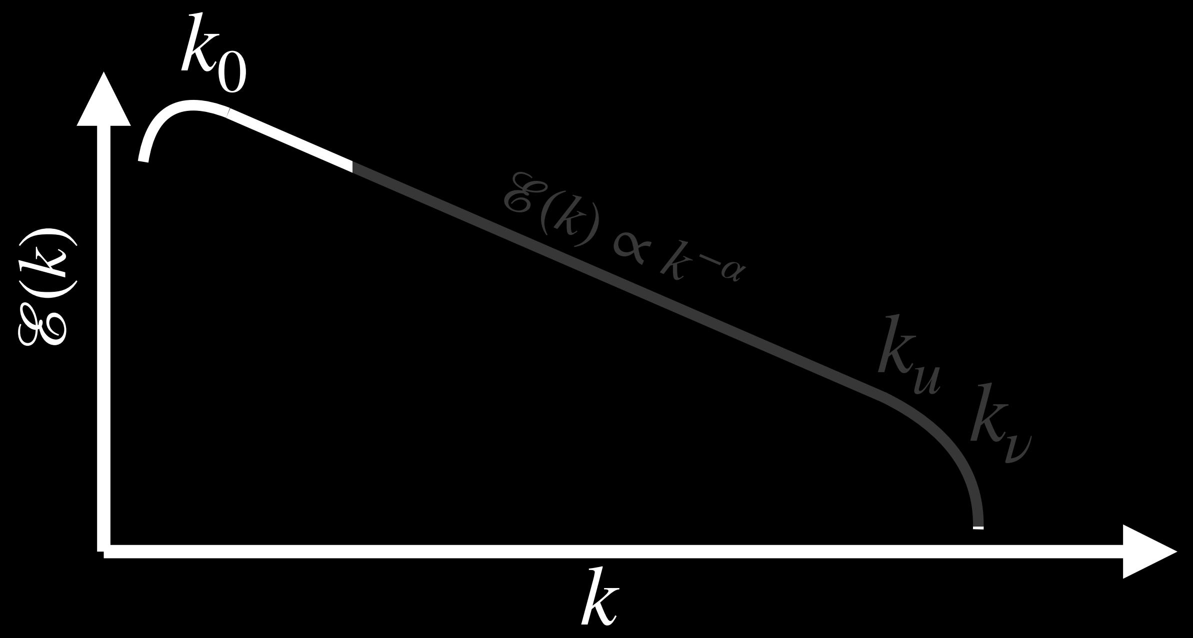


The Whirlpool Galaxy, 2017

$\lambda_{\mathrm{mfp}}/\ell_0 \ll 1$

Turbulence

Cascade



The quadratic nonlinear term

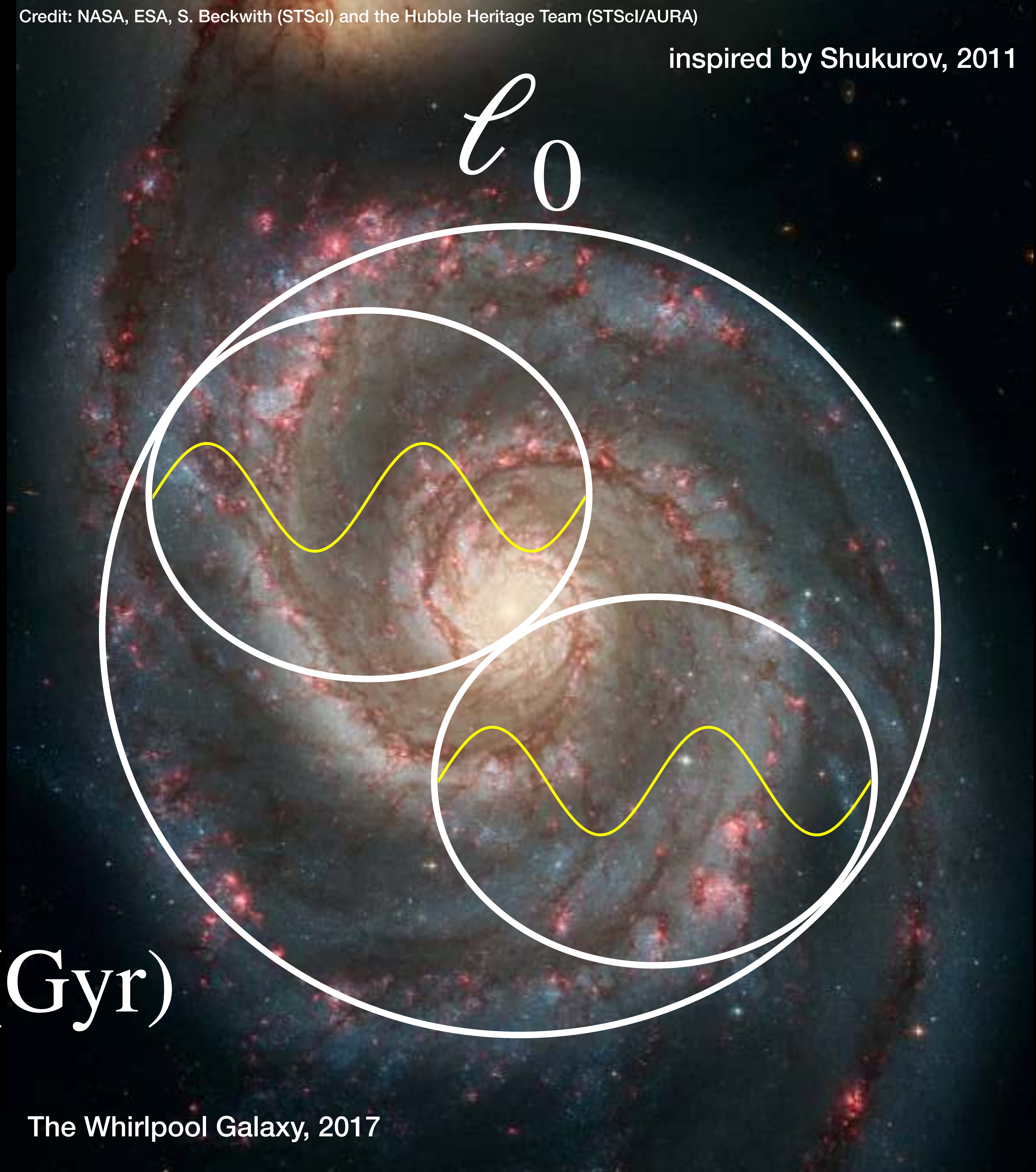
$$|\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})|$$

dominates on large scales, ℓ

Creates new modes on

$$t_{\rm nl} \sim [\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})]^{-1} \sim \mathcal{O}(\text{Gyr})$$

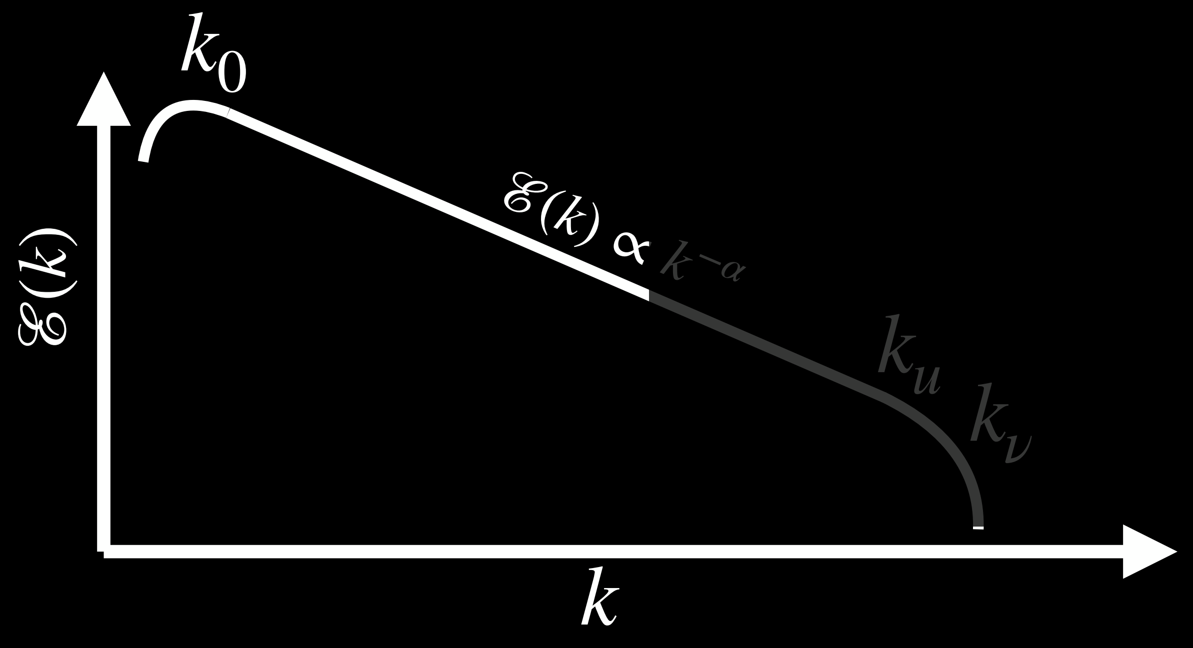
$$\dot{\varepsilon} \sim u_0^3/\ell_0$$



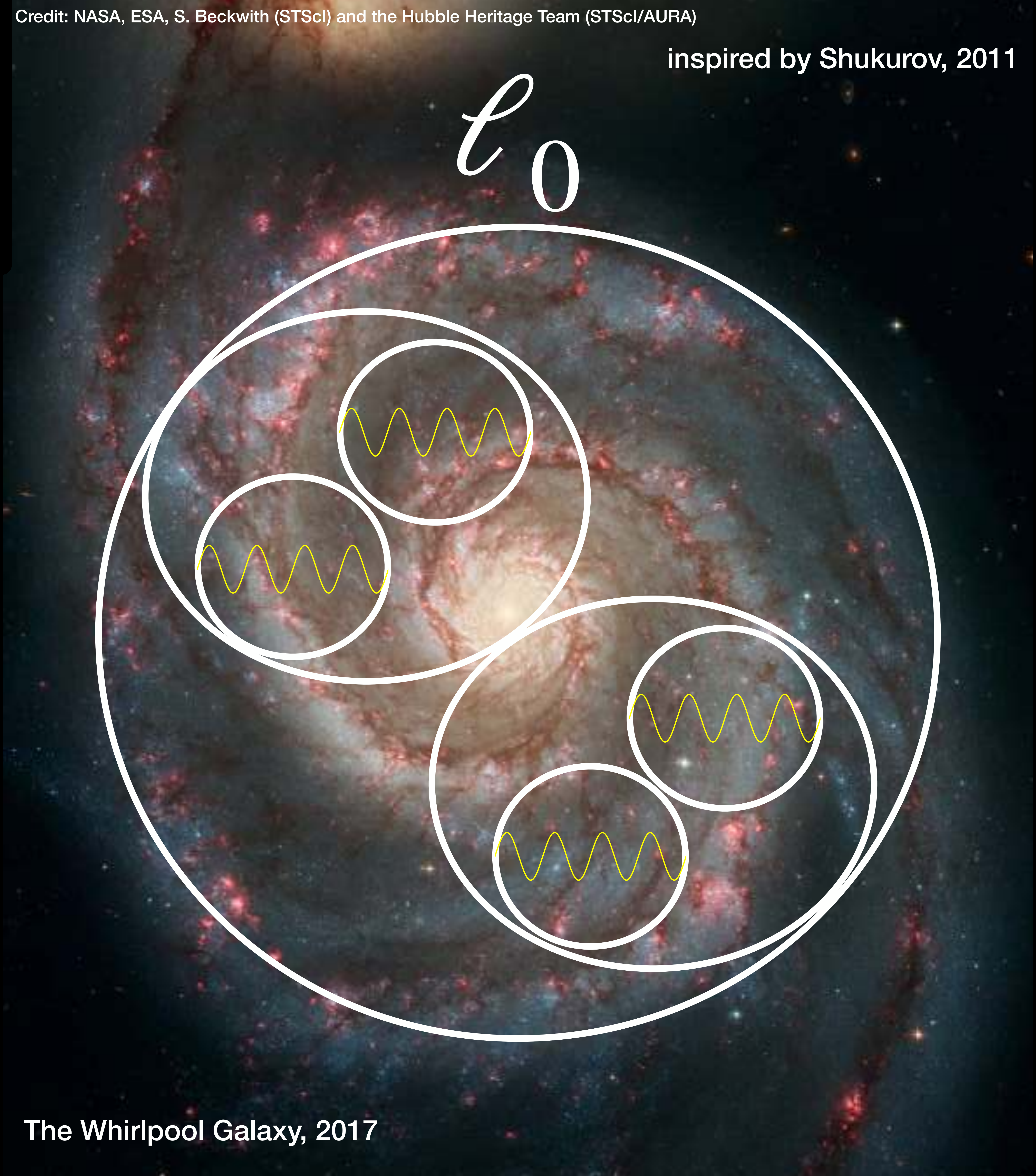
The Whirlpool Galaxy, 2017

Turbulence

Cascade

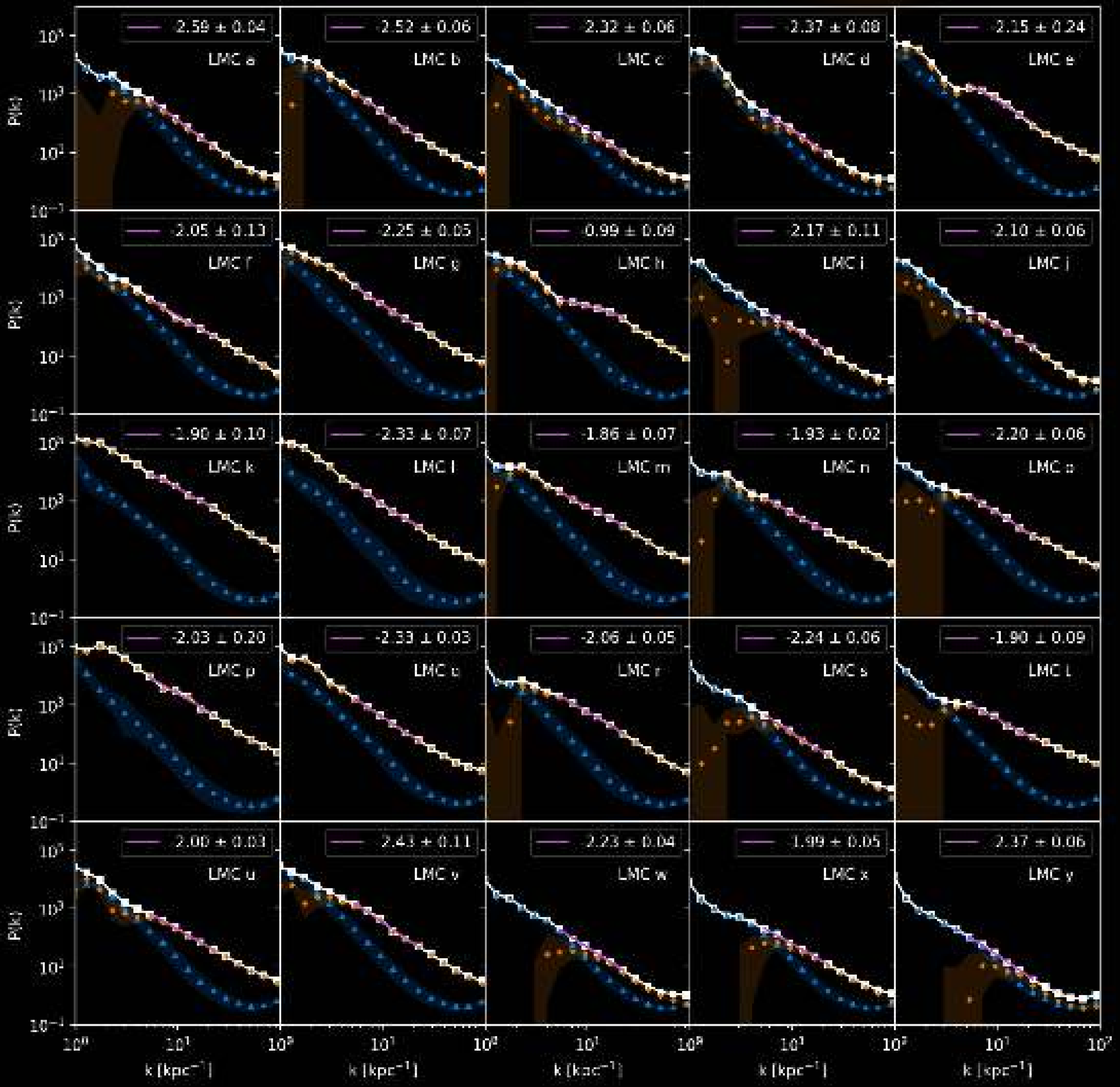


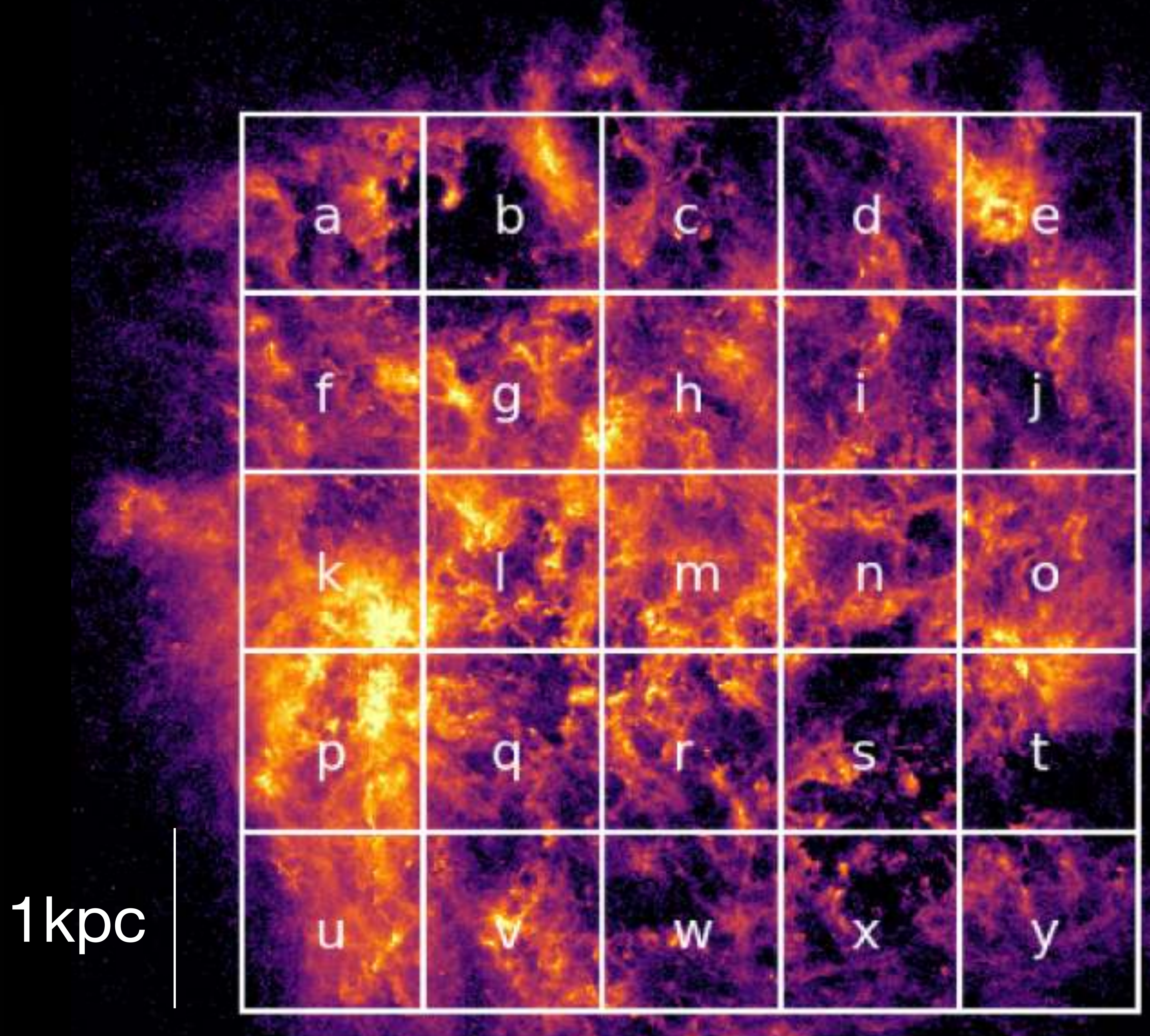
inside of the cascade



Turbulence

LMC: 500μm, *Herschel* (processing Gordon et al. 2014)

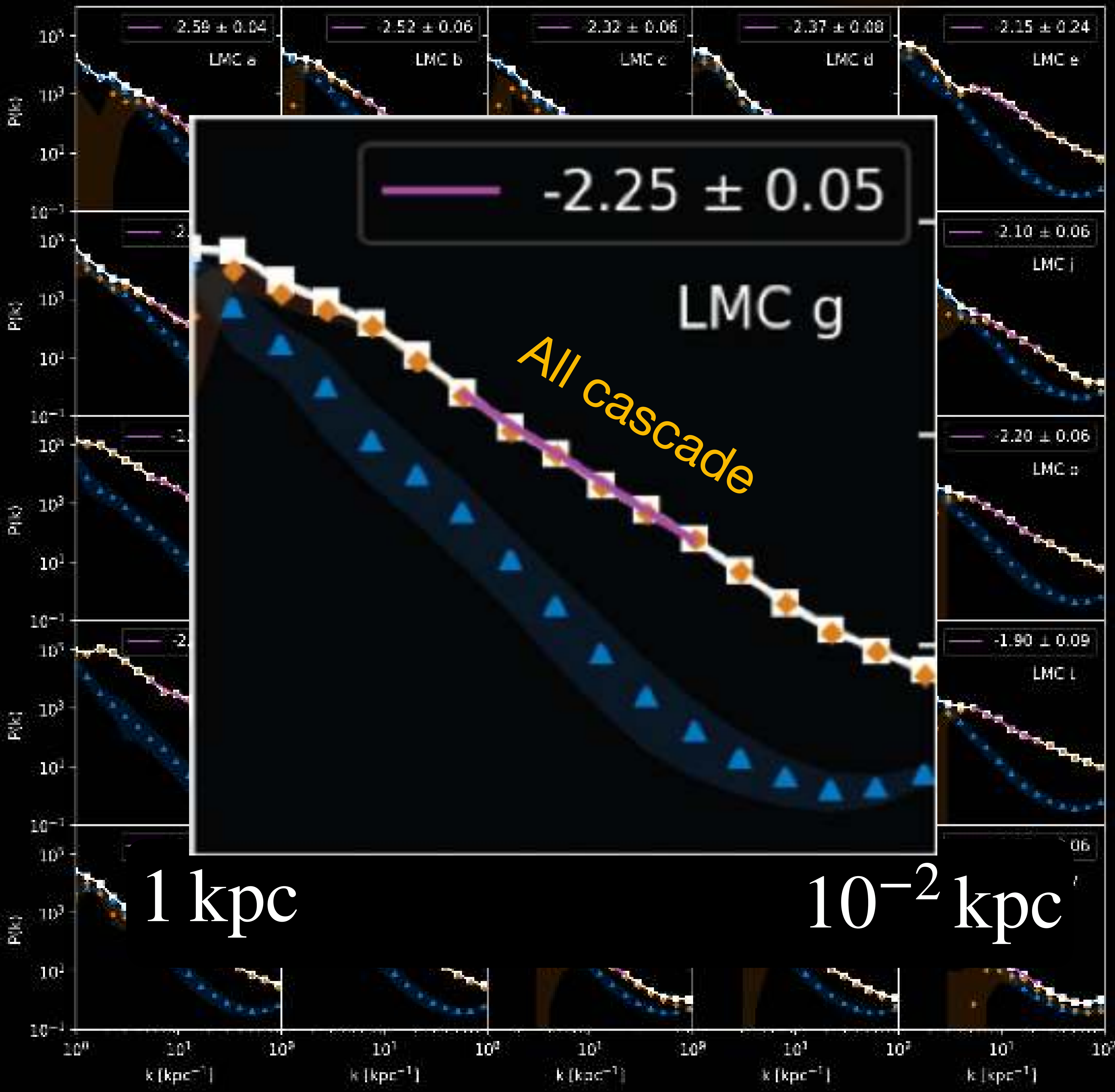


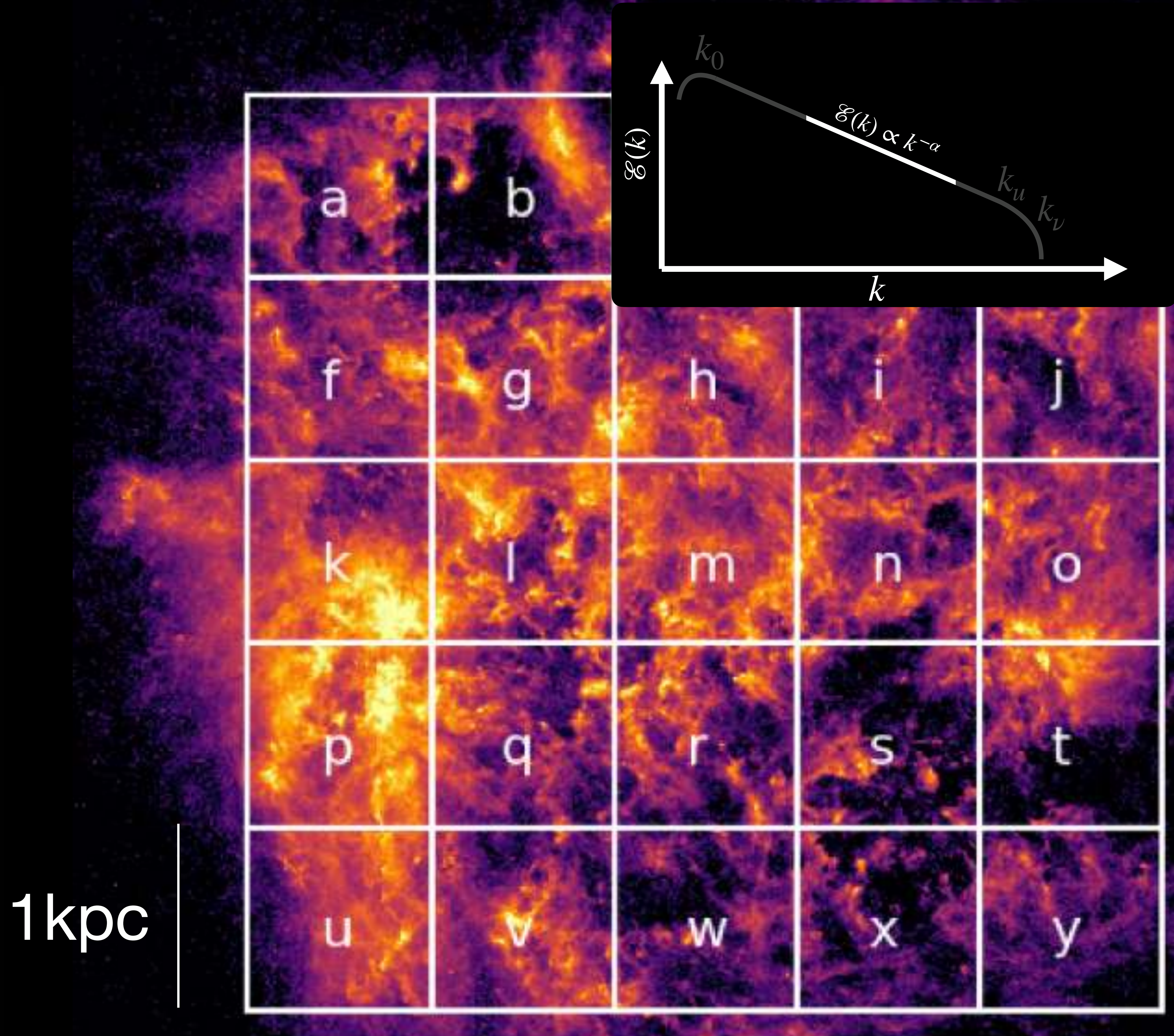


Colman, et al., 2022, MNRAS

Turbulence

LMC: 500μm, *Herschel* (processing Gordon et al. 2014)

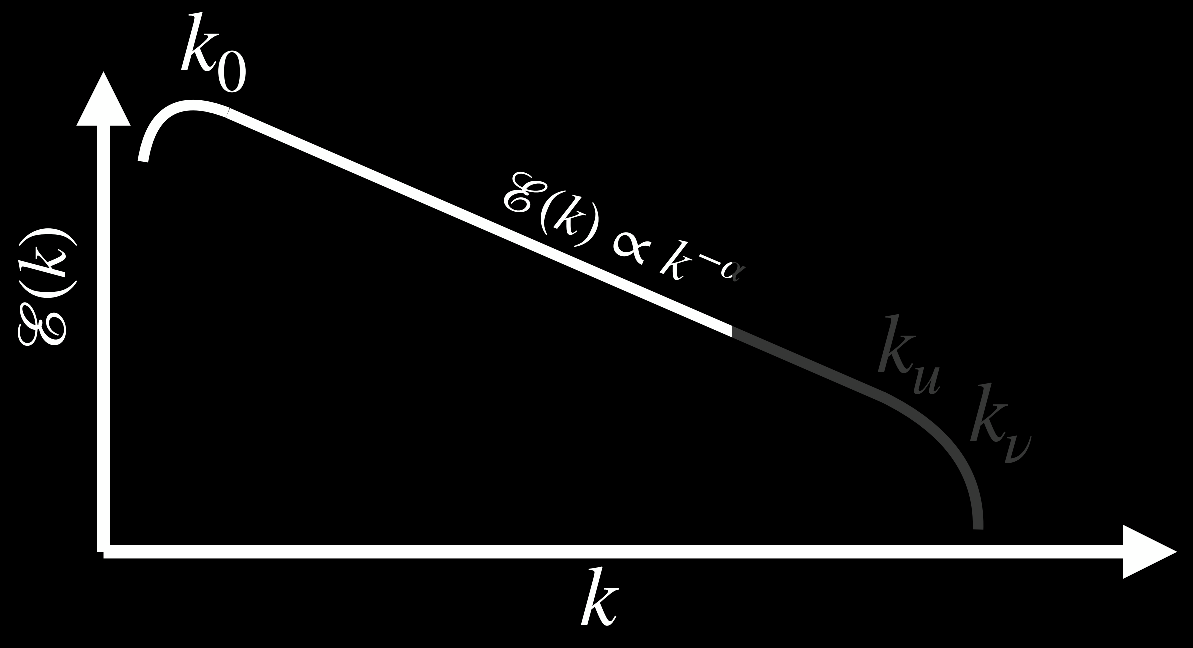




Colman, et al., 2022, MNRAS

Turbulence

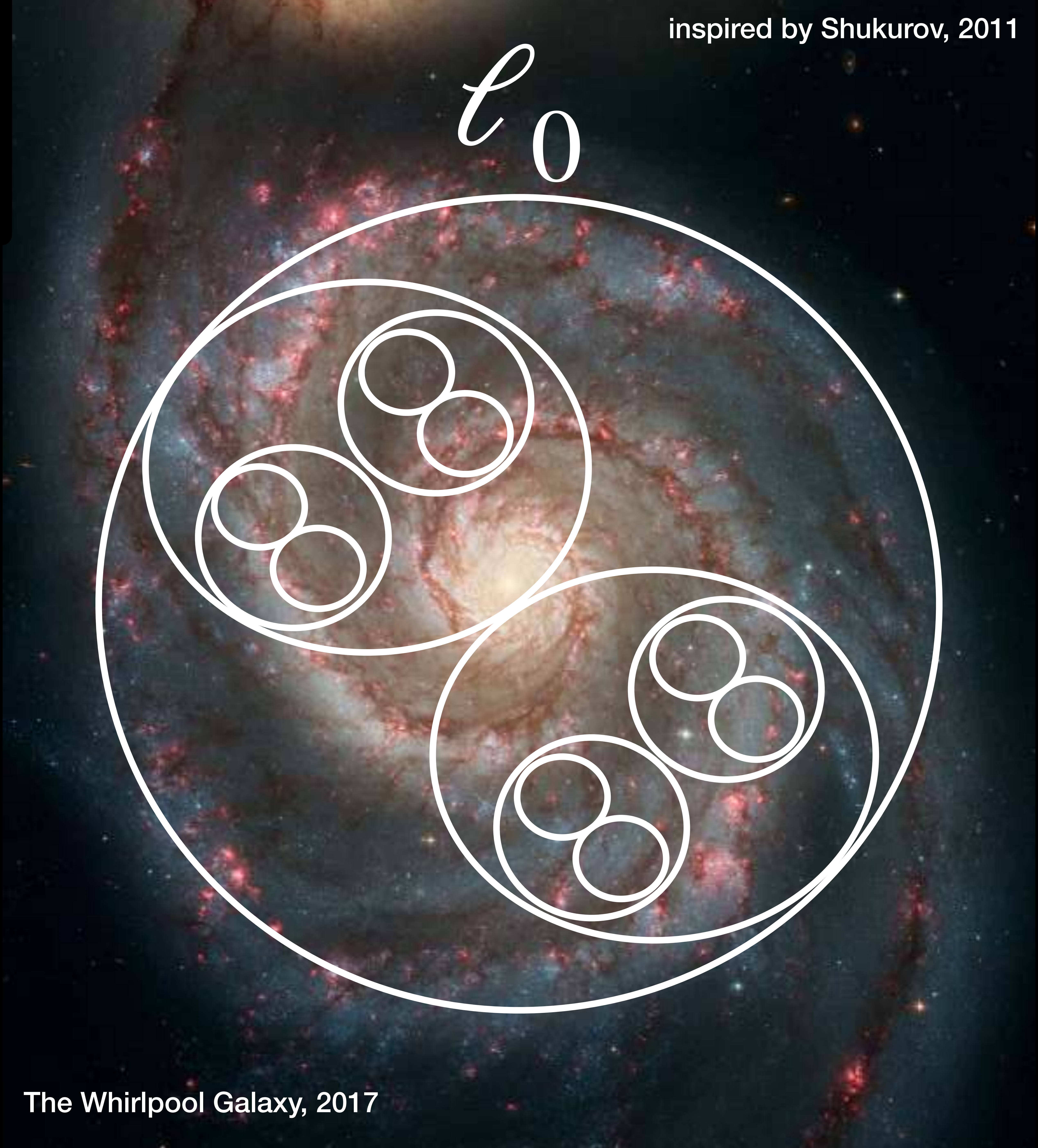
Cascade



deeper into
the cascade

Credit: NASA, ESA, S. Beckwith (STScI) and the Hubble Heritage Team (STScI/AURA)

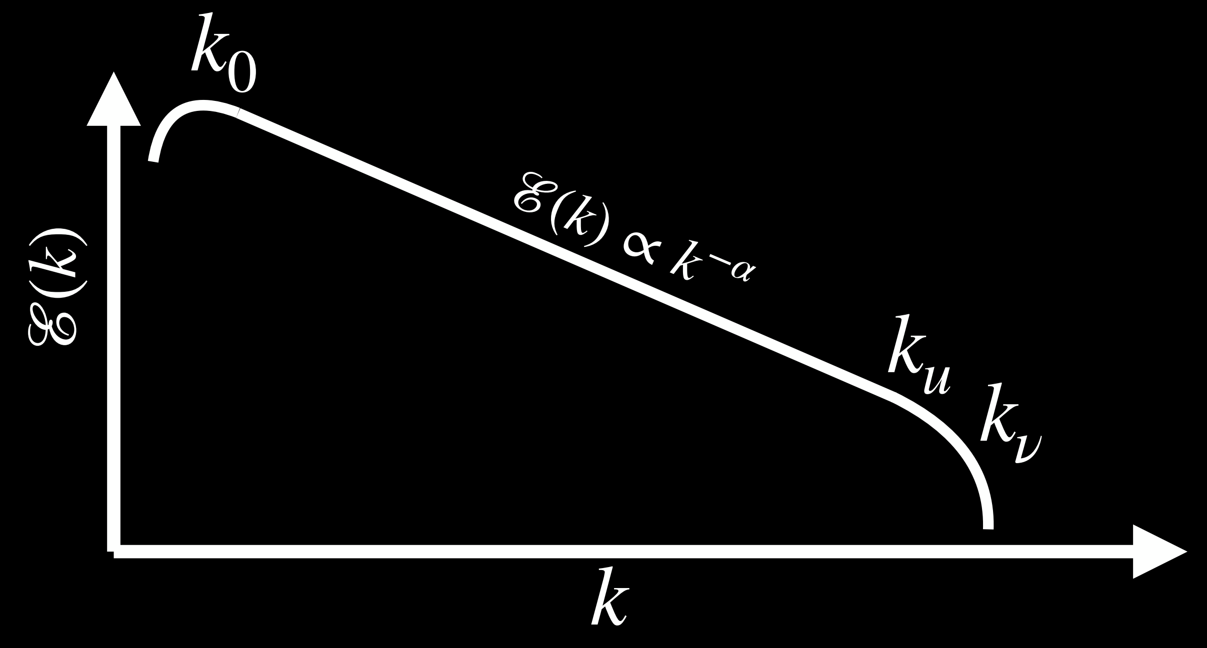
inspired by Shukurov, 2011



The Whirlpool Galaxy, 2017

Turbulence

Cascade



until

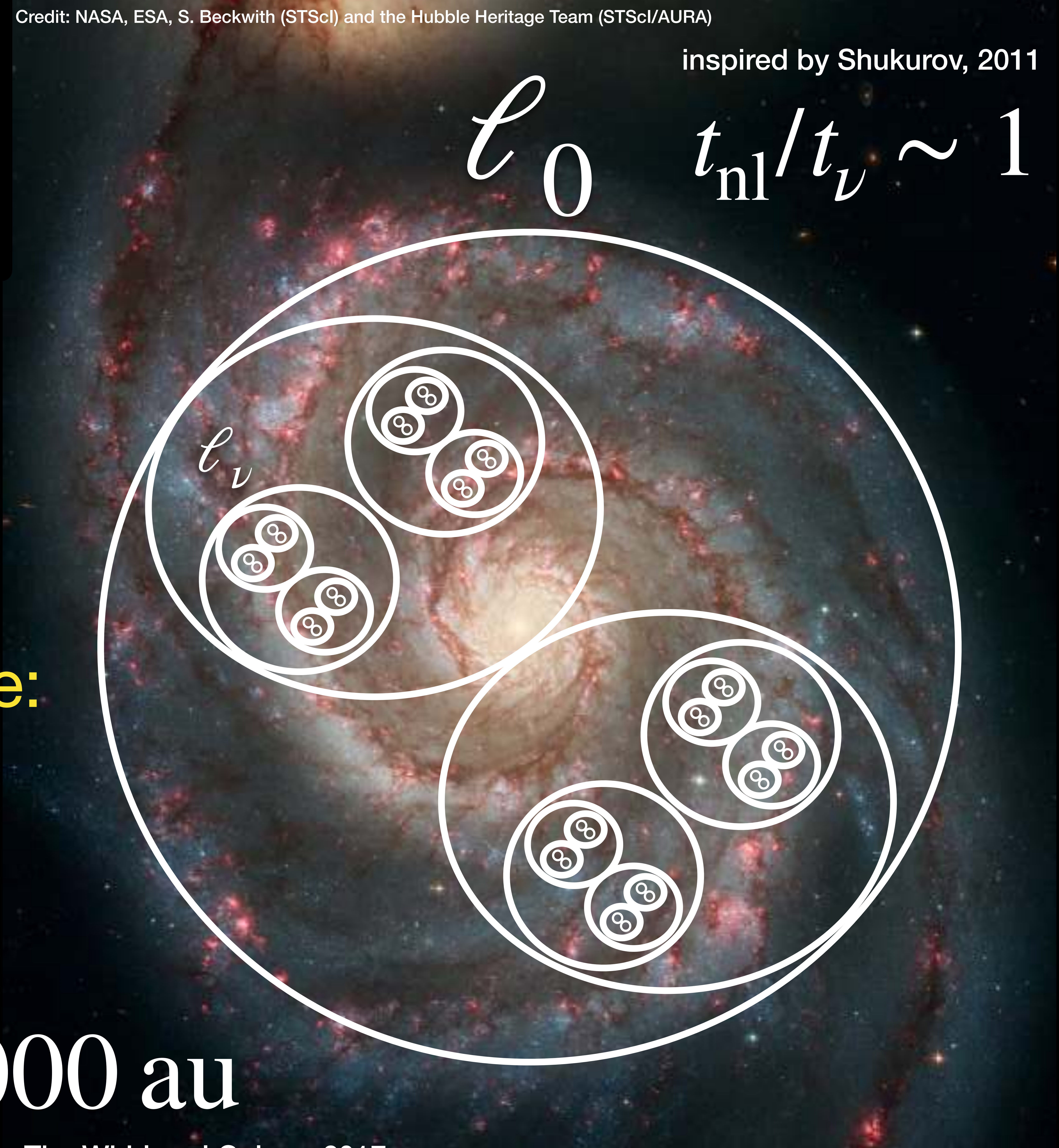
$$\mathrm{Re} = \frac{|\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})|}{|2\nu \nabla \cdot (\rho \mathbb{S})|} = 1$$

dissipation scales for WIM cascade:

$$\ell_\nu \sim \mathrm{Re}^{-3/4} \ell_0$$

WIM: $\mathrm{Re} \sim 10^7$

$$\ell_0 \sim 1\,\mathrm{kpc} \implies \ell_\nu \sim 1000\,\mathrm{au}$$



The Whirlpool Galaxy, 2017

Two great insights from Kolmogorov

- Constant energy flux between modes
- No magnetic field
- No inhomogeneities
- No density fluctuations
- Isotropic

$$\varepsilon \sim u_\ell^3/\ell \sim \text{const}\,.$$

Kolmogorov (1941)

Two great insights from Kolmogorov

- Constant energy flux between modes
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$$\varepsilon \sim u_\ell^3/\ell \sim \text{const}\,.$$

$$u_\ell \sim (\varepsilon\ell)^{1/3}$$

Kolmogorov (1941)

Two great insights from Kolmogorov

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$$\varepsilon \sim u_\ell^3/\ell \sim \text{const}\,.$$

$$u_\ell \sim (\varepsilon\ell)^{1/3}$$

$$u_\ell \sim (\varepsilon\ell)^{1/3} \iff u^2(k) \sim k^{-5/3}dk$$

Kolmogorov (1941)

Two great insights from Kolmogorov

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- No inhomogeneities
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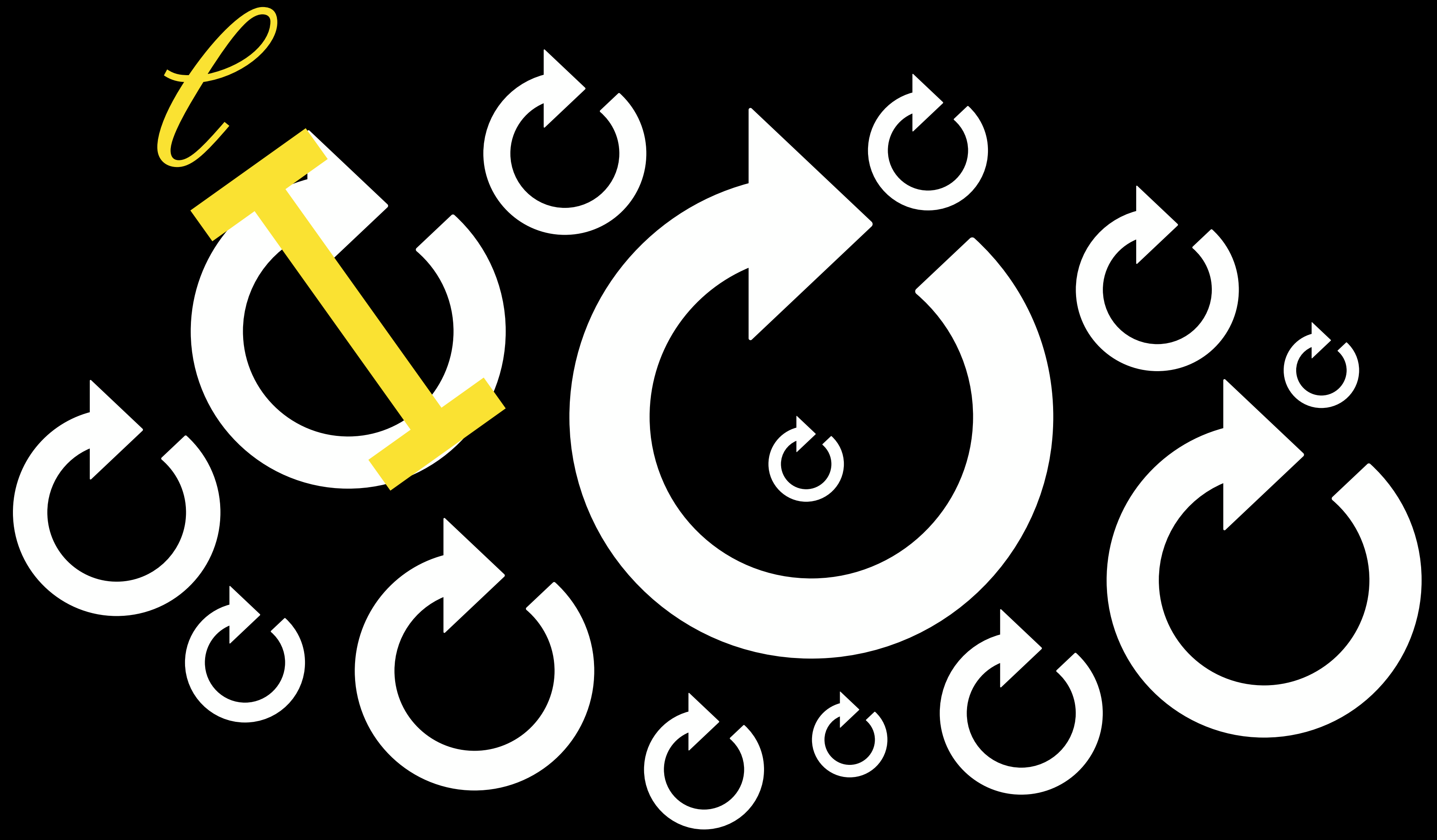
- Isotropic

$$u_\ell \sim (\varepsilon \ell)^{1/3}$$

"structure function"

$$u(\ell) = \langle \, | \, u(\mathbf{x}) - u(\mathbf{x} + \ell) \cdot \ell \, | \, \rangle_{\mathbf{x}} \sim \ell^{1/3}$$

ℓ picks out an eddy



Kolmogorov (1941)

Two great insights from Kolmogorov

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$$u_\ell \sim (\varepsilon \ell)^{1/3}$$

$$u(\ell) = \langle \, | \, u(\mathbf{x}) - u(\mathbf{x} + \ell) \cdot \ell \, | \, \rangle_{\mathbf{x}} \sim \ell^{1/3}$$

$$u(\ell) = \langle \, | \, \delta u_L \, | \, \rangle_{\mathbf{x}} \sim \ell^{1/3}$$

Kolmogorov (1941)

Two great insights from Kolmogorov

- Constant energy flux between modes
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- No inhomogeneities
- No density fluctuations
- Isotropic

$$u^p(\ell) = \left\langle \, | \delta u_L |^p \, \right\rangle_{\mathbf{x}} \sim \ell^{\zeta_p}$$

$$\zeta_p \sim p/3$$

Kolmogorov (1941)
Kolmogorov (1962)

Two great insights from Kolmogorov

- Constant energy flux between modes
- No magnetic field
- No inhomogeneities
- No density fluctuations
- Isotropic

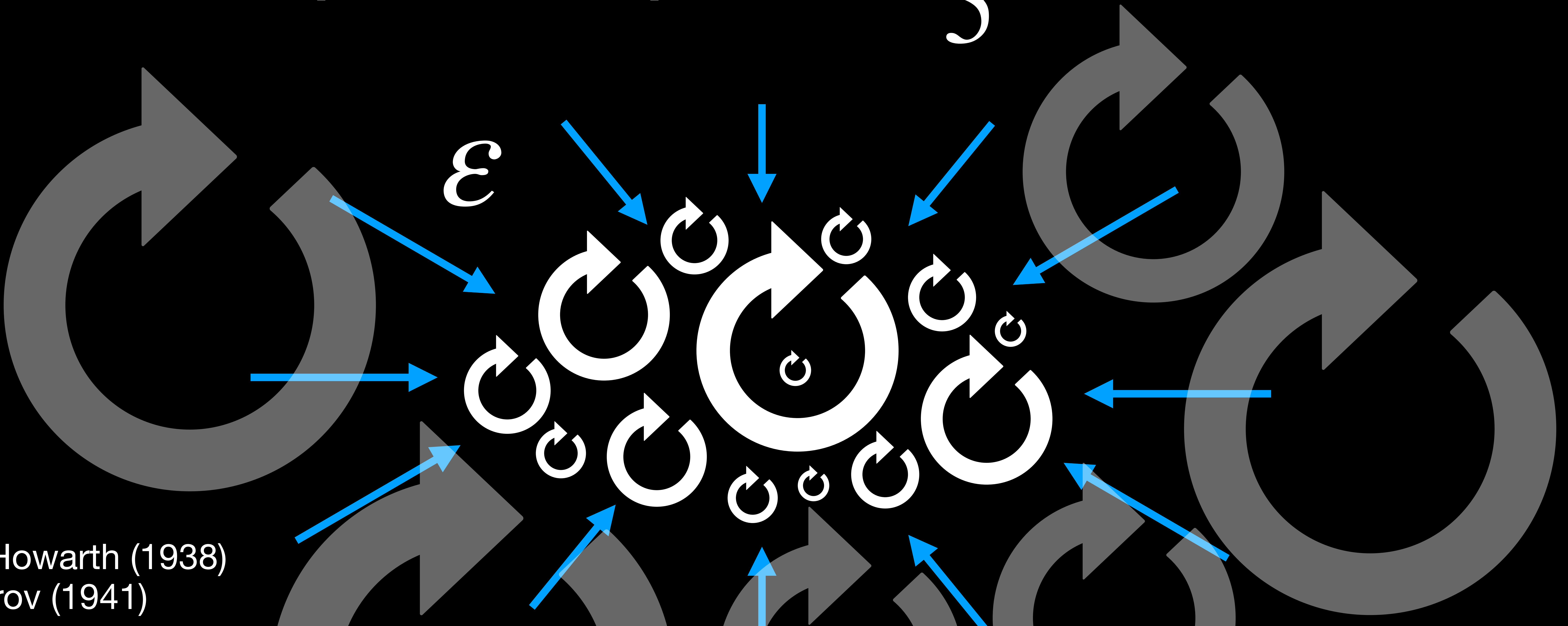
$$u^3(\ell) = \left\langle \, | \delta u_L |^3 \right\rangle_{\mathbf{x}} \sim \ell$$

$$u^3/\ell = \text{const}\,.$$

Kolmogorov (1941)
Kolmogorov (1962)

An exact relation for turbulence
Kolmogorov's 4/5 law

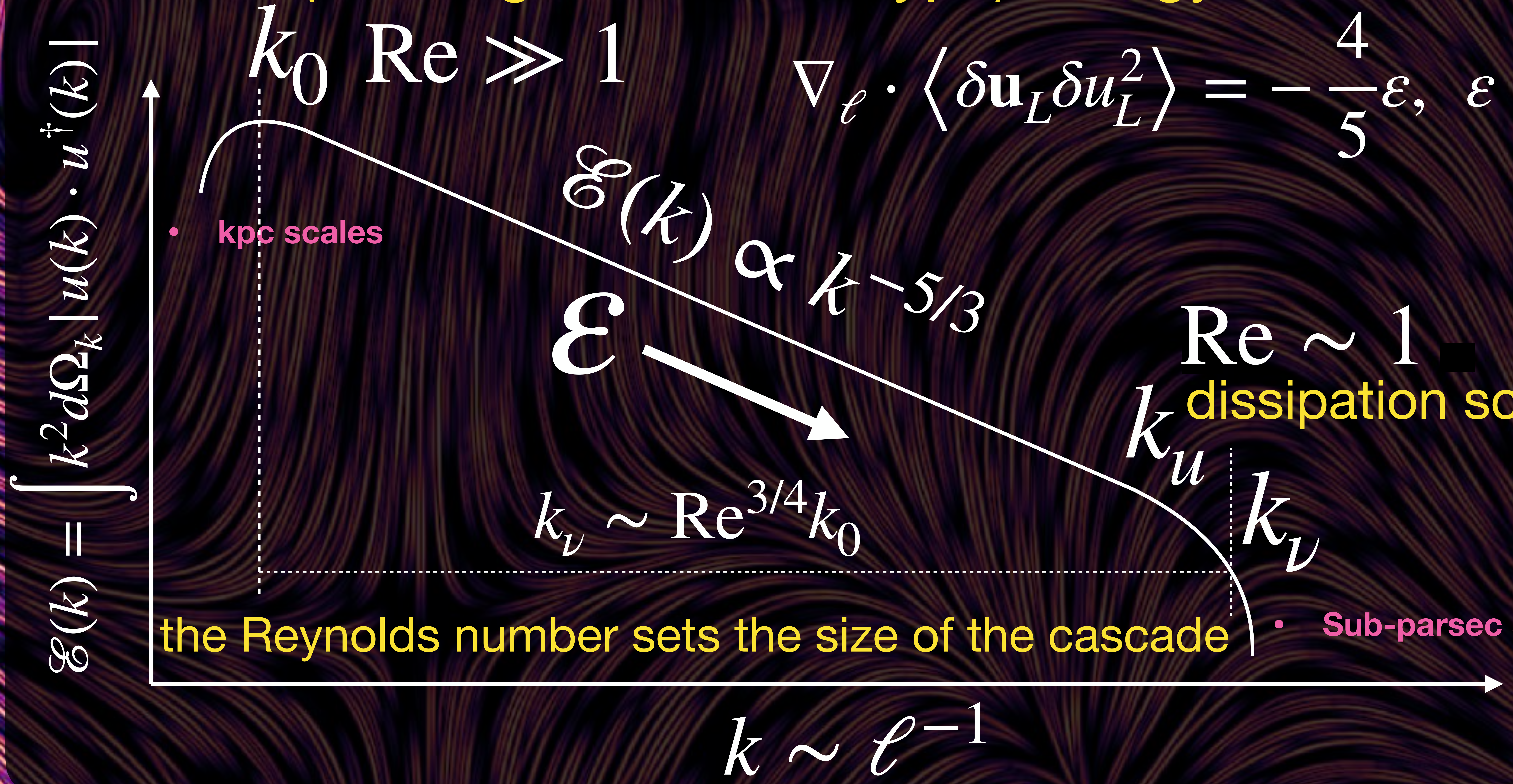
$$\nabla_{\ell} \cdot \left\langle \delta \mathbf{u}_L \delta u_L^2 \right\rangle = -\frac{4}{5}\varepsilon, \ \ \varepsilon > 0$$



Kármán–Howarth (1938)
Kolmogorov (1941)

The (Kolmogorov, 1941 -type) energy cascade

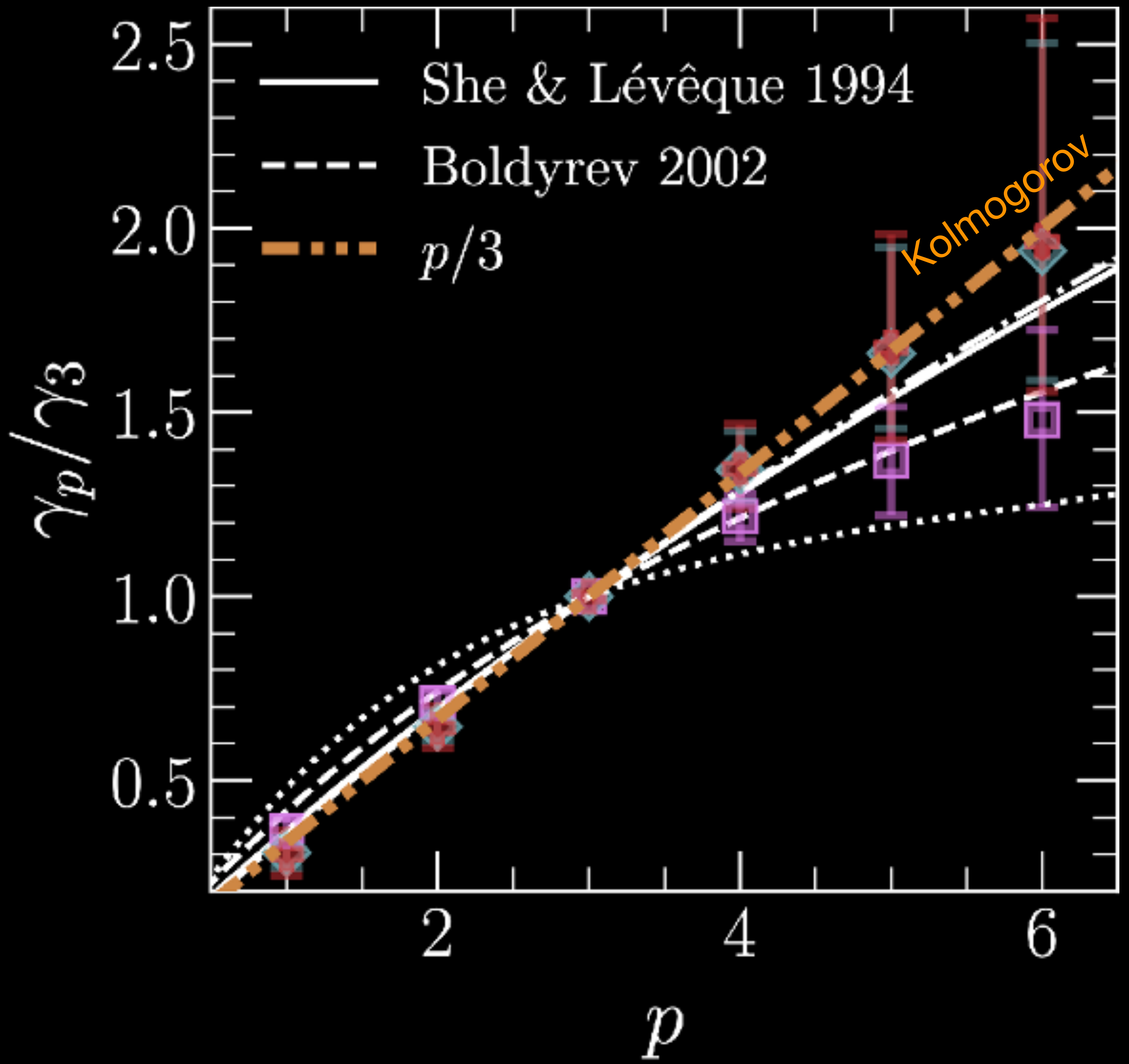
$$\nabla_\ell \cdot \langle \delta \mathbf{u}_L \delta u_L^2 \rangle = -\frac{4}{5}\varepsilon, \quad \varepsilon > 0$$



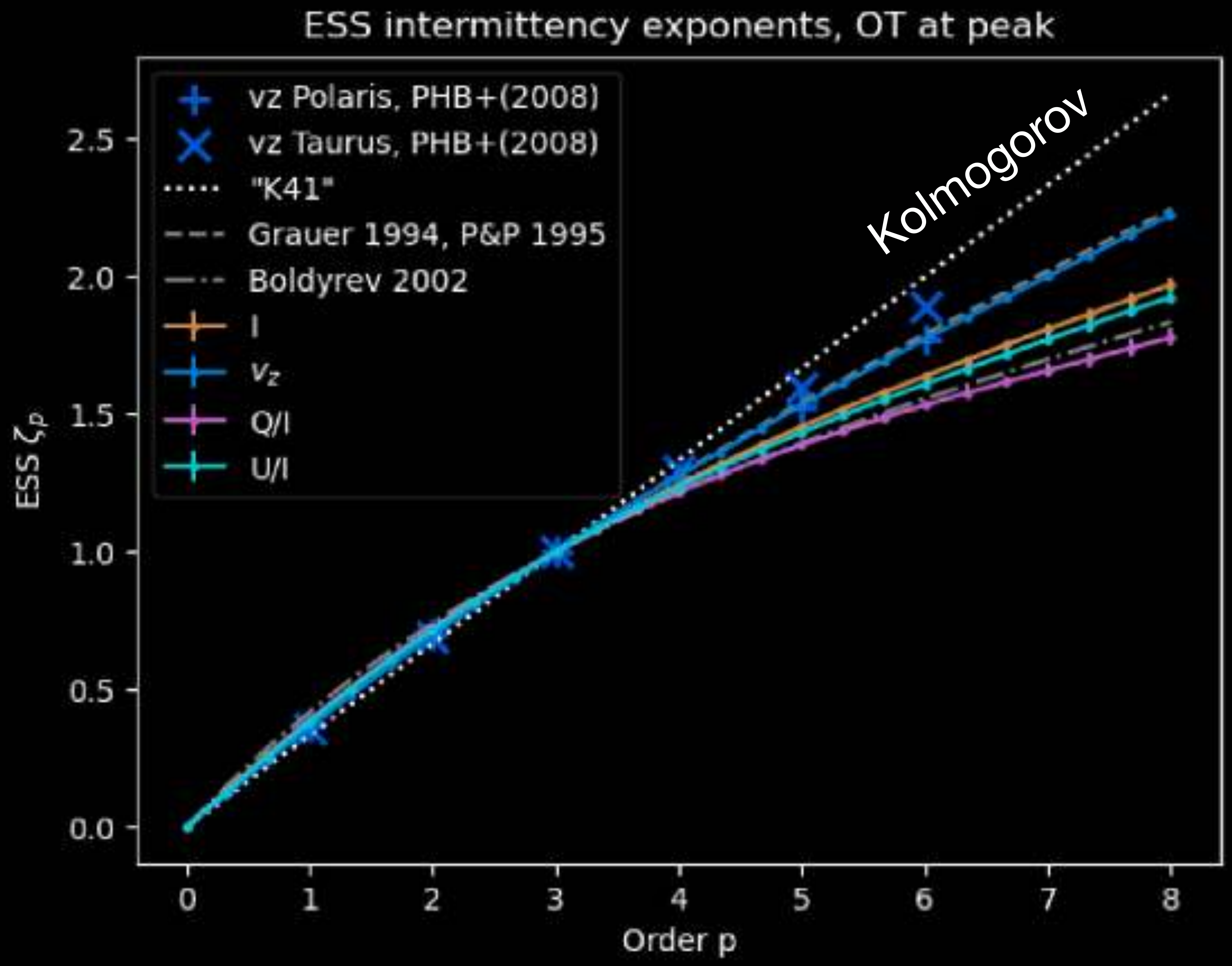
Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Then we started measuring…

CGM; quasars sources; Chen+2024

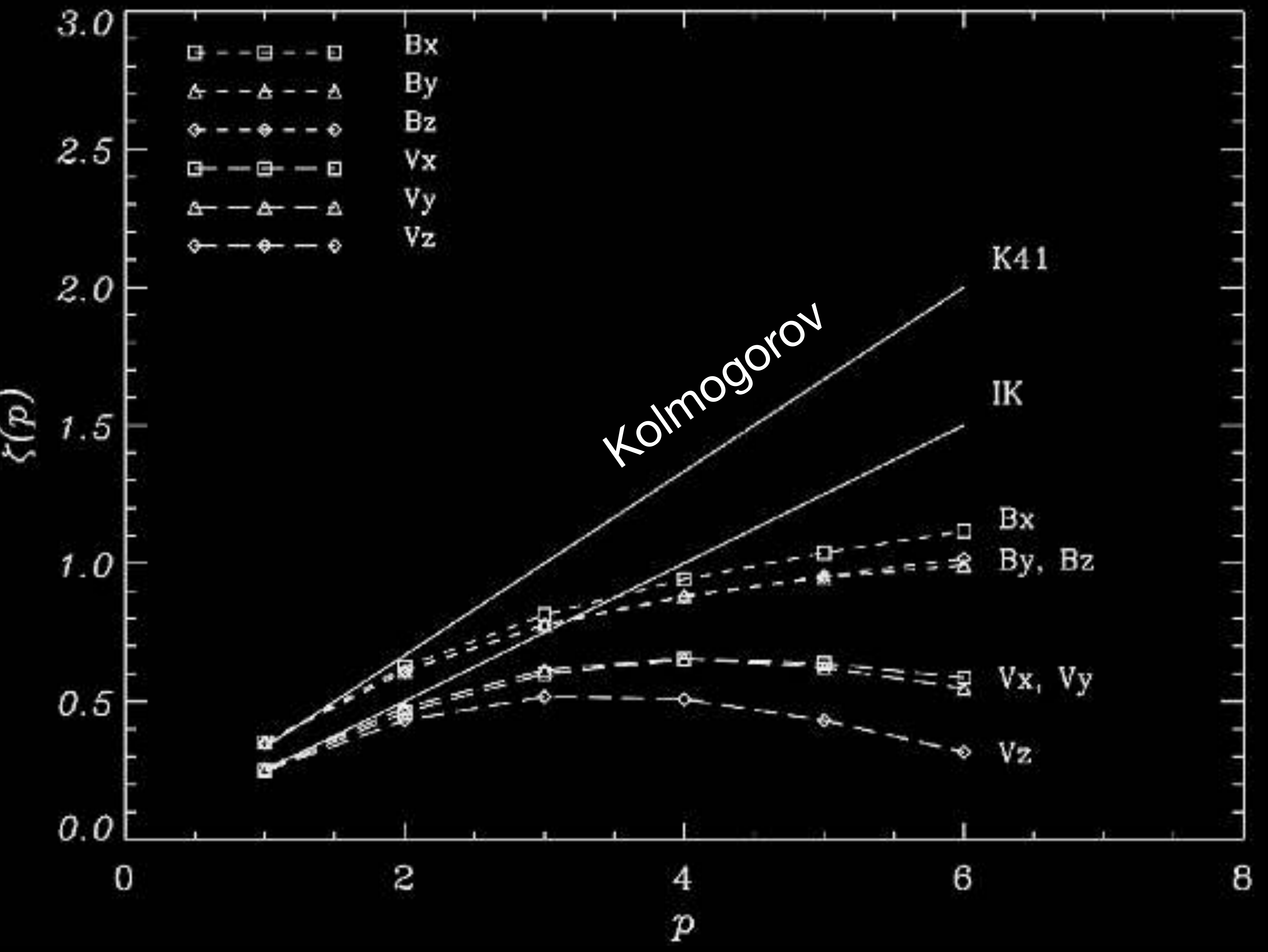


ISM; cold cloud sources; Lesaffra+2024

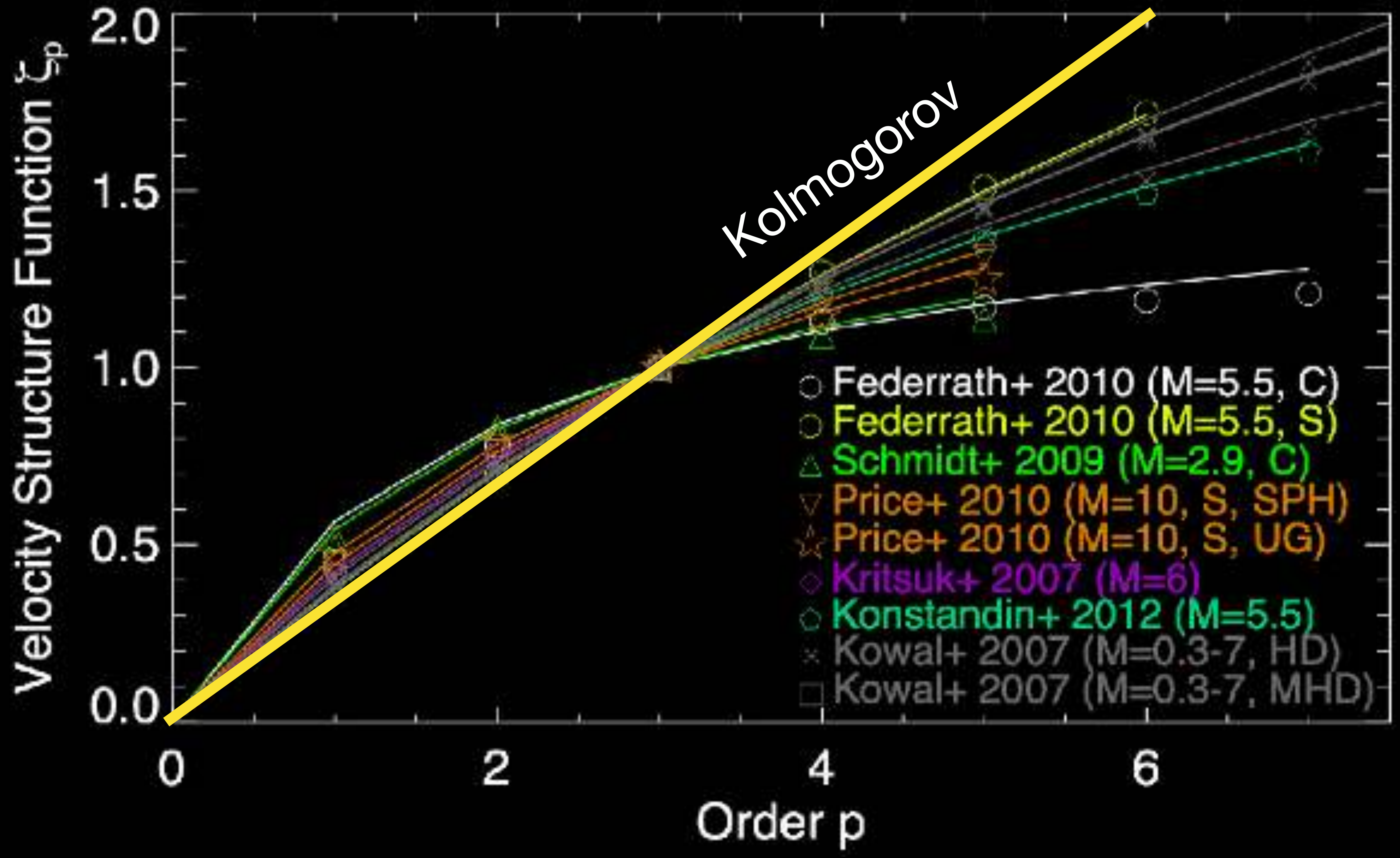


Then we started measuring…

Solar wind; WIND spacecraft; Salem+2009

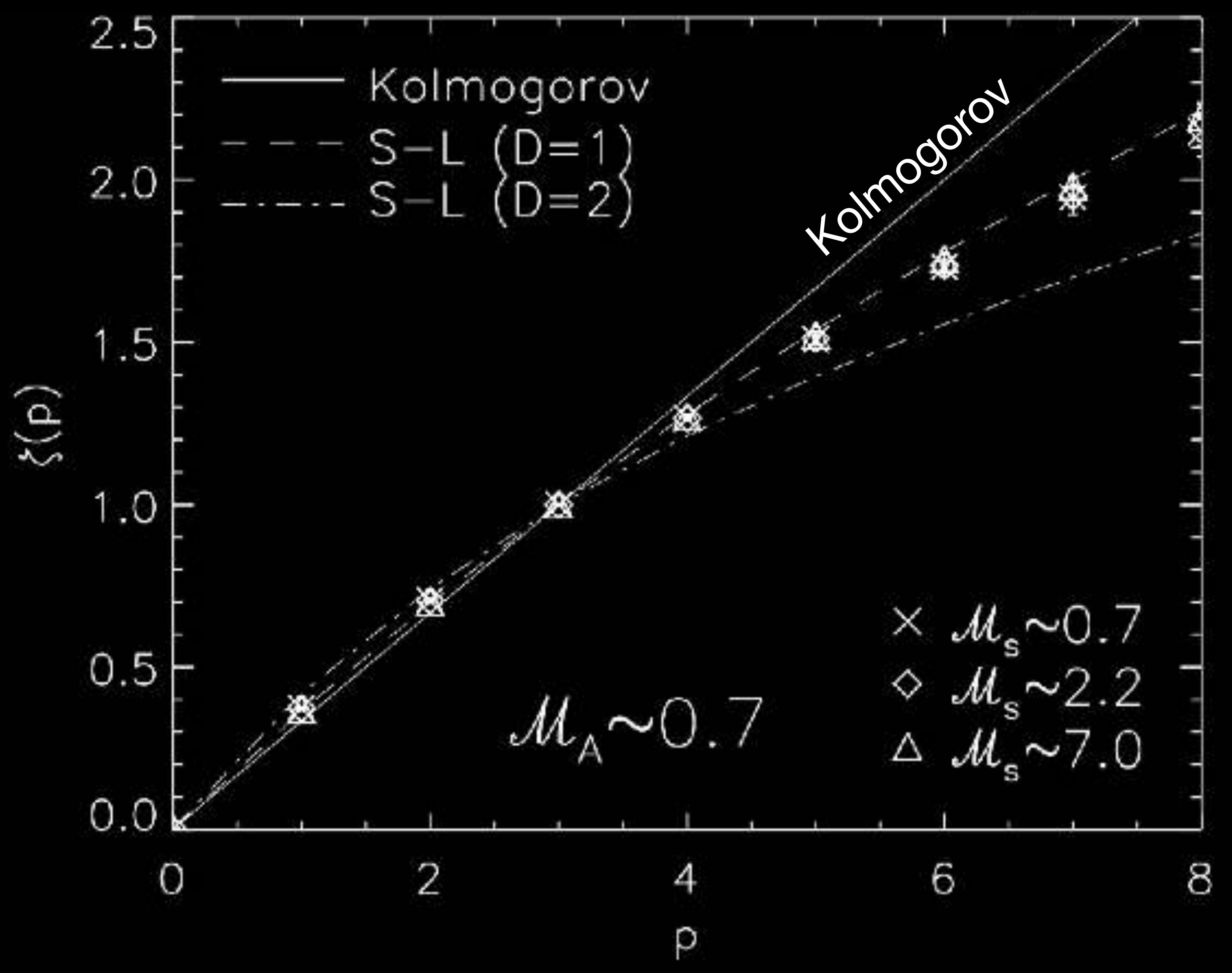


Turbulence simulations; Hopkins 2009

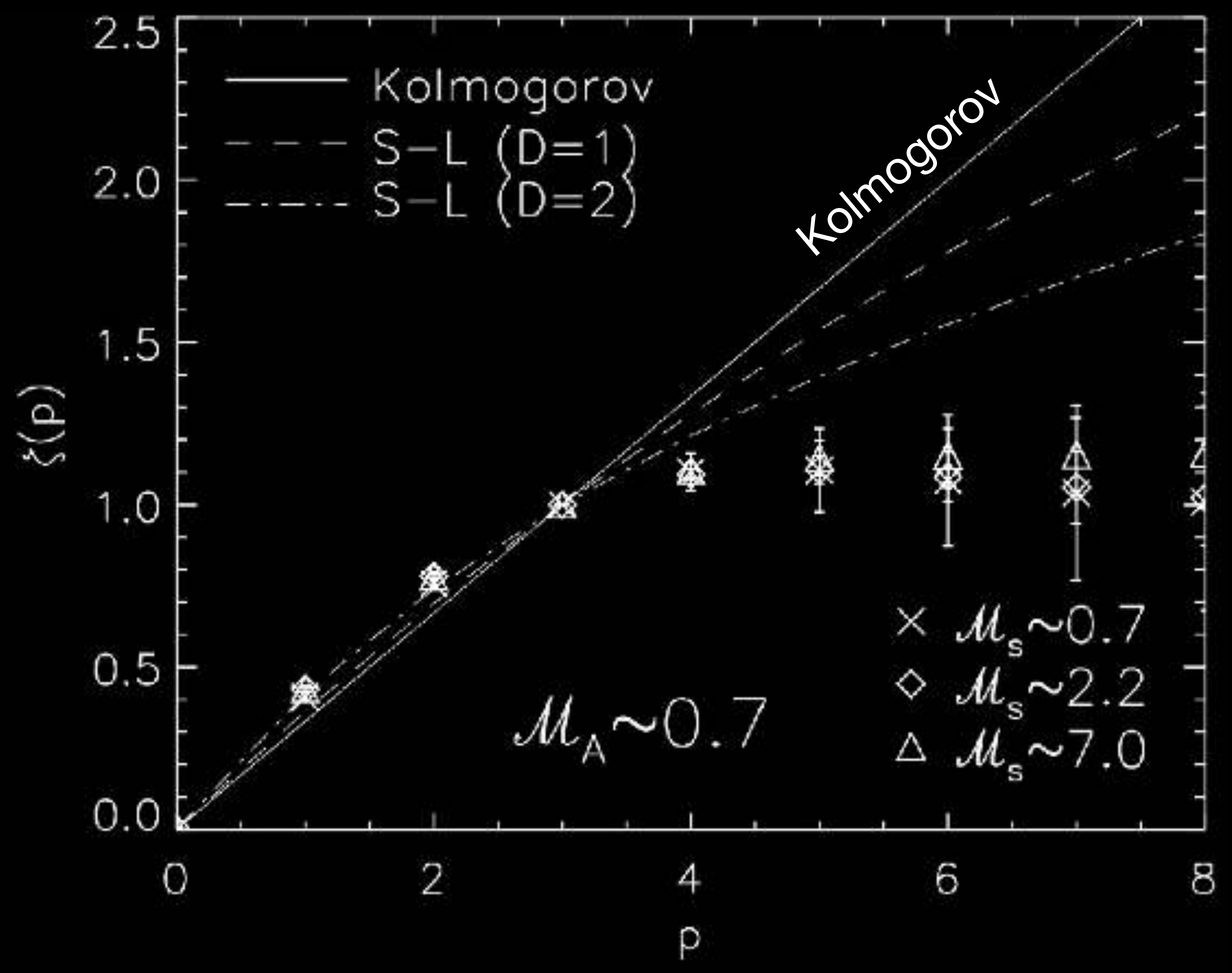


Then we started measuring…

Incompressible modes



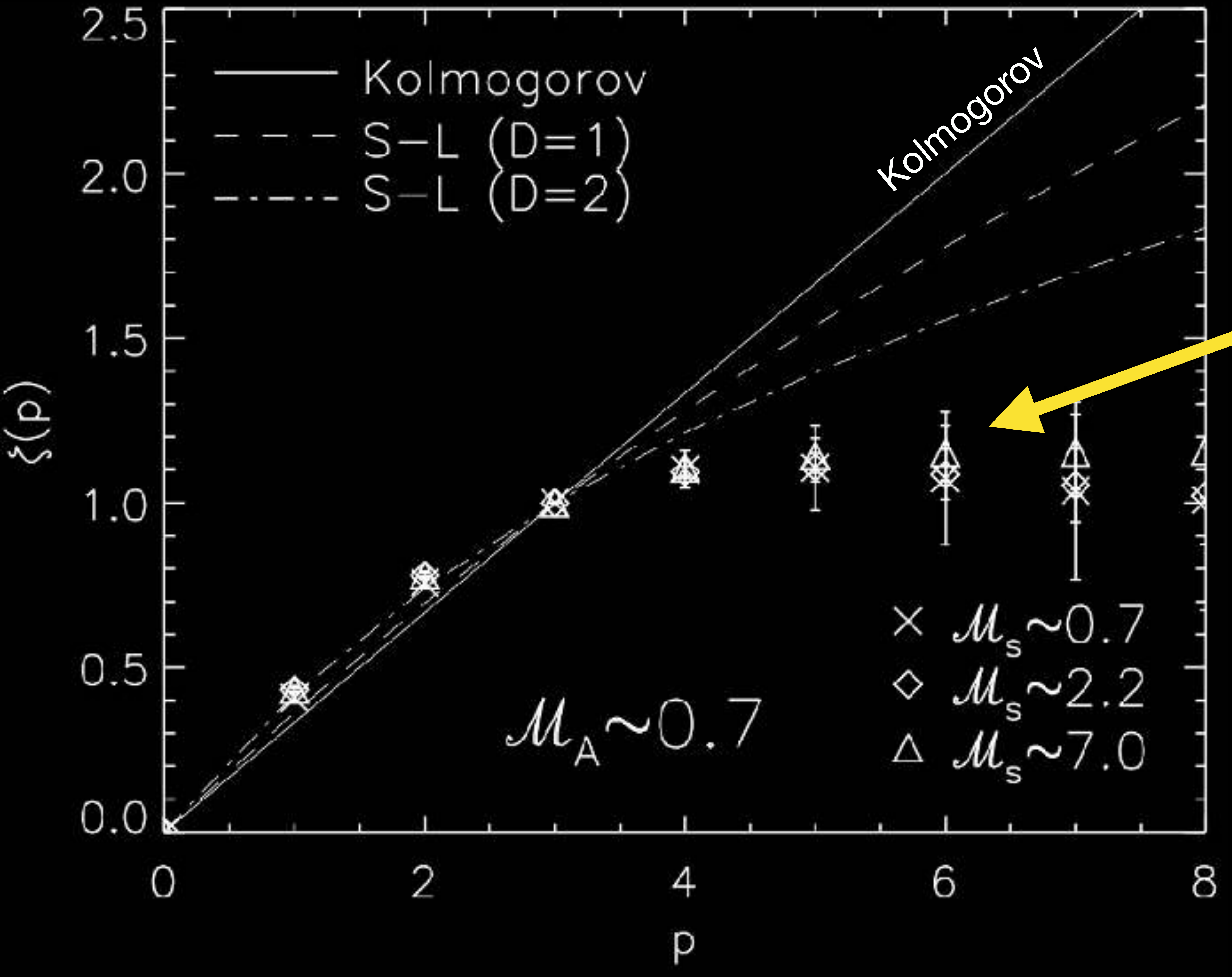
Compressible modes



the things that cause $\nabla\rho$

MHD turbulence simulations; Kowal+2018

The issue… for everywhere…



Higher-order structure functions too shallow…

too much energy in the higher order moments at small scales for K41

Two schools of thought for intermittency modeling

Agreement on non-K41 behavior being related to intense gradients in the turbulence.

$\nabla \mathbf{u}$ $\nabla \cdot \mathbf{u}$ $\nabla \times \mathbf{u}$ $\nabla \mathbf{B}$ $\nabla \times \mathbf{B}$

velocity shear

vortex tubes / sheets

current sheets

shocks
discontinuities

magnetic shear

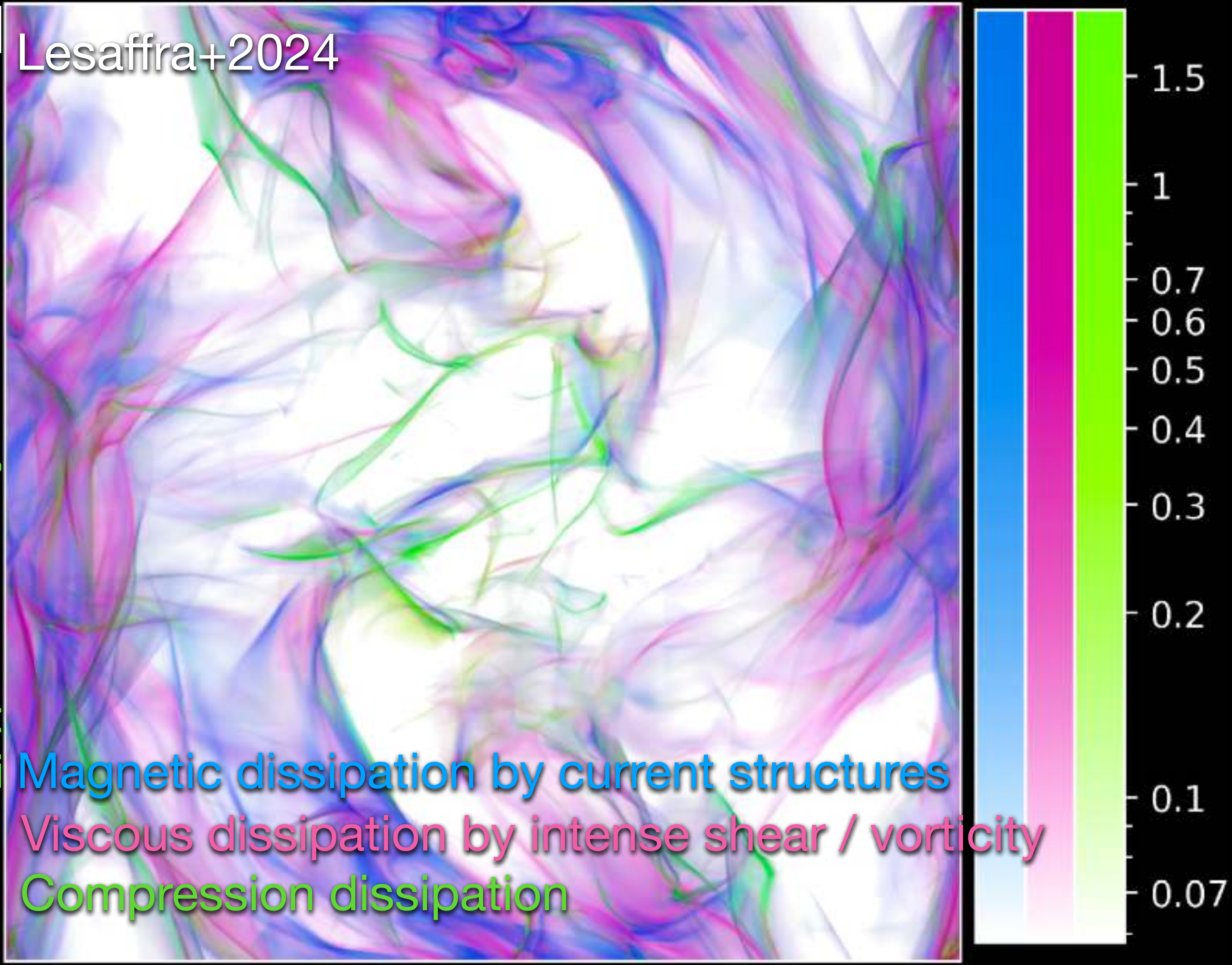
Two schools of t

Agreement on non-K4
turbulence.

$\nabla \mathbf{u}$ $\nabla \cdot$

velocity shear

shoc
disconti



Two schools of thought for intermittency modeling

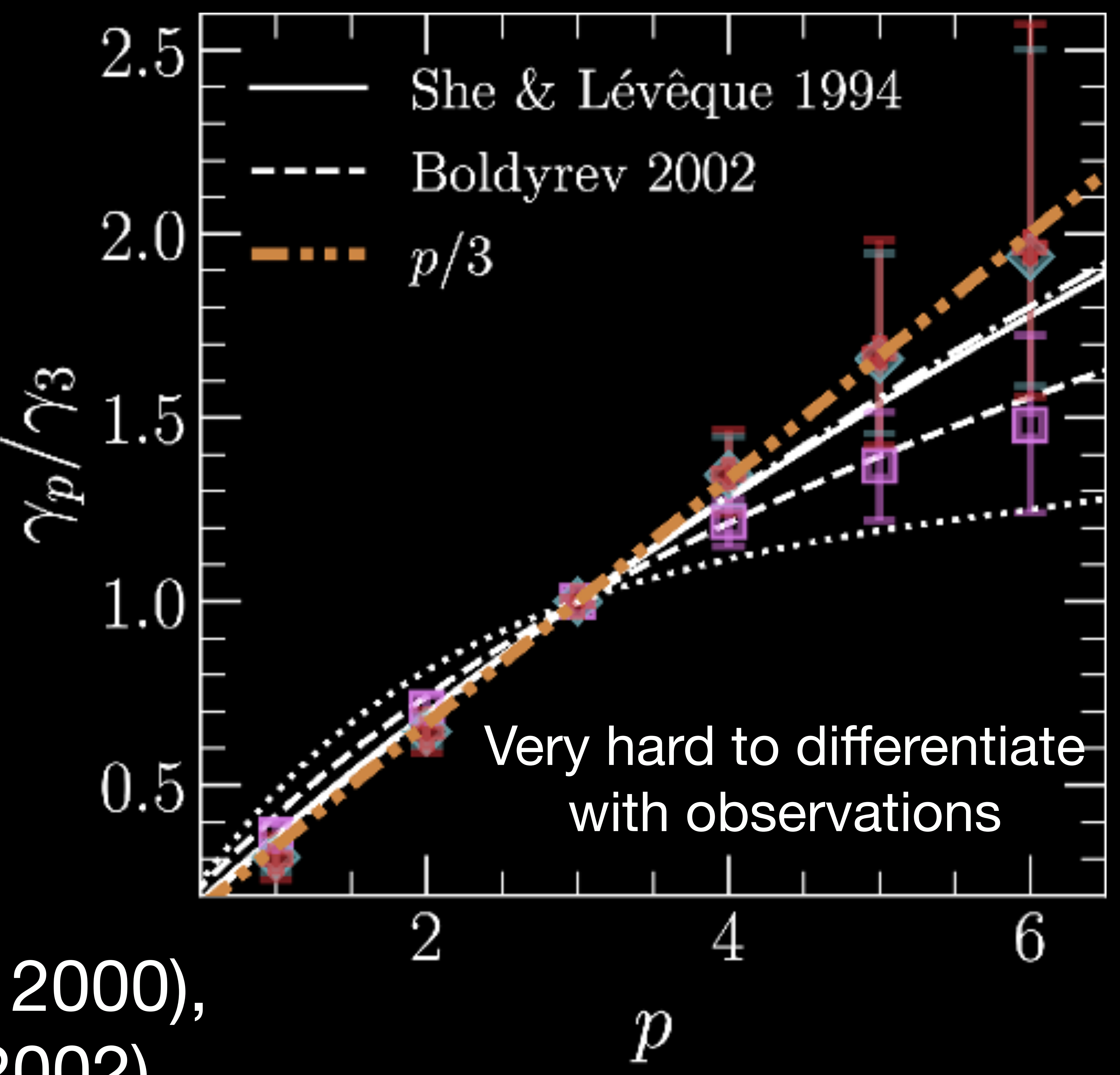
1) modify all moments… including power spectrum exponent

By modeling the moments of the dissipative structures (1D vortex tubes) She & Leveque (1994) found a K41 correction:

$$\zeta_p = \frac{p}{9} + 2\left[1 - \left(\frac{2}{3}\right)^{p/3}\right],$$

(correction of order 0.03 to low-order structure function…)

CGM; quasar sources; Chen+2024



Can do the same for MHD (Biskamp-Muller 2000), and compressible / supersonic (Boldeyrev 2002)

Two schools of thought for intermittency modeling

1) modify all moments… including power spectrum exponent

Federrath, Klessen, Ipachio & Beattie (2021)
Nature Astronomy

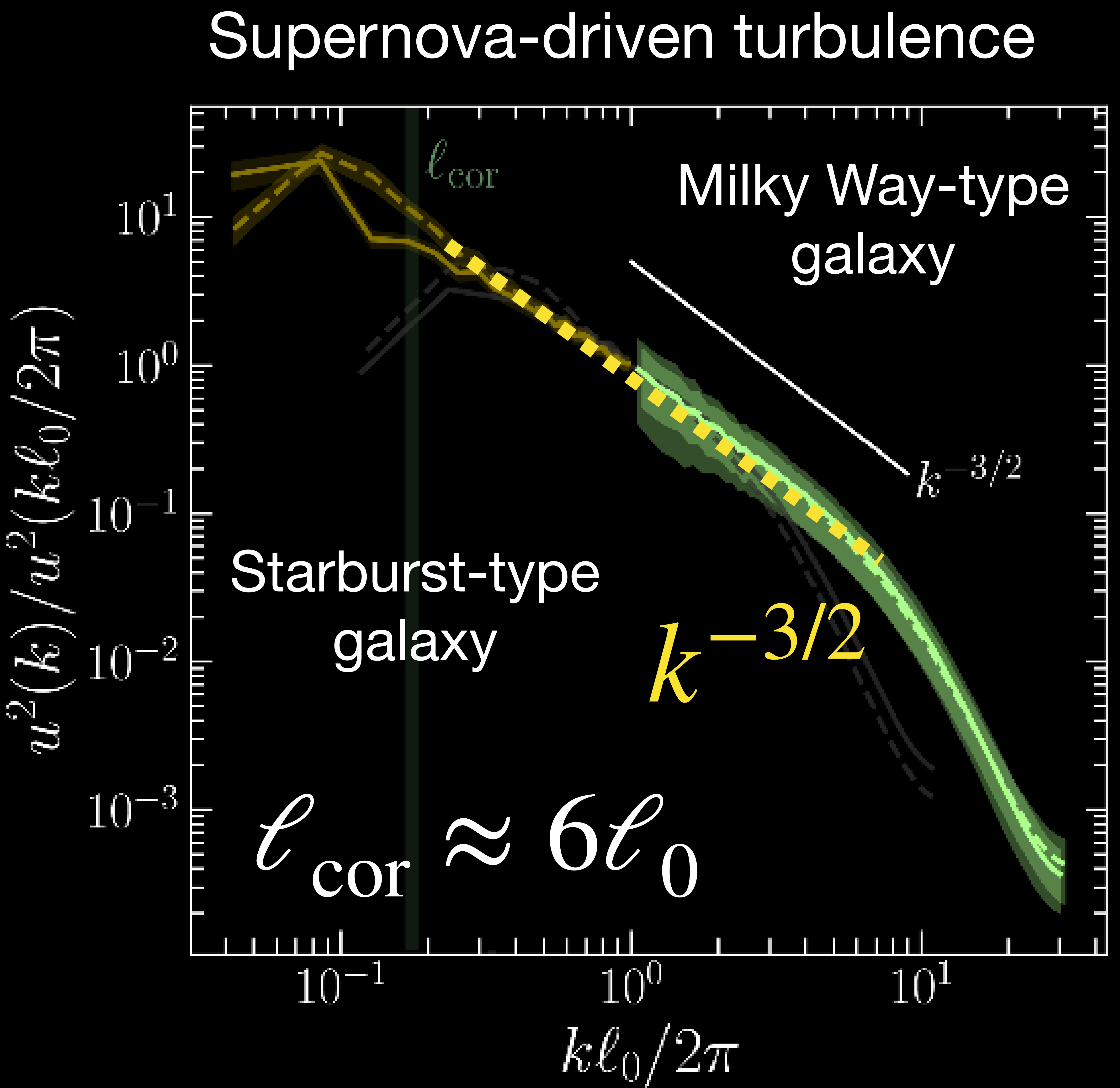
But even at the level of the spectrum for ISM…

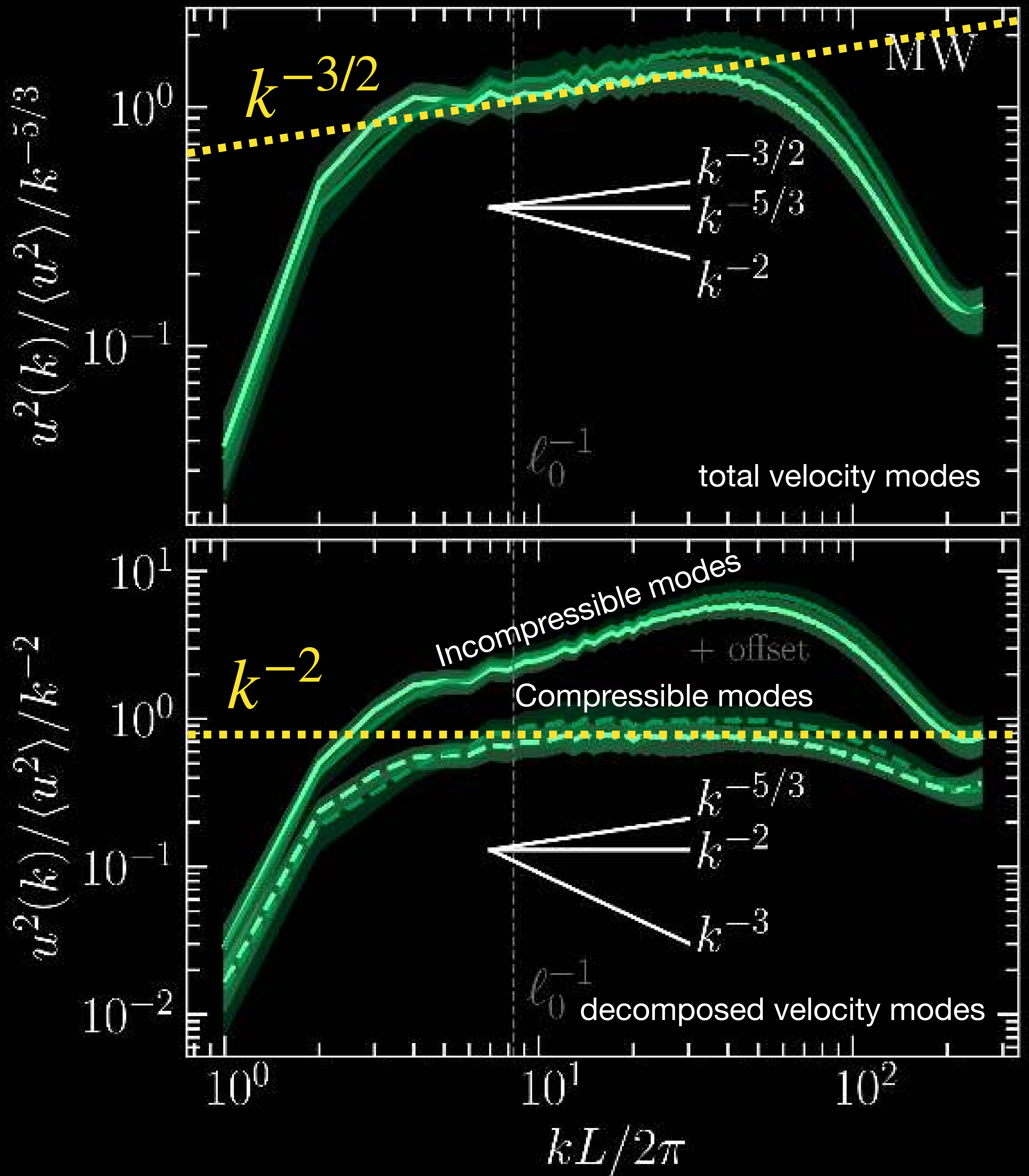
Two power laws

One power law, only emerging at very high Rm

Beattie+ 2025 (*Nature Astronomy*). The spectrum of magnetized turbulence in the interstellar medium

But even at the level of the spectrum for ISM…

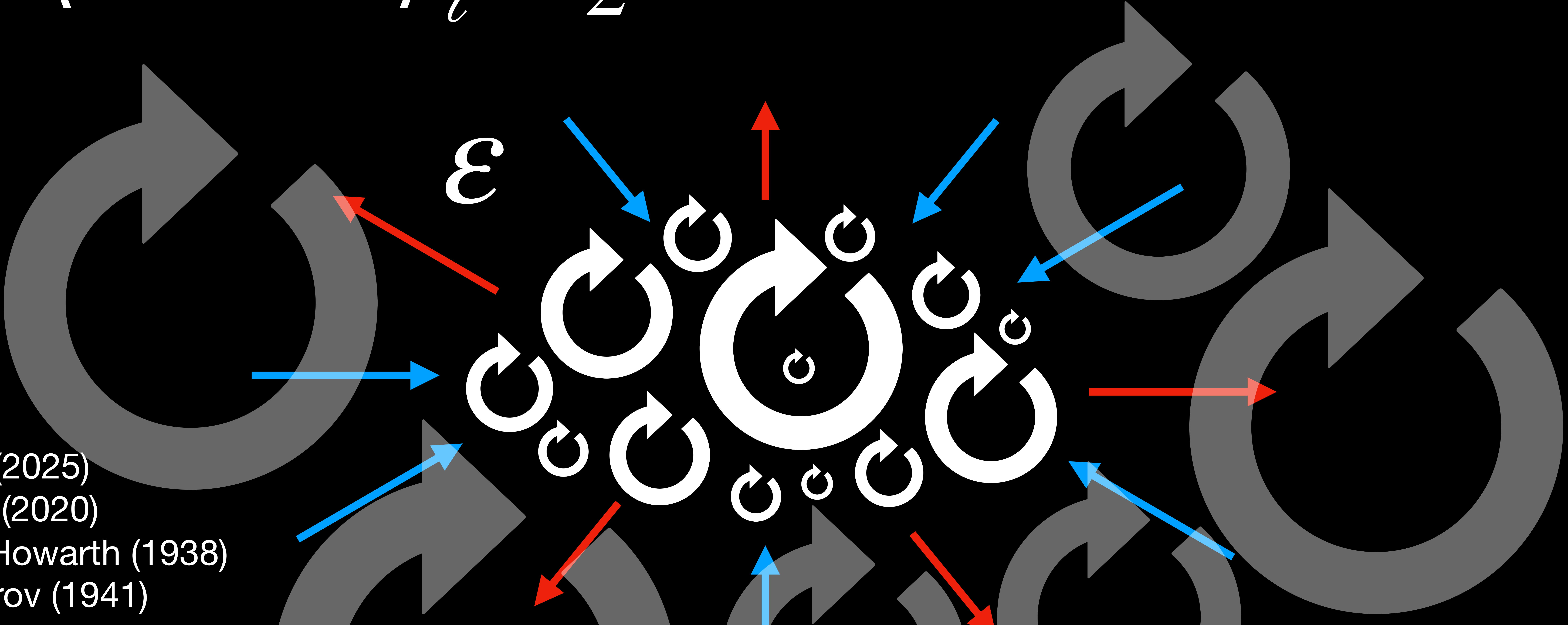




An exact relation for compressible turbulence
Ferrand+ 4 law

$$\theta = \nabla \cdot \delta \mathbf{u}$$

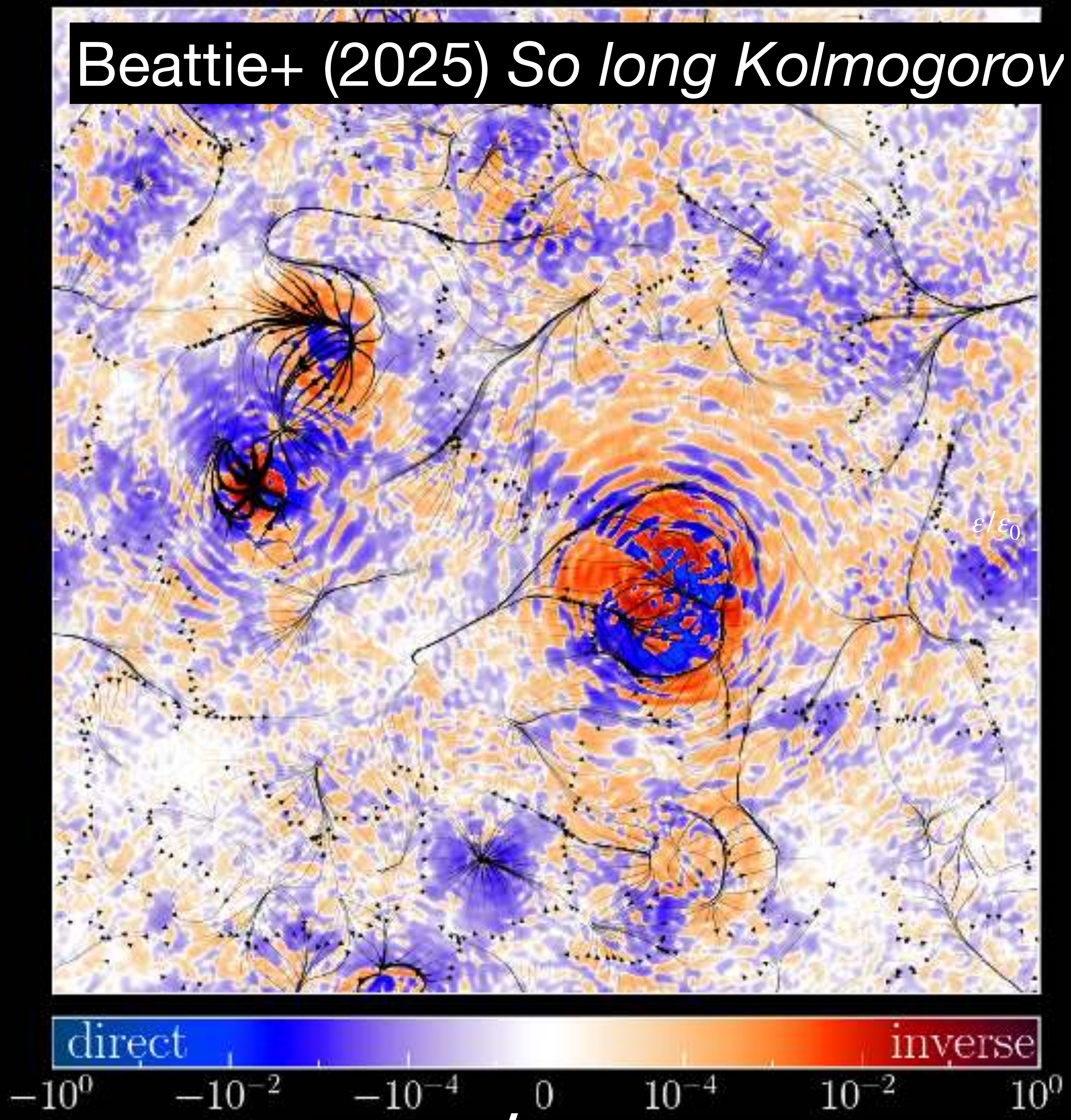
$$\nabla_\ell \cdot \left\langle \overline{\delta\rho\, \delta u^2\, \delta \mathbf{u}} \right\rangle_\ell - \frac{1}{2} \left\langle (\rho\theta' + \rho'\theta)\, \delta u^2 \right\rangle_\ell = -4\,\varepsilon$$



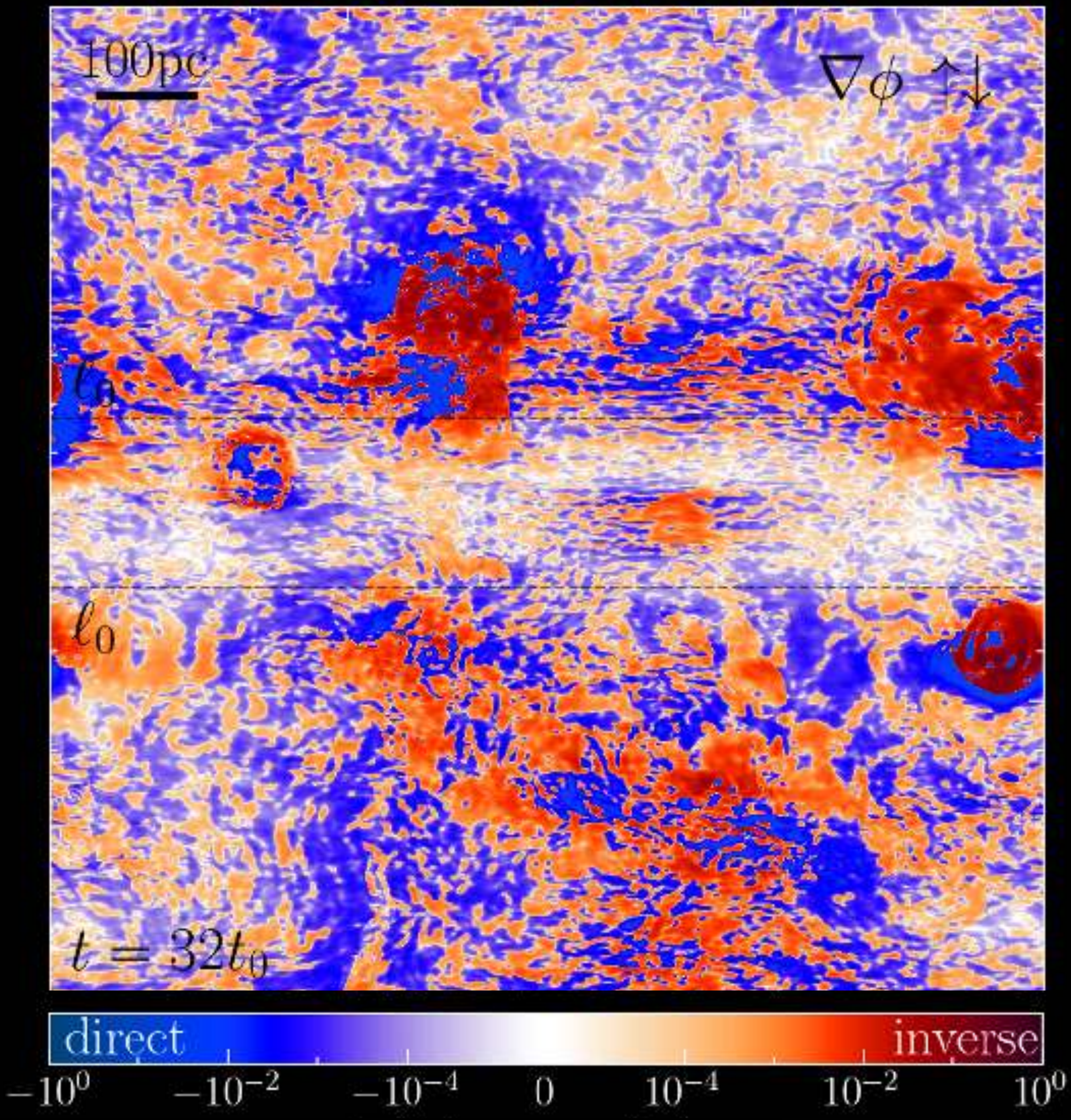
Beattie+ (2025)
Ferrand+ (2020)
Kármán–Howarth (1938)
Kolmogorov (1941)

An exact relation for compressible turbulence

Beattie+ (2025) *So long Kolmogorov*



$\varepsilon/\varepsilon_0$

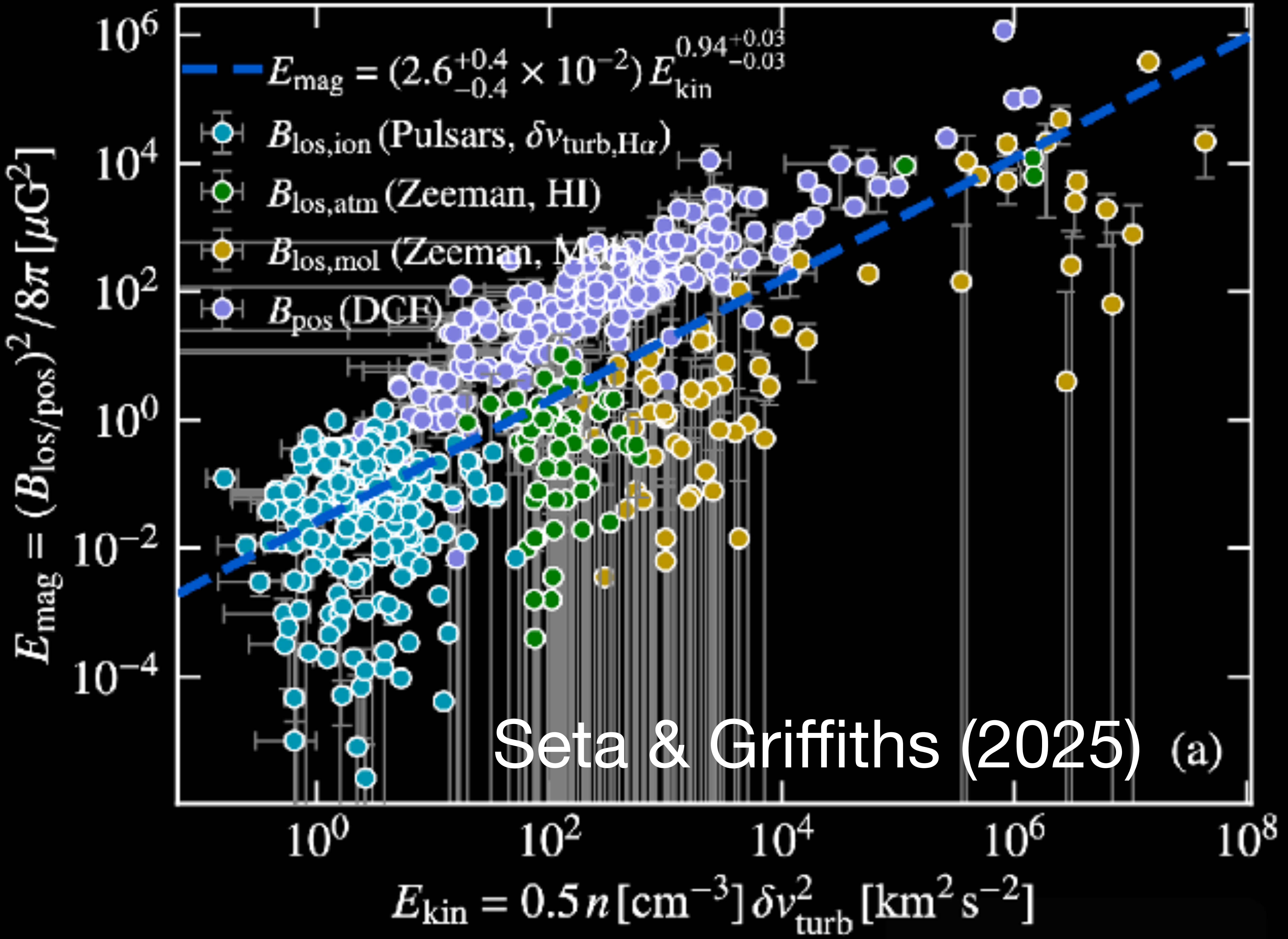


$\varepsilon/\varepsilon_0$

Two schools of thought for intermittency modeling

2) intermittent structures modify the cascade at small scales

What is an eddy in MHD?



Two schools of thought for intermittency modeling

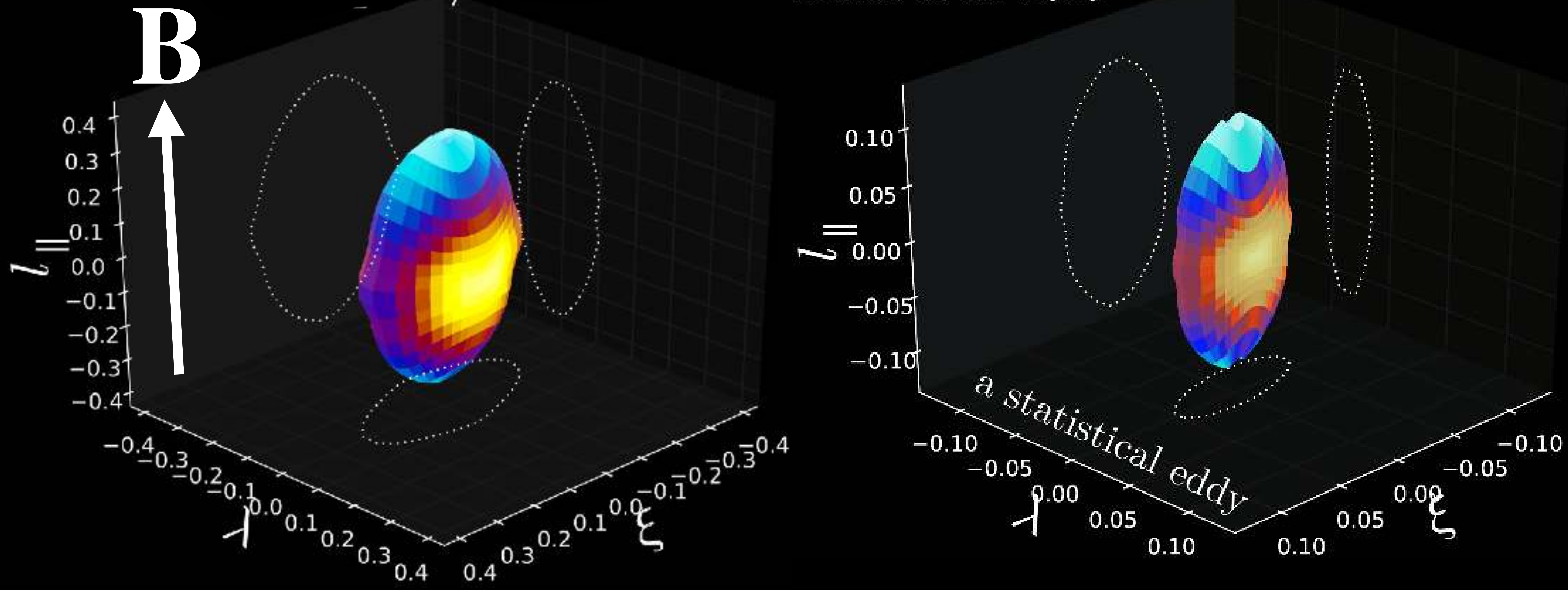
2) intermittent structures modify the cascade at small scales

We can measure!!! Iso-contours of the 3D structure function!

Large perpendicular scales

Mallet et al. 2016

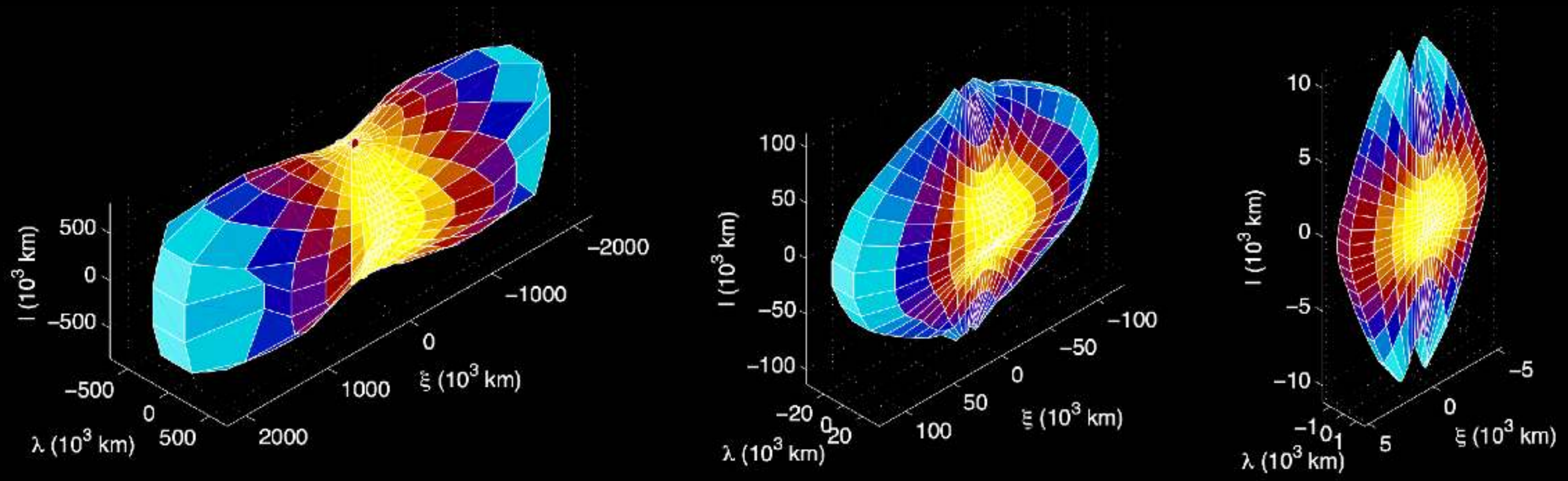
Smaller perpendicular scales



Two schools of thought for intermittency modeling

2) intermittent structures modify the cascade at small scales

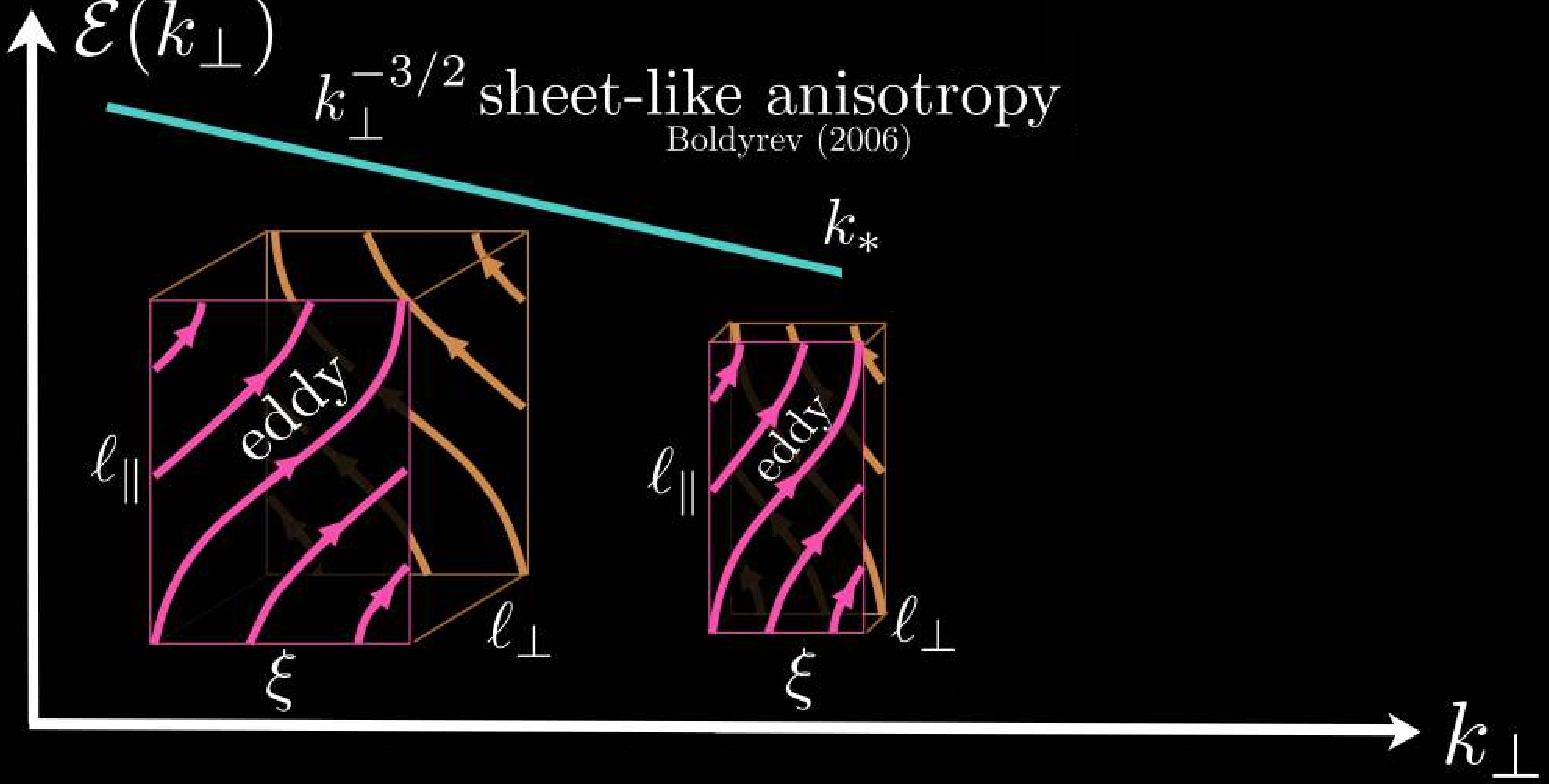
We can measure!!! From observations:



Chen+2012. *Three-dimensional structure of solar wind turbulence.*

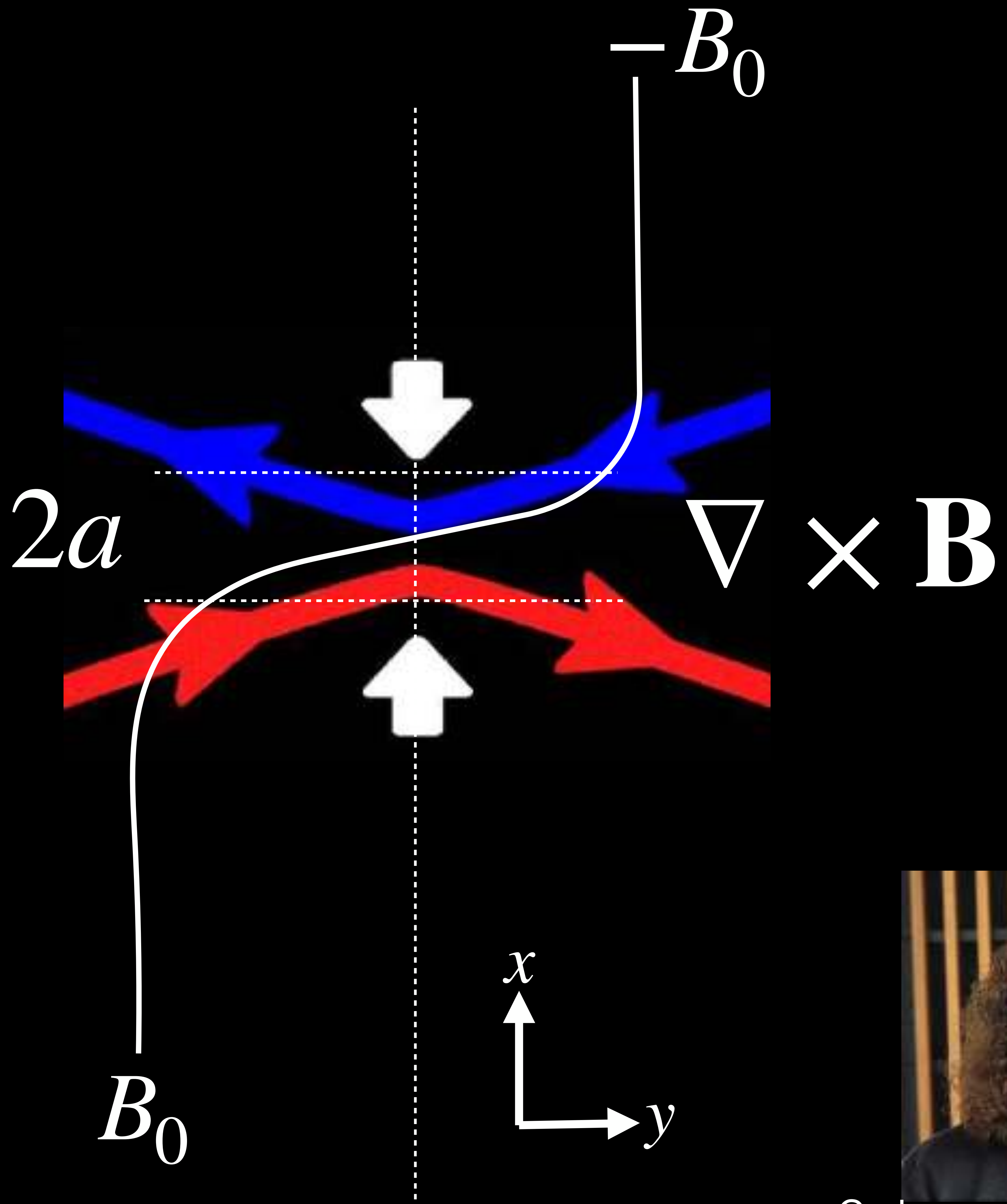
Two schools of thought for intermittency modeling

2) intermittent structures modify the cascade at small scales



Two schools of thought for intermittency modeling

Current sheet 101 — one dimension



In pressure equilibrium

$$\nabla P = \mathbf{J} \times \mathbf{B}$$

A Harris sheet

$$B_y(x) = B_0 \tanh(x/a)$$

has an equilibrium current profile

$$J_z(x,t) = \sqrt{\frac{B_0^2}{4\pi^2 \eta t}} \exp\left(-\frac{\pi x^2}{\eta t}\right)$$

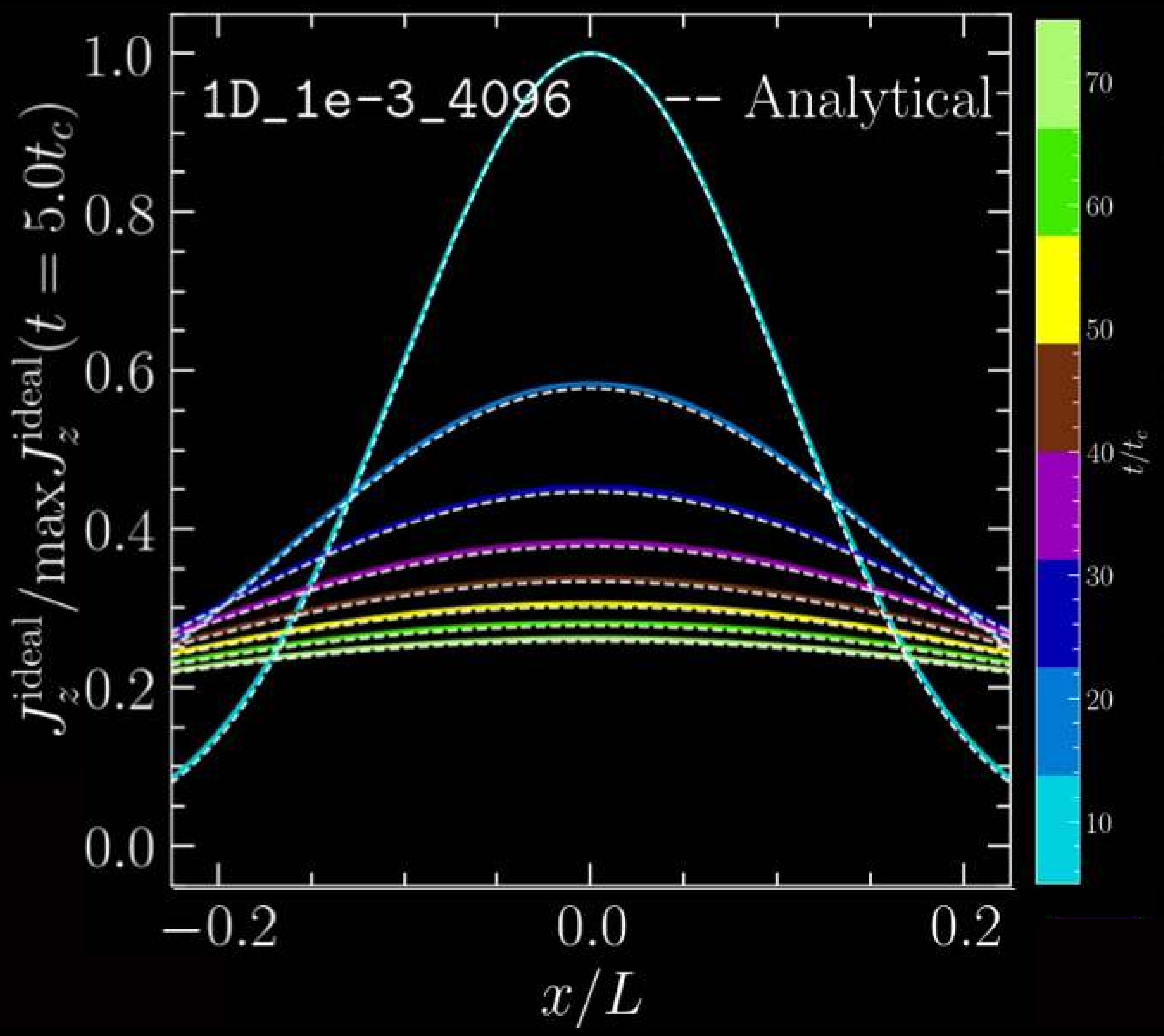
Grehan, grad. student (CITA)

Grehan+ incl. Beattie (2025)

η is the Ohmic resistivity of the plasma

Two schools of thought for intermittency modeling

Current sheet 101 — one dimension



just diffusing… quite boring

In pressure equilibrium

$$\nabla P = \mathbf{J} \times \mathbf{B}$$

A Harris sheet

$$B_y(x) = B_0 \tanh(x/a)$$

has an equilibrium current profile

$$J_z(x,t) = \sqrt{\frac{B_0^2}{4\pi^2 \eta t}} \exp\left(-\frac{\pi x^2}{\eta t}\right)$$

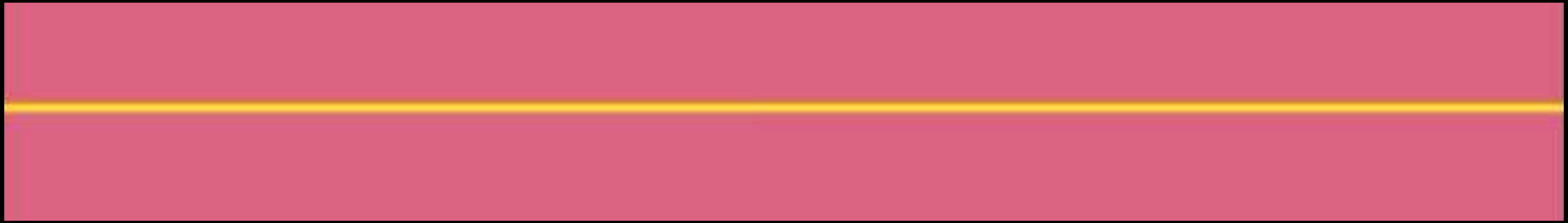
Grehan+ incl. Beattie (2025)

η is the Ohmic resistivity of the plasma

Two schools of thought for intermittency modeling

Current sheet 101 — two dimensions

$$S < S_{\rm crit} \sim 10^5$$



Work with Anna Tsai (CITA graduate) on reconnection in a compressible medium / CS screens for scintillation

$$S > S_{\rm crit}$$

$$S \sim \ell_{\rm sheet}/a$$

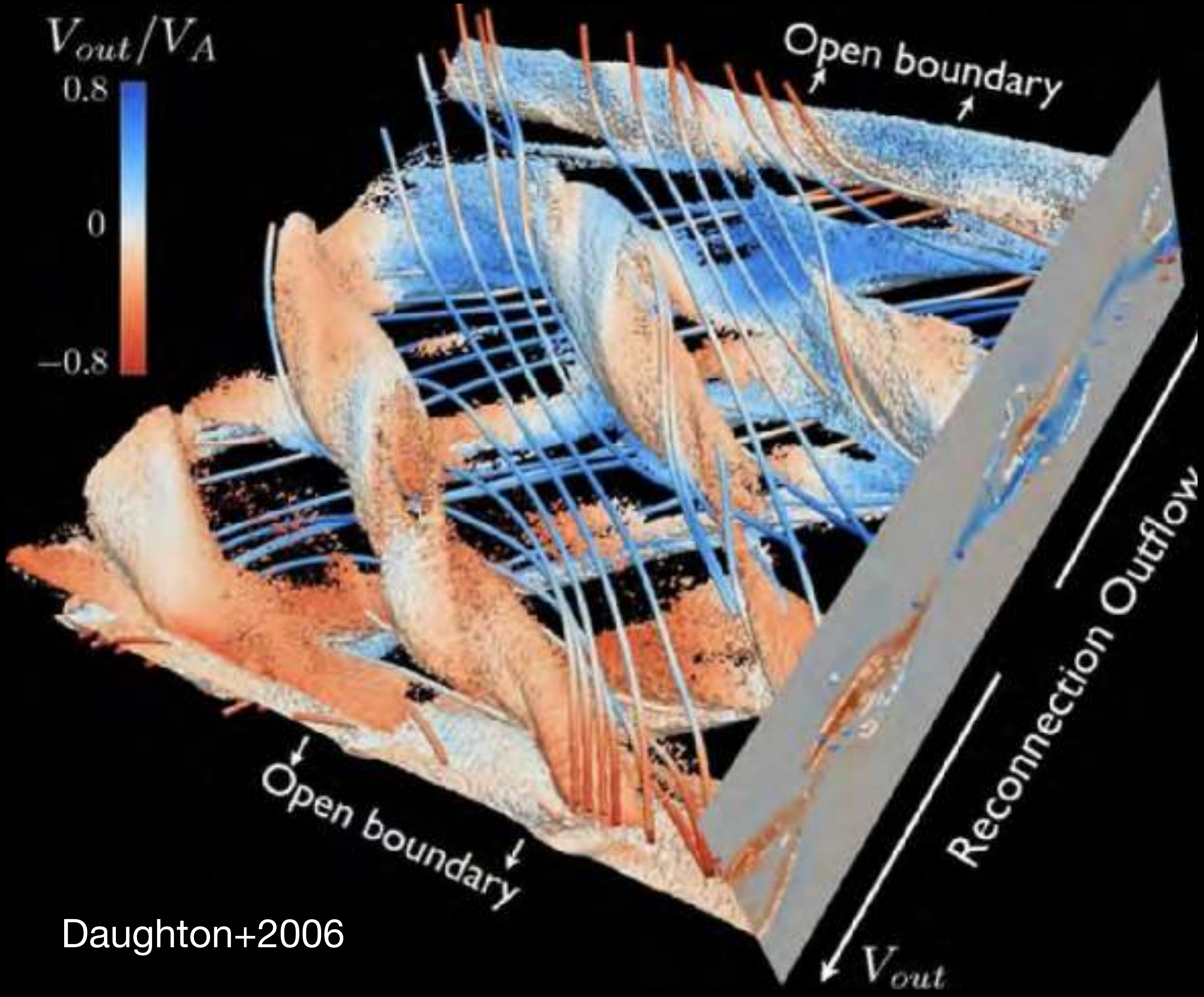
tearing instability breaks sheet into secondary/tertiary/etc. sheets

Visualization by Michael Grehan

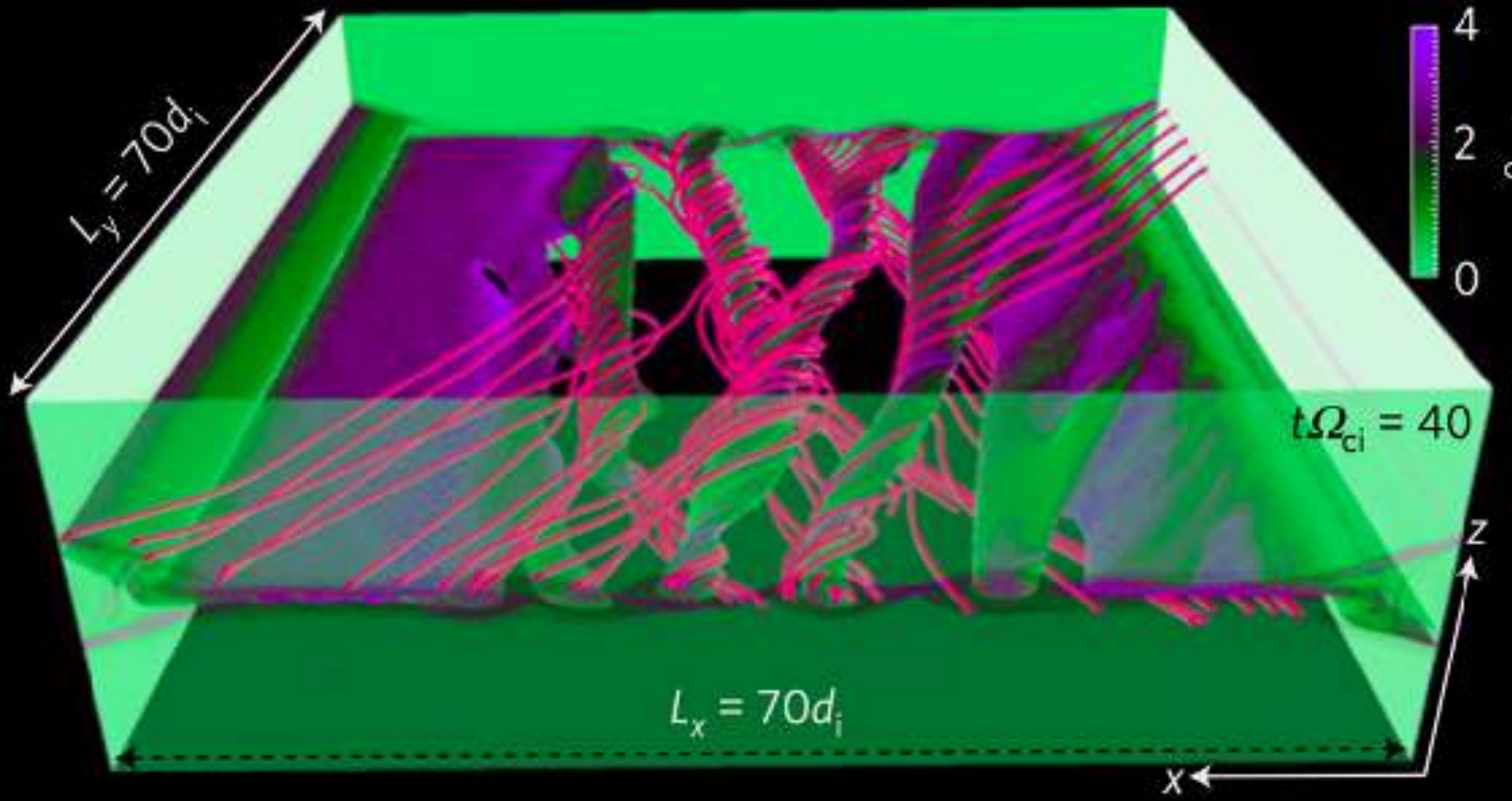
Two schools of thought for intermittency modeling

Current sheet 101 — three dimensions

$$S > S_{\mathrm{crit}}$$



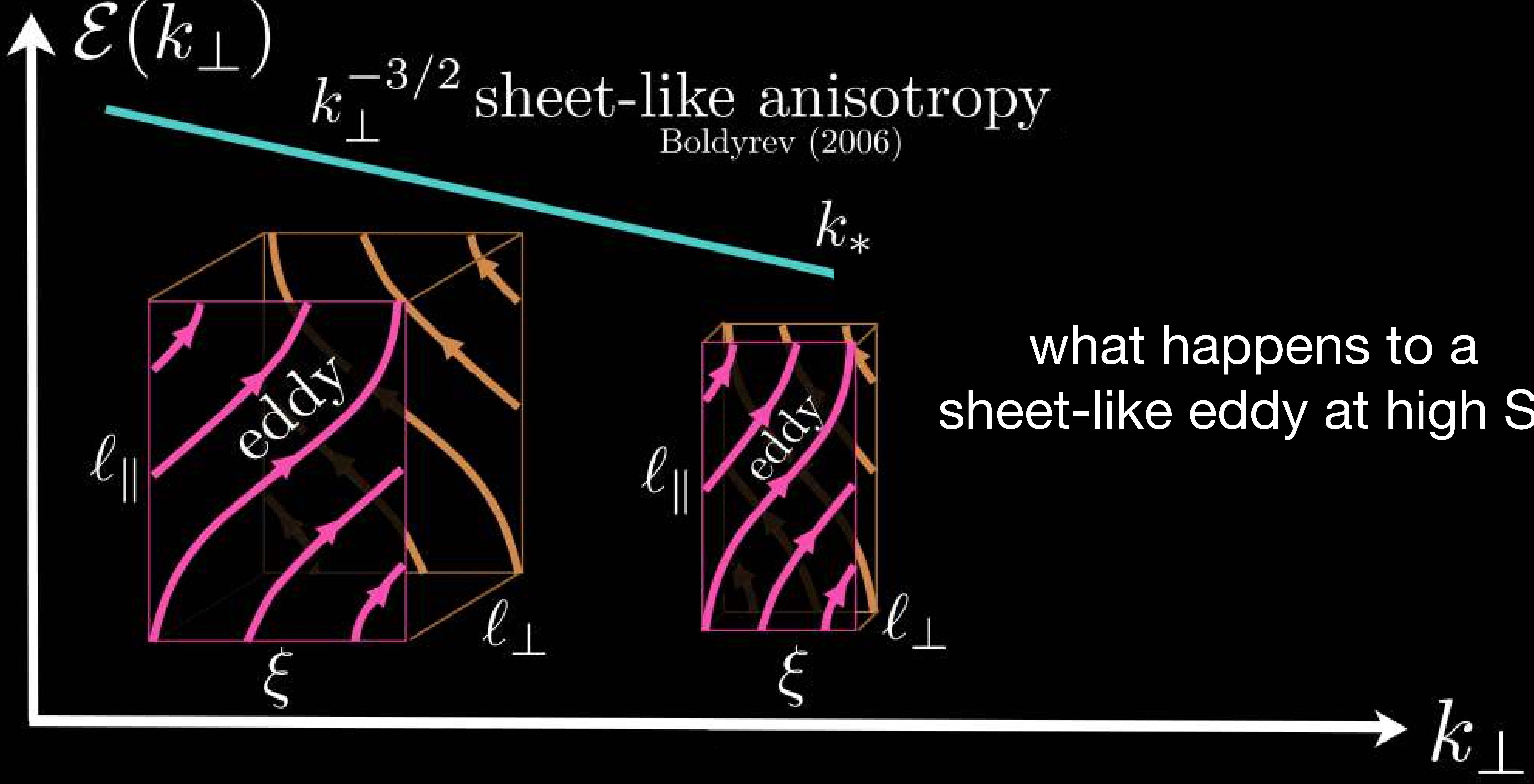
Daughton+2006



Daughton+2010

Two schools of thought for intermittency modeling

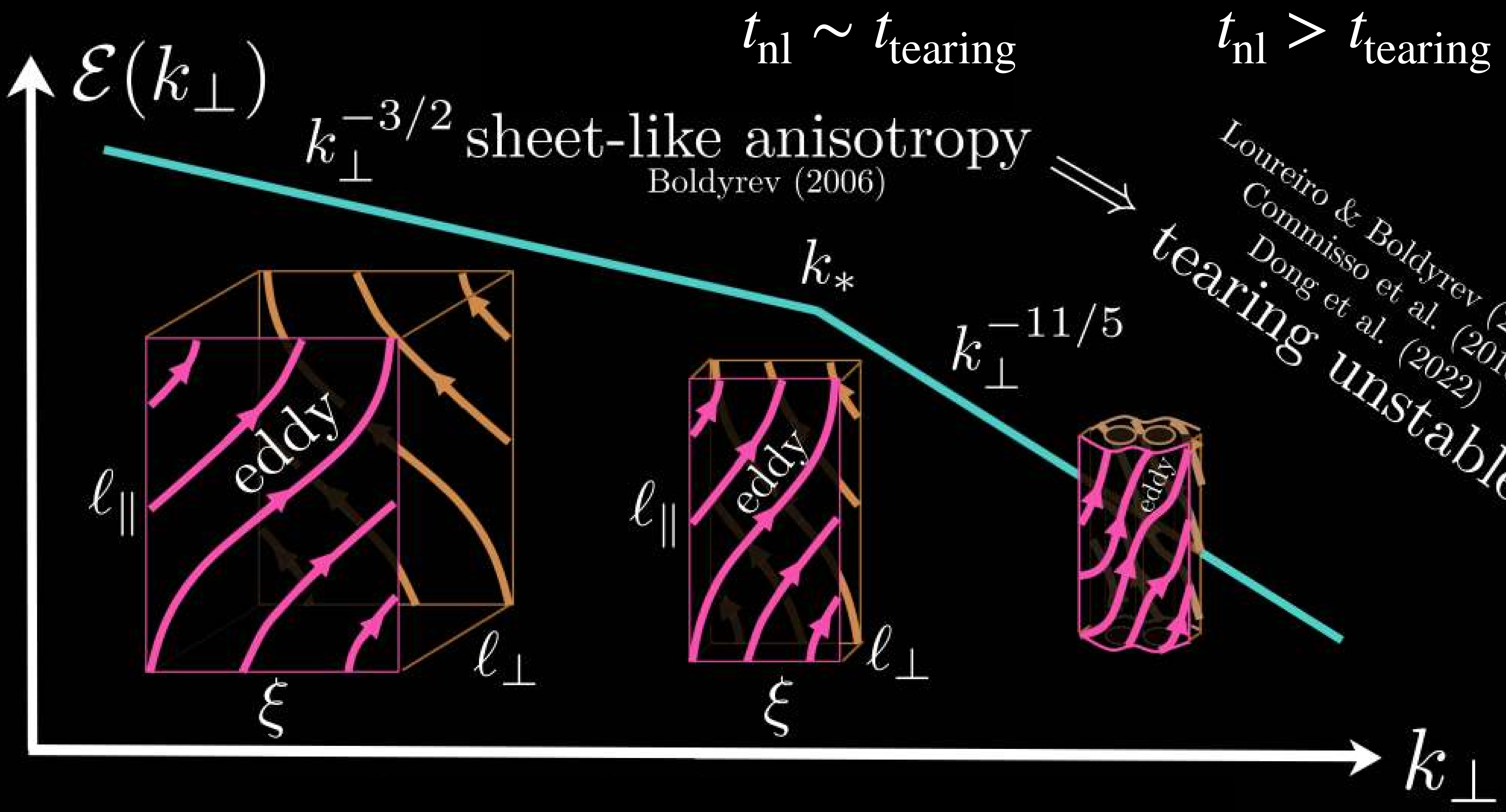
2) intermittent structures is just the cascade changing at small scales



what happens to a sheet-like eddy at high S?

Two schools of thought for intermittency modeling

2) intermittent structures is just the cascade changing at small scales



Re, Rm, S Landscape

Rincon (2019)

$$S \sim \mathrm{Rm}$$

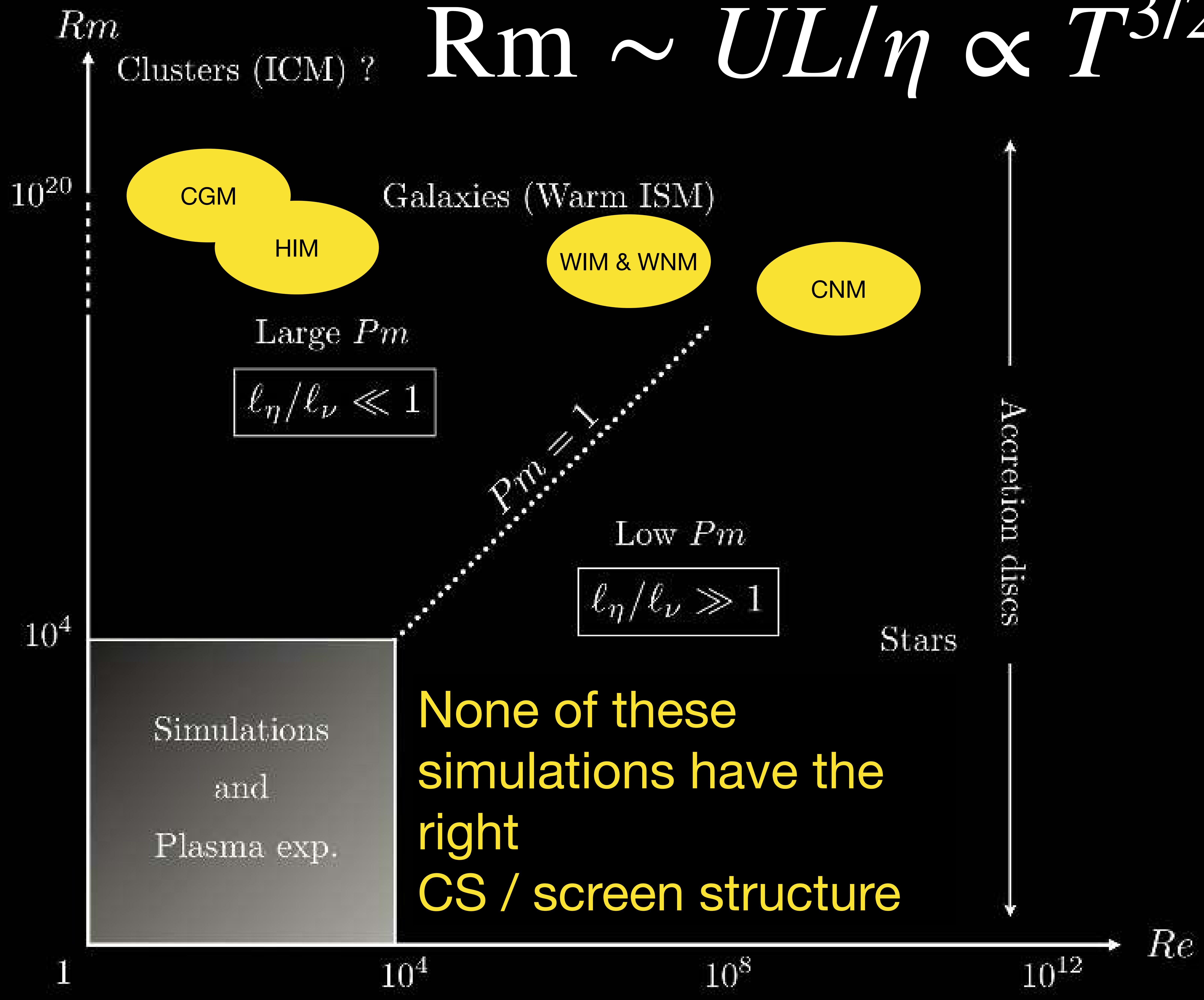
WIM: $\mathrm{Rm} \sim 10^{18}$

WNM: $\mathrm{Rm} \sim 10^{18}$

CNM: $\mathrm{Rm} \sim 10^{15}$

Ferrière, 2020; Plasma Physics and Controlled Fusion

$$\mathrm{Rm} \sim UL/\eta \propto T^{3/2}$$



None of these simulations have the right CS / screen structure

$10{,}080^3$ magnetized supersonic turbulence simulation

Beattie, Federrath, Klessen, Cielo & Bhattacharjee

1. What is the scaling of the energy cascade in compressible MHD turbulence with no net flux?
2. How are the characteristic scales organized in the compressible supersonic turbulence?
3. What are the saturation physics of the compressible turbulent dynamo?

PI of a three total 190<u>million core-hour</u> projects on superMUC-NG

ILES of compressible MHD turbulence

Turbulence: $\sigma_V / c_s \approx 4,\ \ell_0 = L/2$

Magnetic fields: $B = b_{\rm turb},\ \mathscr{M}_{\rm A} \approx 2$

Three experiments for convergence tests:

- LOW-RES: $2{,}520^3$ (0.3Mcore-h, 8,640cores)
- MID-RES: $5{,}040^3$ (4.0Mcore-h, 34,560cores)
- HIGH-RES: $10{,}080^3$ (80.0Mcore-h, 148,240cores)

3.45PB in data products $\mathrm{Rm} \sim \mathrm{Re} \gtrsim 10^6, \quad \mathrm{Pm} \sim 1 - 2$

Beattie+ 2025 (*Nature Astronomy*). The spectrum of magnetized turbulence in the interstellar medium

Galishnikova+ (2022) *PRX*

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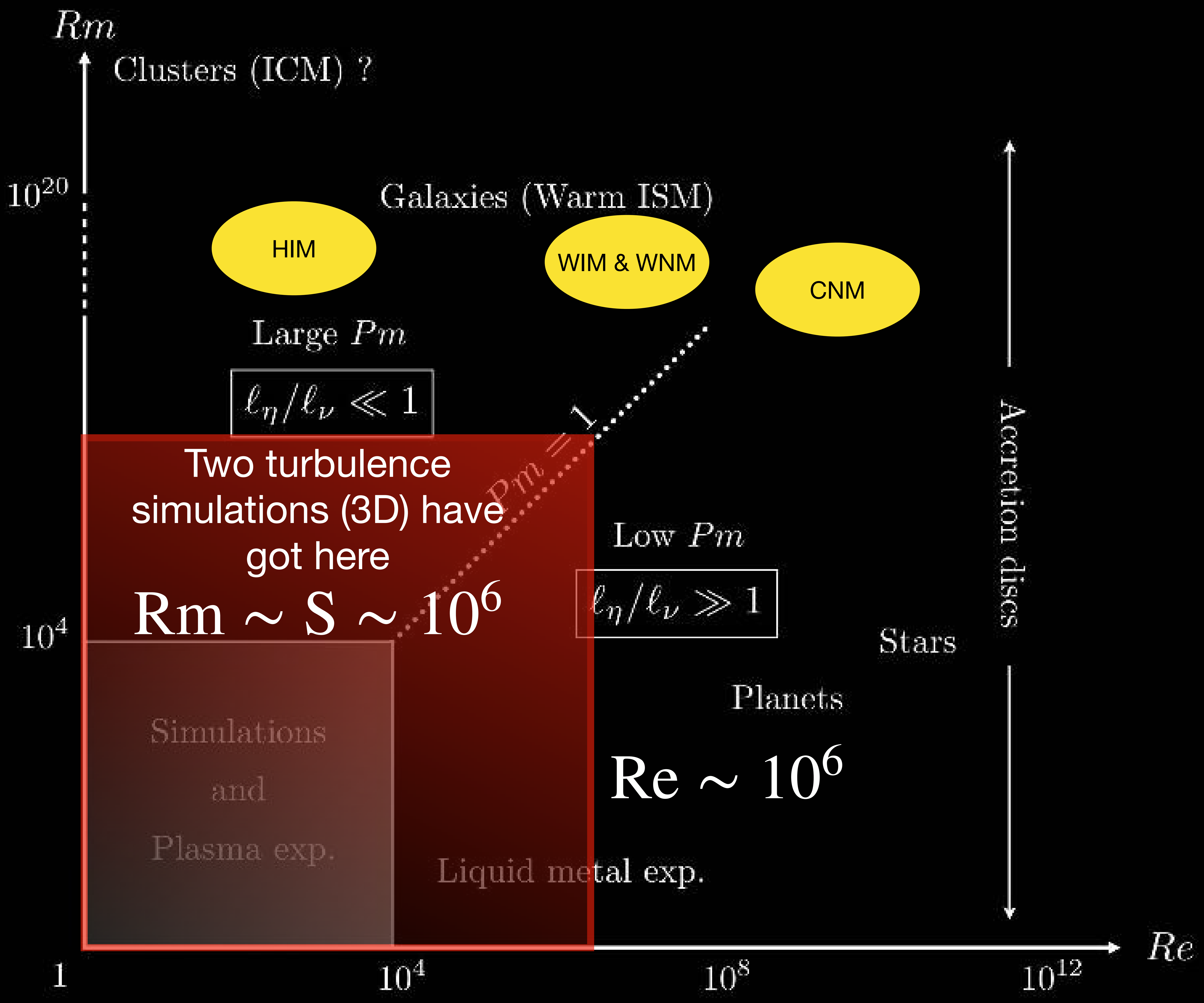

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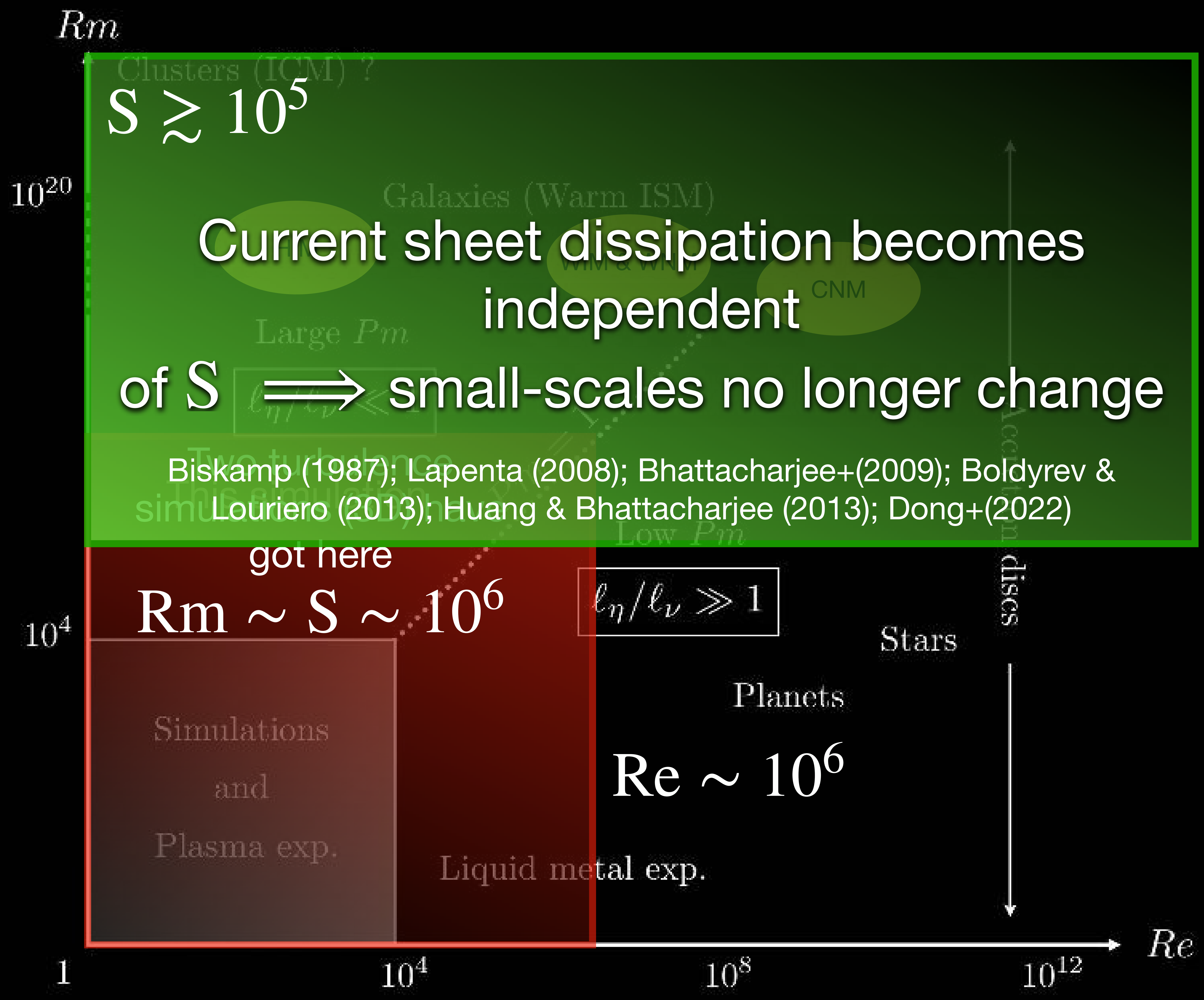
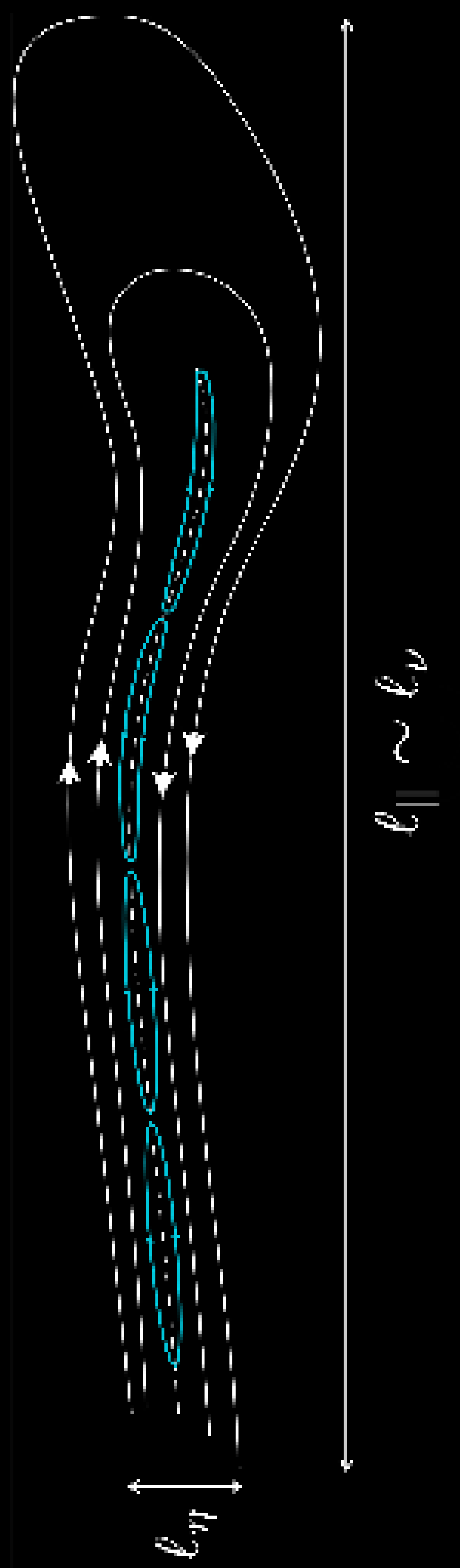
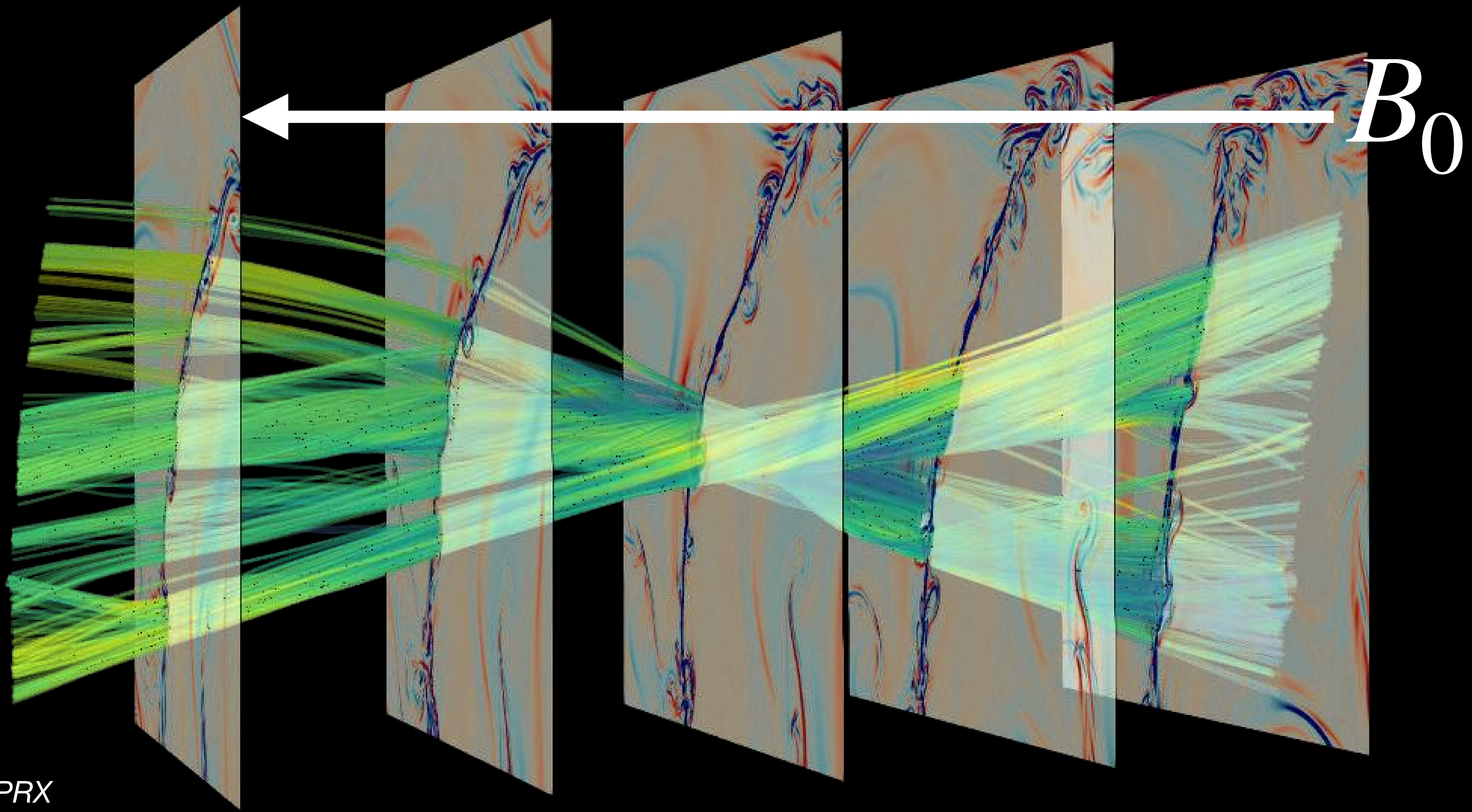


Current sheets in 3D MHD turbulence at high S

Dong+(2022) *Science Advances*



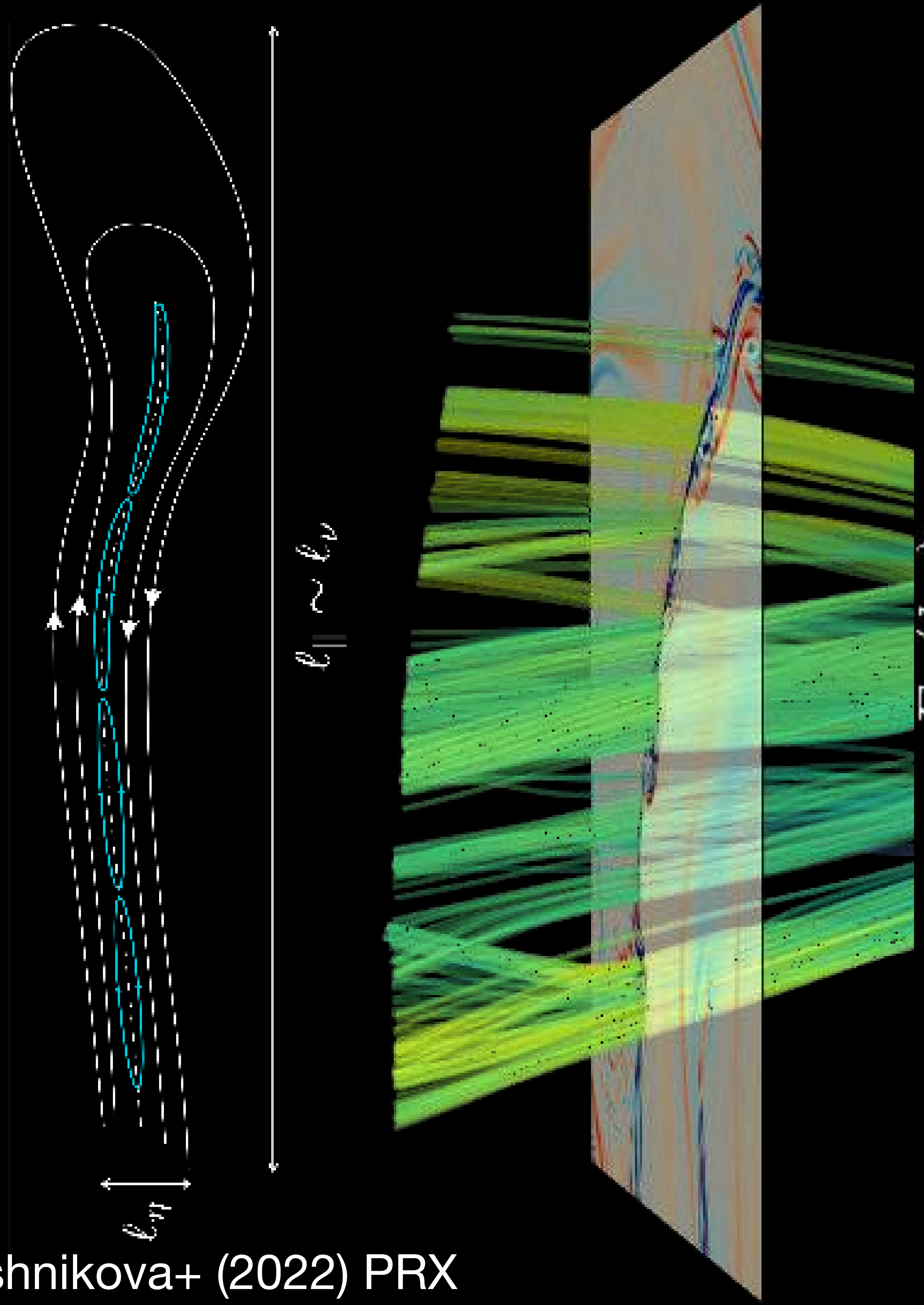
Galishnikova+ (2022) *PRX*



tearing instabilities growing inside a 3D current sheet

Current sheets in 3D MHD turbulence at high S

Dong+(2022) *Science Advances*



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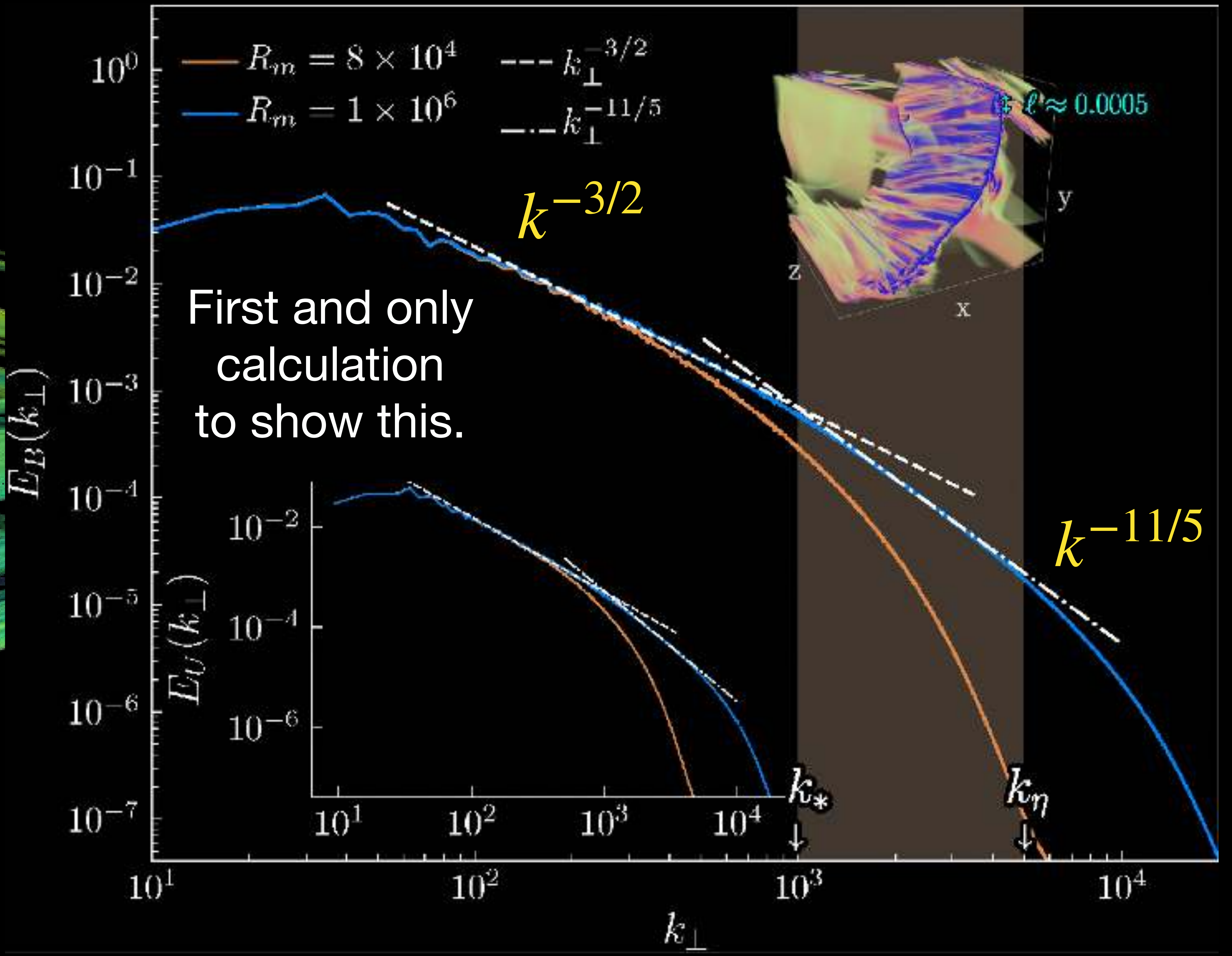


First and only calculation to show this.

3D current sliced into 2D

Beattie+ 2025 (*Nature Astronomy*). The spectrum of magnetized turbulence in the interstellar medium

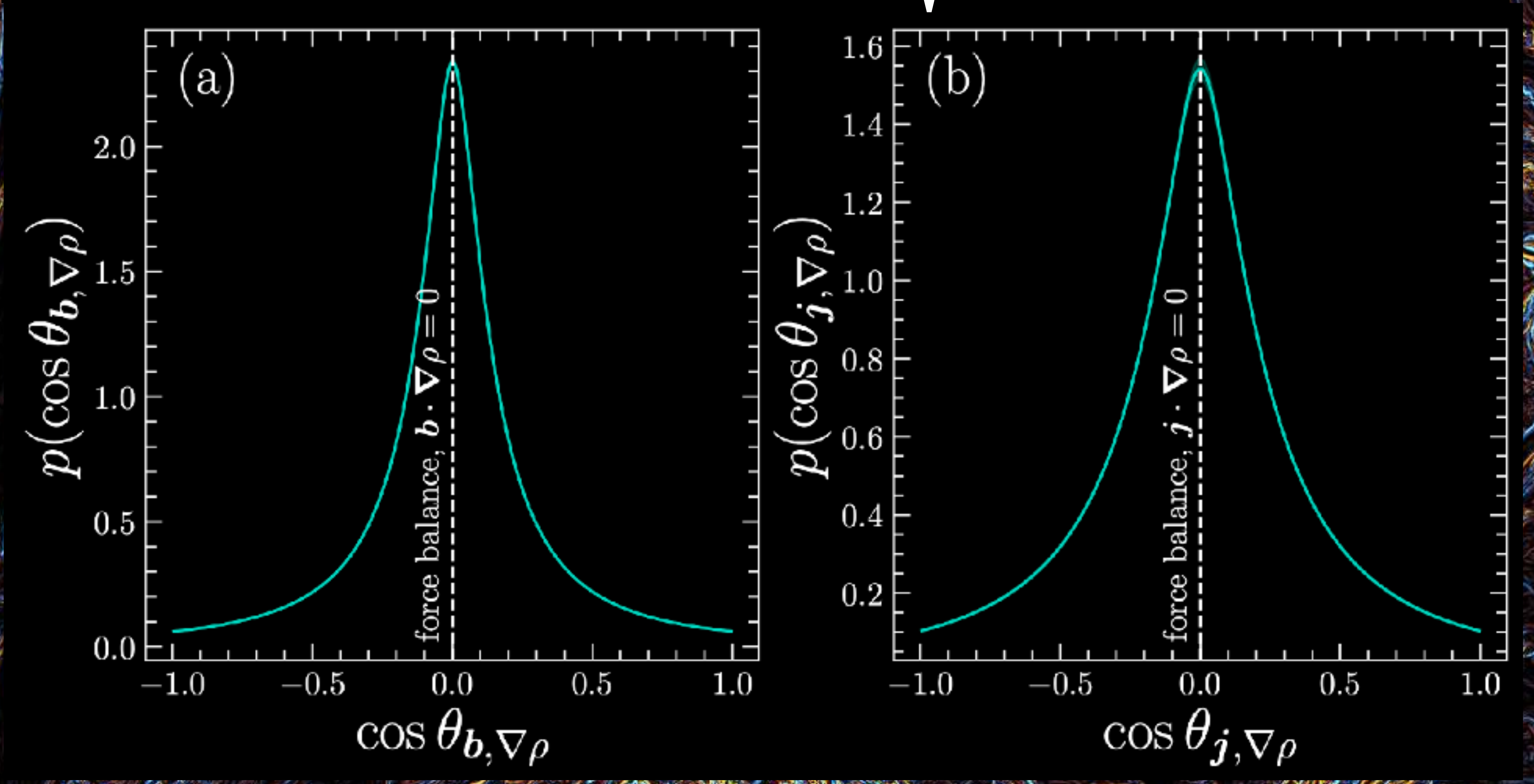
Component of the current along the LOS

Beattie+ 2025 (*Nature Astronomy*). The spectrum of magnetized turbulence in the interstellar medium


Component of the current along the LOS
Long, coherent sheet,
with tearing
Beattie+ 2025 (*Nature Astronomy*). The spectrum of magnetized turbulence in the interstellar medium

Beattie+ 2025 (*Nature Astronomy*). The spectrum of magnetized turbulence in the interstellar medium

$$\nabla P = \mathbf{J} \times \mathbf{B} \qquad J_z(x,t) = \sqrt{\frac{B_0^2}{4\pi^2 \eta t}} \exp\left(-\frac{\pi x^2}{\eta t}\right)$$

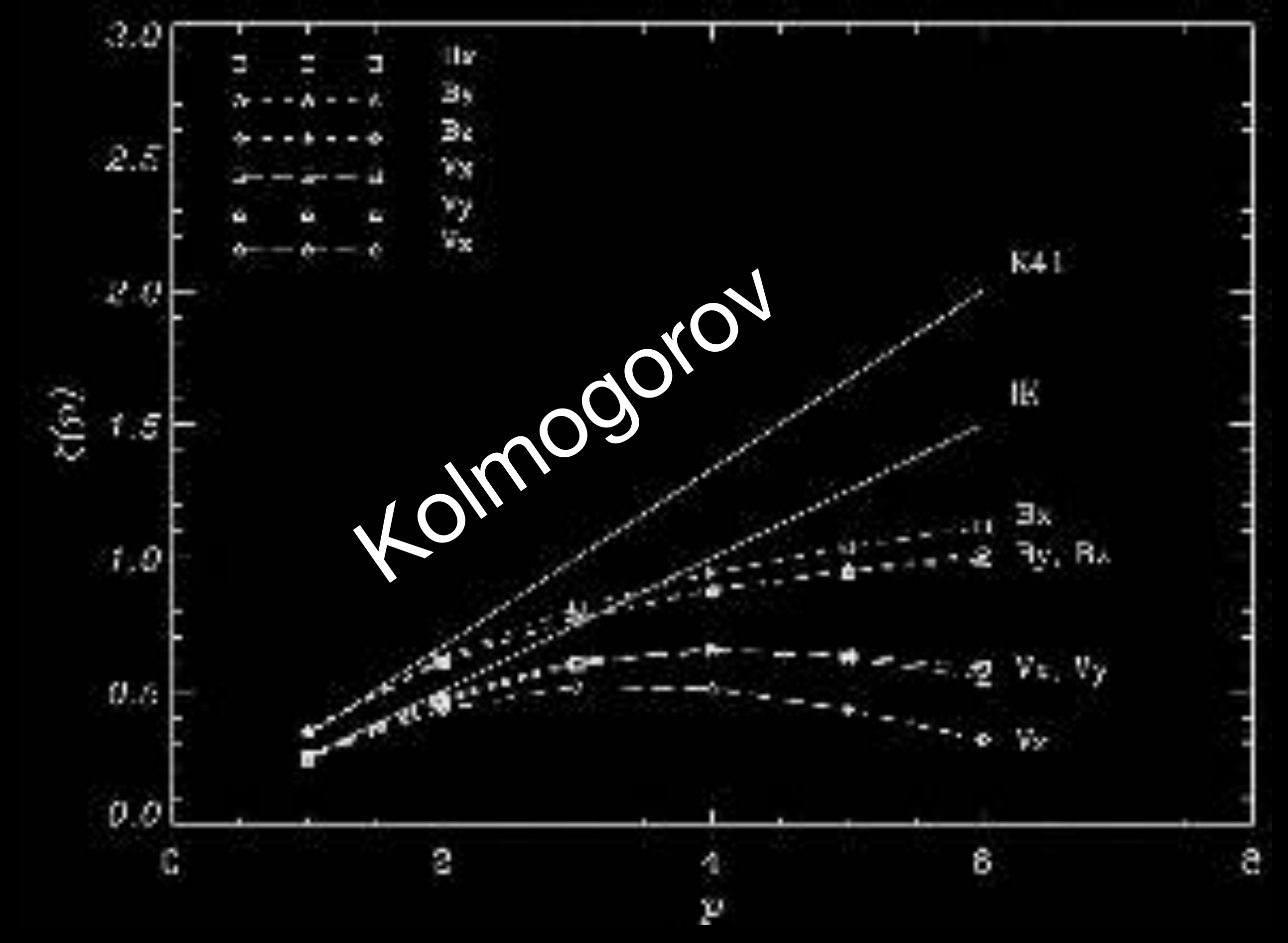


Two schools of thought for intermittency modeling

Non-local modification to the cascade

She & Leveque (1994)

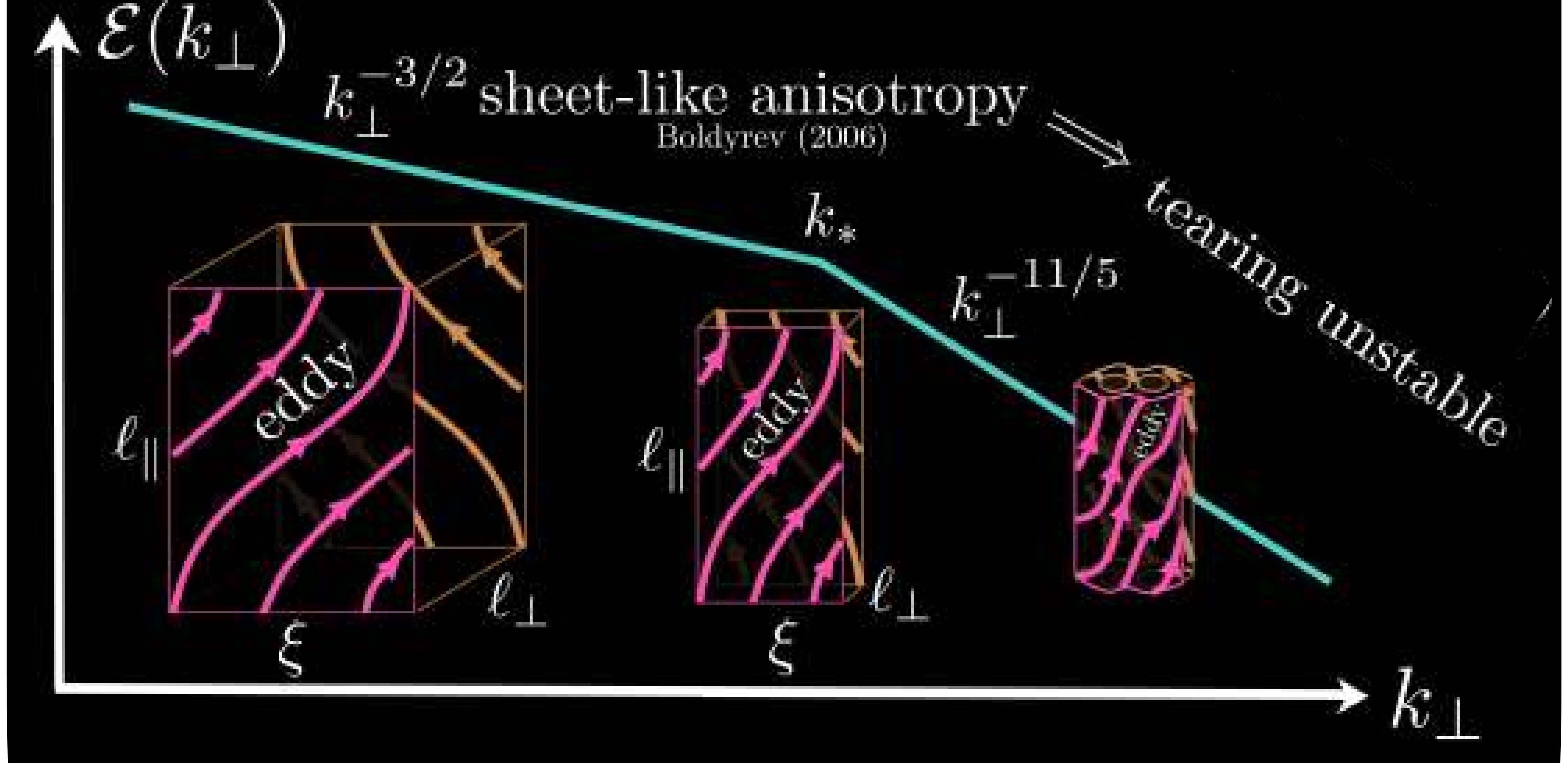
$$\zeta_p = \frac{p}{9} + 2\left[1 - \left(\frac{2}{3}\right)^{p/3}\right],$$



Sheets form out of the turbulence

Local modification to the cascade

Loureiro & Boldyrev (1994)



Sheets are the turbulence

Two schools of thought for intermittency modeling

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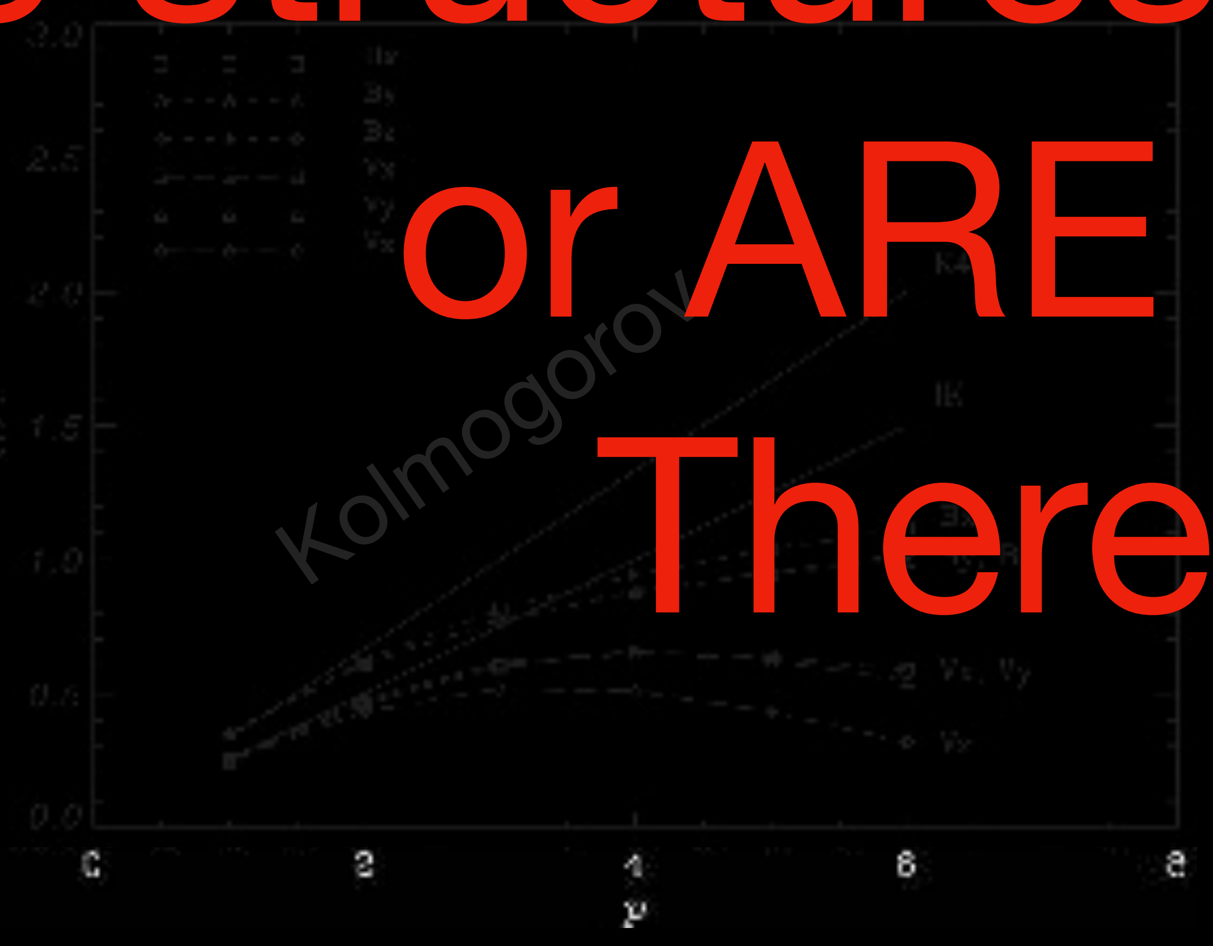
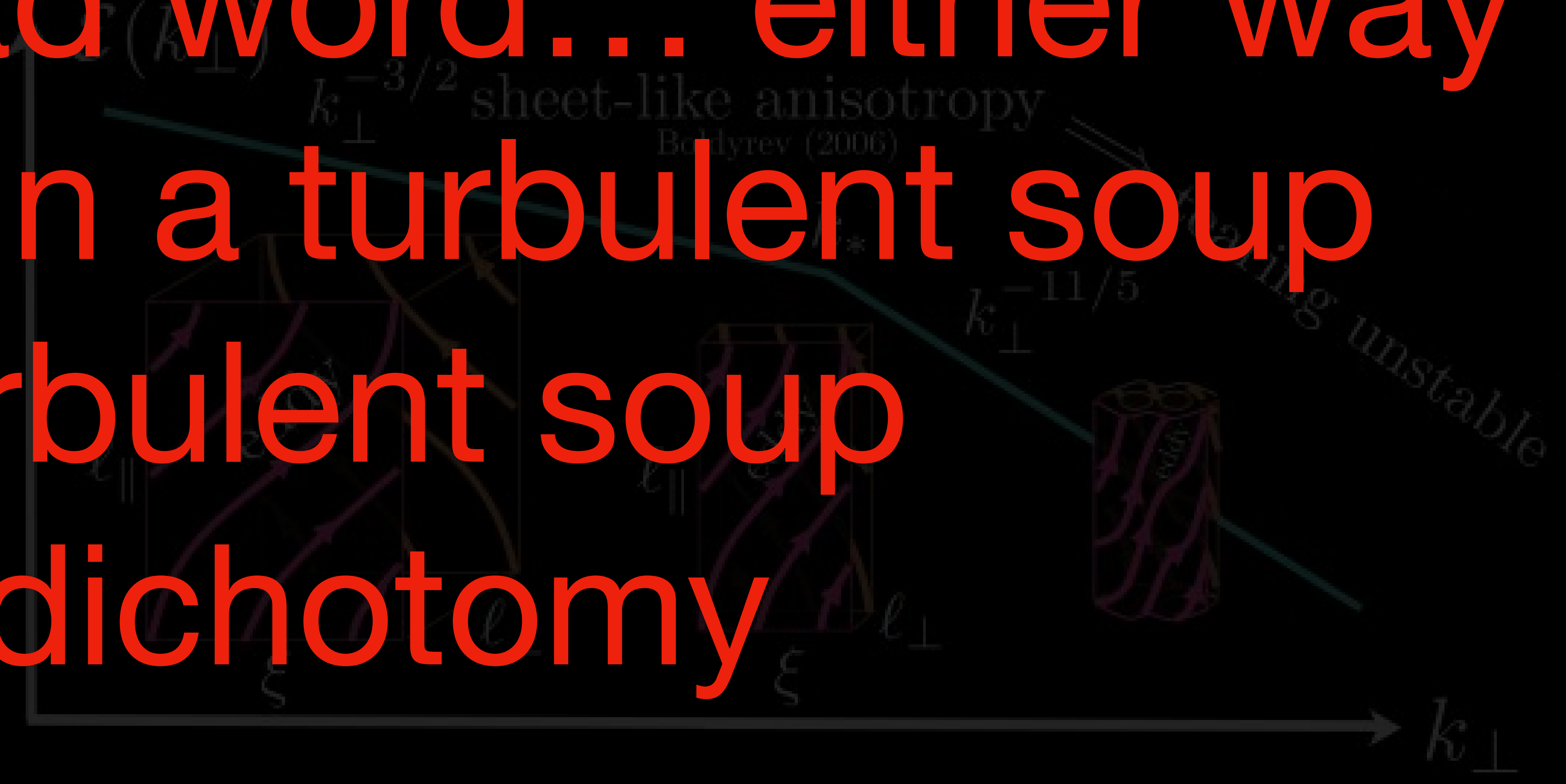


Sheets form out of the turbulence

Local modification to the cascade

Loureiro & Boldyrev (1994)



Sheets are the turbulence

Turbulence is not a bad word… either way
the structures exist in a turbulent soup
or ARE the turbulent soup
There is no dichotomy

Two schools of thought for intermittency modeling

Non-local modification to the cascade

She & Leveque (1994)

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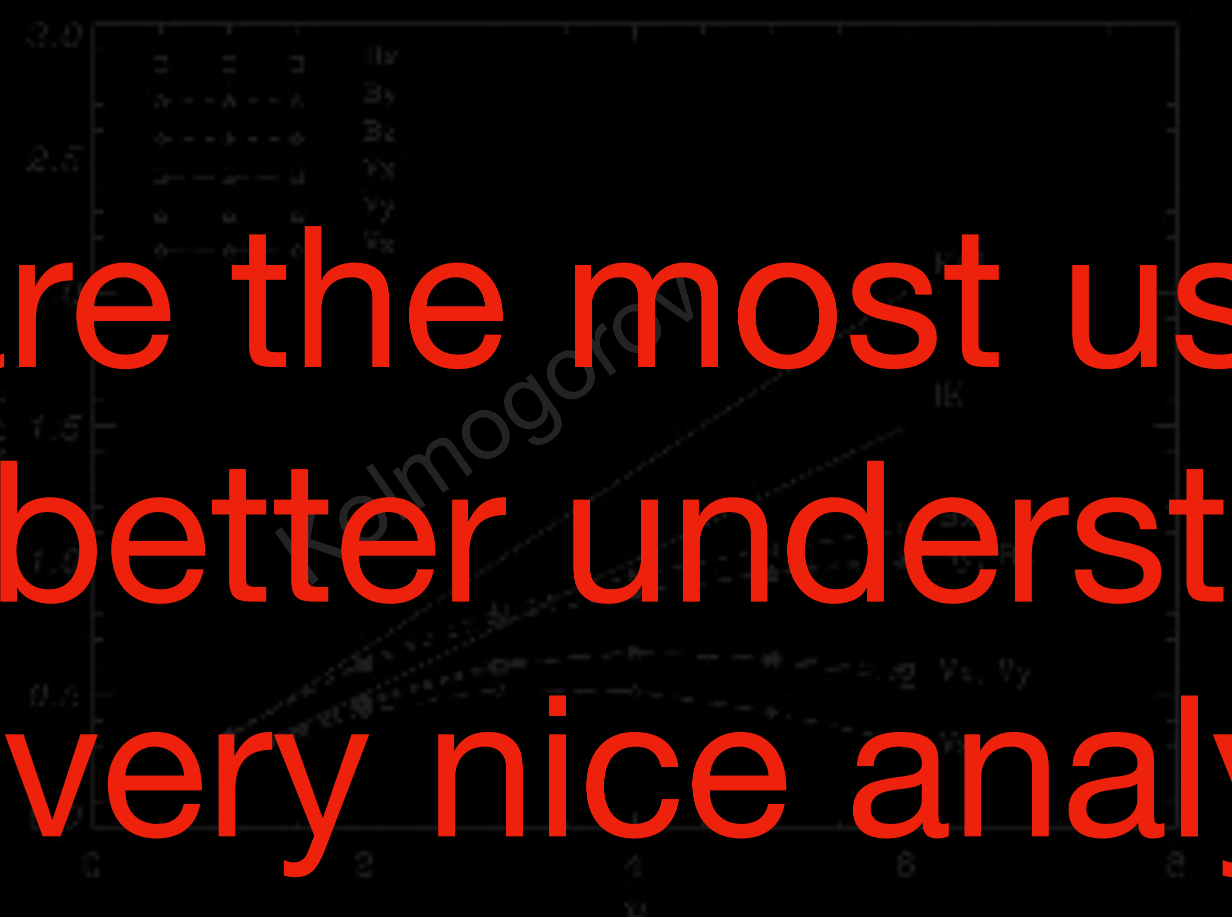

Local modification to the cascade

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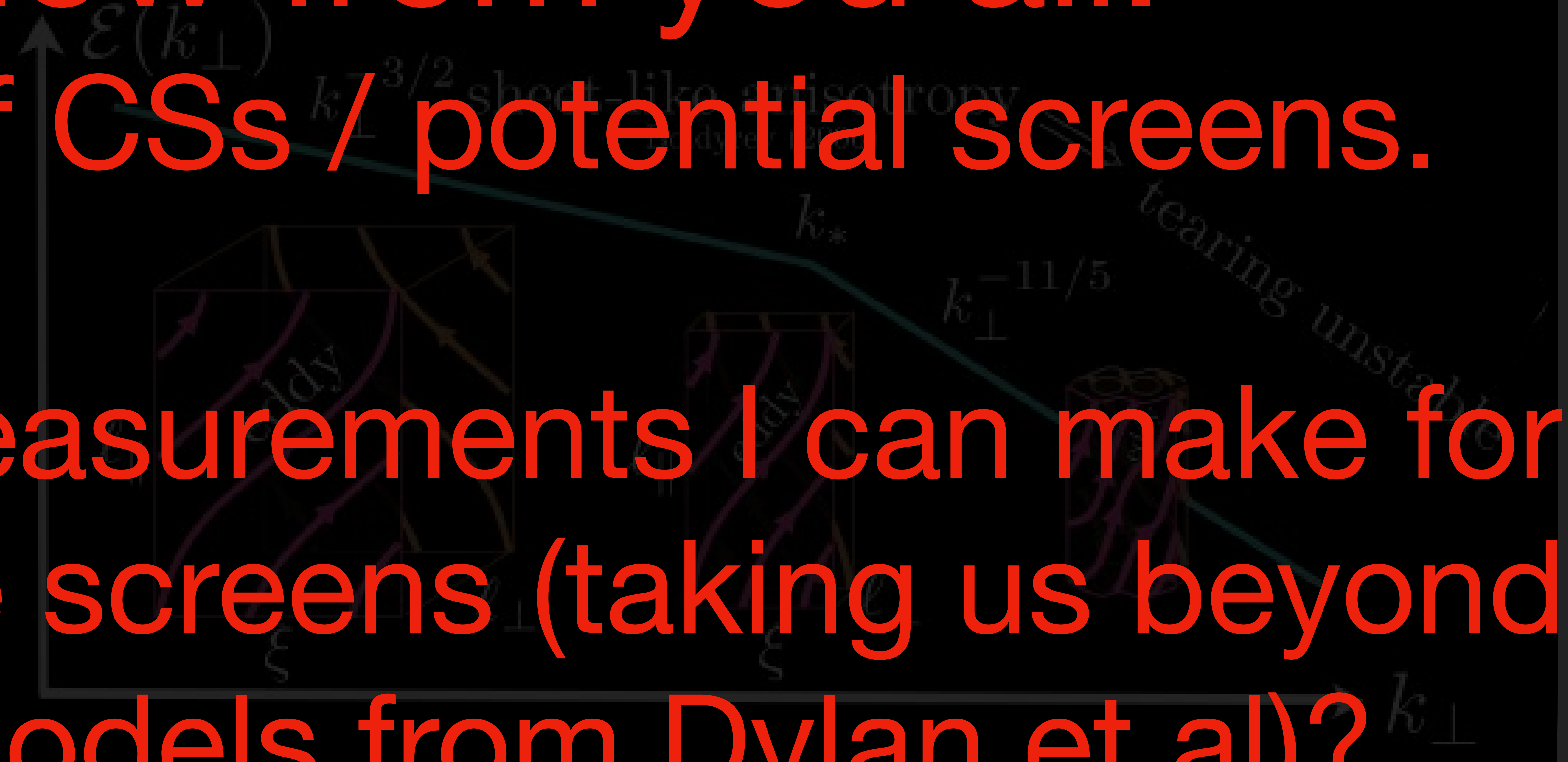


What I want to know from you all:

I have huge populations of CSs / potential screens.

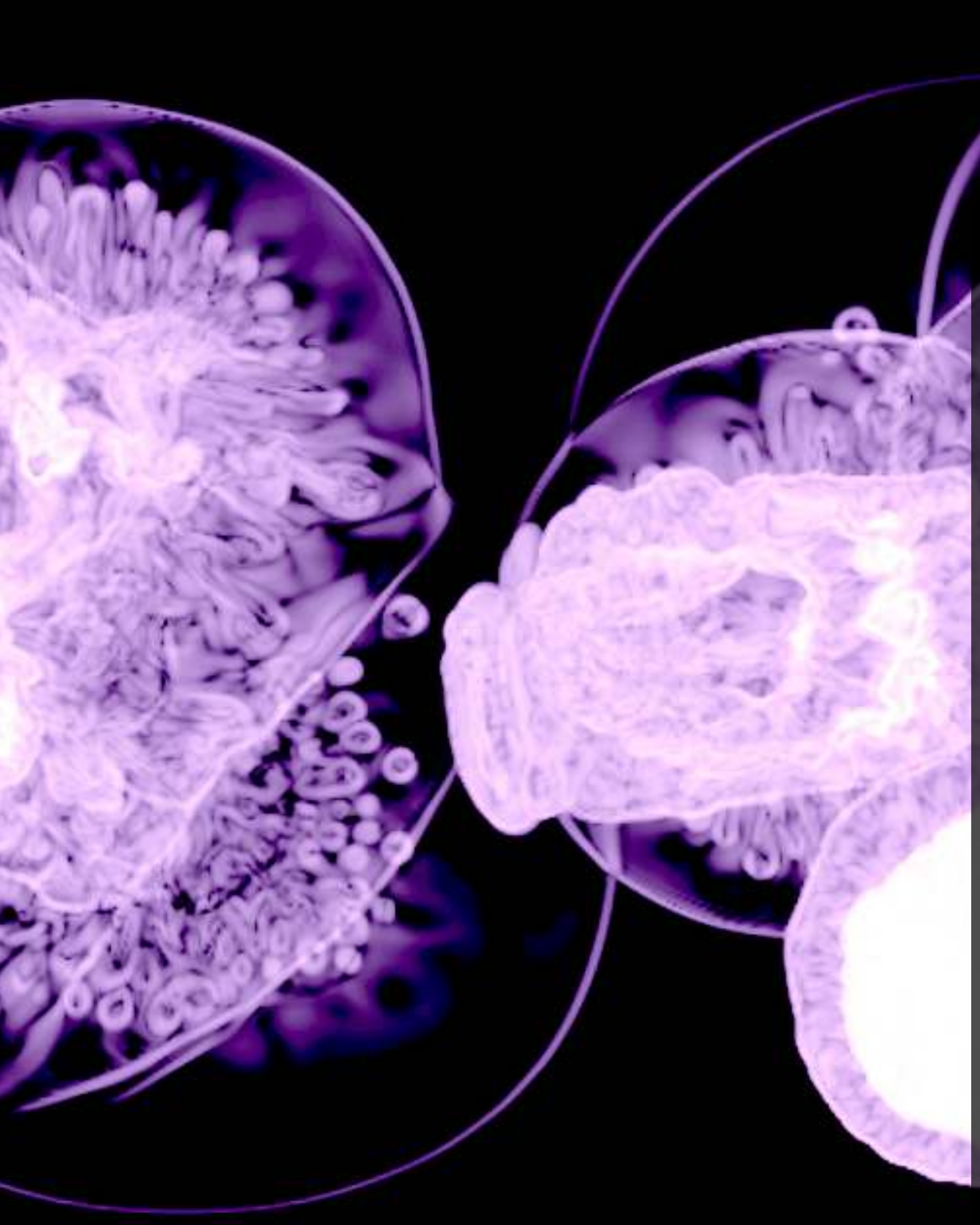
What are the most useful measurements I can make for you to better understand the screens (taking us beyond the very nice analytical models from Dylan et al)?

Sheets form out of the turbulence

Sheets are the turbulence

Thanks, questions?

james.beattie@princeton. @astro_magnetism



- There is no reason to *a priori* assume the ISM is $\sim$ Kolmogorov.
- Structures, like CSs, are part of the turbulence.

Come chat to me about magnetized turbulence and dynamo!

Beattie+ (*Nature Astronomy, 2025*). The spectrum of magnetized turbulence in the interstellar medium

Beattie & *Bhattacharjee* (*submitted PRL, 2025*). Scale-dependent alignment in compressible magnetohydrodynamic turbulence

Beattie+ (MNRAS, *2025*). Taking control of compressible modes: bulk viscosity and the turbulent dynamo

Kriel, **Beattie+** (*submitted PRL, 2025*). Scale-dependent alignment in compressible magnetohydrodynamic turbulence

Sampson, **Beattie+** (*submitted ApJL, 2025*). Cosmic ray and plasma coupling for isothermal supersonic turbulence in the magnetized interstellar medium

Turbulence (and why you need to keep thinking about it)

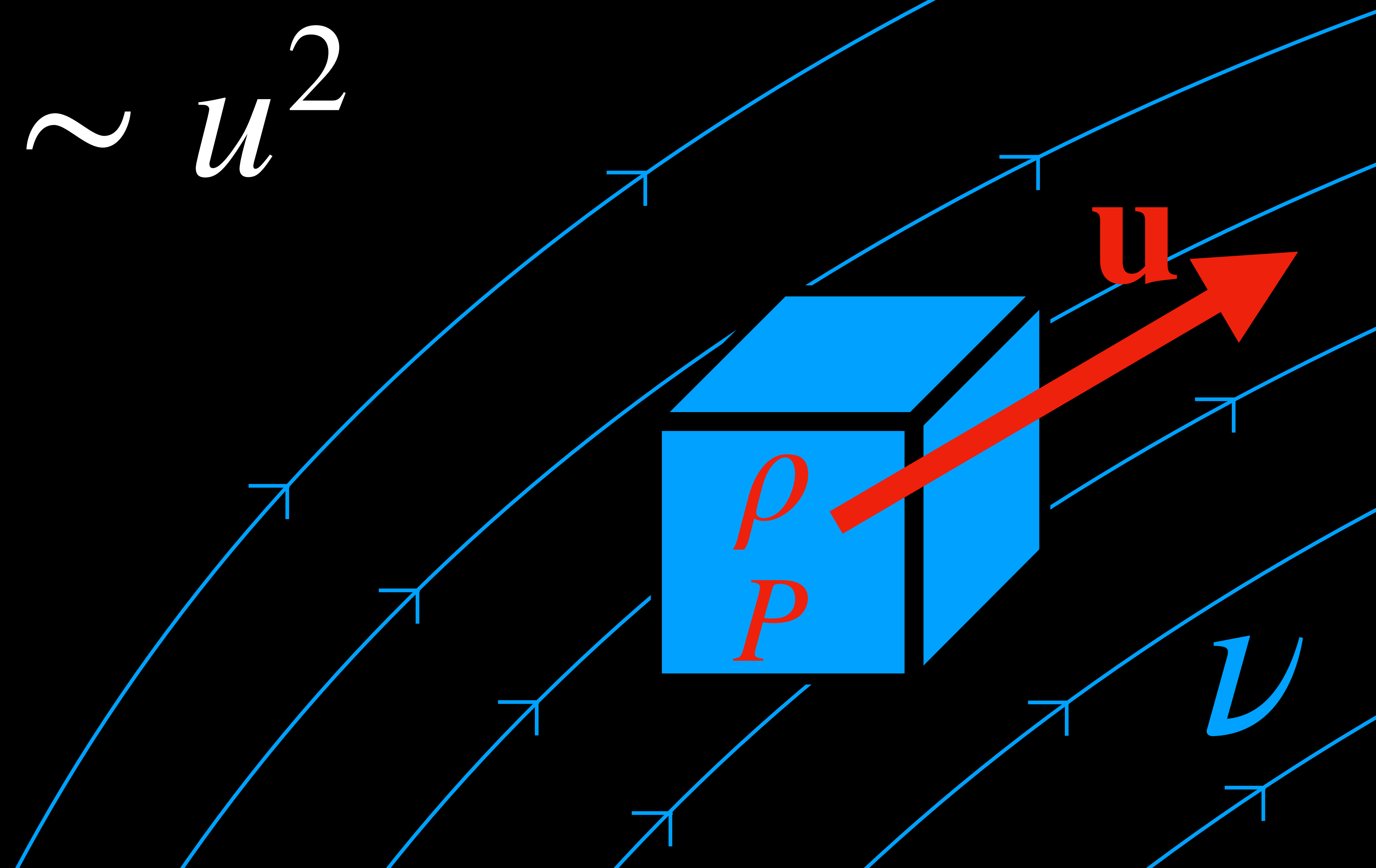
What is it (hand wavey)?

Momentum conservation for a hydrodynamical fluid element

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) = -\nabla \cdot P\mathbb{I} + 2\nu \nabla \cdot (\rho \mathcal{S})$$

$$\mathbf{u} \otimes \mathbf{u} = u_i u_j \sim u^2$$

$$\lambda_{\mathrm{mfp}}/L \ll 1$$



Turbulence (and why you need to keep thinking about it)

What is it (hand wavey)?

Reynolds stress

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) = - \nabla \cdot P\mathbb{I} + 2\nu \nabla \cdot (\rho \mathcal{S})$$

$$\mathbf{u} \otimes \mathbf{u} = u_i u_j \sim u^2$$

Viscous stress

Turbulence (and why you need to keep thinking about it)

What is it (hand wavey)?

quadratic nonlinearity

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) = - \nabla \cdot P\mathbb{I} + 2\nu \nabla \cdot (\rho \mathcal{S})$$

$$\mathbf{u} \otimes \mathbf{u} = u_i u_j \sim u^2$$

Smooths out nonlinear things in the fluid

Turbulence (and why you need to keep thinking about it)

What is it (hand wavey)?

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) = - \nabla \cdot P\mathbb{I} + 2\nu\nabla \cdot (\rho \mathcal{S})$$

Creating nonlinear things in the fluid

$$\mathrm{Re} = \frac{|\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})|}{|2\nu\nabla \cdot (\rho \mathbb{S})|} \sim \frac{UL}{\nu} \propto \frac{n_e}{T^{5/2}}$$

Smoothing out nonlinear things in the fluid

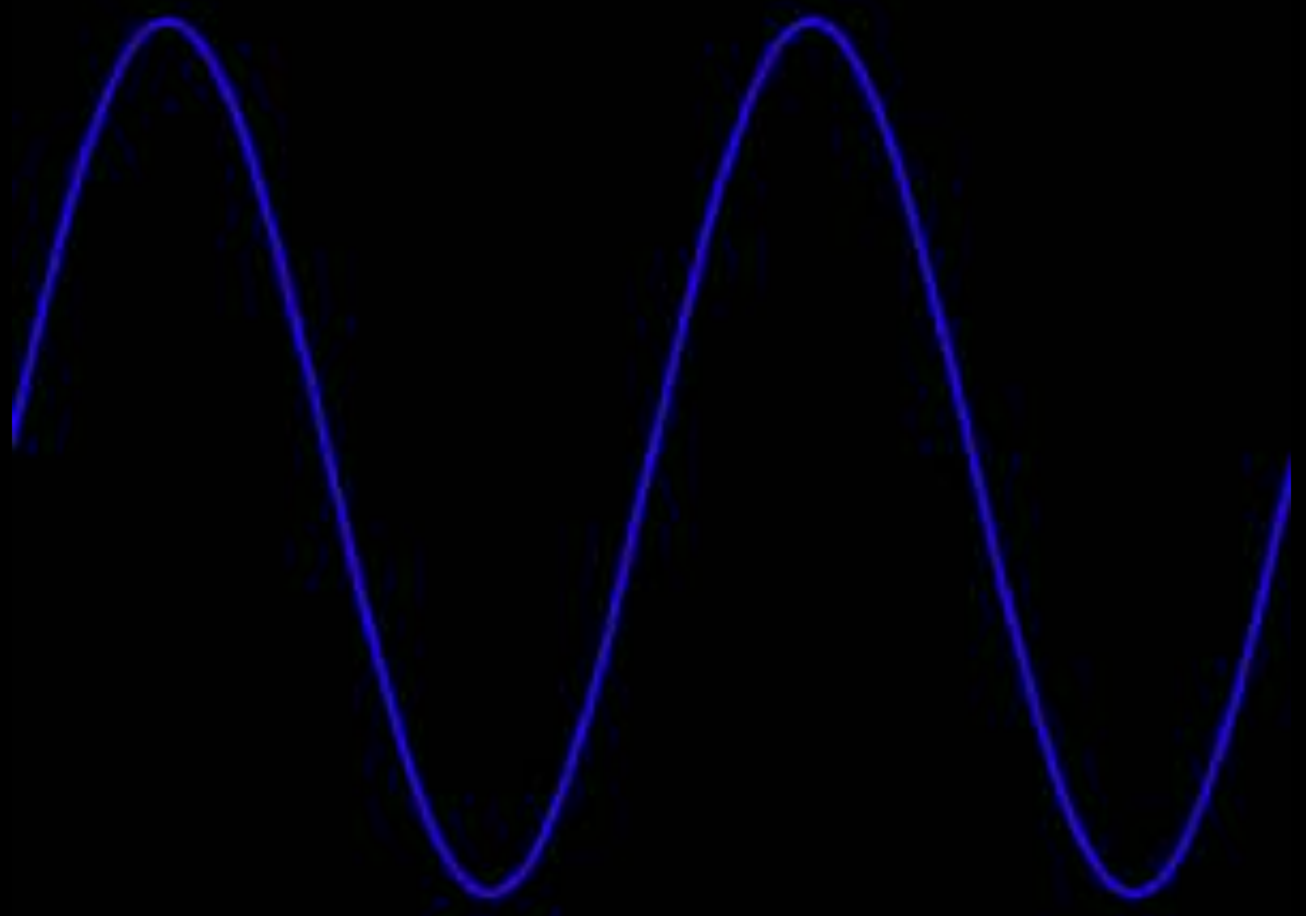
Turbulence

What does $\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})$ want to do?

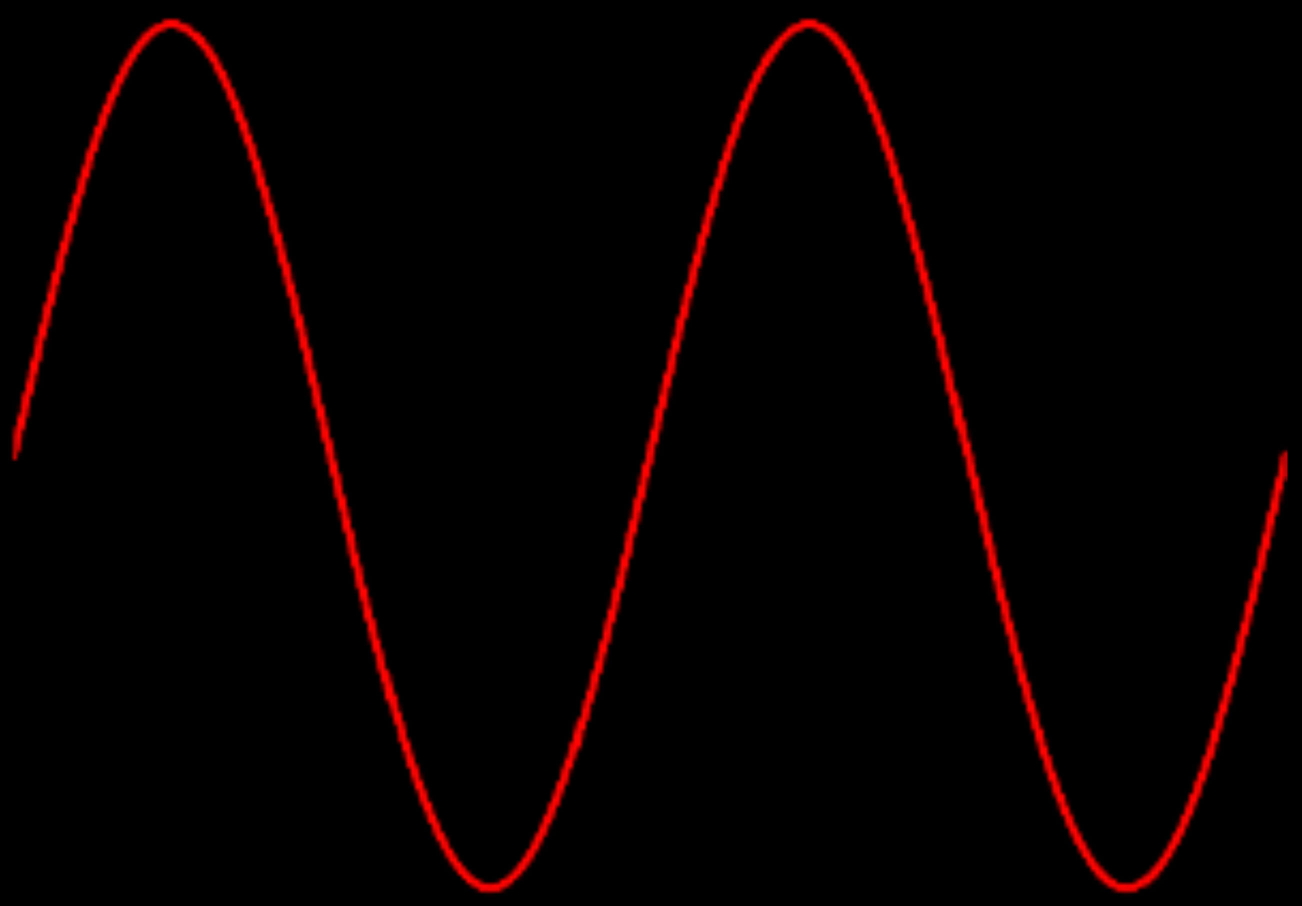
$$k_1 = k_2$$

Consider two waves

$$u_1(x) = \sin(k_1 x)$$
$$u_2(x) = \sin(k_2 x)$$



$u_1(x)$



$u_2(x)$

really these should be "eddies"
isotropic packets of correlation, that are local in k space

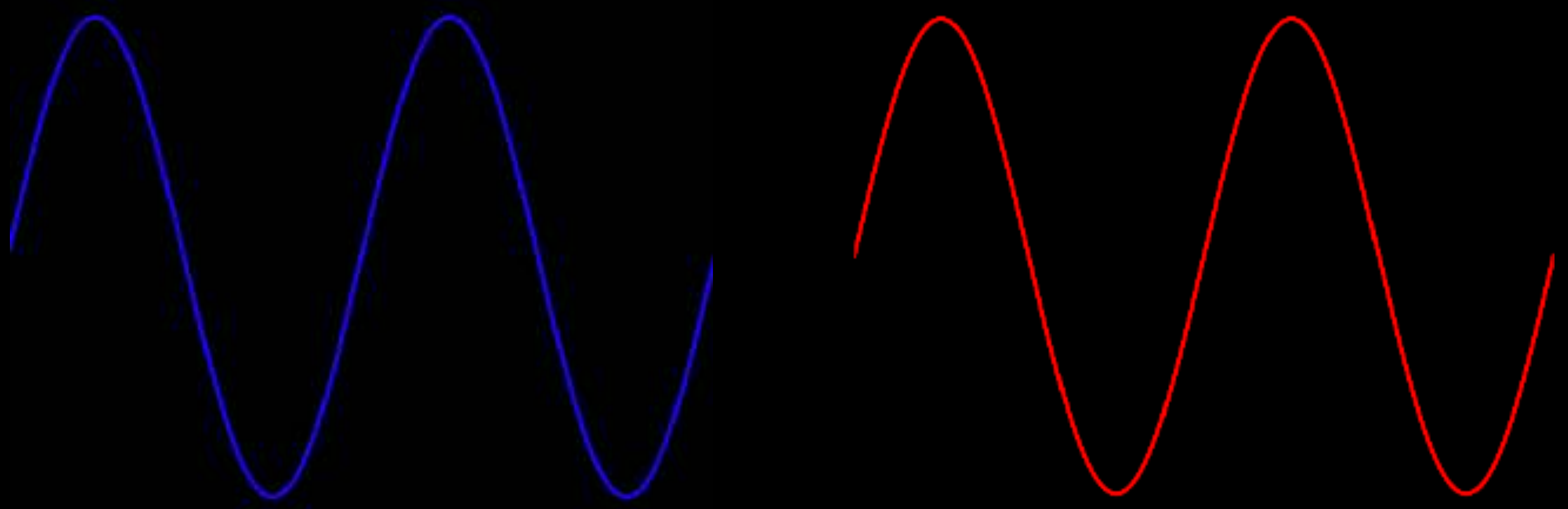
Turbulence

What does $\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})$ want to do?

$$k_1 = k_2$$

Consider two waves

$$u_1(x) = \sin(k_1 x)$$
$$u_2(x) = \sin(k_2 x)$$



The nonlinear term creates a new wave

$$\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) \sim u_1 \partial_x u_2 = k_2 \sin(k_1 x) \cos(k_2 x) \propto \sin(k_3 x)$$

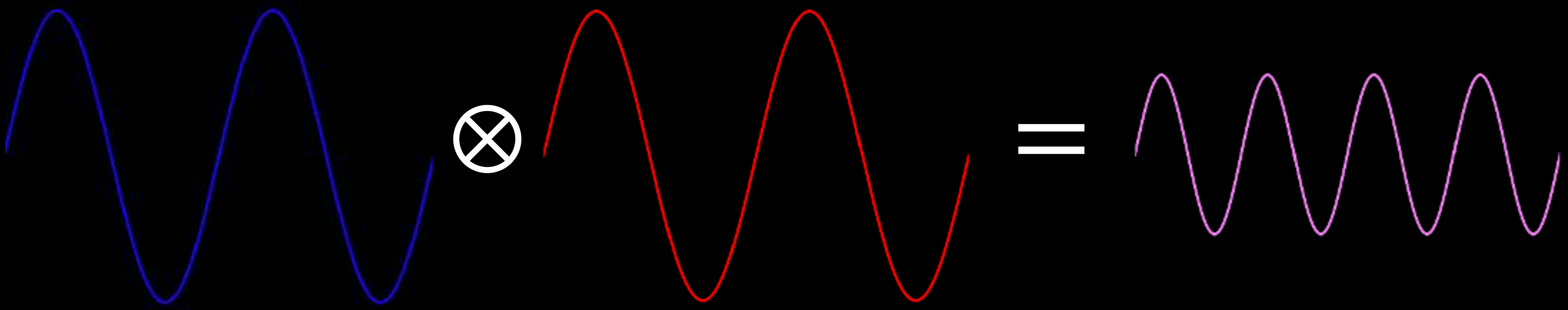
$$k_1 + k_2 = k_3$$

(momentum conservation)

Turbulence

What does $\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})$ want to do — cascade!!!

$$\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})$$

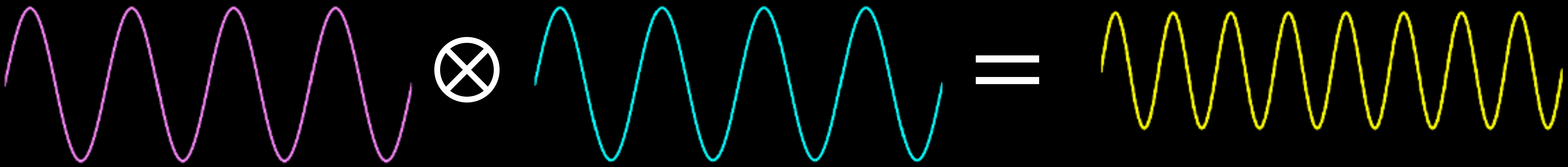


$$k_1 + k_2 = k_3$$

Turbulence

What does $\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})$ want to do — cascade!!!

$$\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})$$

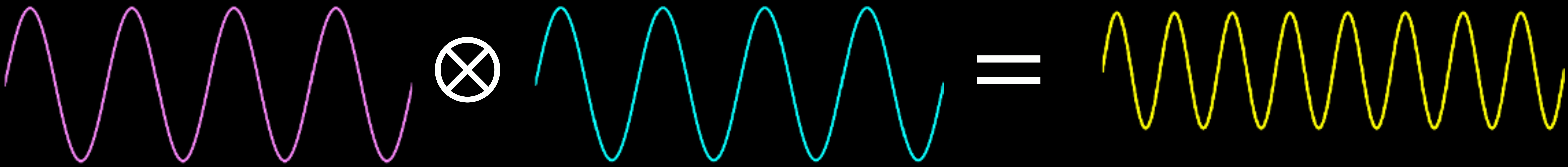


$$k_3 + k_4 = k_5$$

Turbulence

What does $\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})$ want to do — cascade!!!

$$\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})$$



$$k_3 + k_4 = k_5$$

New eddies created on nonlinear timescales:

$$t_{\rm nl} \sim [\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})]^{-1} \sim \ell / u$$