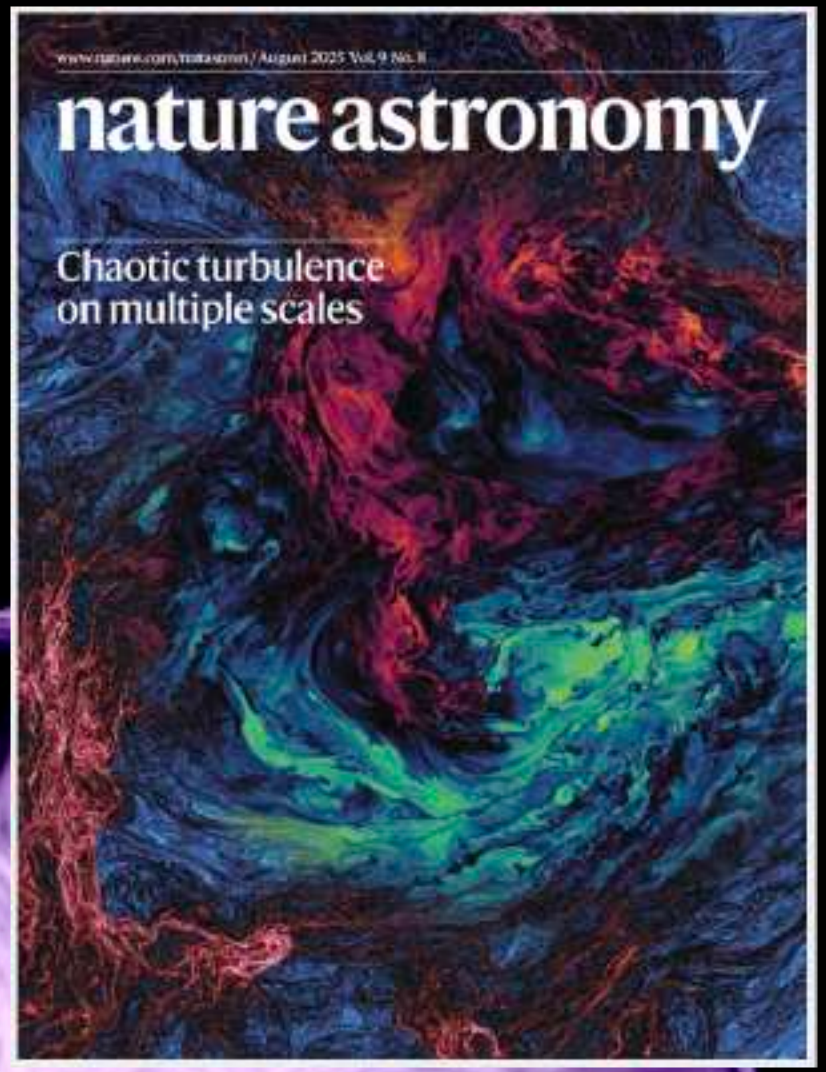

Beattie, *Federrath, Klessen, Cielo, Bhattacharjee,* 2025 (*Nature Astronomy*). The spectrum of magnetized turbulence in the interstellar medium

Supernova remnants as engines of non-Kolmogorov galactic turbulence

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Astro Seminar, Michigan State University
James Beattie
Fellow / Research Associate
Canadian Institute for Theoretical Astrophysics / Princeton

In collaboration: Isabelle Connor (UCSC), Anne Noer Kolborg (UCSC), Enrico Ramiez Ruiz (UCSC), Amitava Bhattacharjee (Princeton), Christoph Federrath (ANU), Ralf Klessen (UH), Salvatore Cielo (LRZ)

Beattie + (2025; ApJ) So long Kolmogorov: the forward and backwards cascades in a supernovae-driven, multiphase ISM
Connor, **Beattie** + (2025, ApJ) Cascading from the winds to the disk: universality of supernovae-driven turbulence in different galactic ISMs
Beattie (2026; submitted ApJL) Supernovae drive large-scale incompressible turbulence from small-scale instabilities

Roadmap for this seminar

1. Discuss K41 turbulence in a pedagogical manner.

2. Turbulence in our galaxy.

3. Supernova-driven turbulence — how to fit them into the cascade? Simulations.

4. Spectra, cascade directions.

5. Where does the incompressible turbulence come from — shell instabilities and spectra model.

Turbulence

What is it (hand wavey)?

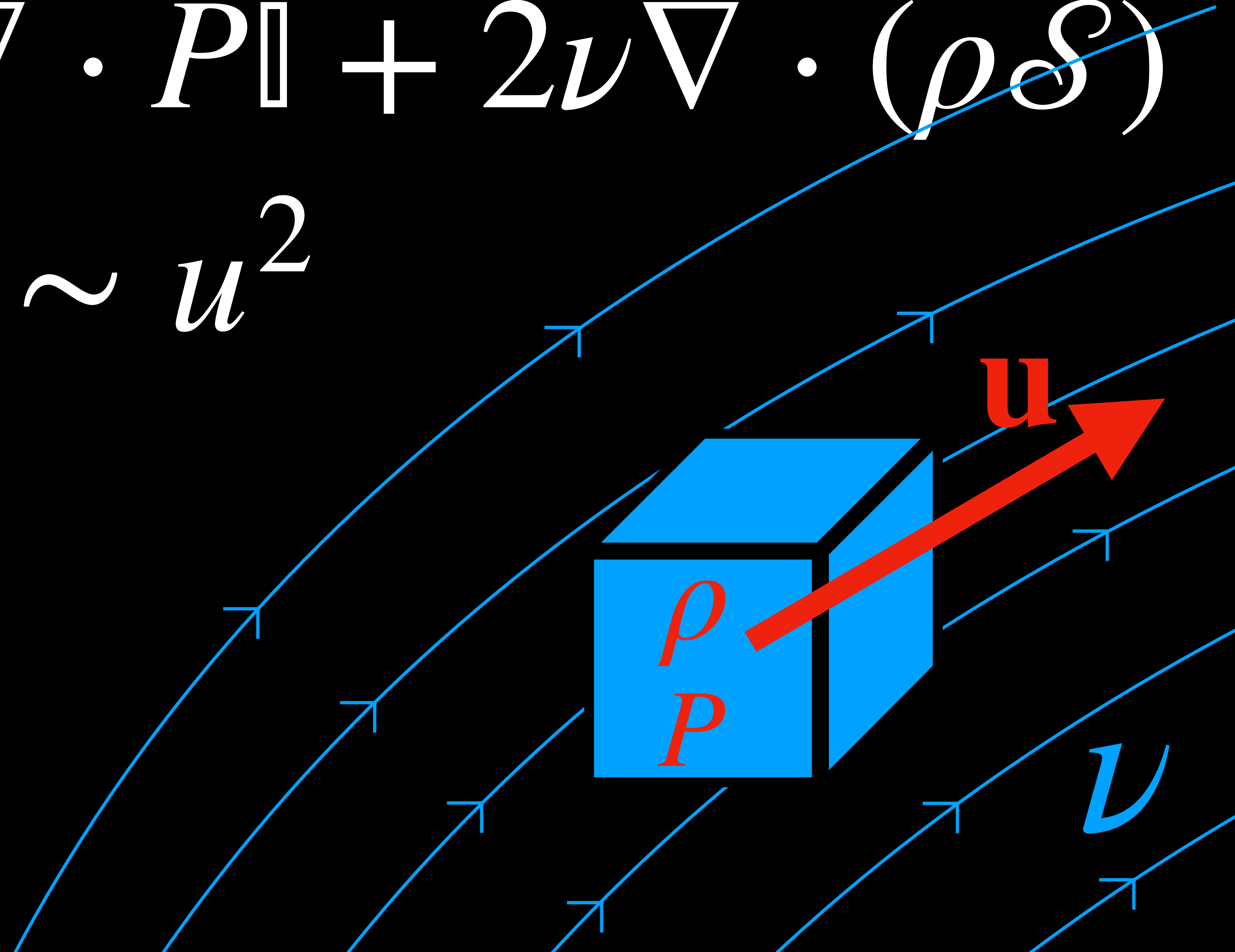
Momentum conservation for a hydrodynamical fluid element

(first moment of Boltzmann eq. for monoatomic plasma slightly out of LTE)

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) = -\nabla \cdot P\mathbb{I} + 2\nu\nabla \cdot (\rho \mathcal{S})$$

$$\mathbf{u} \otimes \mathbf{u} = u_i u_j \sim u^2$$

$$\lambda_{\mathrm{mfp}}/L \ll 1$$



Turbulence

What is it (hand wavey)?

Reynolds stress

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) = -\nabla \cdot P\mathbb{I} + 2\nu\nabla \cdot (\rho \mathcal{S})$$

$$\mathbf{u} \otimes \mathbf{u} = u_i u_j \sim u^2$$

Viscous stress

Turbulence

What is it (hand wavey)?

quadratic nonlinearity

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) = -\nabla \cdot P\mathbb{I} + 2\nu \nabla \cdot (\rho \mathcal{S})$$

$$\mathbf{u} \otimes \mathbf{u} = u_i u_j \sim u^2$$

Smooths out nonlinear things in the fluid

Turbulence

What is it (hand wavey)?

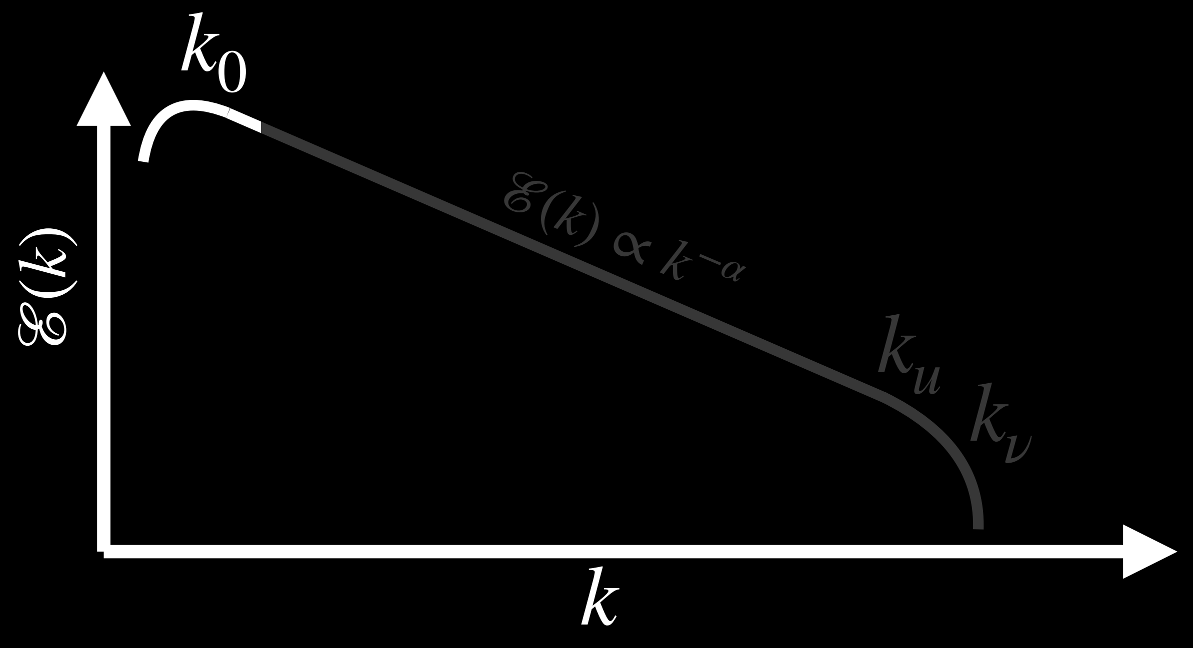
$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) = - \nabla \cdot P\mathbb{I} + 2\nu \nabla \cdot (\rho \mathcal{S})$$

Creating nonlinear things in the fluid

$$\mathrm{Re} = \frac{|\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})|}{|2\nu \nabla \cdot (\rho \mathcal{S})|} \sim \frac{UL}{\nu}$$

Smoothing out nonlinear things in the fluid

Turbulence
Cascade



WIM: $\mathrm{Re} \sim 10^7$ $\lambda_{\mathrm{mfp}} \sim 5\ \mathrm{R}_\oplus$

WNM: $\mathrm{Re} \sim 10^7$ $\lambda_{\mathrm{mfp}} \sim 1.3\ \mathrm{AU}$

CNM: $\mathrm{Re} \sim 10^{10}$ $\lambda_{\mathrm{mfp}} \sim 0.3\ \mathrm{pc}$

Ferrière, 2020;

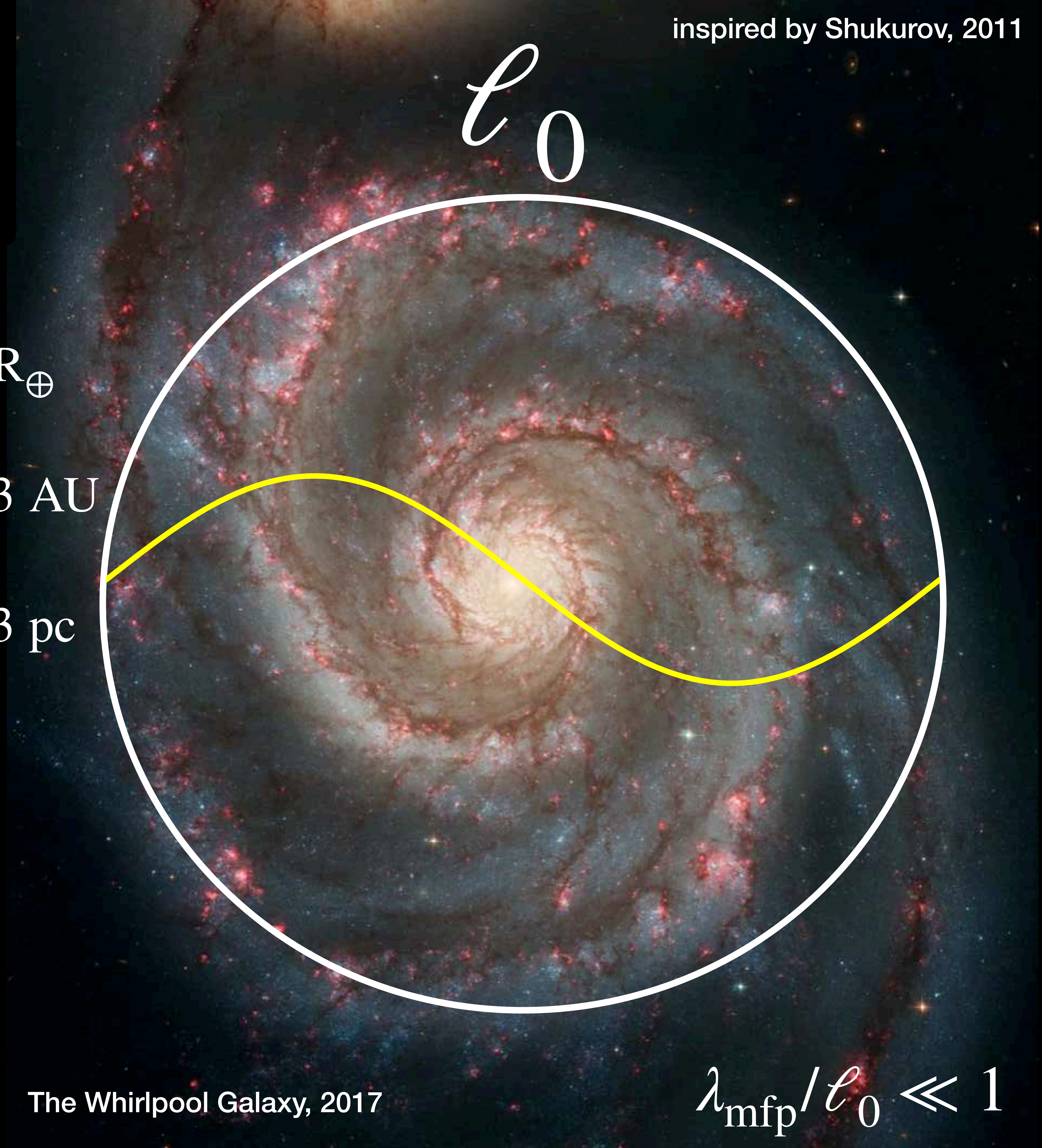
The quadratic nonlinear term

$$|\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})|$$

dominates on large scales, ℓ

Credit: NASA, ESA, S. Beckwith (STScI) and the Hubble Heritage Team (STScI/AURA)

inspired by Shukurov, 2011



The Whirlpool Galaxy, 2017

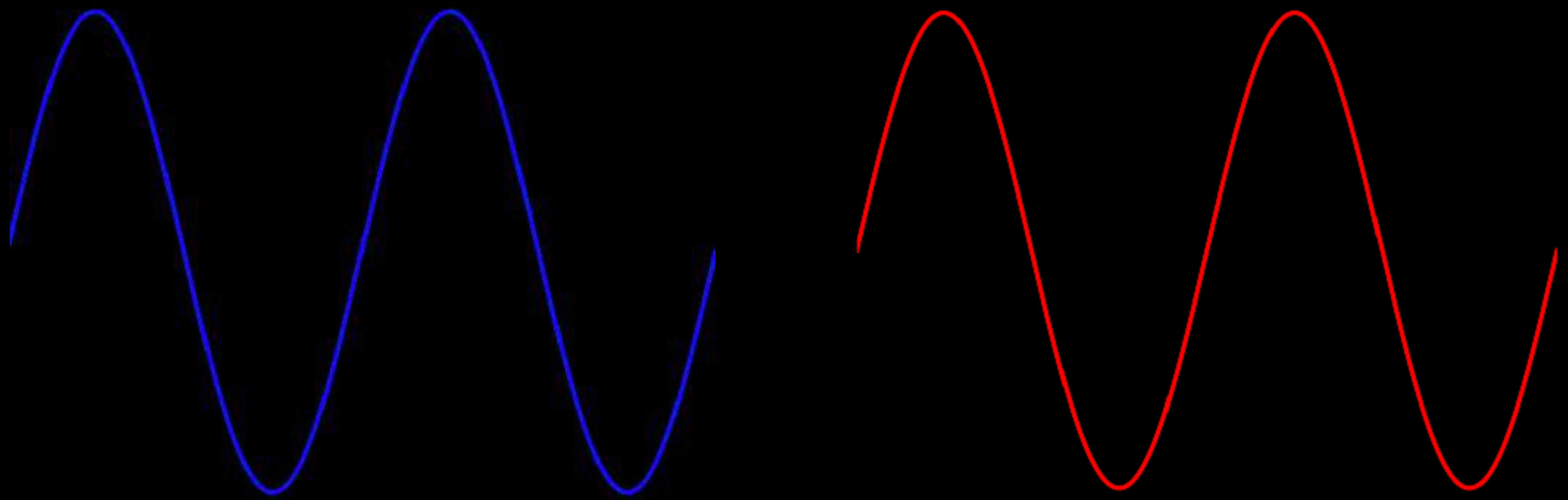
Turbulence

What does $\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})$ want to do?

$$k_1 = k_2$$

Consider two waves

$$u_1(x) = \sin(k_1 x)$$
$$u_2(x) = \sin(k_2 x)$$



The nonlinear term creates a new wave

$$\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) \sim u_1 \partial_x u_2 = k_2 \sin(k_1 x) \cos(k_2 x) \propto \sin(k_3 x)$$

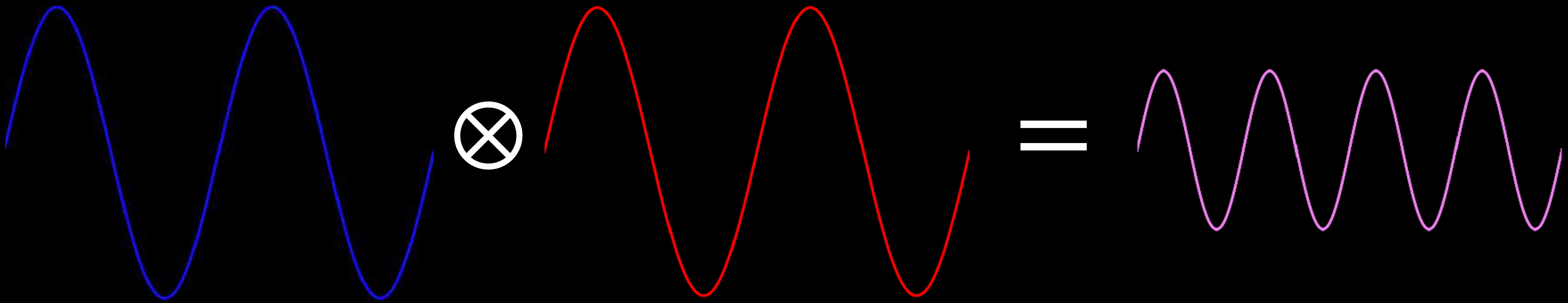
$$k_1 + k_2 = k_3$$

(momentum conservation)

Turbulence

What does $\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})$ want to do — cascade!!!

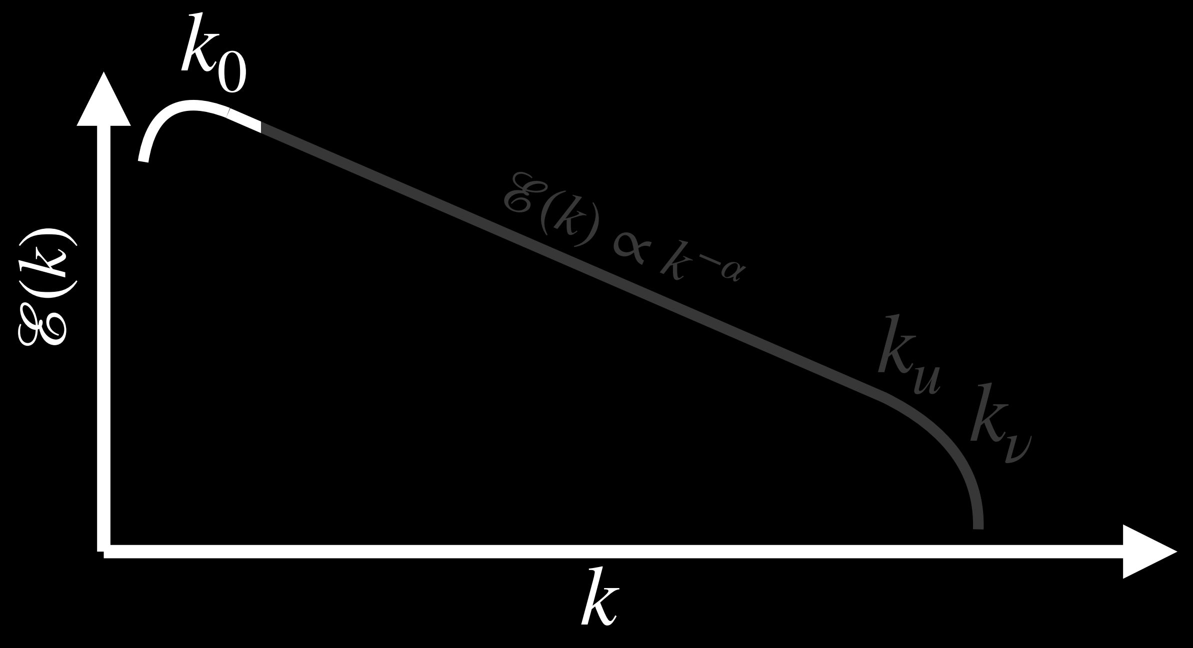
$$\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})$$



$$k_1 + k_2 = k_3$$

Turbulence

Cascade



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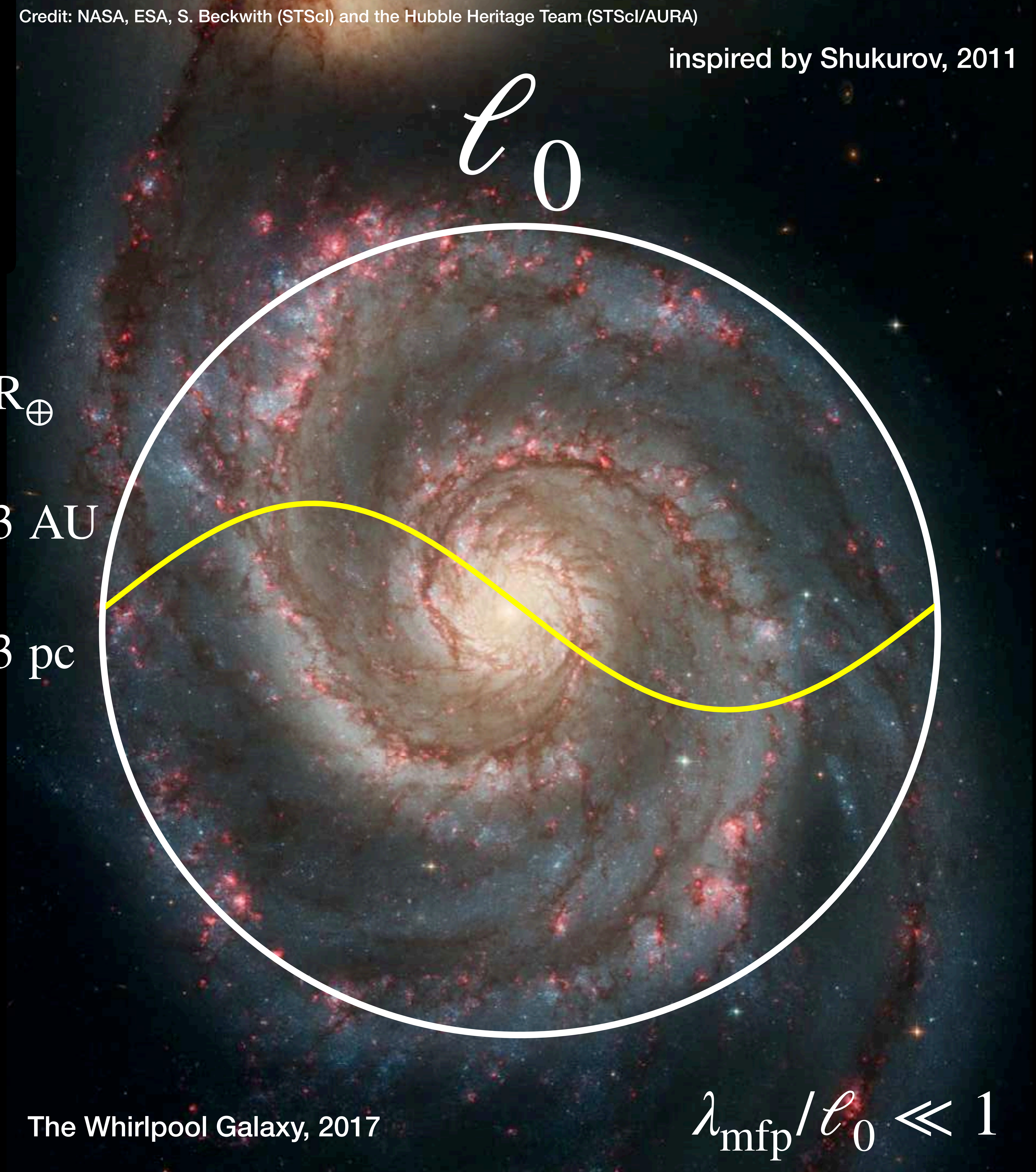
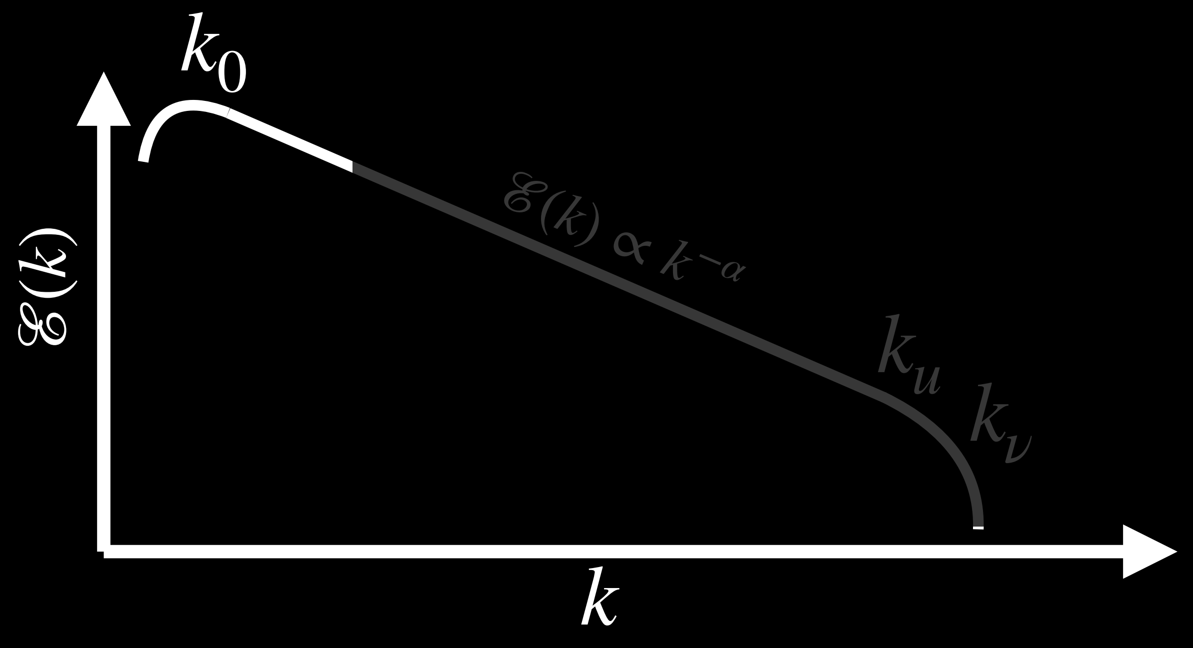


The Whirlpool Galaxy, 2017

Turbulence

Cascade



The quadratic nonlinear term

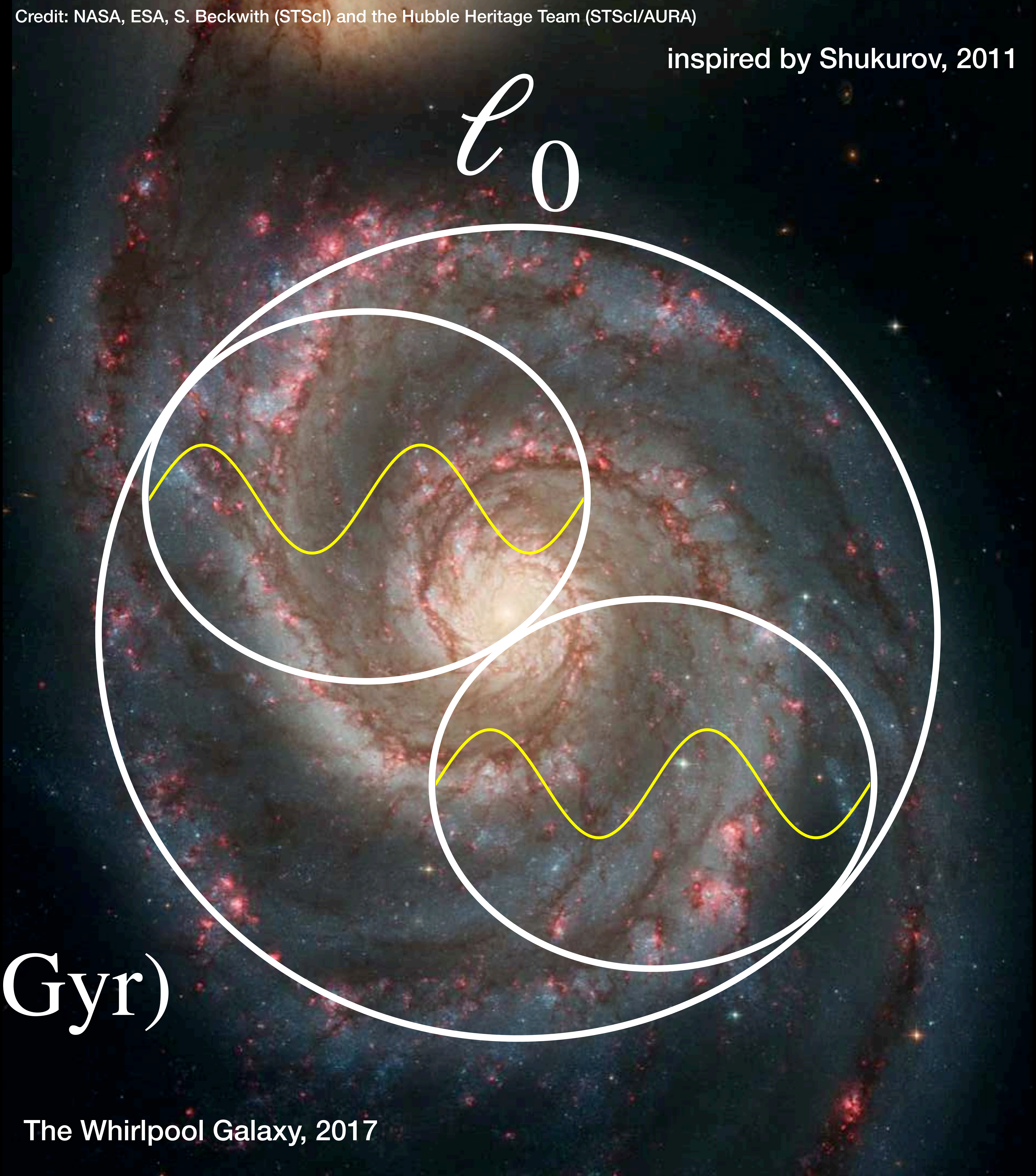
$$|\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})|$$

dominates on large scales, ℓ

Creates new modes on

$$t_{\rm nl} \sim [\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})]^{-1} \sim \mathscr{O}(\text{Gyr})$$

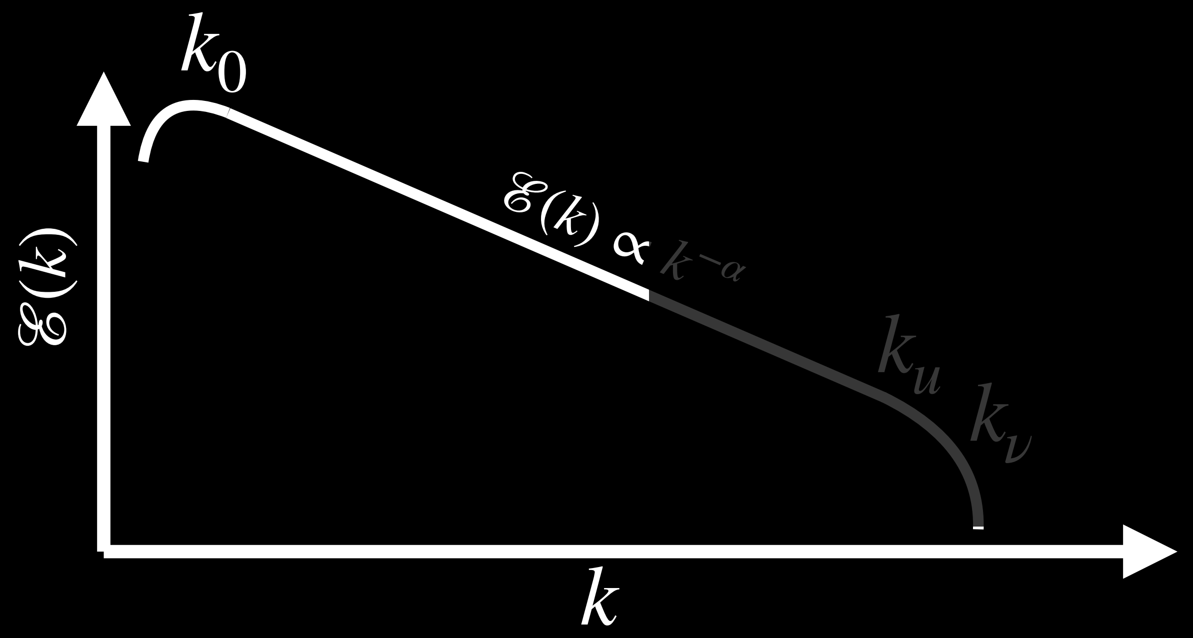
$$\dot{\varepsilon} \sim u_0^3/\ell_0$$



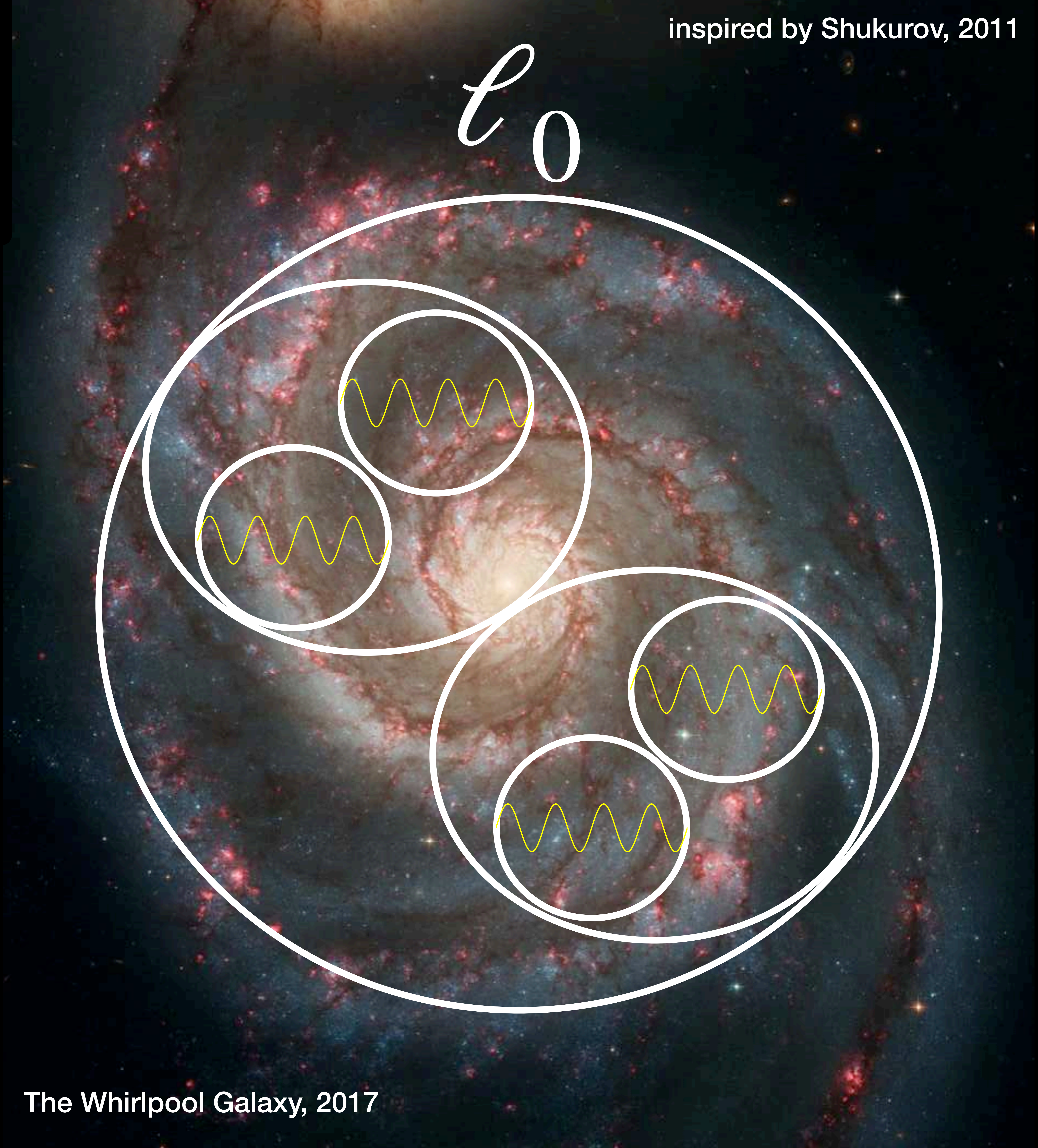
The Whirlpool Galaxy, 2017

Turbulence

Cascade

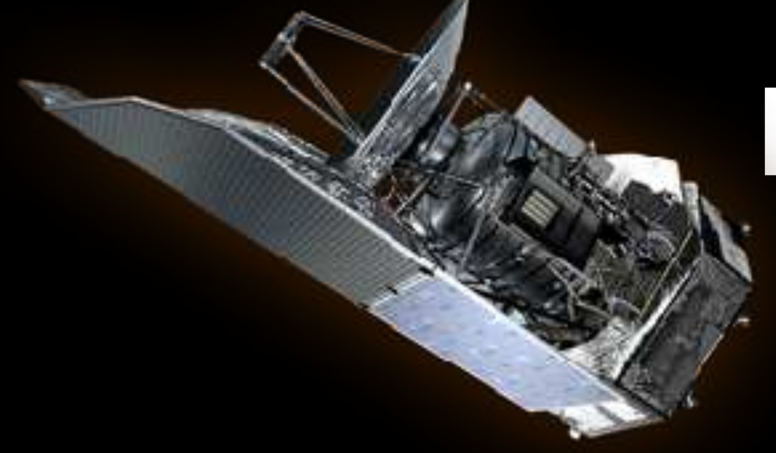


inside of the cascade

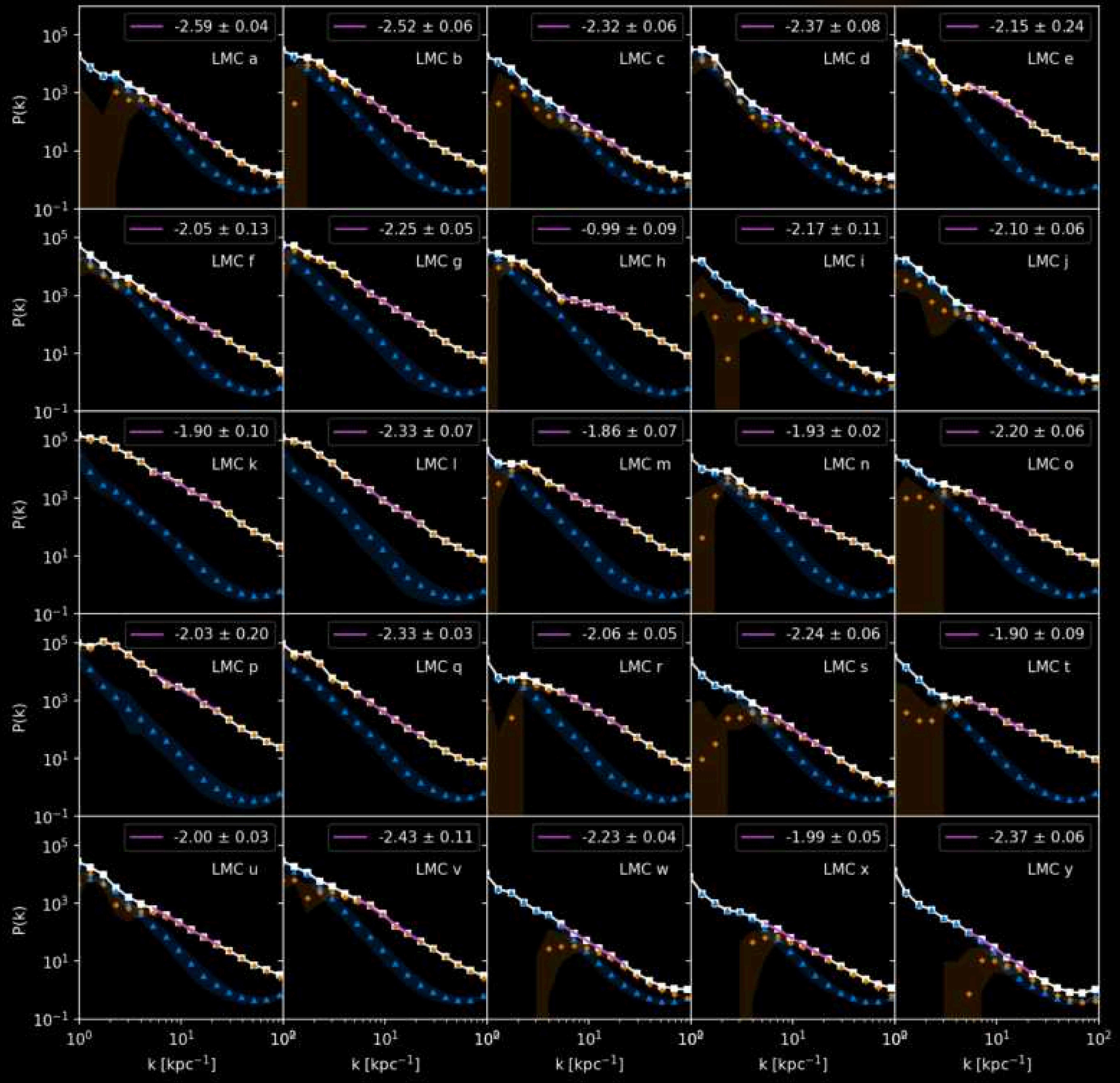


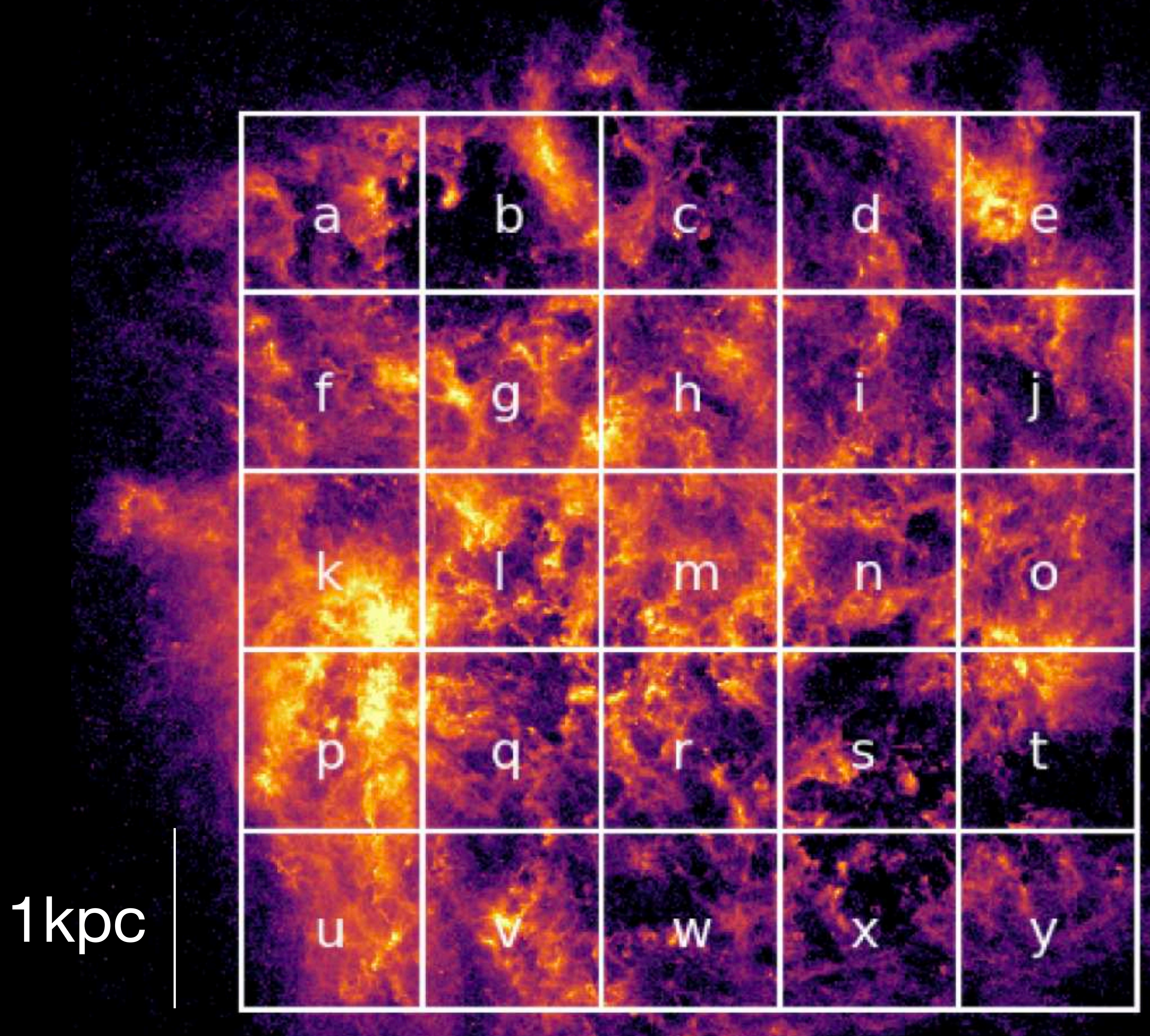
The Whirlpool Galaxy, 2017

Turbulence



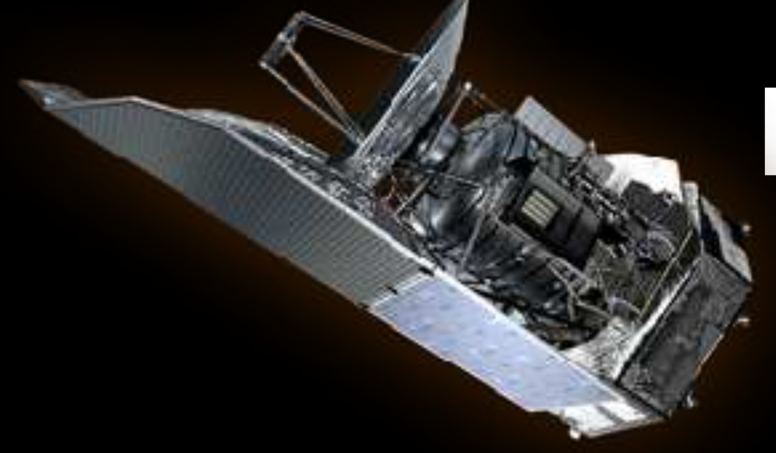
LMC: 500μm, *Herschel* (processing Gordon et al. 2014)





Colman, et al., 2022, MNRAS

Turbulence



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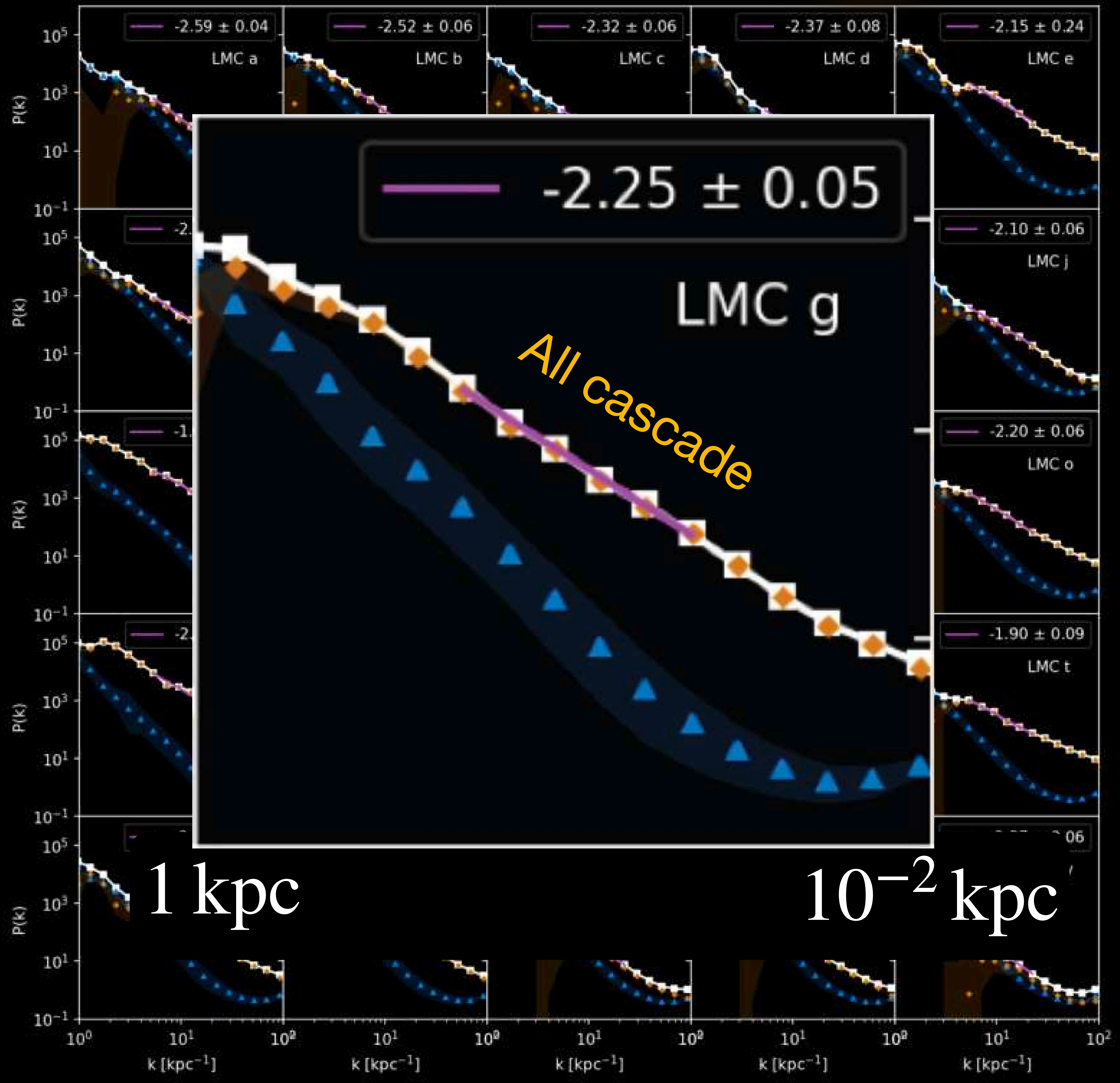
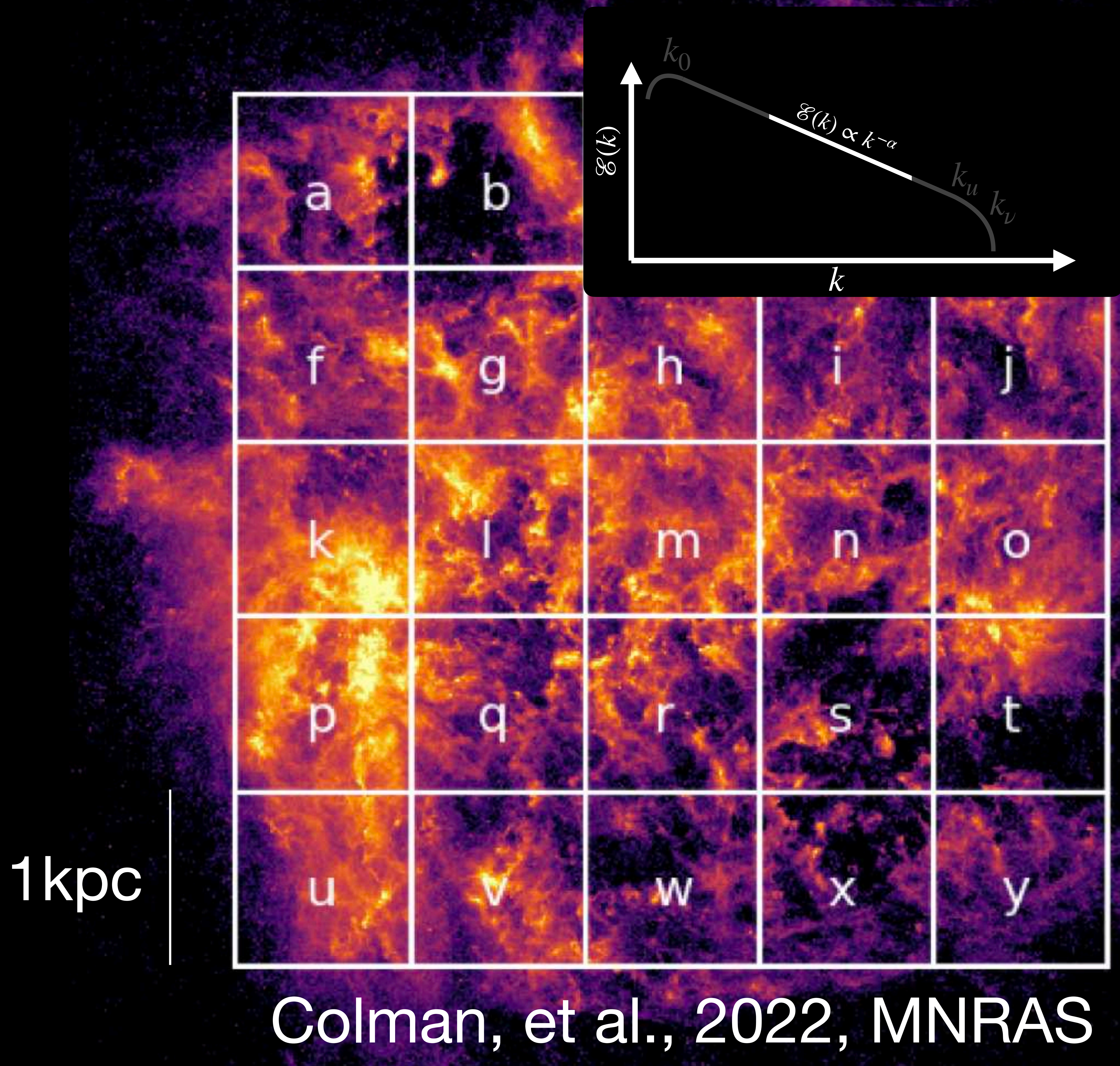
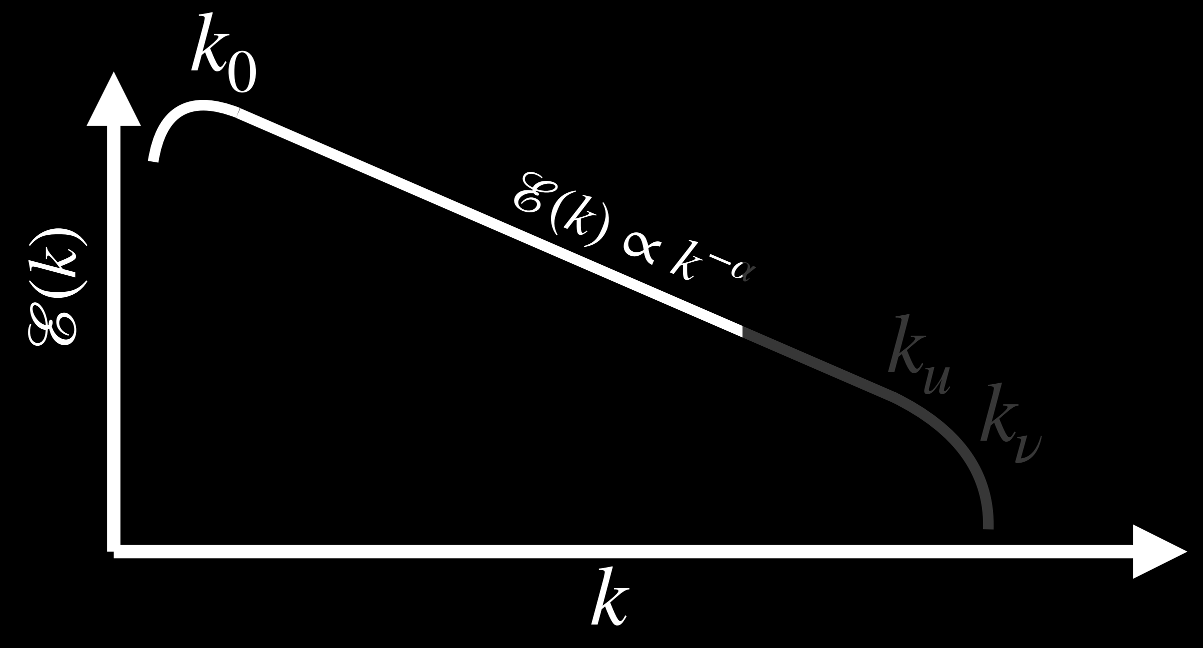




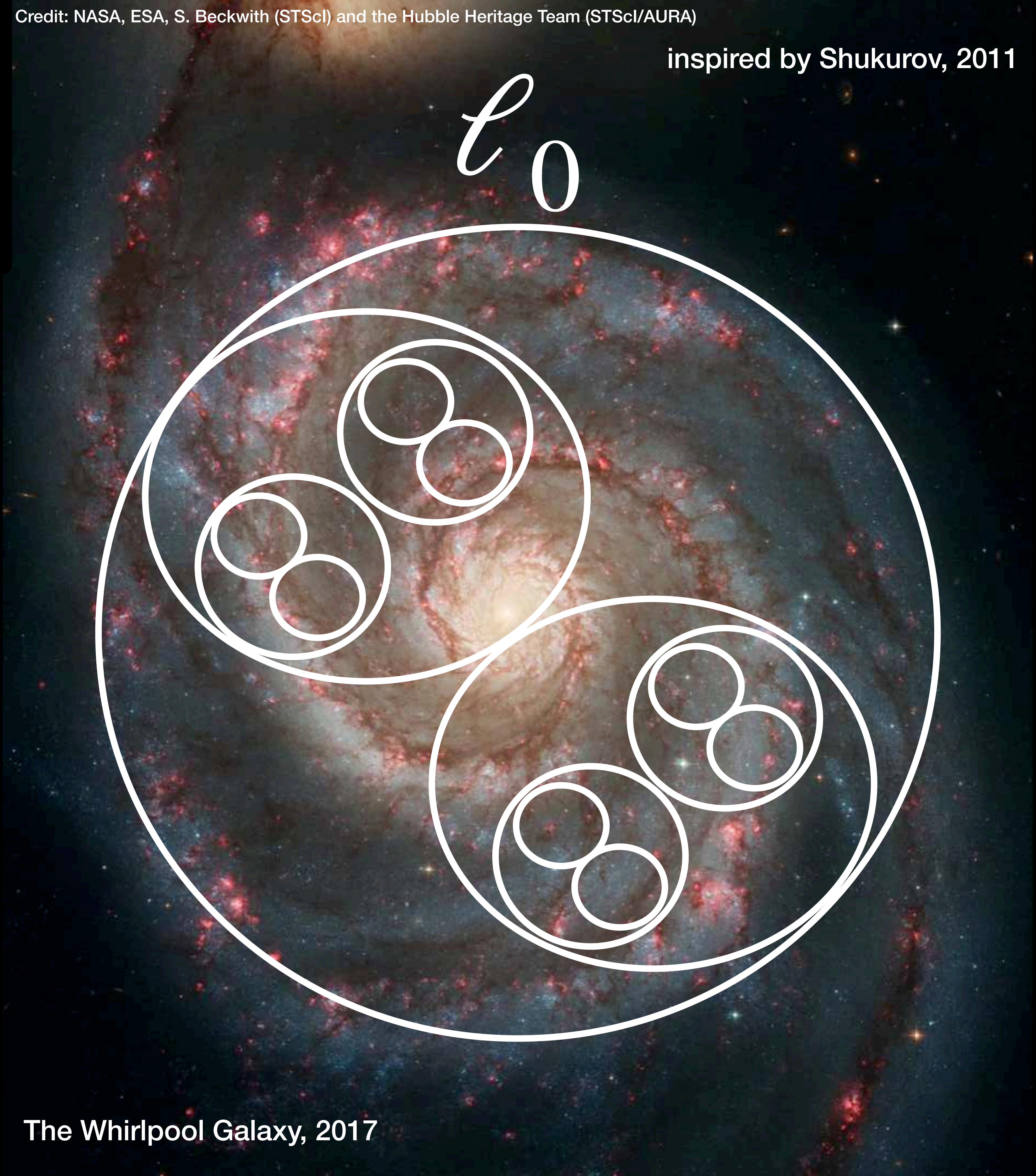
Colman, et al., 2022, MNRAS

Turbulence

Cascade

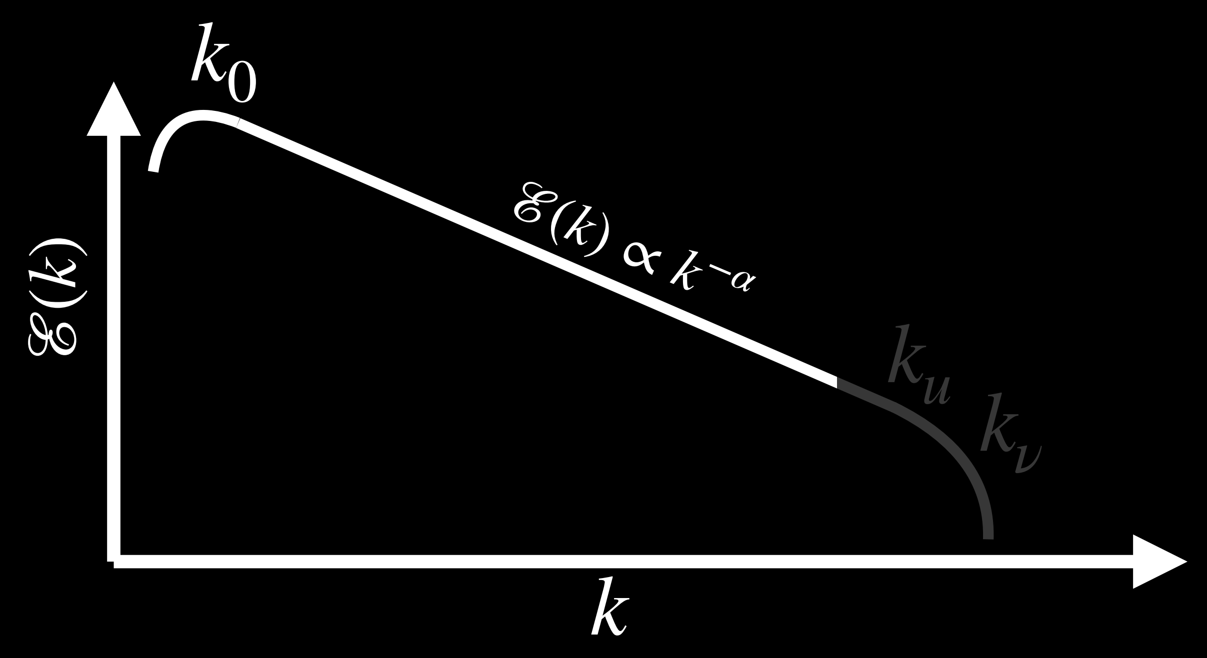


deeper into
the cascade



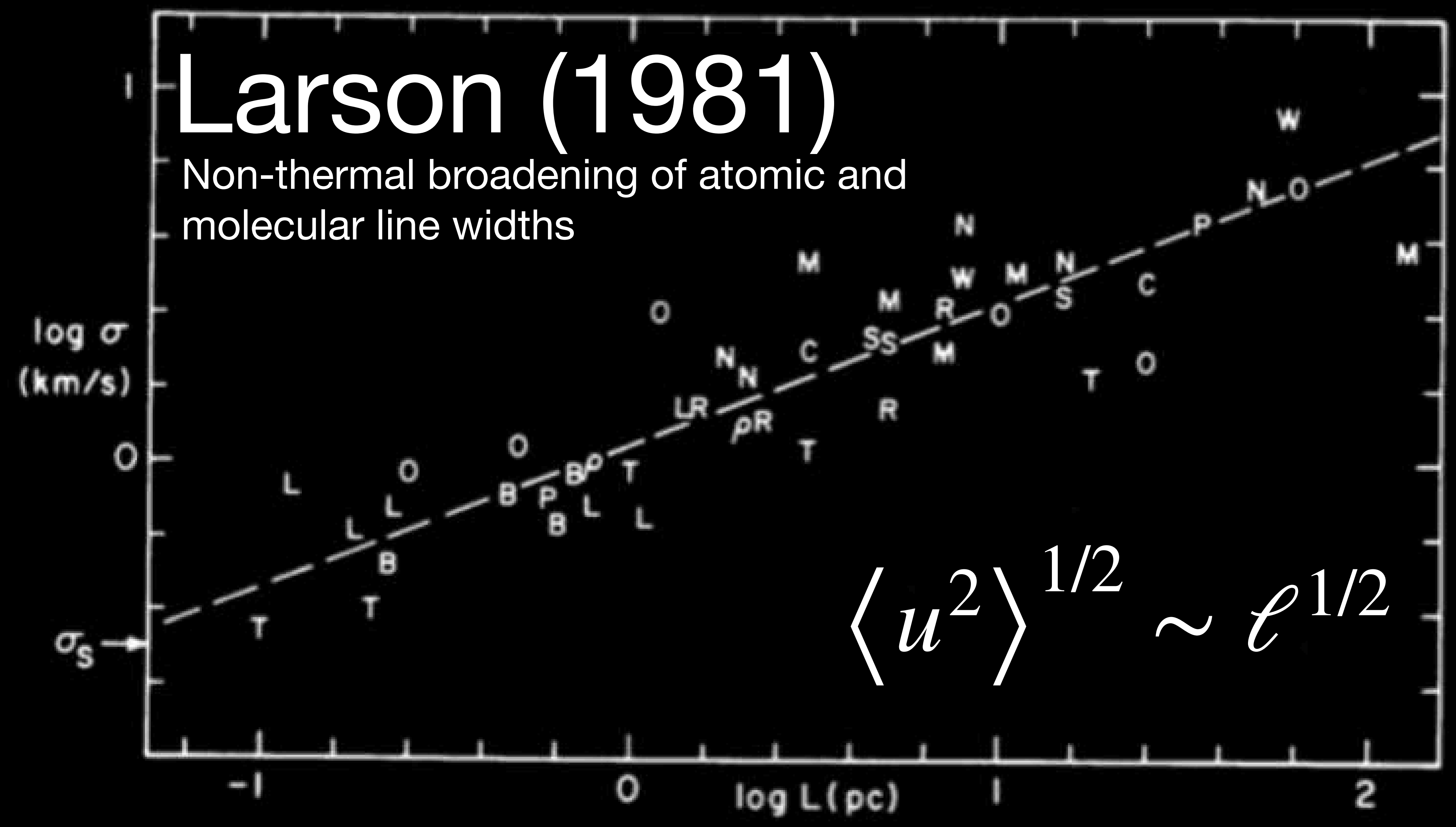
Turbulence

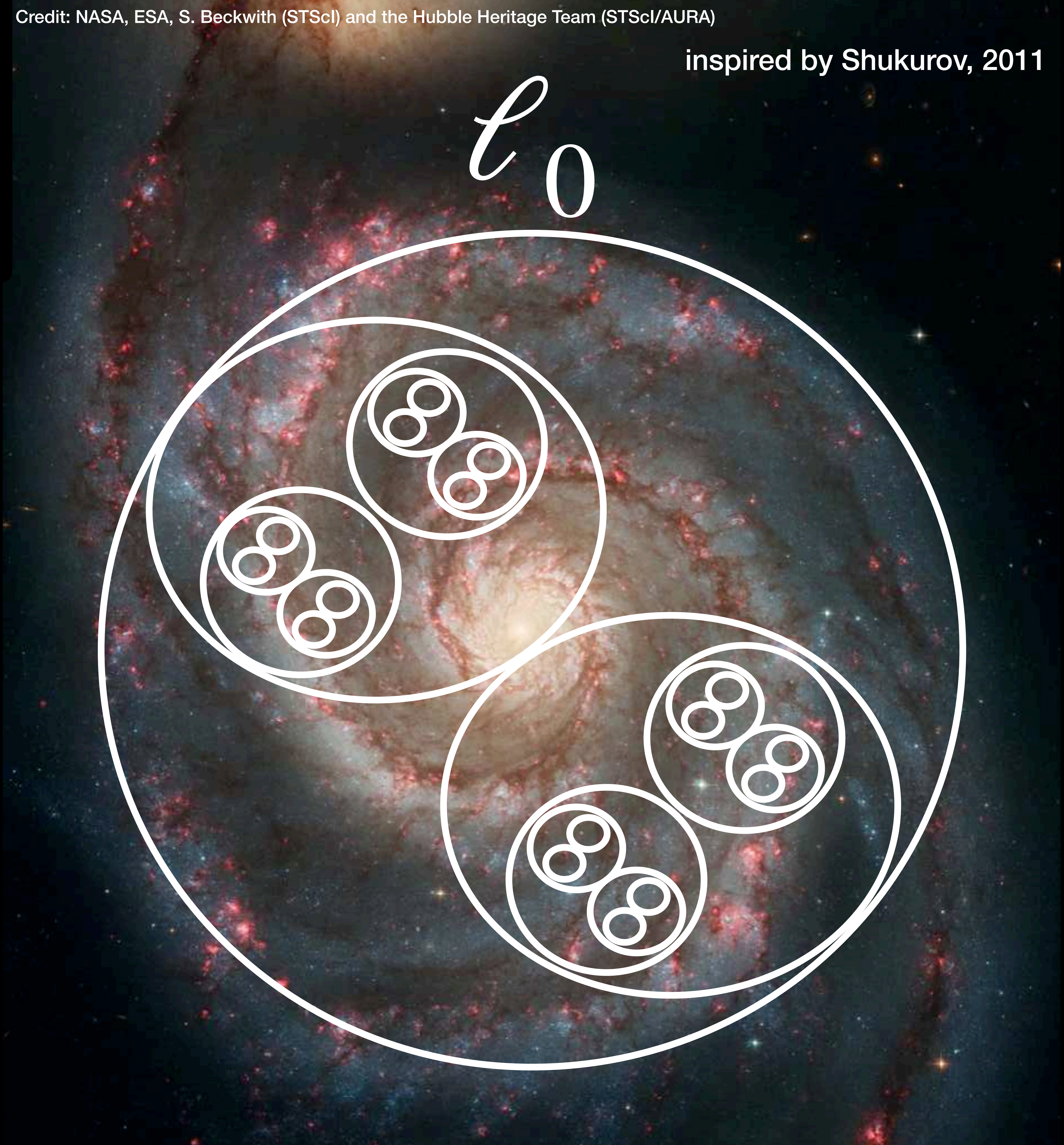
Cascade



Reynolds number is shrinking

$$\mathrm{Re} \sim \langle u^2 \rangle^{1/2} \ell / \nu \sim \ell^{3/2} / \nu$$

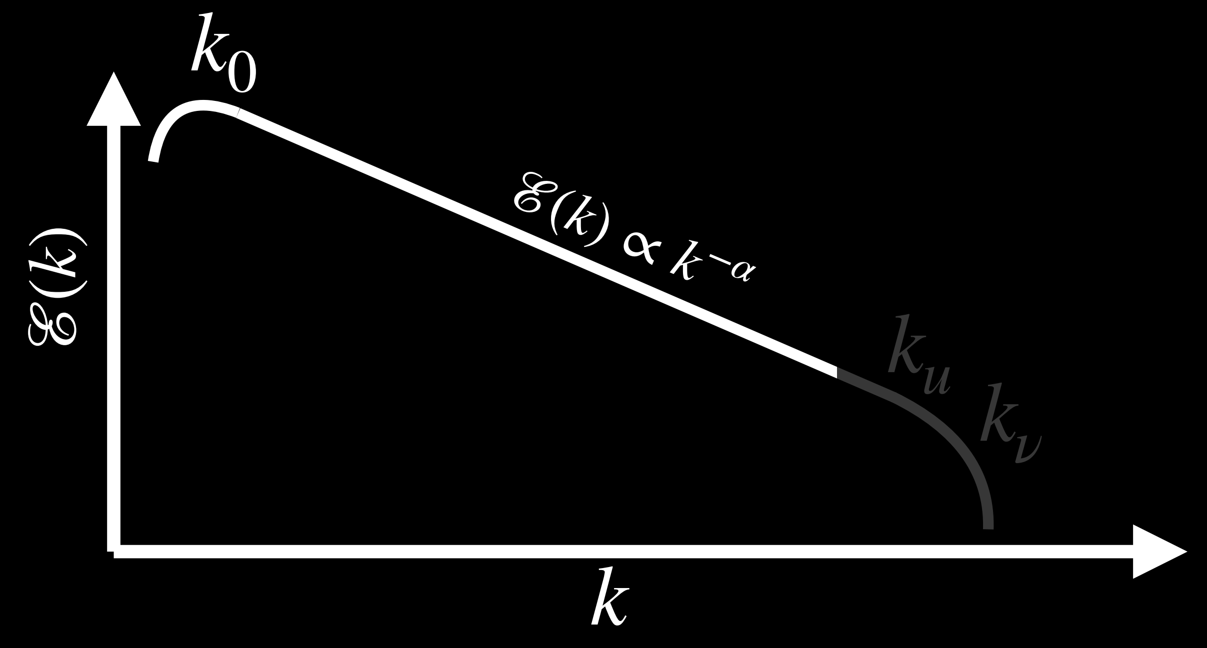




The Whirlpool Galaxy, 2017

Turbulence

Cascade



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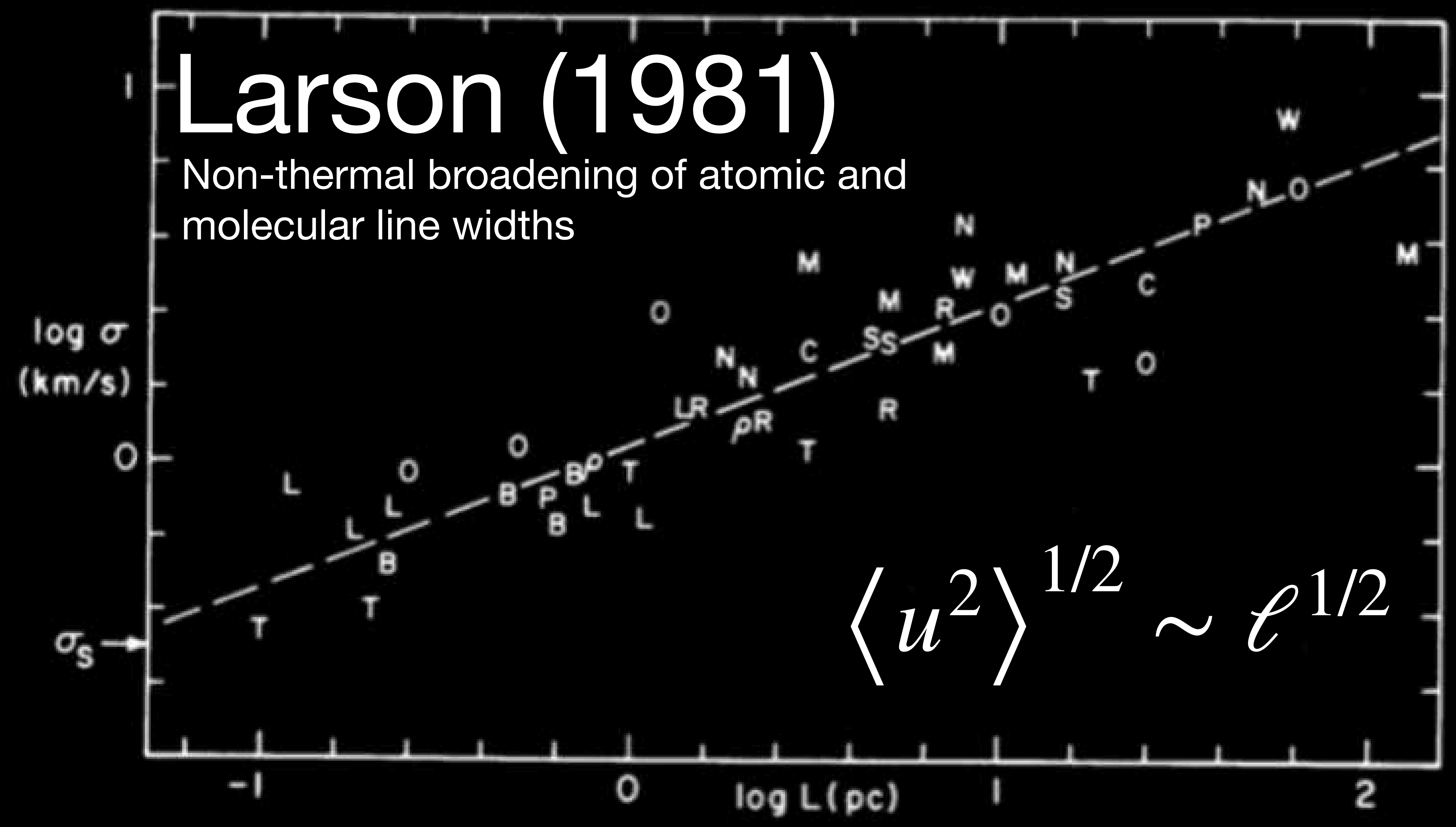

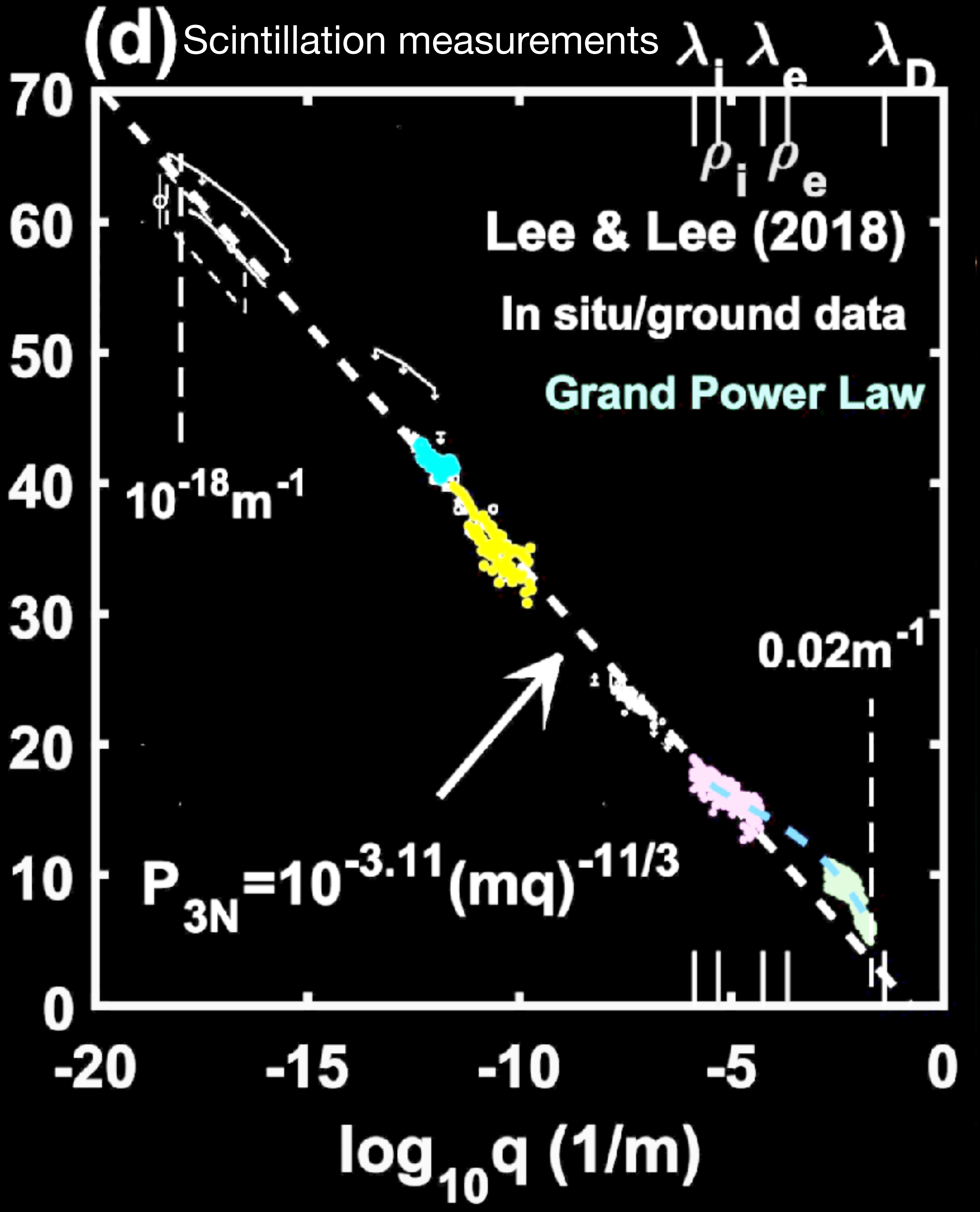

Two great insights from Kolmogorov

- Constant energy flux between modes
- No magnetic field
- No inhomogeneities
- No density fluctuations
- Isotropic

$$\varepsilon \sim u_\ell^3/\ell \sim \text{const}\,.$$

Kolmogorov (1941)

Two great insights from Kolmogorov

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$$\varepsilon \sim u_\ell^3/\ell \sim \text{const}\,.$$

$$u_\ell \sim (\varepsilon \ell)^{1/3}$$

Kolmogorov (1941)

Two great insights from Kolmogorov

- Constant energy flux between modes
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- Isotropic

$$\varepsilon \sim u_\ell^3/\ell \sim \text{const.}$$

$$u_\ell \sim (\varepsilon \ell)^{1/3}$$

$$u_\ell \sim (\varepsilon \ell)^{1/3} \iff u^2(k) \sim k^{-5/3} dk$$

Kolmogorov (1941)

Two great insights from Kolmogorov

- Constant energy flux between modes
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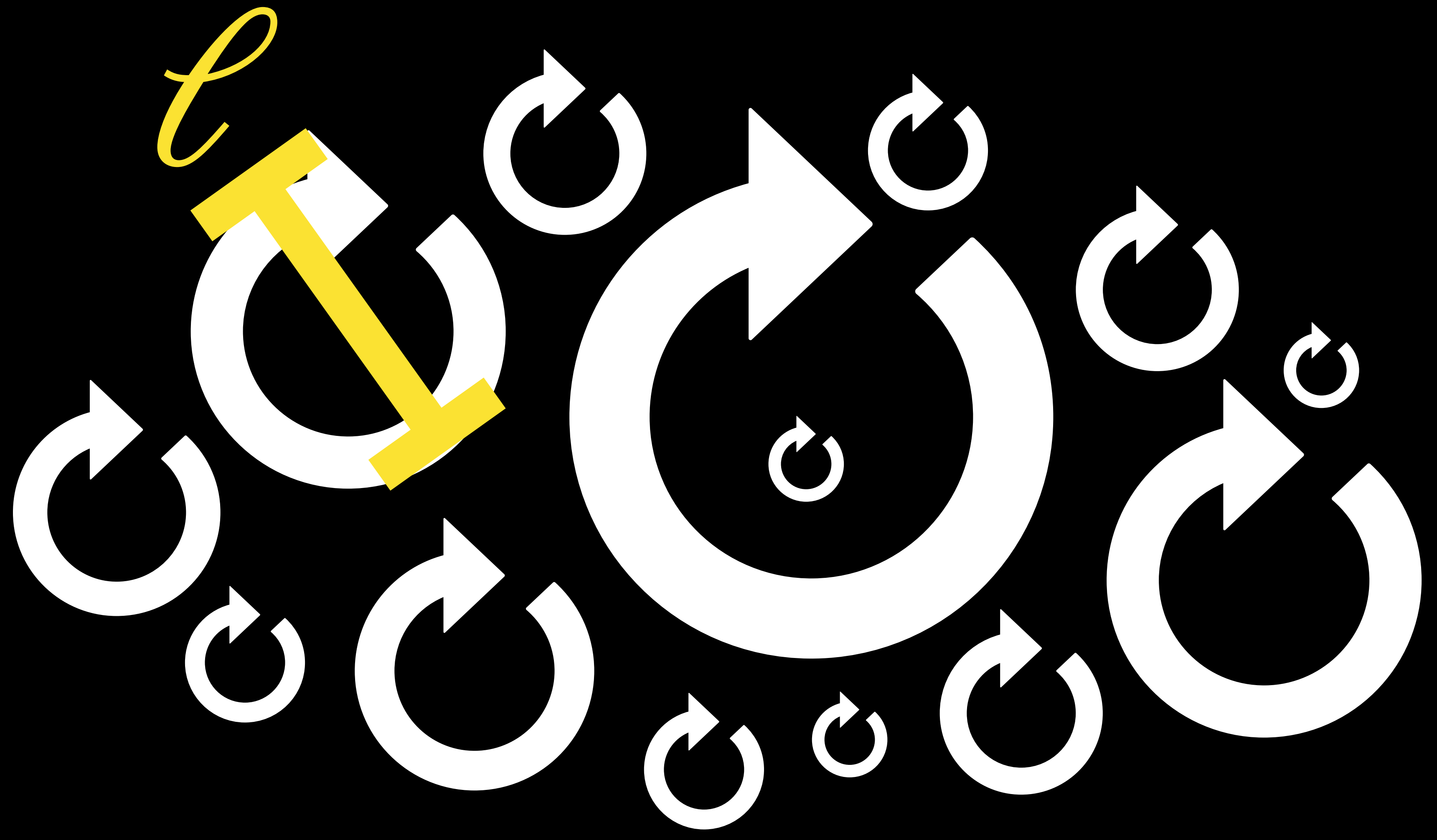
- No inhomogeneities
- No density fluctuations
- Isotropic

$$u_\ell \sim (\varepsilon \ell)^{1/3}$$

"structure function"

$$u(\ell) = \langle \, | \, u(\mathbf{x}) - u(\mathbf{x} + \ell) \cdot \ell \, | \, \rangle_{\mathbf{x}} \sim \ell^{1/3}$$

ℓ picks out an eddy



Kolmogorov (1941)

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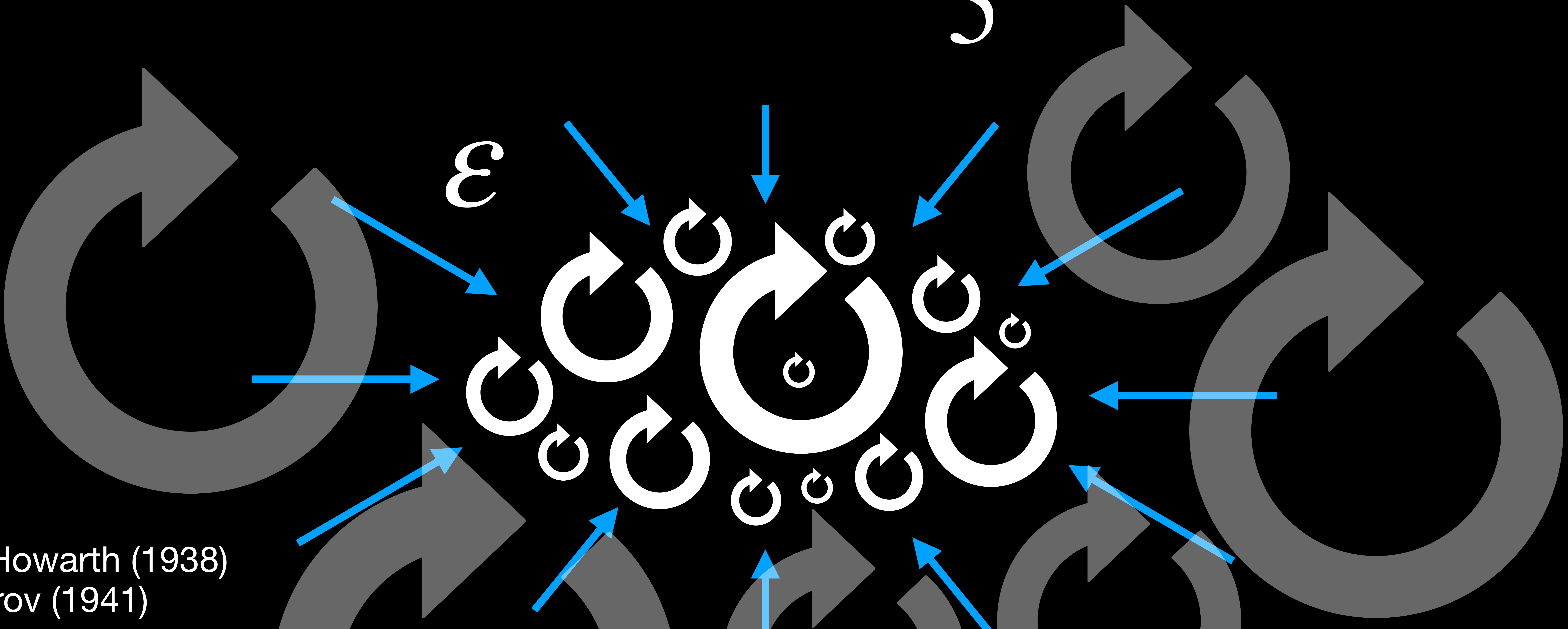
$$u(\ell) = \langle \, | \, u(\mathbf{x}) - u(\mathbf{x} + \ell) \cdot \ell \, | \, \rangle_{\mathbf{x}} \sim \ell^{1/3}$$

$$u(\ell) = \left\langle \, | \, \delta u_L \, | \, \right\rangle_{\mathbf{x}} \sim \ell^{1/3}$$

Kolmogorov (1941)

An exact relation for turbulence
Kolmogorov's 4/5 law

$$\nabla_\ell \cdot \left\langle \delta \mathbf{u}_L \delta u_L^2 \right\rangle = -\frac{4}{5}\varepsilon, \quad \varepsilon > 0$$



Kármán–Howarth (1938)
Kolmogorov (1941)

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Fact: the galaxy that we reside in is in a state of highly-compressible fluid turbulence

$$E_{\mathrm{SNe}} \approx 10^{51}\,\mathrm{erg}$$

$$\gamma_{\mathrm{SNe}} \approx 0.01 - 0.03\,\mathrm{yr}^{-1}$$ Diehl + (2016)

$$\dot{E}_{\mathrm{SNe}} \approx E_{\mathrm{SNe}}\gamma_{\mathrm{SNe}} \approx 10^{41}\mathrm{erg\,s}^{-1}$$

$$E_{\mathrm{turb}} \approx 10^{54}\,\mathrm{erg}$$

Ferriere + (2020

$$t_{\mathrm{turb}} \approx \ell_0/\langle u^2\rangle^{1/2} \approx 10^7\,\mathrm{yr}$$

$$\dot{E}_{\mathrm{turb}} \approx E_{\mathrm{turb}}/t_{\mathrm{turb}} \approx 10^{39}\mathrm{erg\,s}^{-1}$$

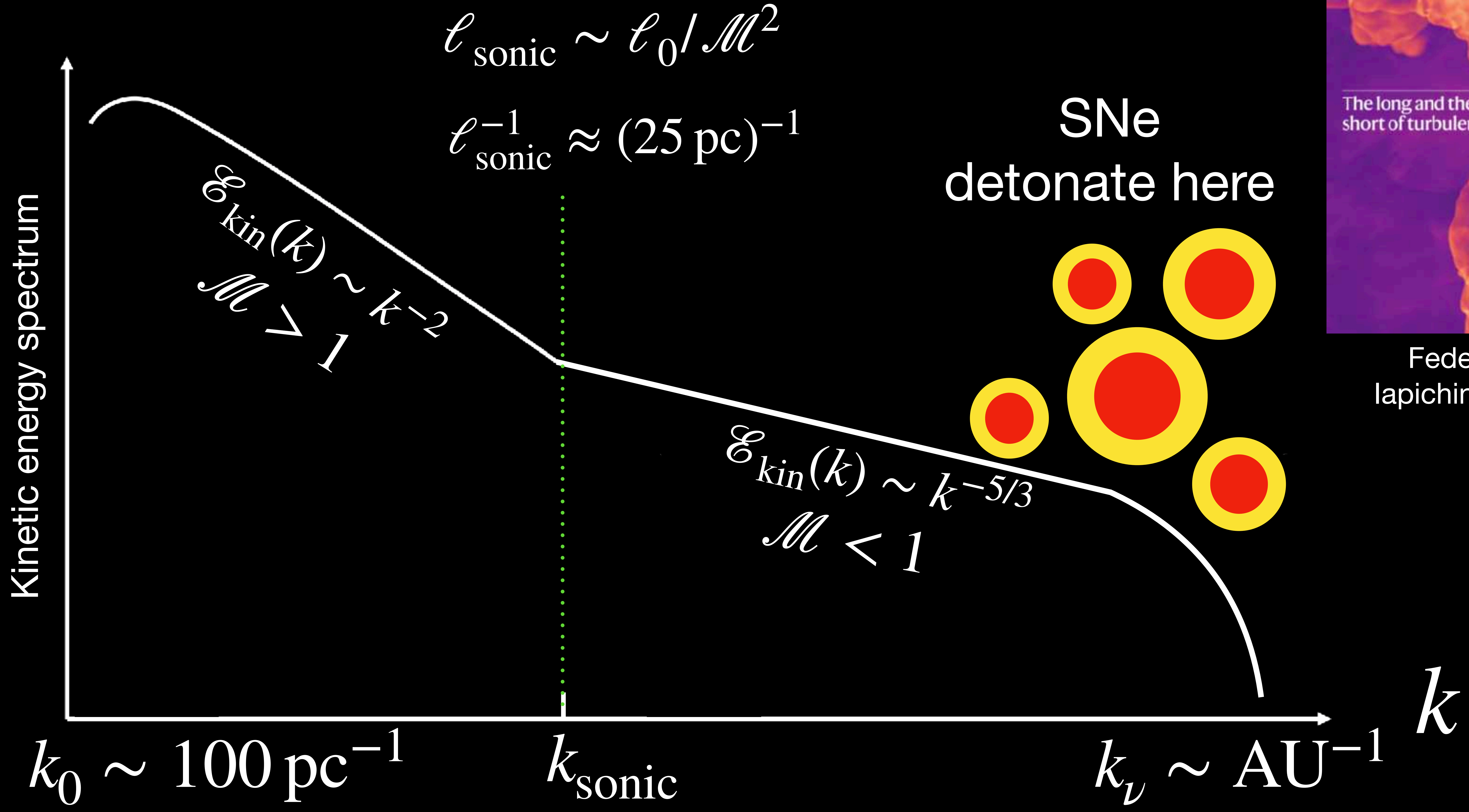
$$\dot{E}_{\mathrm{turb}}/\dot{E}_{\mathrm{SNe}} \approx 10^{-2}$$

CC-supernovae alone provide enough energy flux to fuel a continuously driven cascade

Connor, **Beattie** + (2025, ApJ) Cascading from the winds to the disk: universality of supernovae-driven turbulence in different galactic ISMs

Simulation from: Goldbaum, N. J., Krumholz, M. R., & Forbes, J. C. 2016

The naive ($\mathscr{M} \approx 2$) warm ionized medium spectrum (the volume-filling spectrum)



Federrath, Klessen, Iapichino, **Beattie** (2021)

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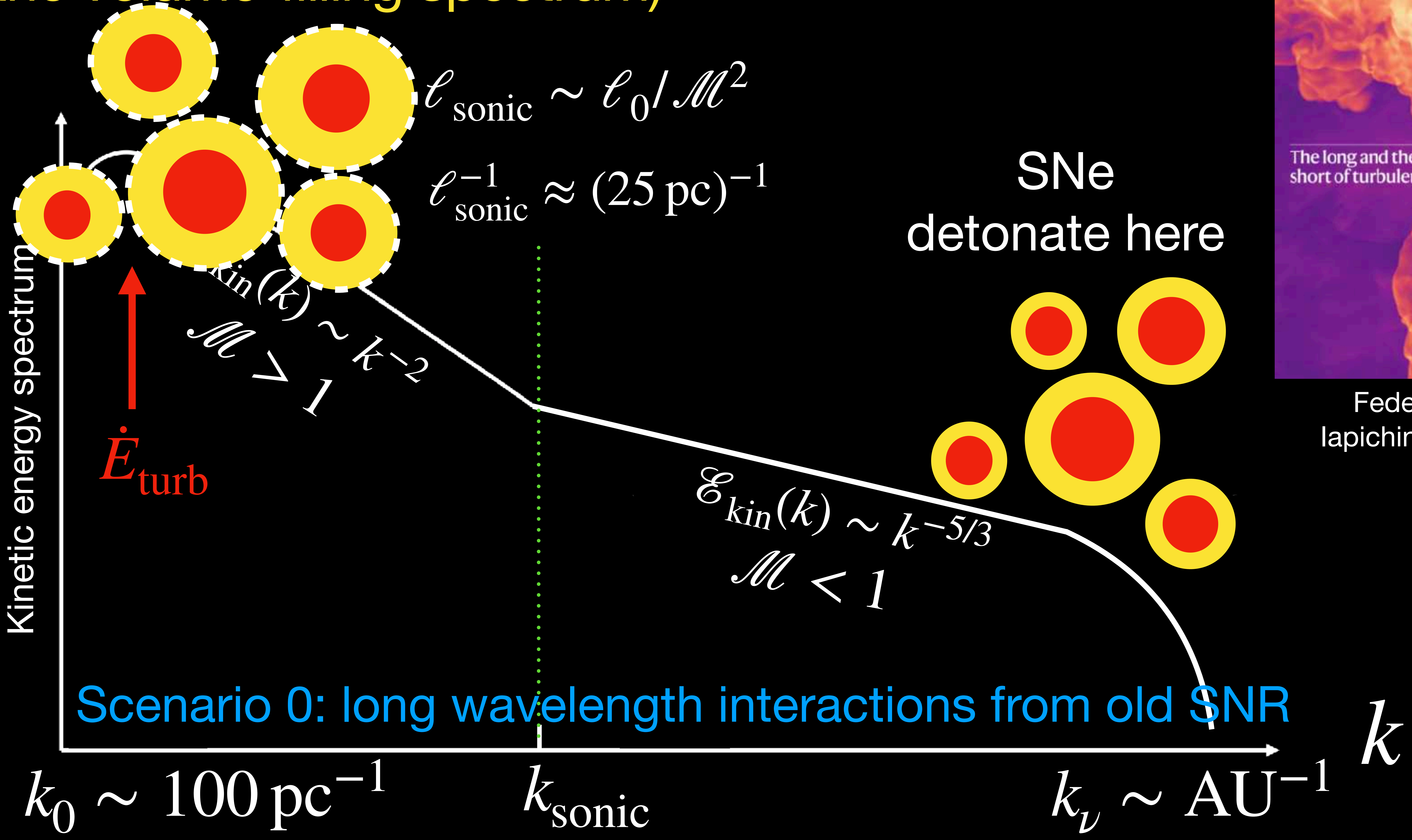

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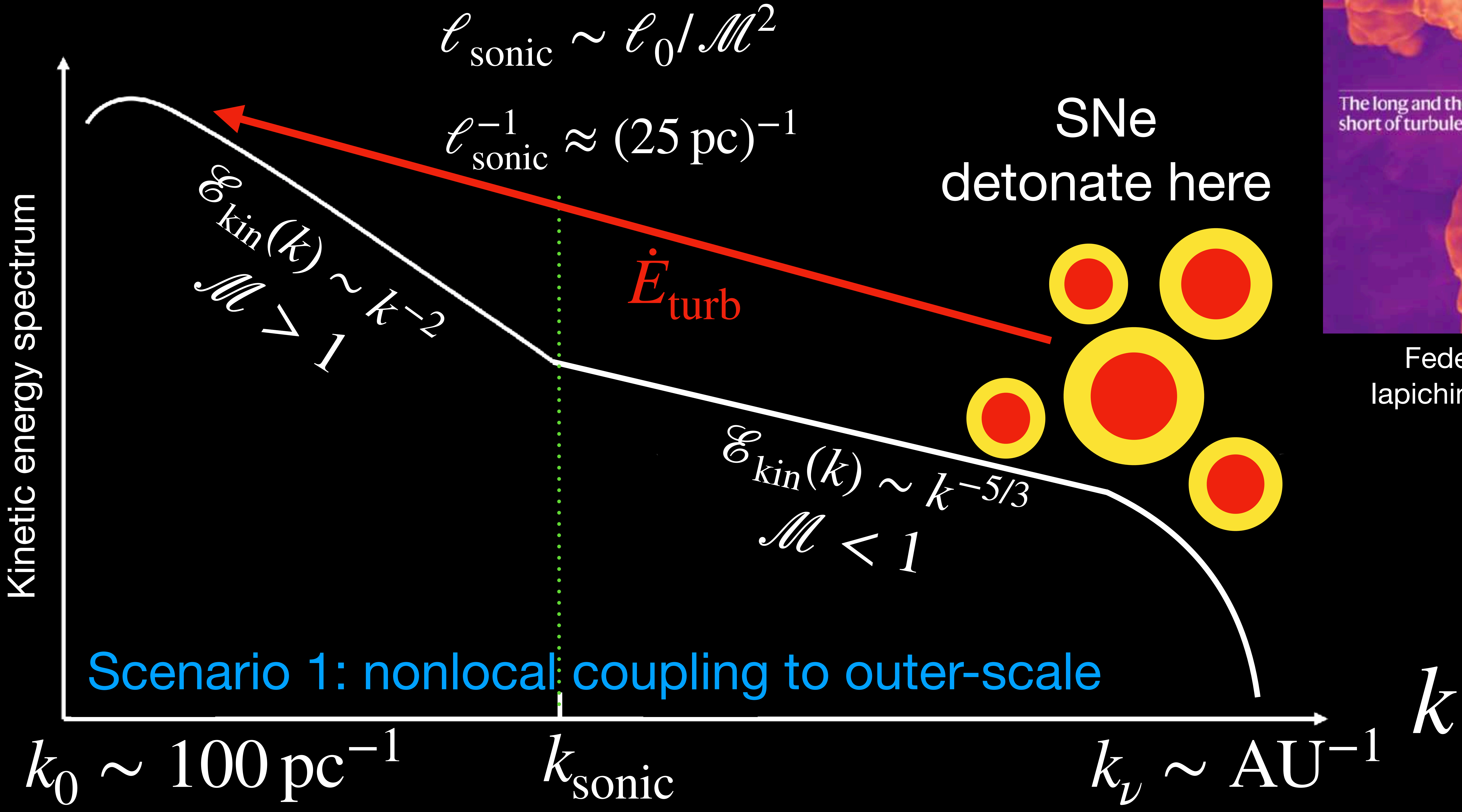

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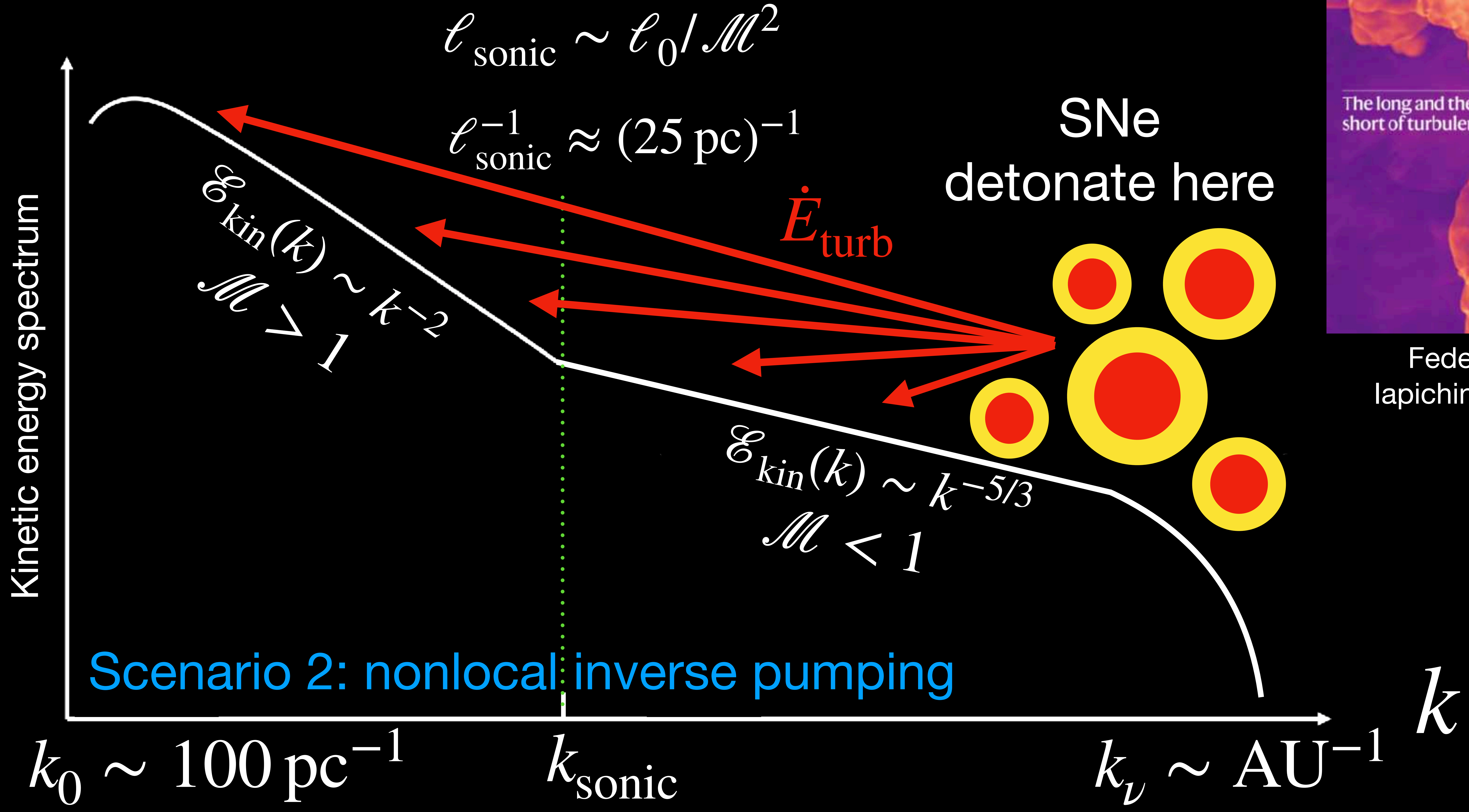

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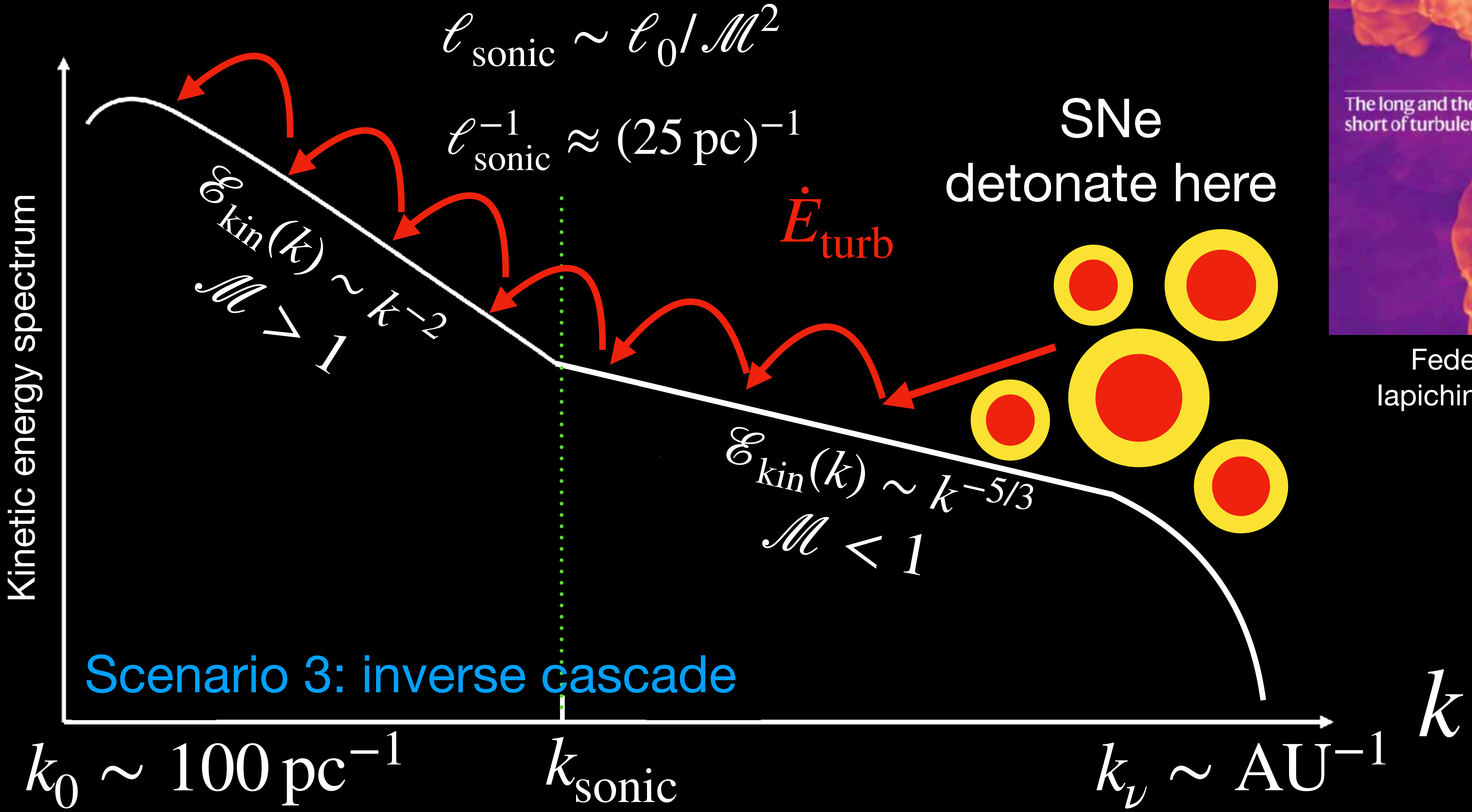
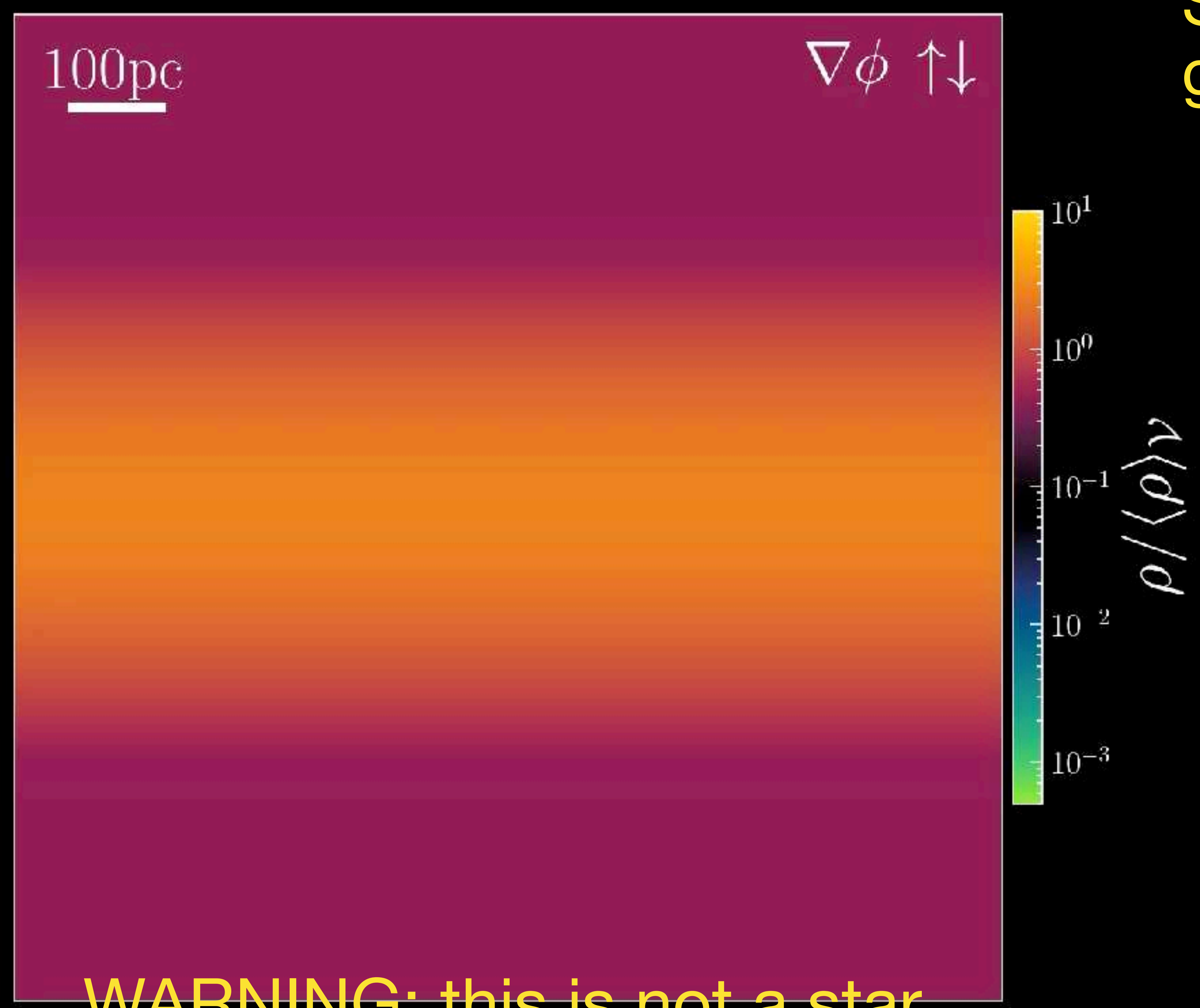


Federrath, Klessen, Iapichino, **Beattie** (2021)

The simplest SNe-driven simulations possible

$L = 1$ kpc



WARNING: this is not a star formation simulation

Martizzi+2016

RAMSES (Teyssier 2002)

Supernova driven gravito-hydro dynamical model

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho \boldsymbol{u}) = \dot{n}_{\rm SNe} M_{\rm ej}, \quad (1)$$

$$\frac{\partial \rho \boldsymbol{u}}{\partial t} + \nabla\cdot(\rho\boldsymbol{u}\otimes\boldsymbol{u} + P\mathbb{I}) = -\,\rho\nabla\phi + \dot{n}_{\rm SNe}\mathbf{p}_{\rm SNe}(Z, n_{\rm H}), \quad (2)$$

$$\frac{\partial \rho e}{\partial t} + \nabla\cdot[\rho\,(e+P)\,\boldsymbol{u}] = -\,n_{\rm H}^2\Lambda - \rho\boldsymbol{u}\cdot\nabla\phi + \dot{n}_{\rm SNe}\left[E_{\rm th,SNe}(Z, n_{\rm H}) + \frac{\mathbf{p}_{\rm SNe}(Z, n_{\rm H})^2}{2(M_{\rm ej}+M_{\rm swept})}\right], \quad (3), (4)$$

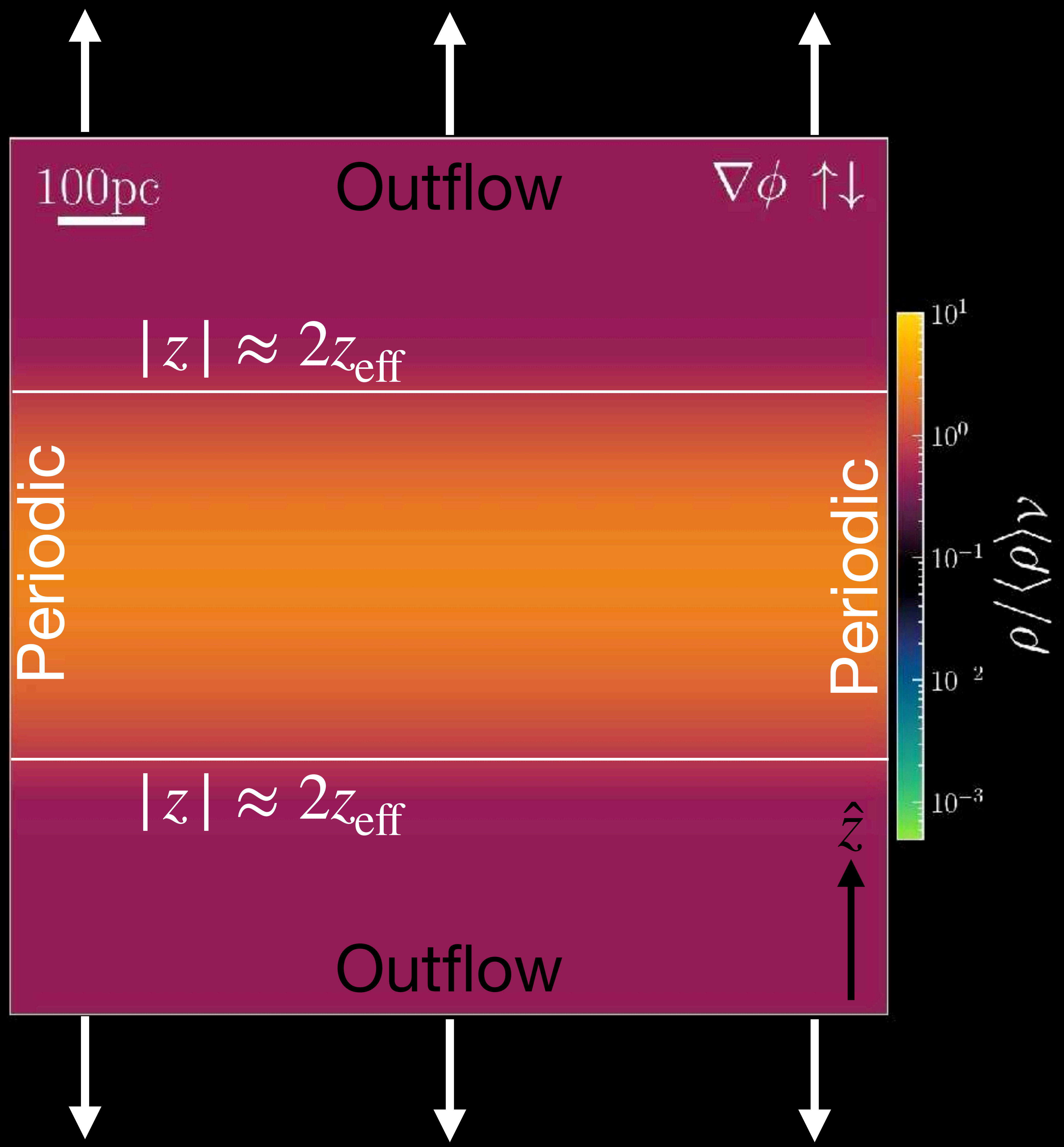
$$e = \epsilon + \frac{u^2}{2}, \quad P = \frac{2}{3}\rho e, \quad (5)$$

$$N_{\rm grid}^3 = 1024^3 \implies \Delta x \sim 1\ {\rm pc}$$

$$\text{numerical diss.} \implies \Delta x \sim 10\ {\rm pc}$$

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

The simplest SNe-driven simulations possible



Static gravitational potential

$$\phi(z) = \underbrace{2\pi G \Sigma_* \left(\sqrt{z^2 - z_0^2} - z_0\right)}_{\text{stratified disk}} + \underbrace{\frac{2\pi G \rho_{\rm halo}}{3} z^2}_{\text{spherical halo}}$$

Supernova driving prescription

(following 1D evolution models for momentum and energy deposition in Martizzi + 2016)

$$\dot{n}_{\rm SNe} = \frac{\dot{\Sigma}_*}{2z_{\rm eff} 100 M_\odot}$$

1 SN per $100 M_\odot$ of SF

$\dot{\Sigma}_* \propto \Sigma^{1.4}$

KS relation

$$p(z) = \begin{cases} \frac{1}{4z_{\rm eff}}, & |z| \le 2z_{\rm eff} \\ 0, & |z| > 2z_{\rm eff} \end{cases}$$

$$p(n\,|\,\lambda) = \lambda^k e^{-n}/n!$$

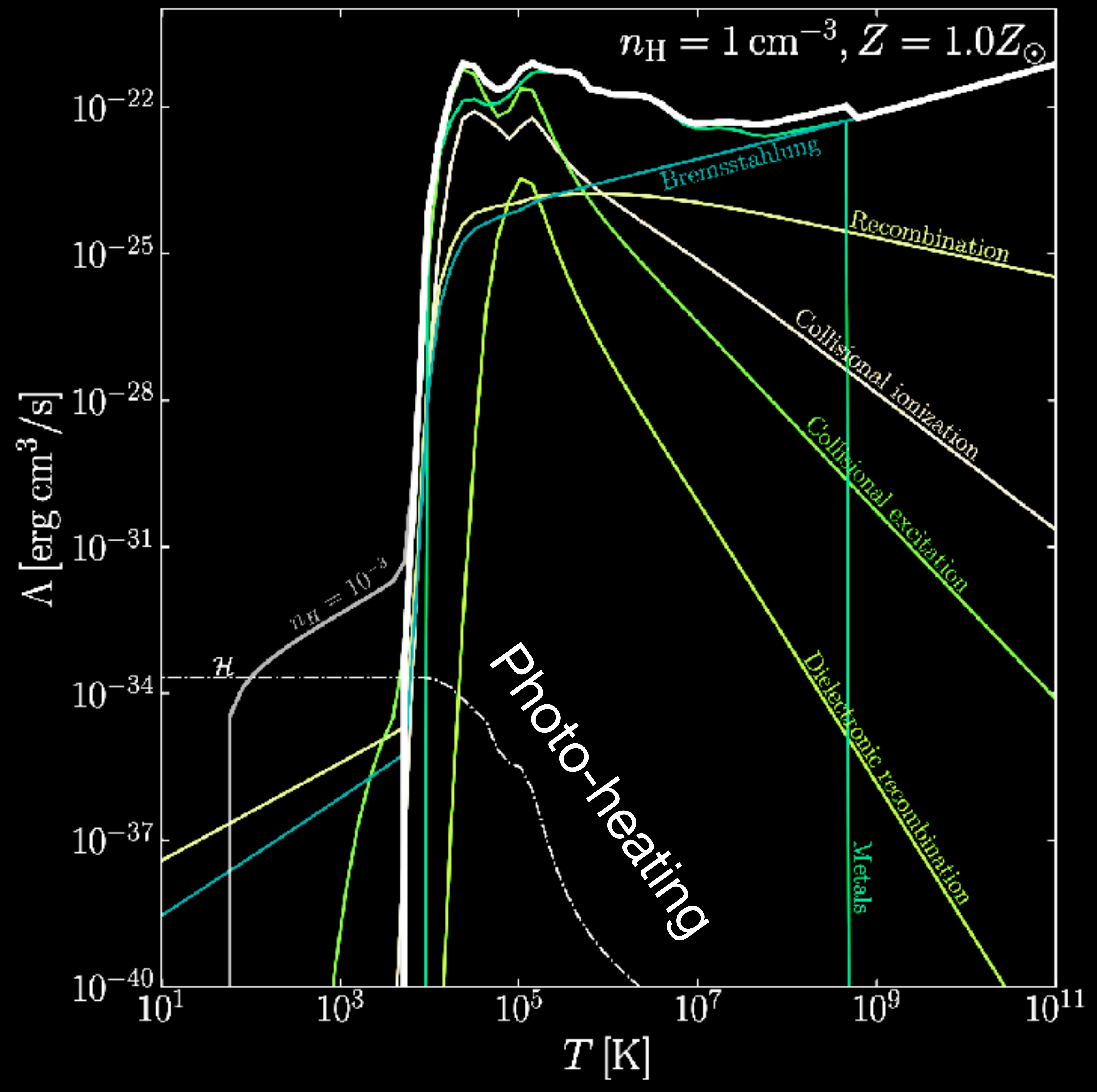
$$\lambda(x, y, z) = \dot{n}_{\rm SNe} \mathscr{V}_{\rm cell} \Delta t$$

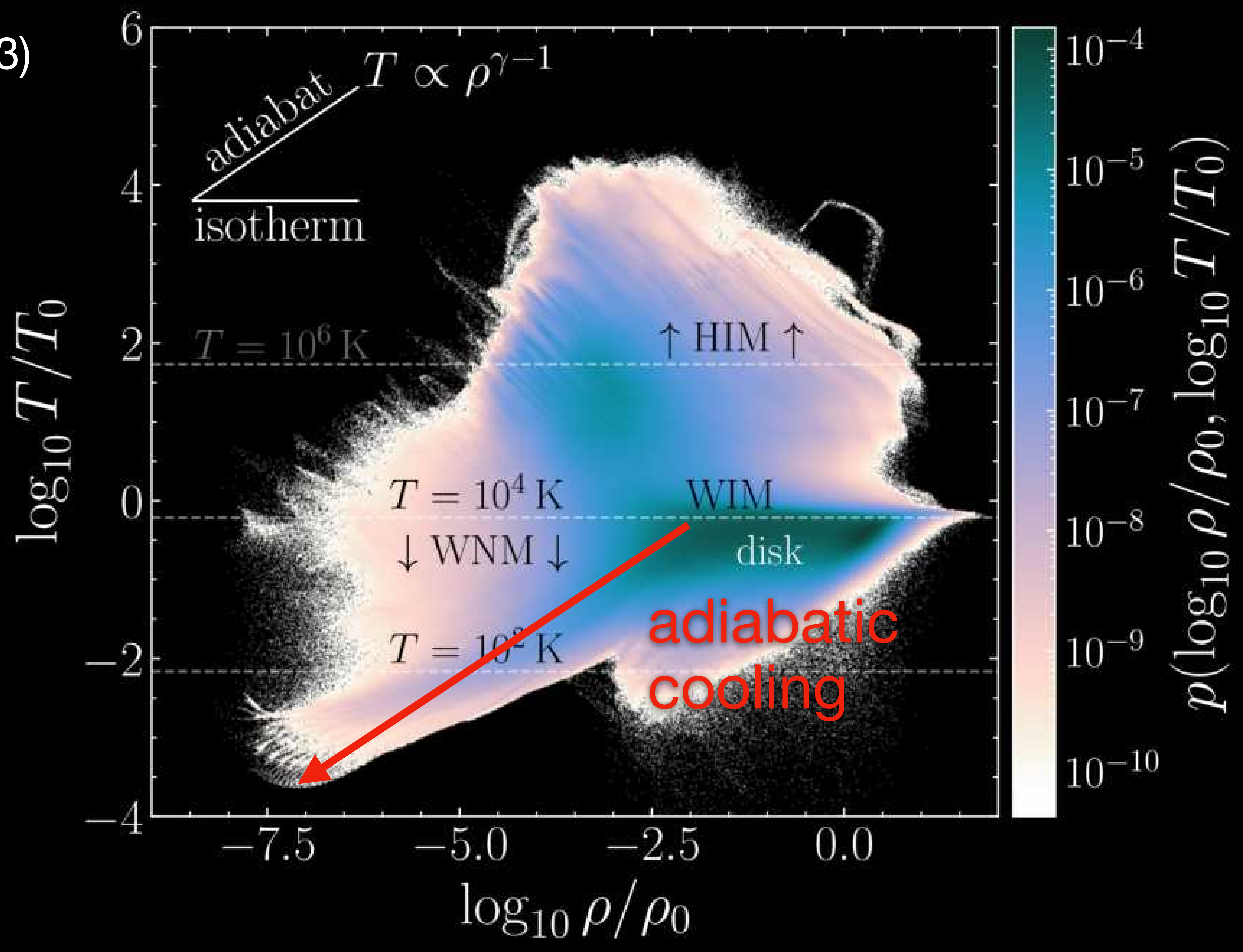
Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

The simplest SNe-driven simulations possible

Cooling function

Theuns+(1998)
Sutherland & Dopita(1993)





HI, HII, HeI, HeII, HeIII and free electrons

$$\rho_0 = 2.1 \times 10^{-24}\ \mathrm{g\ cm^{-3}}$$
$$P_0 = 2.2 \times 10^{-12}\ \mathrm{Ba}$$

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

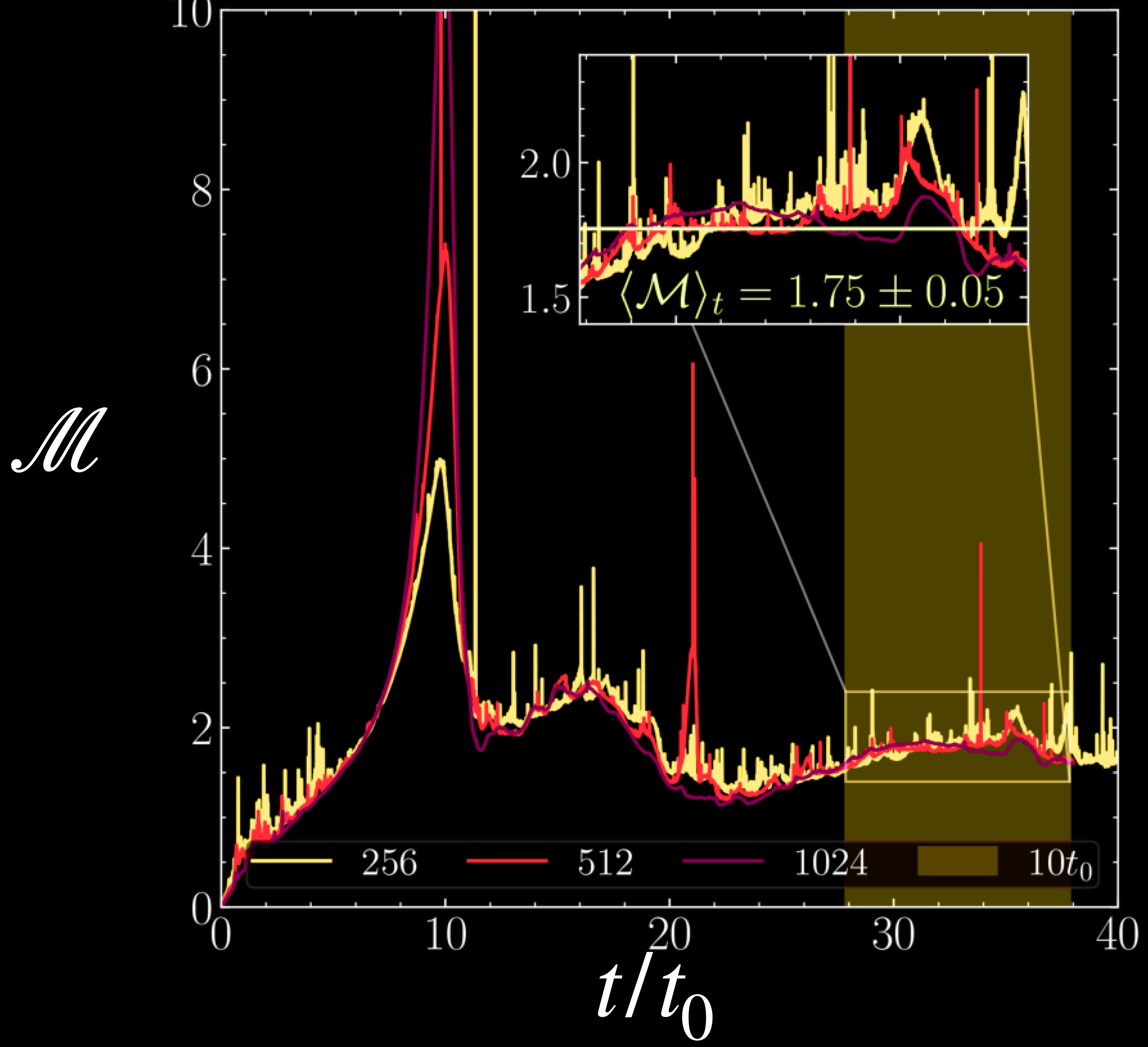
Stationarity and Mach number

$$\mathscr{M} = \left\langle \left(\frac{u}{c_s} \right)^2 \right\rangle^{1/2}$$

$$t_0 \sim \frac{\ell_0}{\langle u^2 \rangle^{1/2}} \sim 80 \text{ Myr}$$

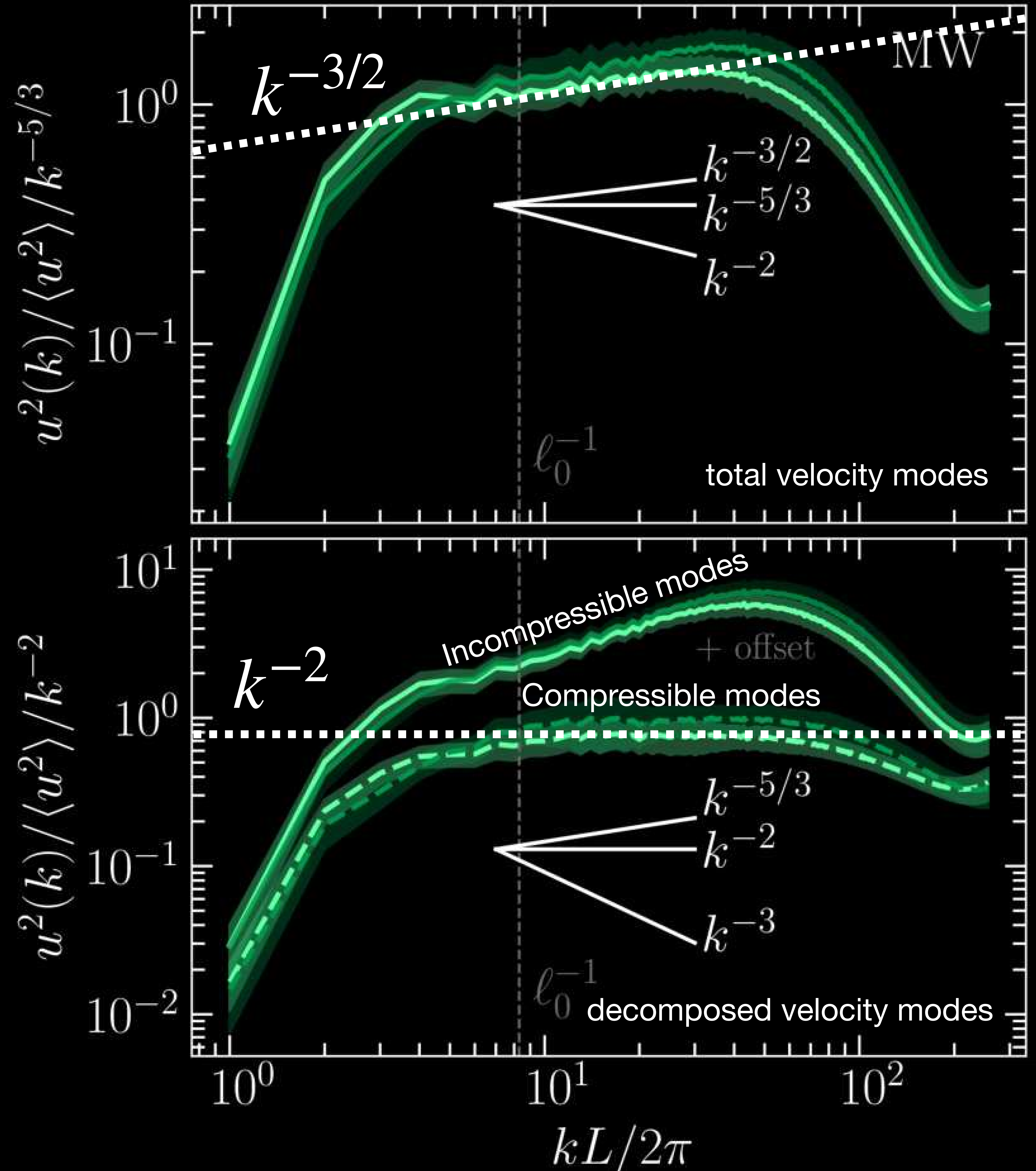
reaches a quasi stationary state

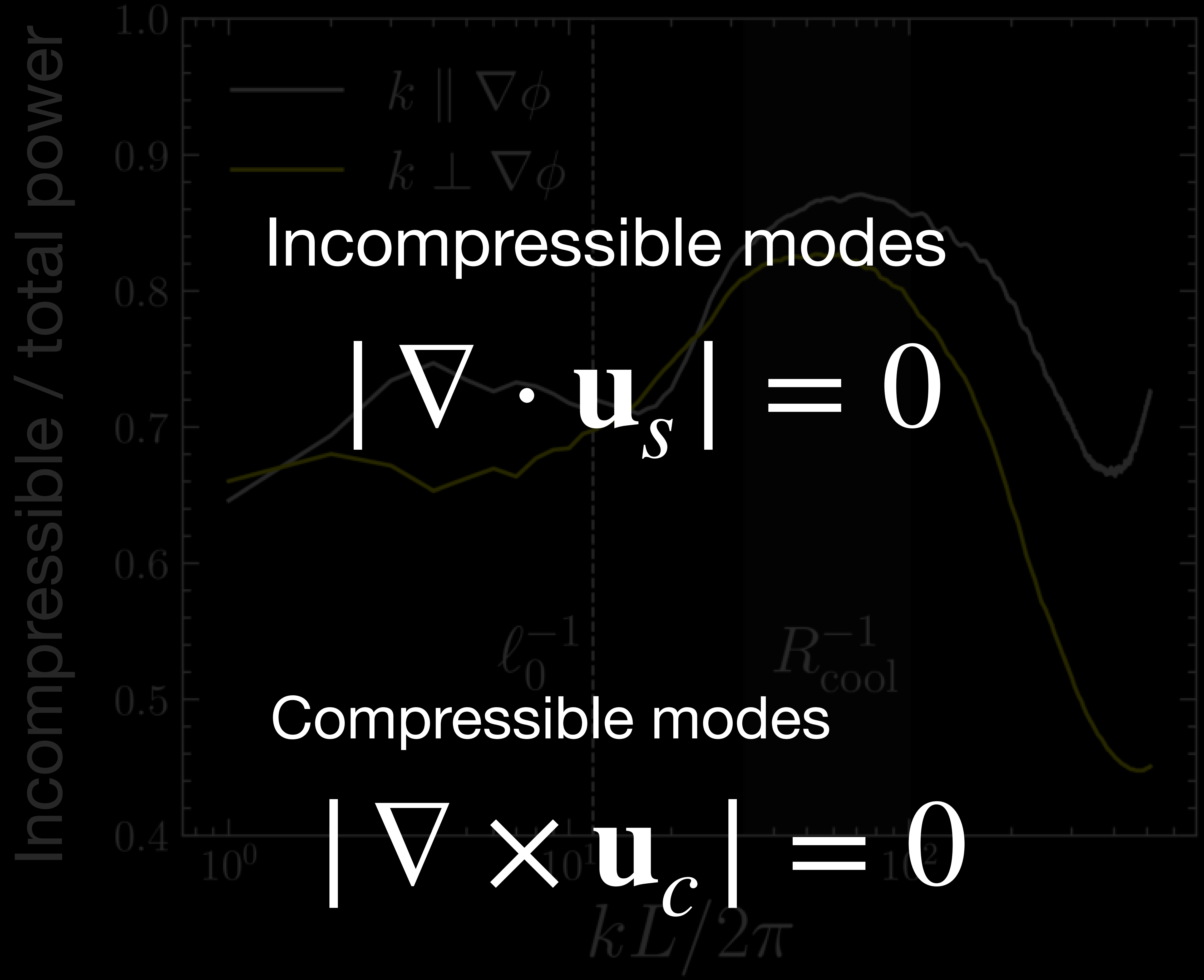
Beattie + (2025, ApJ) So long Kolmogorov.



Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Basic spectral properties





Incompressible modes

$$|\nabla \cdot \mathbf{u}_s| = 0$$

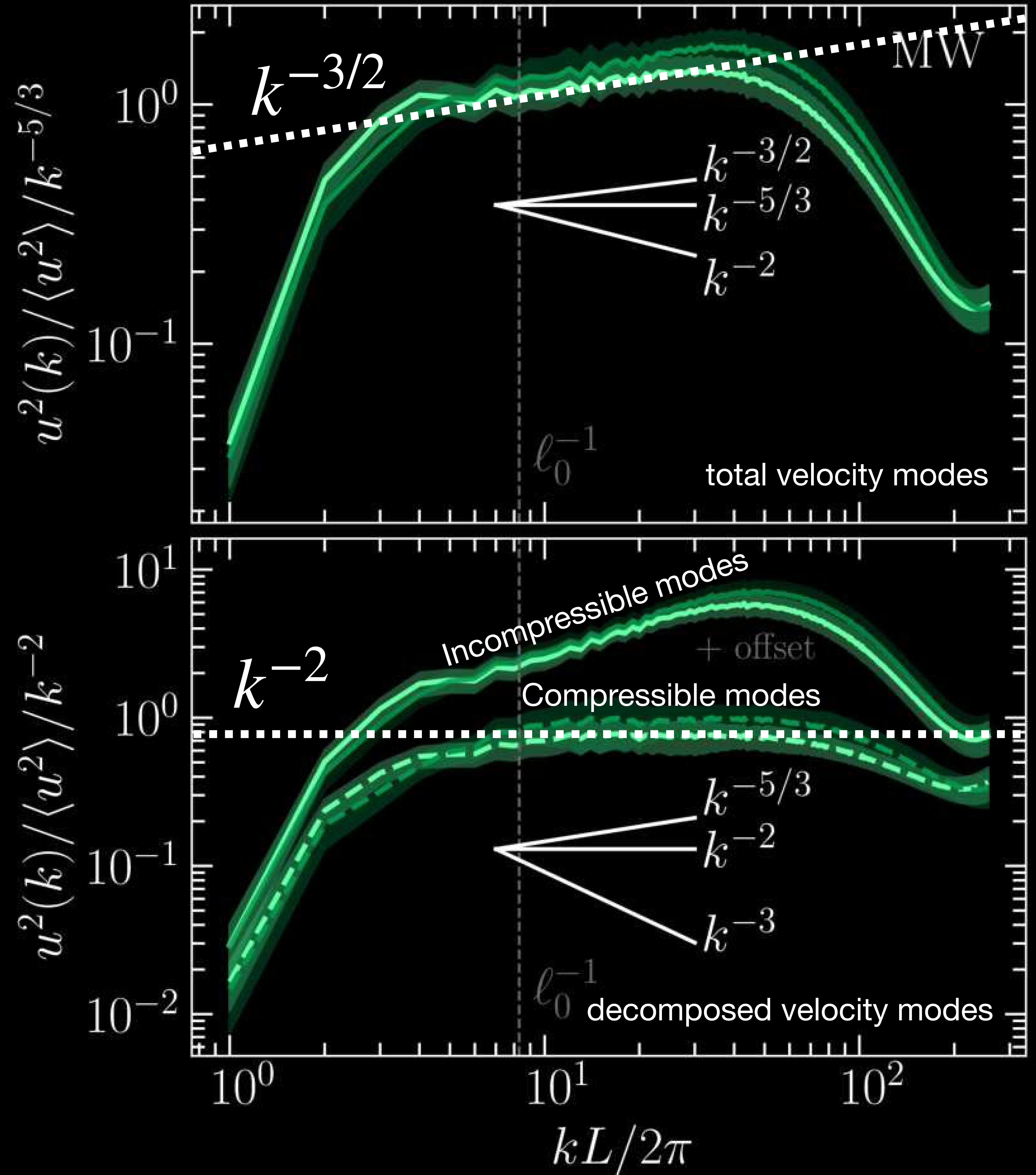
Compressible modes

$$|\nabla \times \mathbf{u}_c| = 0$$

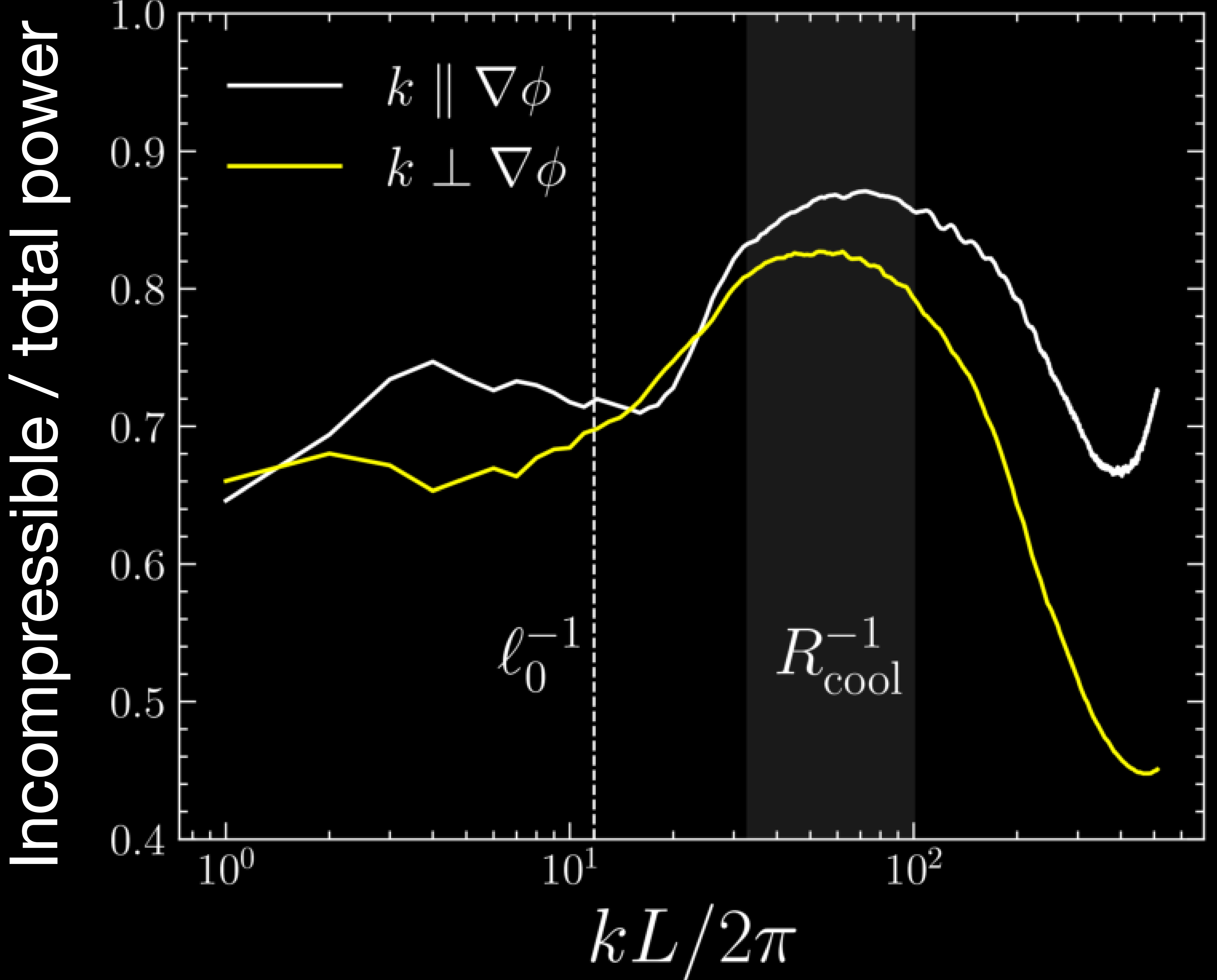
$$R_{\rm cool} \approx 14\,{\rm pc}\left(\frac{\rm E_{SN}}{10^{51}\,{\rm erg}}\right)\left(\frac{0.29}{\rm n_0\,cm^{-3}}\right)^{-0.43}$$

Cioffi + 1998
Blondin + 1998

Basic spectral properties



Beattie + (2025, ApJ) So long Kolmogorov.



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Cioffi + 1998
Blondin + 1998

Phase fluctuations and thermal response is mostly adiabatic

Consider the adiabatic limit

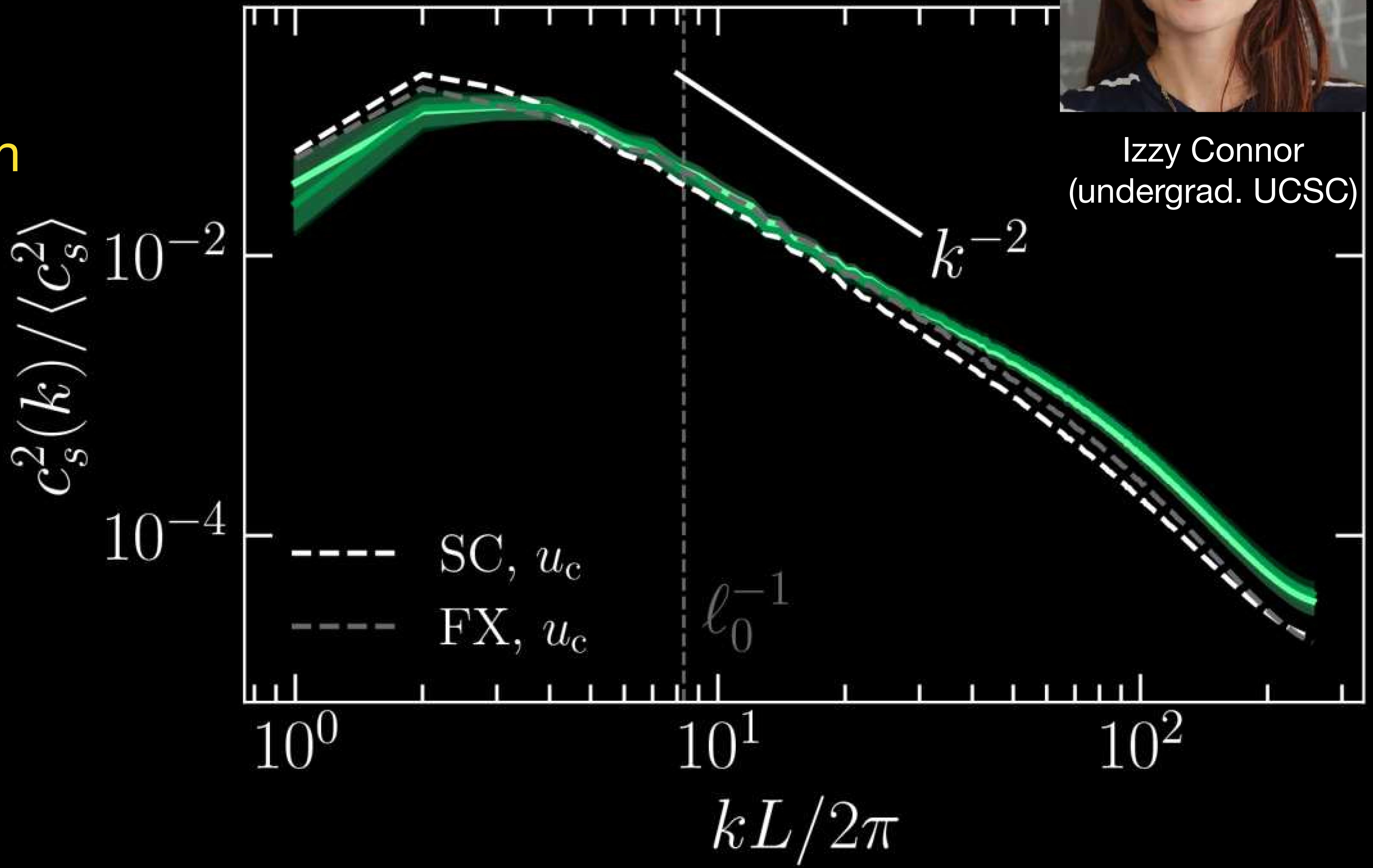
$$t_{\rm turb} \ll t_{\rm cool}$$

via linearized continuity equation

$$\delta\tilde{c}_s \sim c_{s,0}\frac{\gamma - 1}{2\omega}\mathbf{k}\cdot\tilde{\mathbf{u}}_c$$

$$\omega \sim c_{s,0}k$$

$$\delta\tilde{c}_s \sim \frac{\gamma - 1}{2}\tilde{u}_s$$



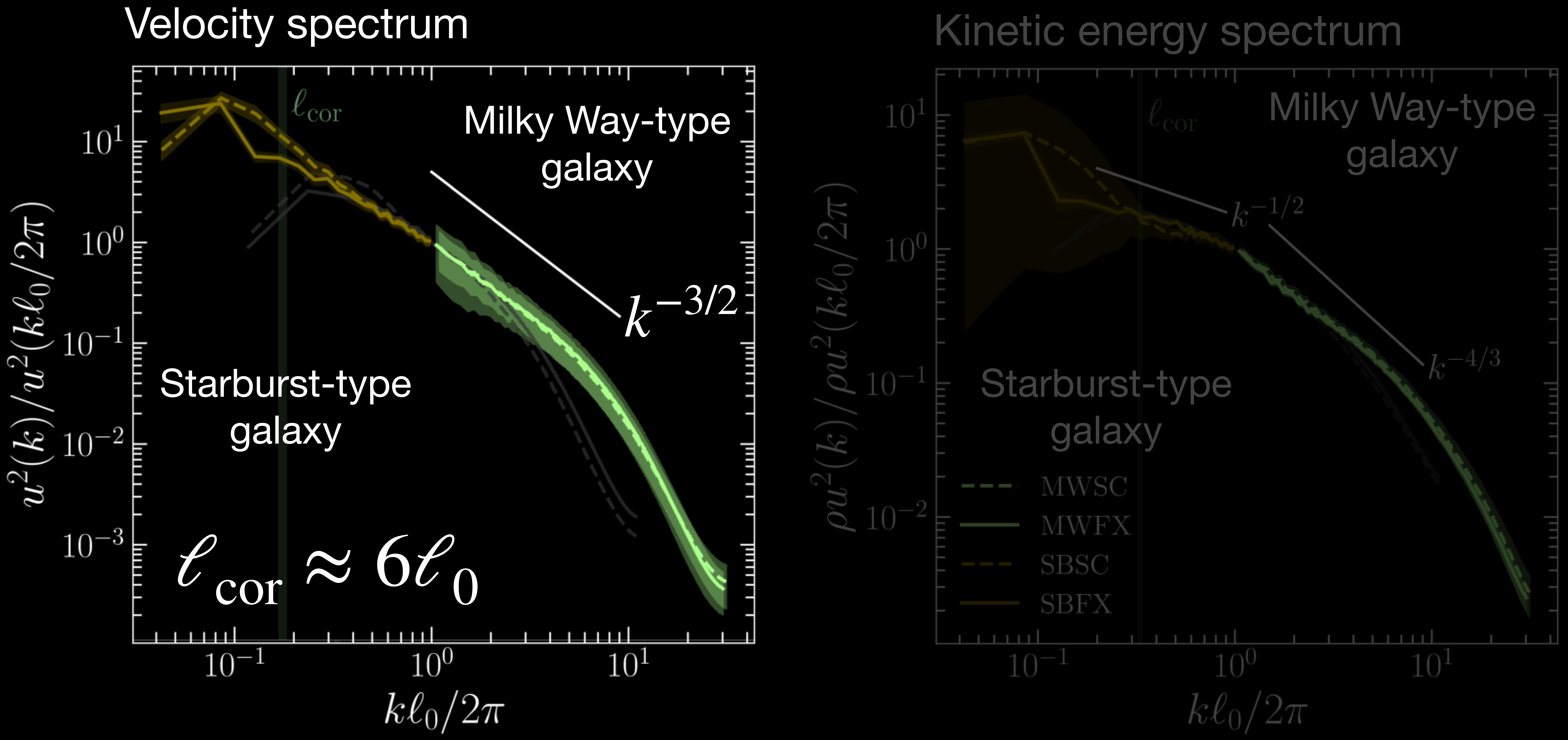
Izzy Connor
(undergrad. UCSC)

thermal response / phase fluctuations controlled by compressible modes

Connor, **Beattie** + (2025, ApJ) Cascading from the winds to the disk: universality of supernovae-driven turbulence in different galactic ISMs

Universality between galactic models

$$\gamma_{\mathrm{SN_e,SB}} = 10\gamma_{\mathrm{SN_e,MW}}$$



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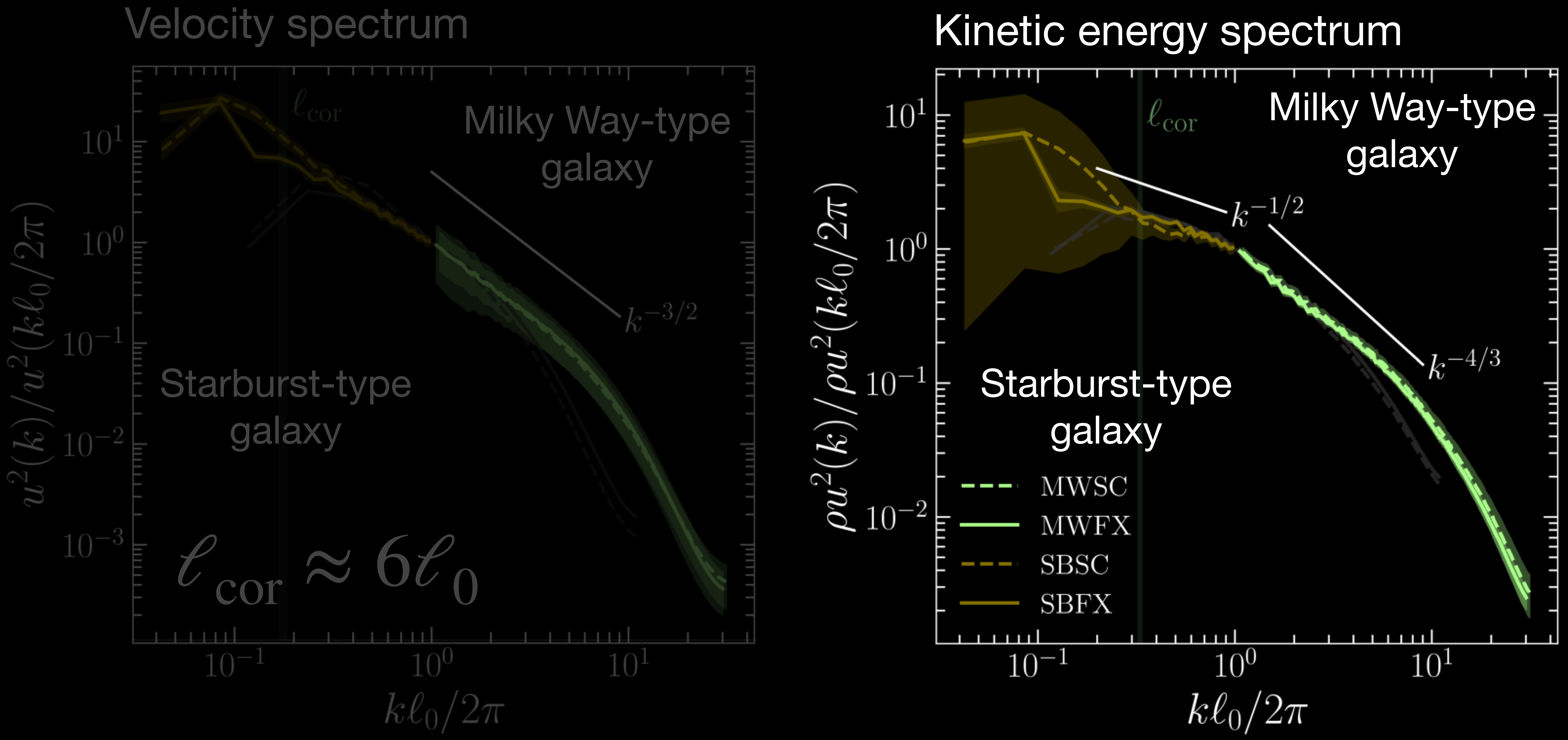
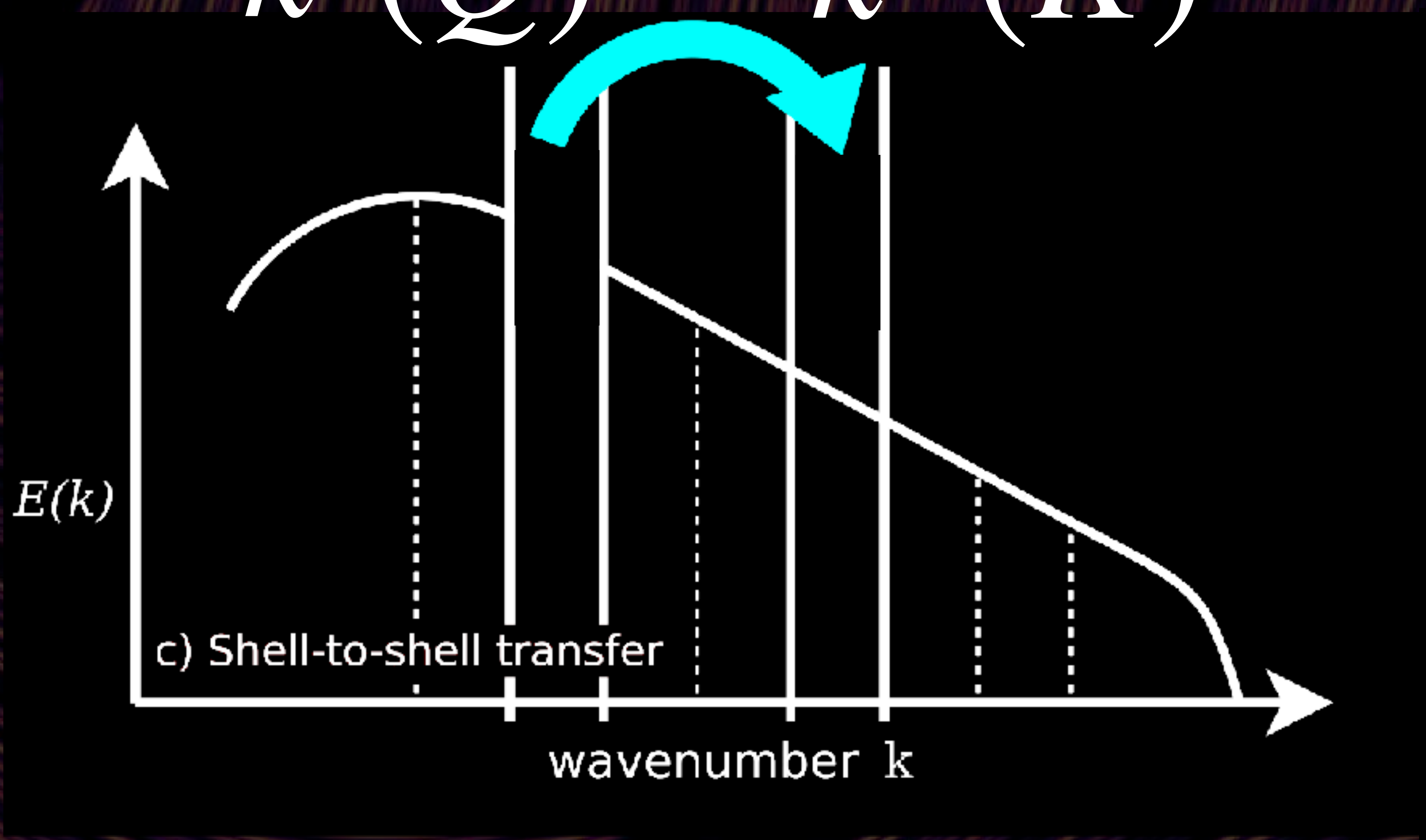
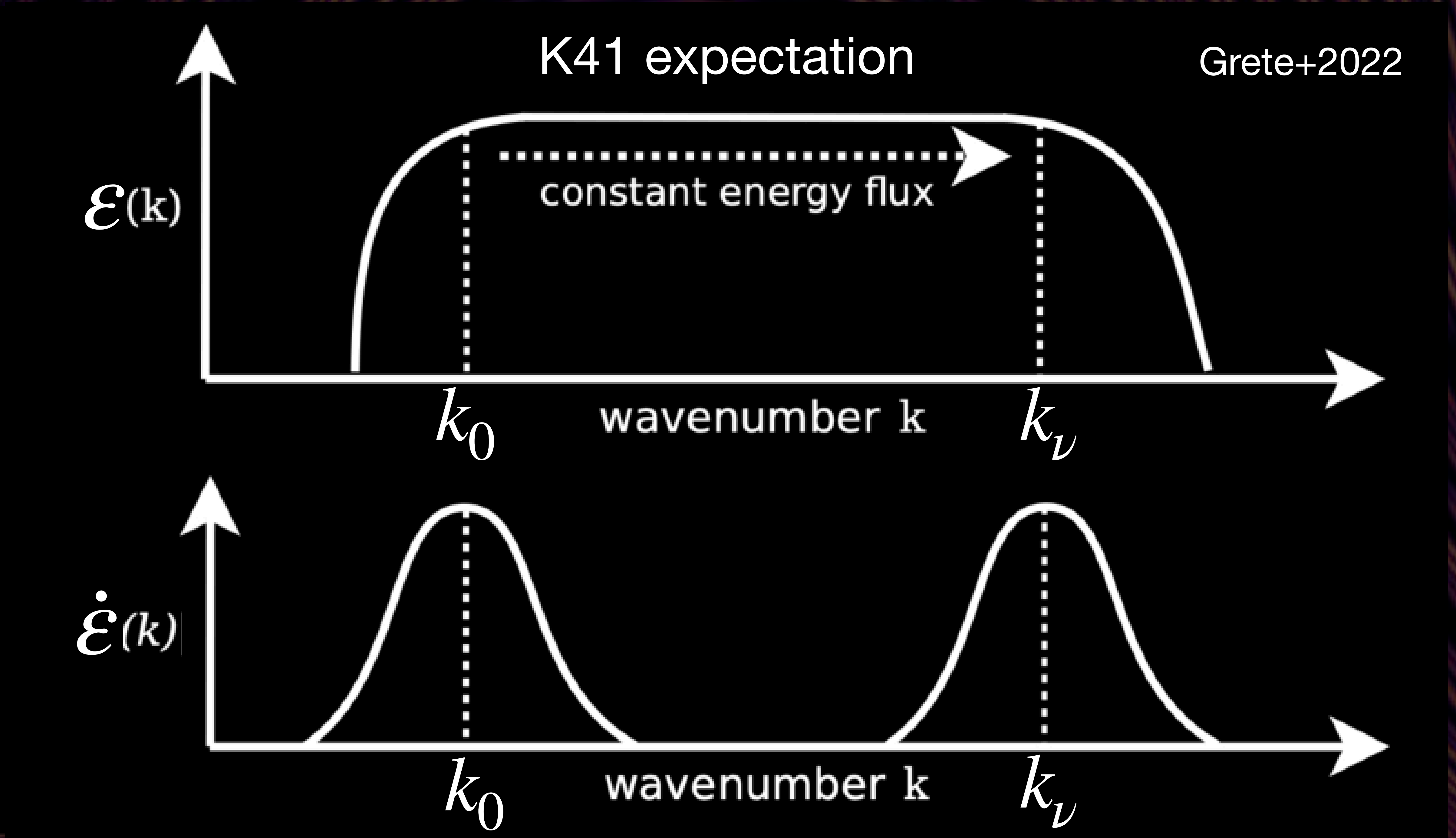


Connor, **Beattie** + (2025, ApJ) Cascading from the winds to the disk: universality of supernovae-driven turbulence in different galactic ISMs

But to understand the turbulence we must understand the flux rates from mode to mode!

$k''(Q)$ ε $k'''(K)$





Grete, O'Shea +2022a,b,23

Dar+2001; Verma+2004; Mininni+2005; Alexakis+2005

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

But to understand the turbulence we must understand the flux rates from mode to mode!

Momentum conservation:

doner receiver

$$Q + P + K = 0$$

mediator

doner receiver

$$Q \xrightarrow{P} K = -K \xrightarrow{P} Q$$

mediator

$$\mathbf{u}^Q = \mathbf{u}(\mathbf{r}^Q) = \int \delta^3(\mathbf{k} - Q)\,\mathbf{u}(\mathbf{k})\exp\{2\pi i\mathbf{k}\cdot\mathbf{r}\}$$

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

But to understand the turbulence we must understand the flux from mode to mode!

$$Q \xrightarrow{P} K$$

kinetic energy density: $\overbrace{\rho \mathbf{u}^Q \cdot \mathbf{u}^K}$

$$\frac{1}{2}\partial_t \overbrace{\rho \mathbf{u}^Q \cdot \mathbf{u}^K} + \mathbf{u}^K \cdot \underbrace{\nabla \cdot \mathbb{F}_{\rho \mathbf{u}}(\mathbf{u}^P, \mathbf{u}^Q)} = 0$$

energy flux rate density
from transport between $\mathbf{u}^Q$ and $\mathbf{u}^K$

$$\mathbf{u}^K \cdot \nabla \cdot \mathbb{F}(\mathbf{u}^P, \mathbf{u}^Q)_{\rho \mathbf{u}} = \mathbf{u}^K \otimes \mathbf{u}^P : \nabla \otimes \mathbf{u}^Q + \ldots$$

$$\varepsilon \sim u^3/\ell$$

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

But to understand the turbulence we must understand the flux from mode to mode!

$$\mathbf{u}^K \cdot \nabla \cdot \mathbb{F}(\mathbf{u}^P, \mathbf{u}^Q)_{\rho \mathbf{u}} = \mathbf{u}^K \otimes \mathbf{u}^P : \nabla \otimes \mathbf{u}^Q + \ldots$$

$$\mathcal{T}_{uu}(Q, K | P) = -\int dV\, \mathbf{u}^K \otimes \mathbf{u}^P : \nabla \otimes \mathbf{u}^Q$$

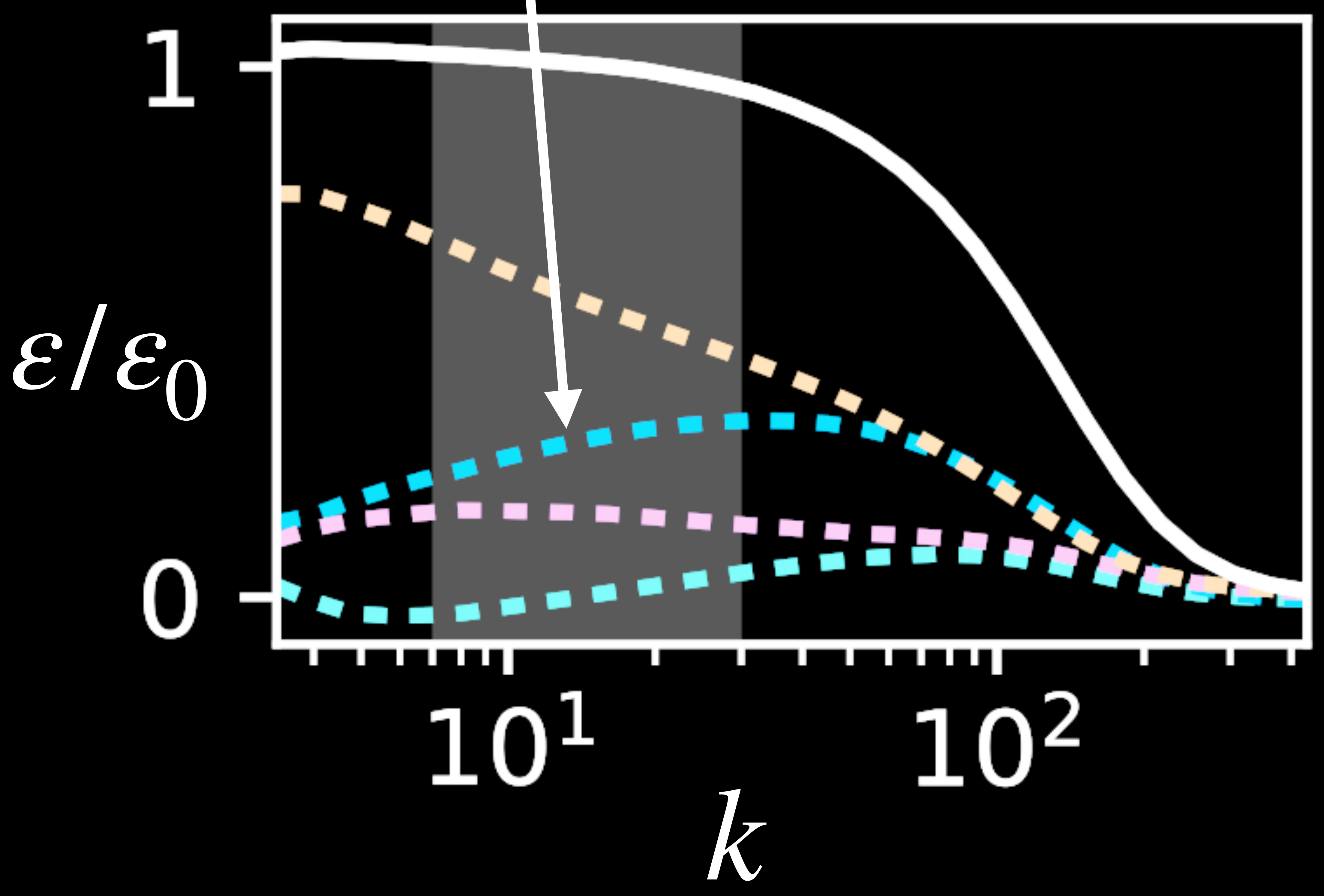
$$\varepsilon \sim u^3/\ell$$

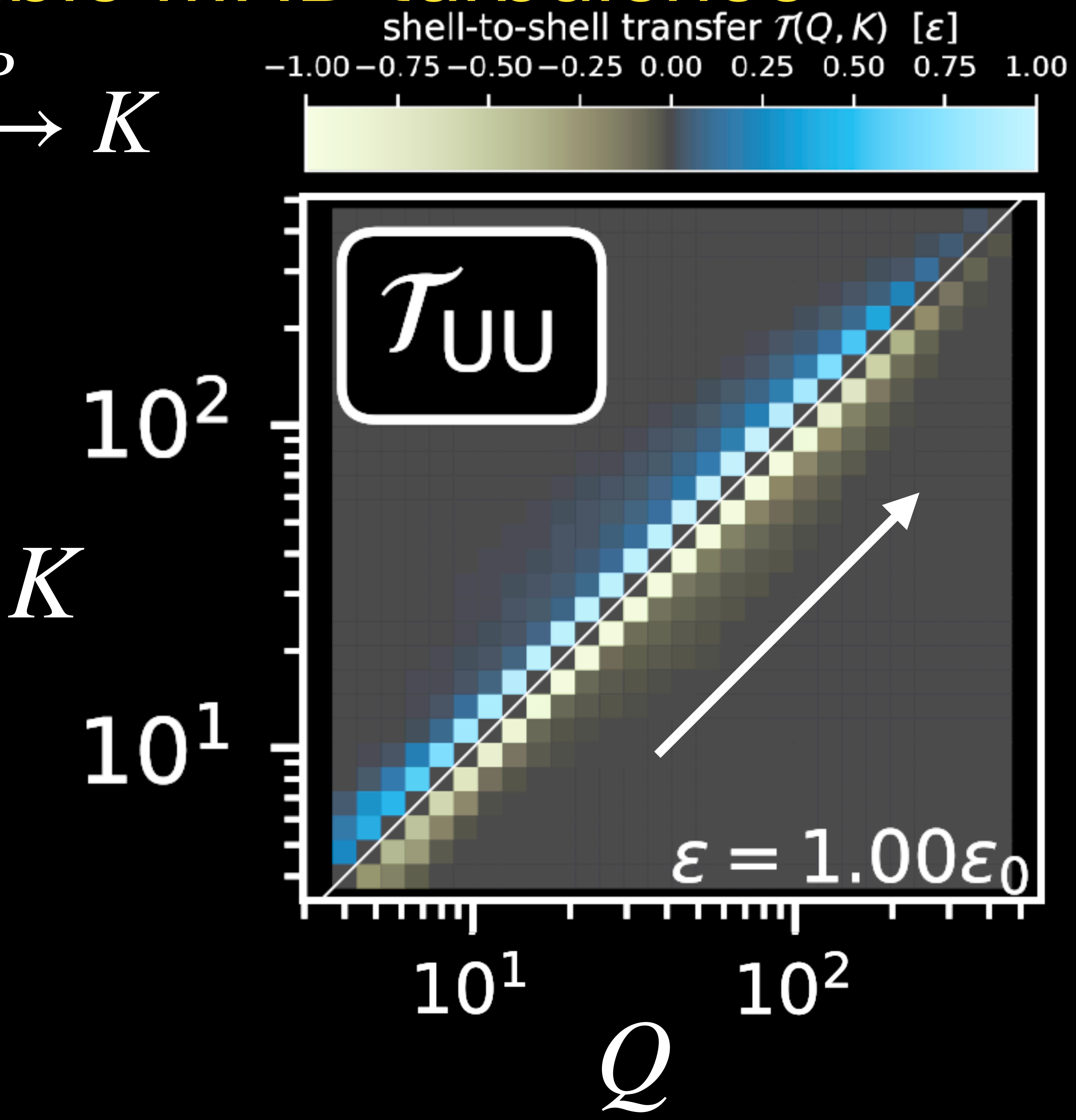
Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Expectations from compressible MHD turbulence in a box

$$Q \xrightarrow{P} K$$

$$\varepsilon \sim u^3/\ell \approx \text{const.} (?)$$





Grete, O'shea + 2022

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Compressible modes and solenoidal modes live very different lives!
We should treat them differently!

$$Q \xrightarrow{P} K$$

$$\mathscr{T}^{c}_{cc}(Q,K) = -\int dV\, \mathbf{u}_c^K \otimes \mathbf{u}_c^P : \nabla \otimes \mathbf{u}_c^Q$$

cascade transfers

$$\mathscr{T}^{s}_{ss}(Q,K) = -\int dV\, \mathbf{u}_s^K \otimes \mathbf{u}_s^P : \nabla \otimes \mathbf{u}_s^Q$$

$$u_s \cdot u_c = 0$$
$$\nabla \cdot u_s = 0$$
$$|\nabla \times u_c| = 0$$
$$u = u_c + u_s$$

$$\vdots$$

$$\mathscr{T}^{s}_{cs}(Q,K) = -\int dV\, \mathbf{u}_s^K \otimes \mathbf{u}_s^P : \nabla \otimes \mathbf{u}_c^Q$$

interaction transfers

Beattie + (2025, ApJ) So long Kolmogorov.

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

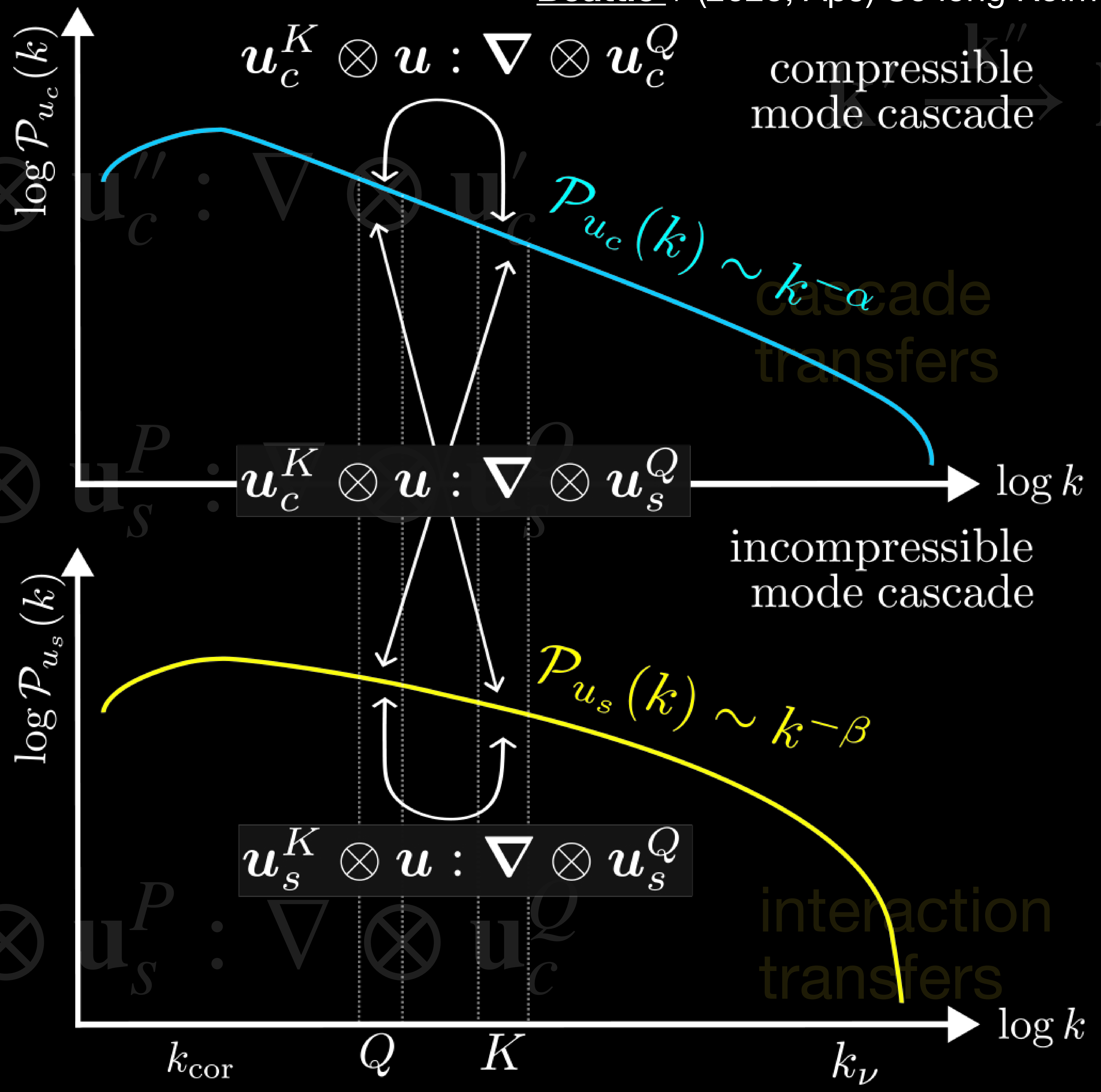
Compressible modes and solenoidal modes live very different lives! We should treat them differently!

Beattie + (2025, ApJ) So long Kolmogorov.

$$\mathcal{T}^{c}_{cc}(Q,K) = -\int dV\, \mathbf{u}'_c$$

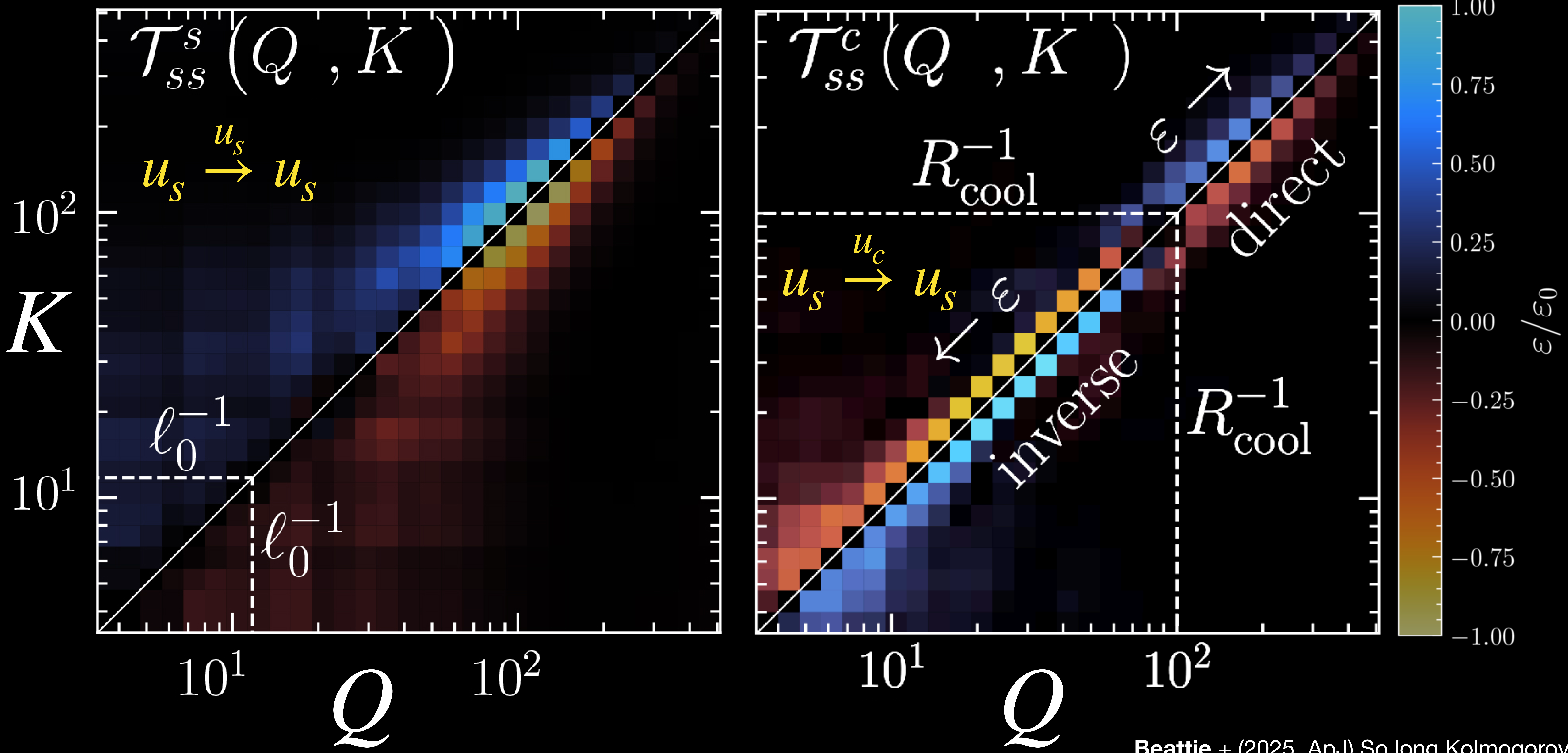
$$\mathcal{T}^{s}_{ss}(Q,K) = -\int dV\, \mathbf{u}^{l}_s$$

$$\mathcal{T}^{s}_{cs}(Q,K) = -\int dV\, \mathbf{u}^{l}_s$$



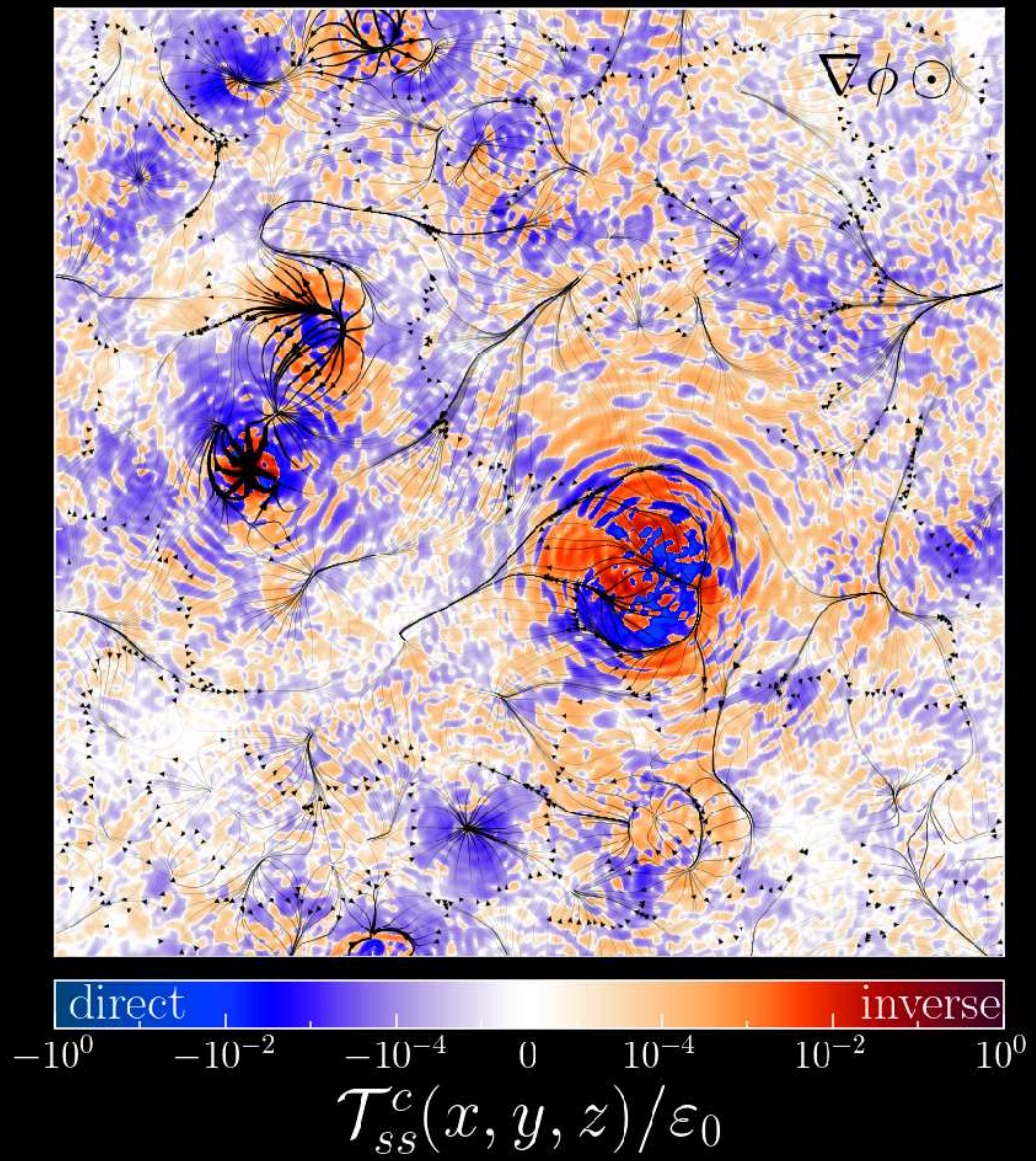
Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

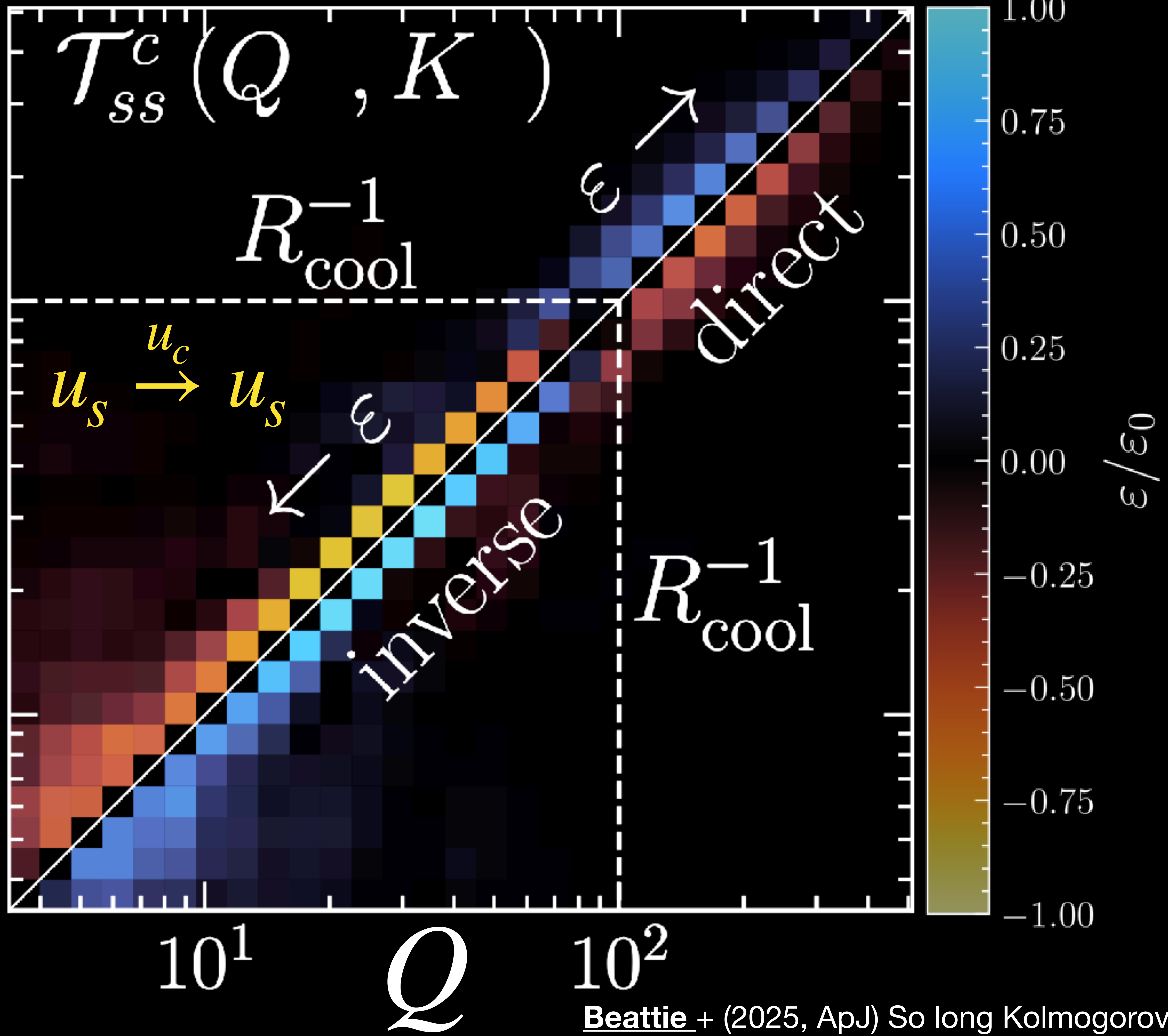
The incompressible turbulence ($u_s \rightarrow u_s$ cascade)



Beattie + (2025, ApJ) So long Kolmogorov.

The incompressible turbulence ($u_s \xrightarrow{u_c} u_s$ cascade)





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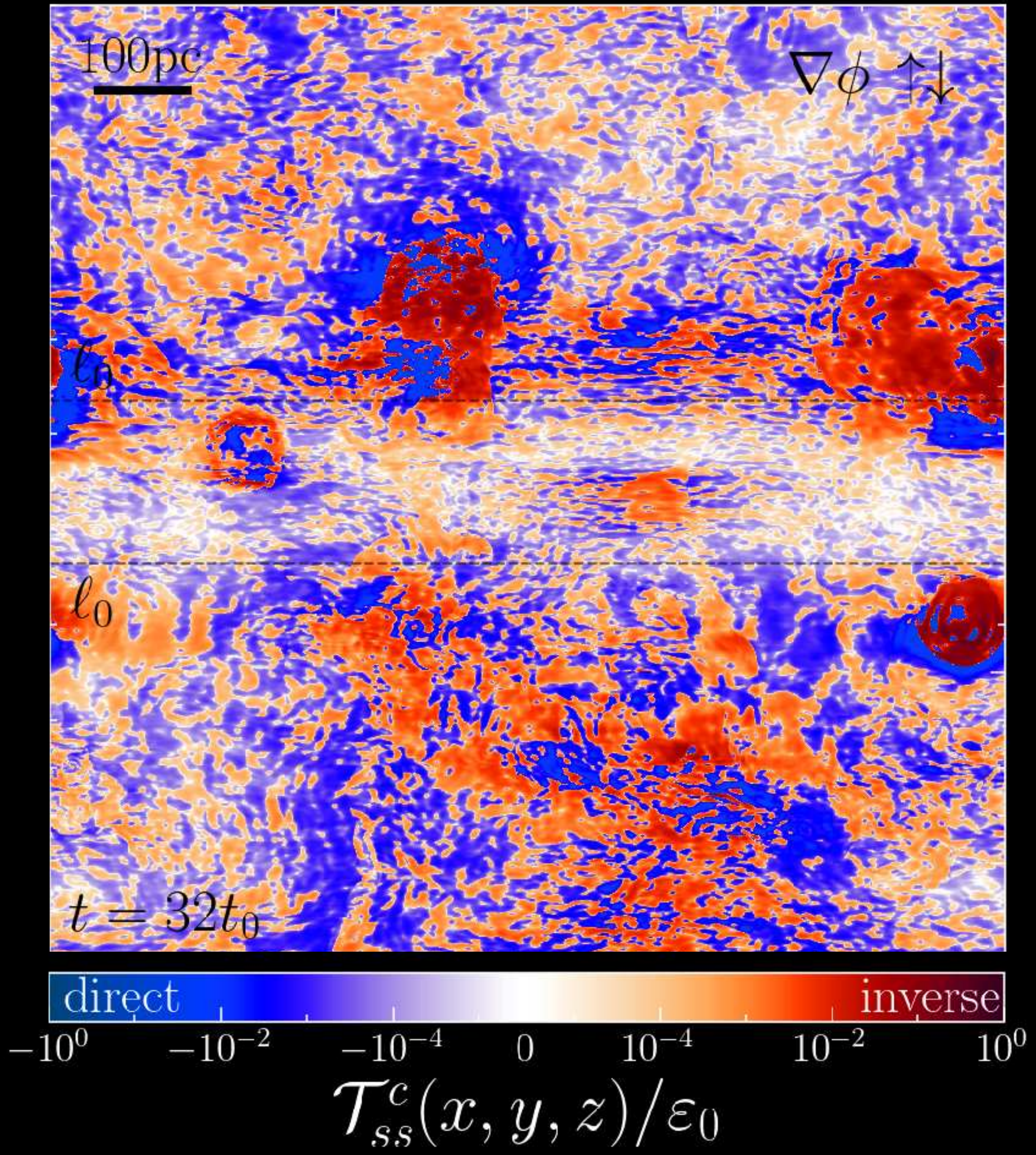
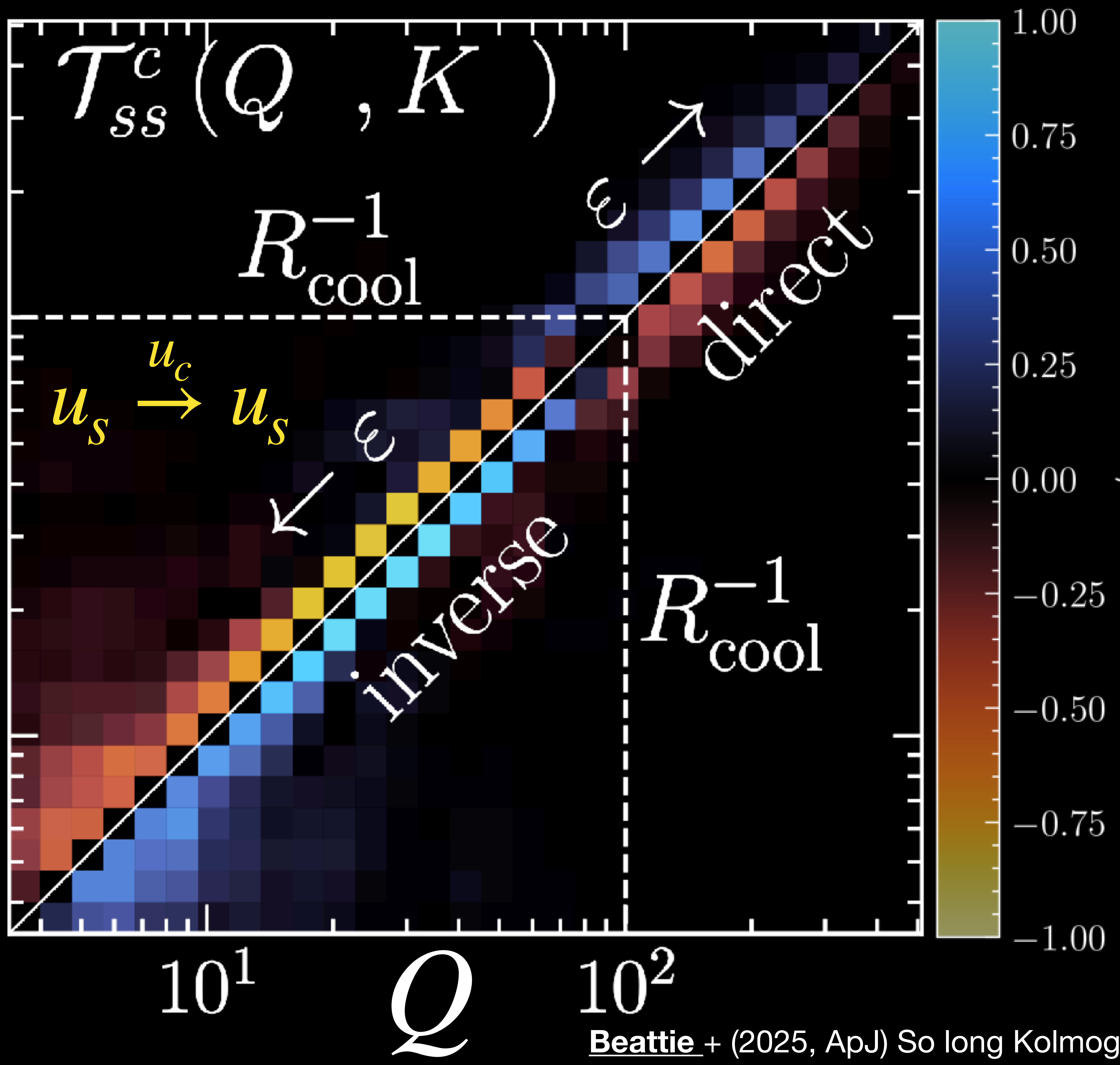
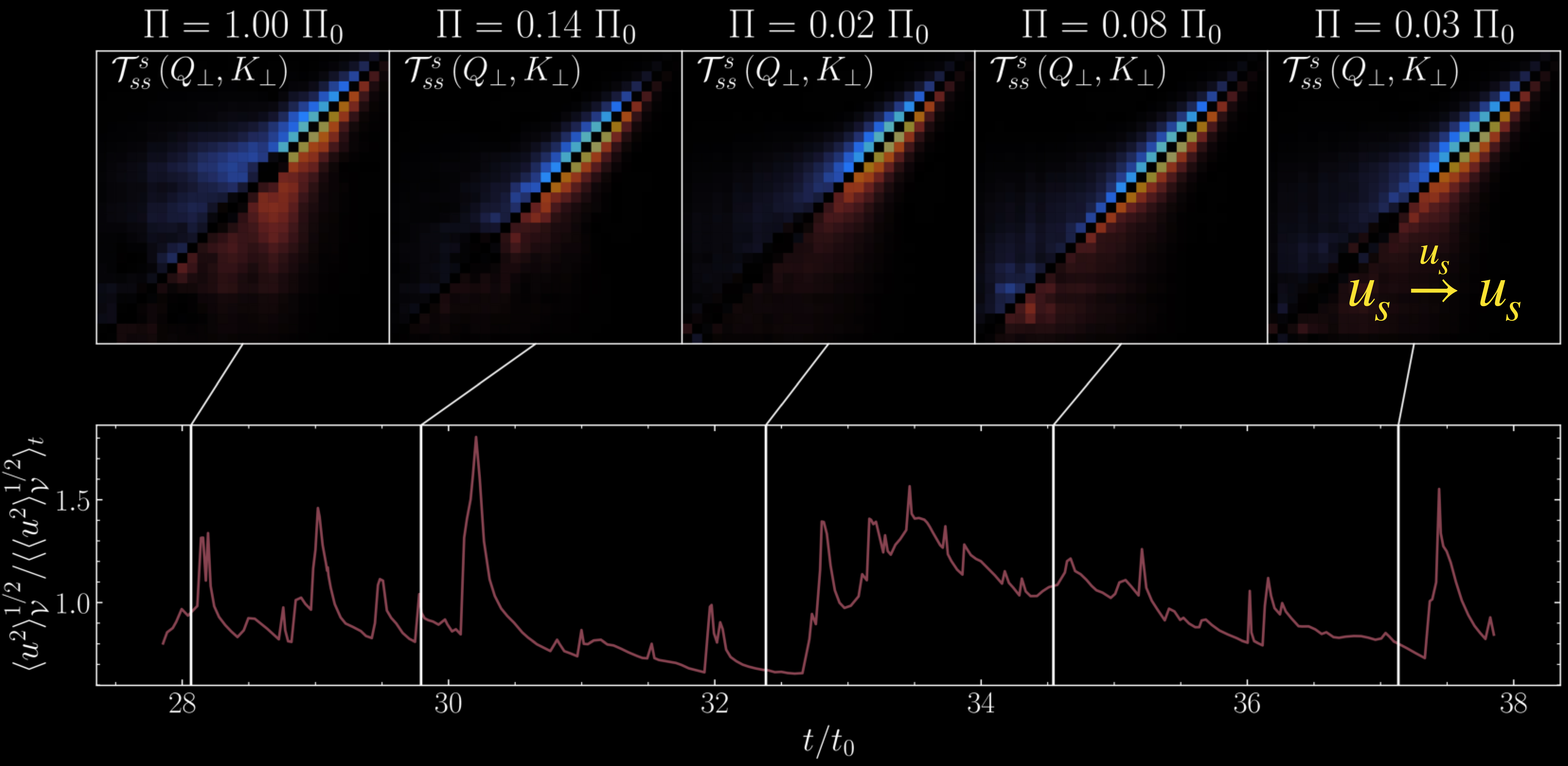




Beattie + (2025, ApJ) So long Kolmogorov.

The entire incompressible cascade comes and goes



Beattie + (2025, ApJ) So long Kolmogorov.

Where does the incompressible turbulence come from?

No vorticity!

$$\frac{\partial \rho \boldsymbol{u}}{\partial t} + \nabla \cdot (\rho \boldsymbol{u} \otimes \boldsymbol{u} + P\mathbb{I}) = -\boxed{\rho \nabla \phi + \dot{n}_{\rm SNe} \mathbf{p}_{\rm SNe}(Z, n_{\rm H})},$$

$$\frac{1}{2}\frac{\partial \omega(\mathbf{k}) \cdot \omega^{\dagger}(\mathbf{k})}{\partial t} = \underbrace{-\frac{d\Pi_{\omega}(\mathbf{k})}{d\mathbf{k}}}_{\text{Cascade}} + \underbrace{\mathrm{Re}\left\{\omega(\mathbf{k}) \cdot (\nabla \rho \times \nabla P/\rho^2)^{\dagger}(\mathbf{k})\right\}}_{\text{Source}} - \underbrace{\mathscr{D}(\mathbf{k})}_{\text{Sink}},$$

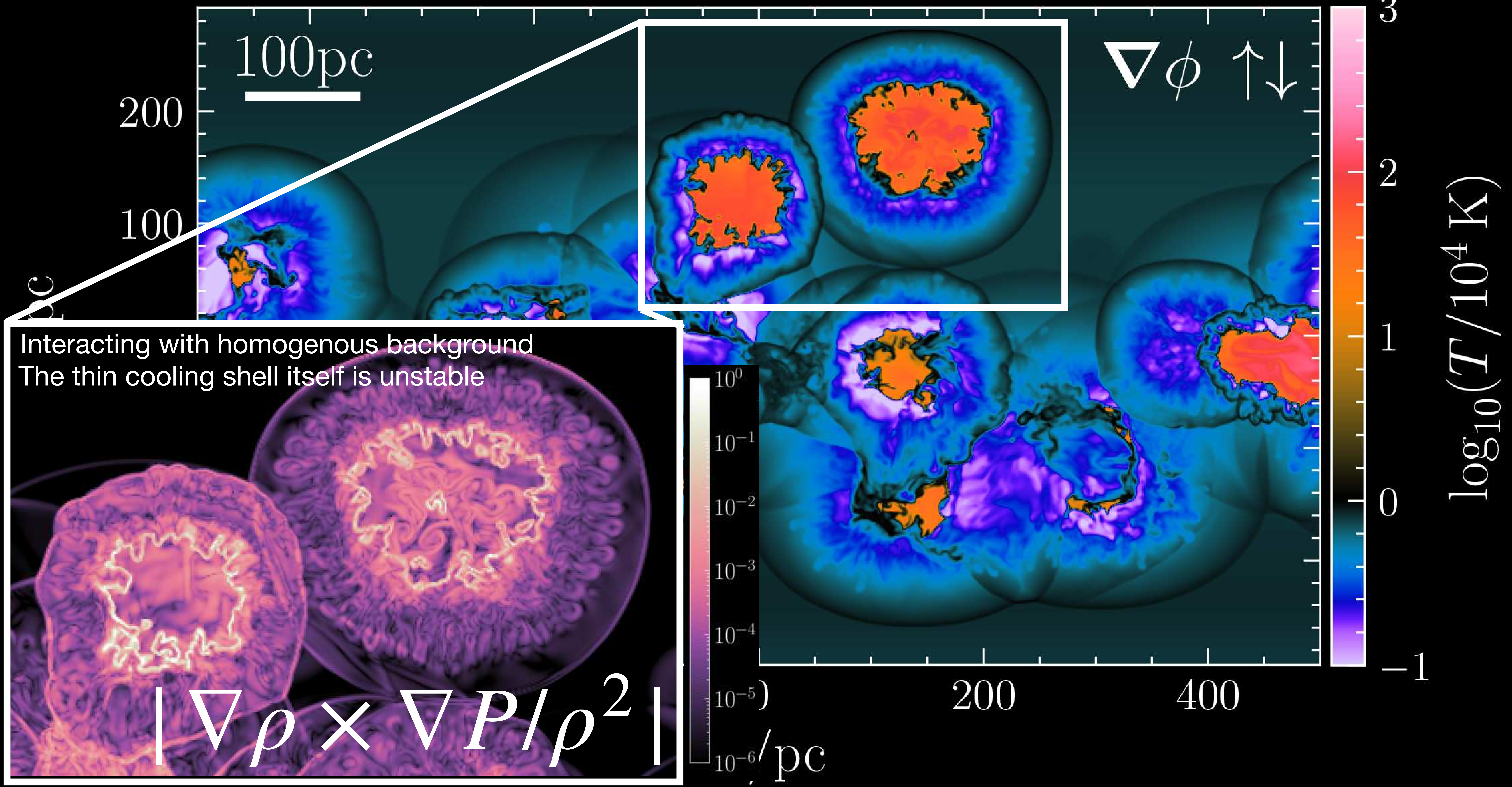
Cascade

Source

Sink

Beattie (2026; submitted ApJL) Supernovae drive large-scale incompressible turbulence from small-scale instabilities

Where does the incompressible turbulence come from?



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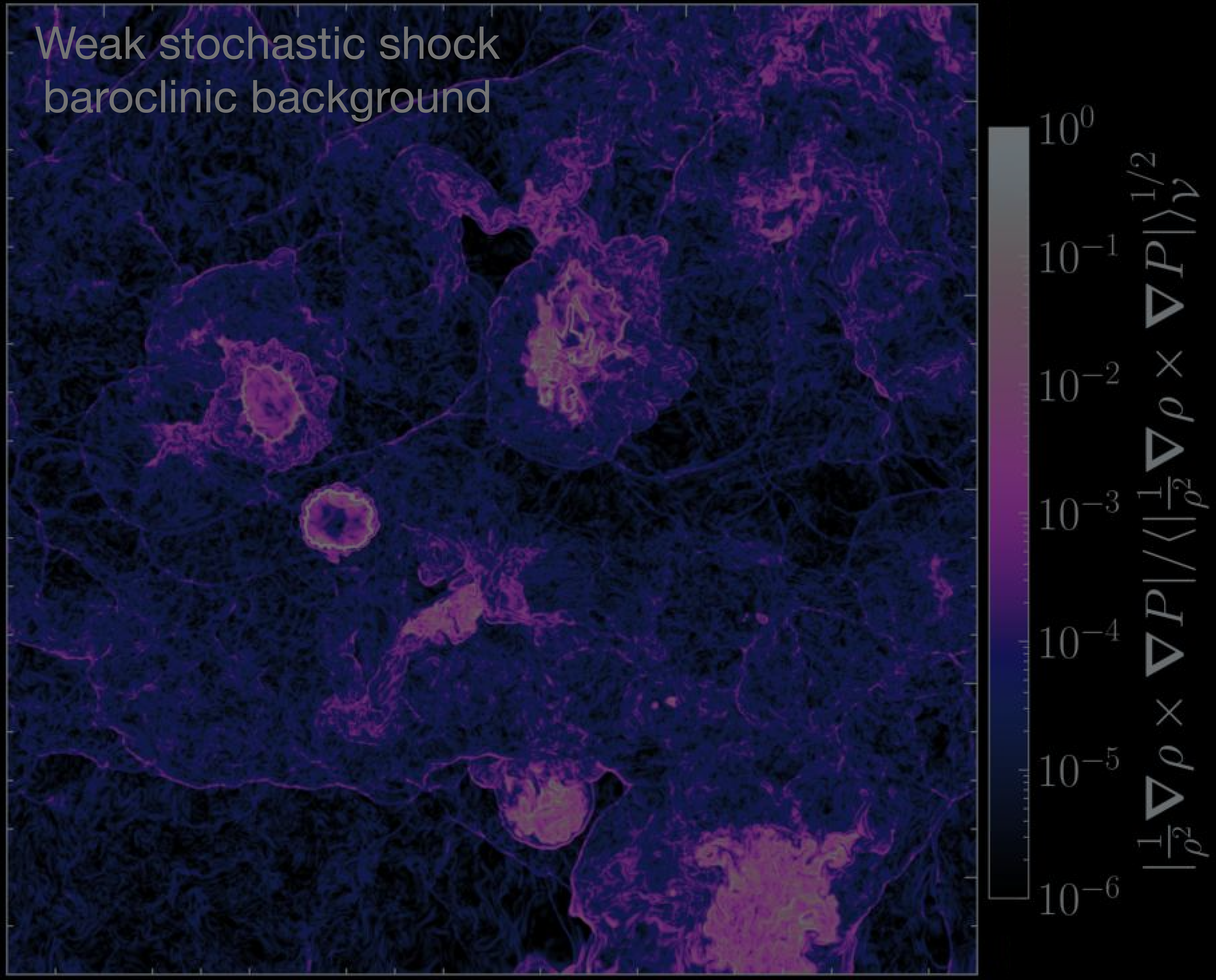
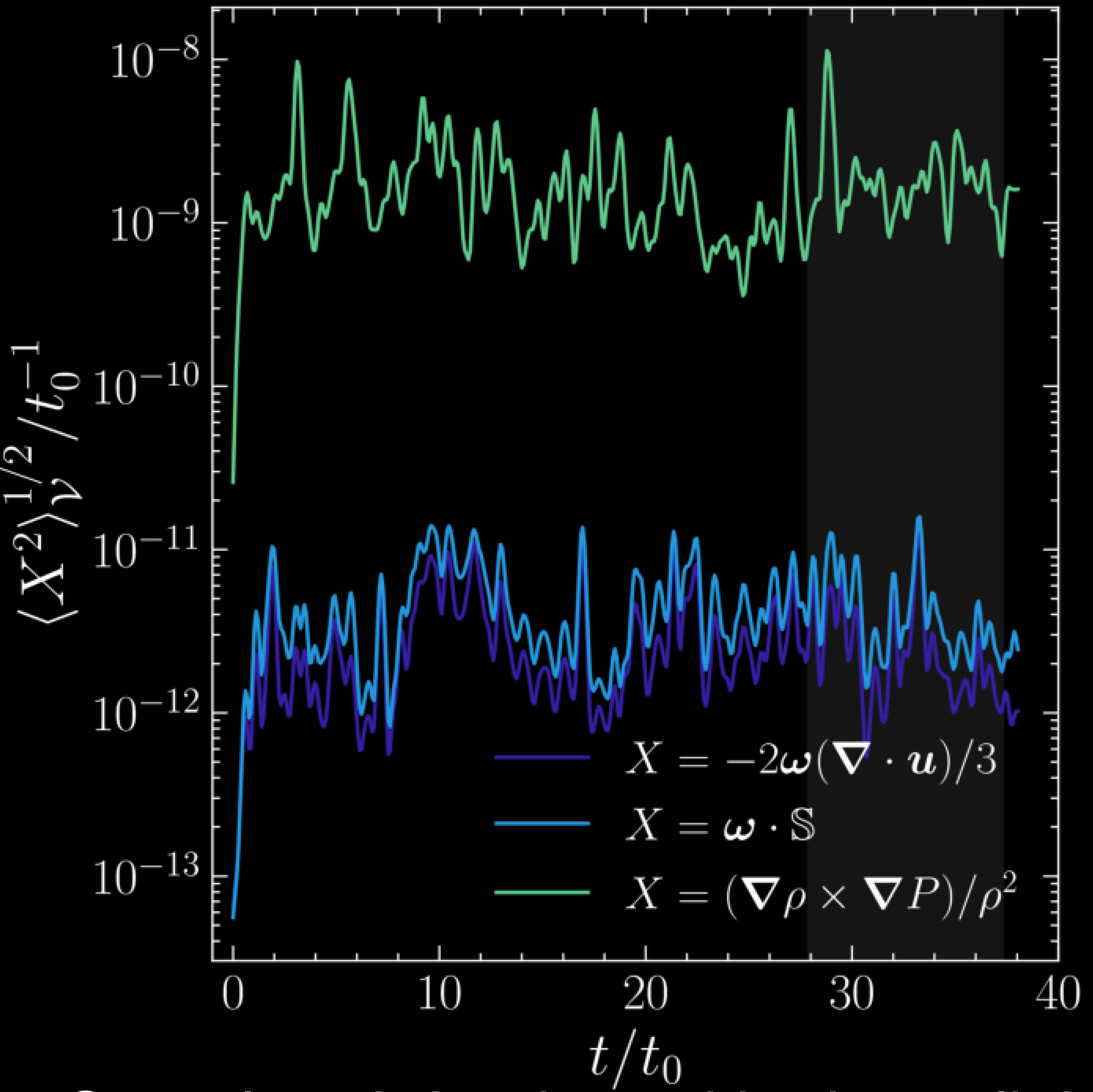
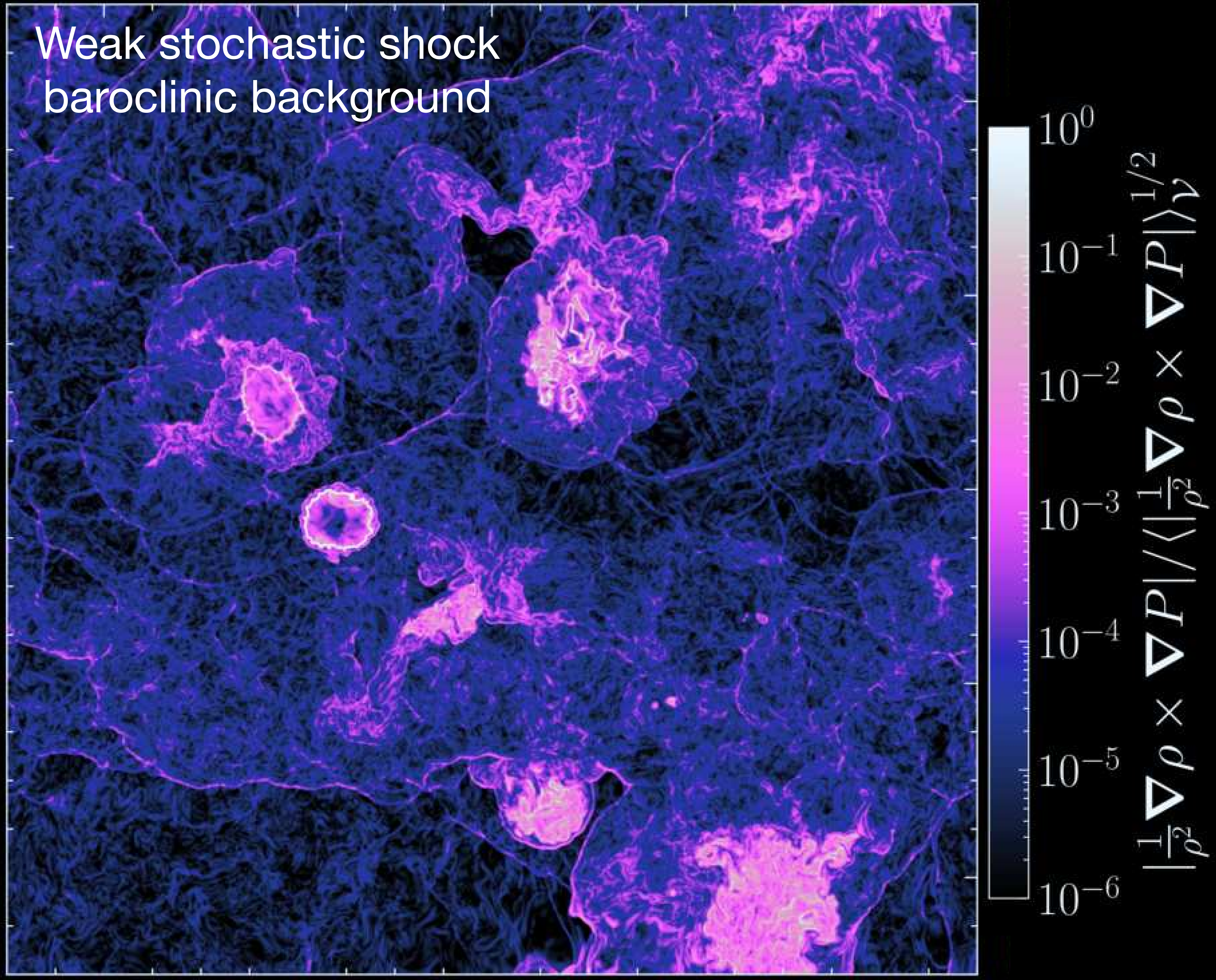


Vorticity sources



Completed dominated by baroclinic battery at all times.

Where does the incompressible turbulence come from?



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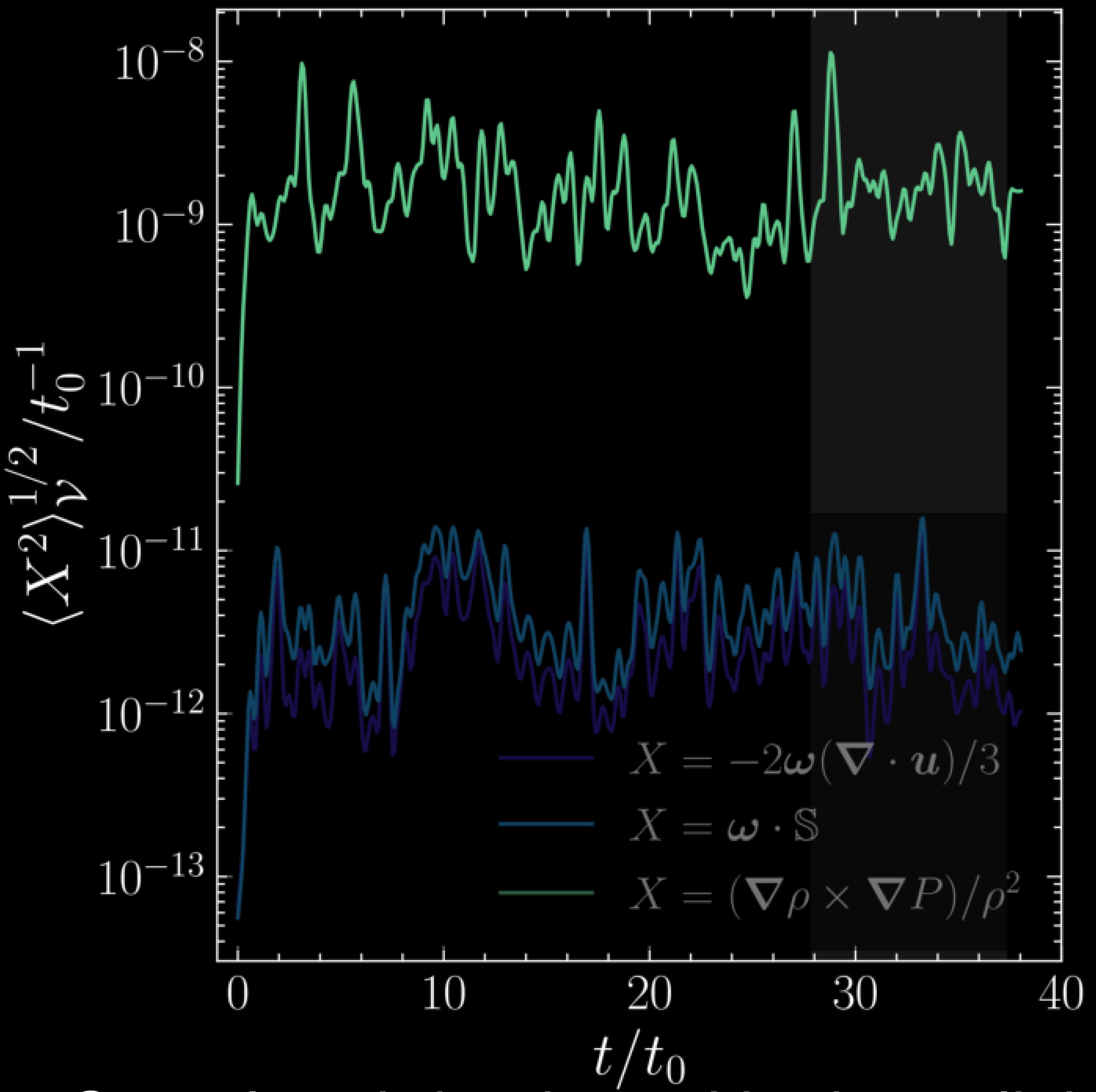

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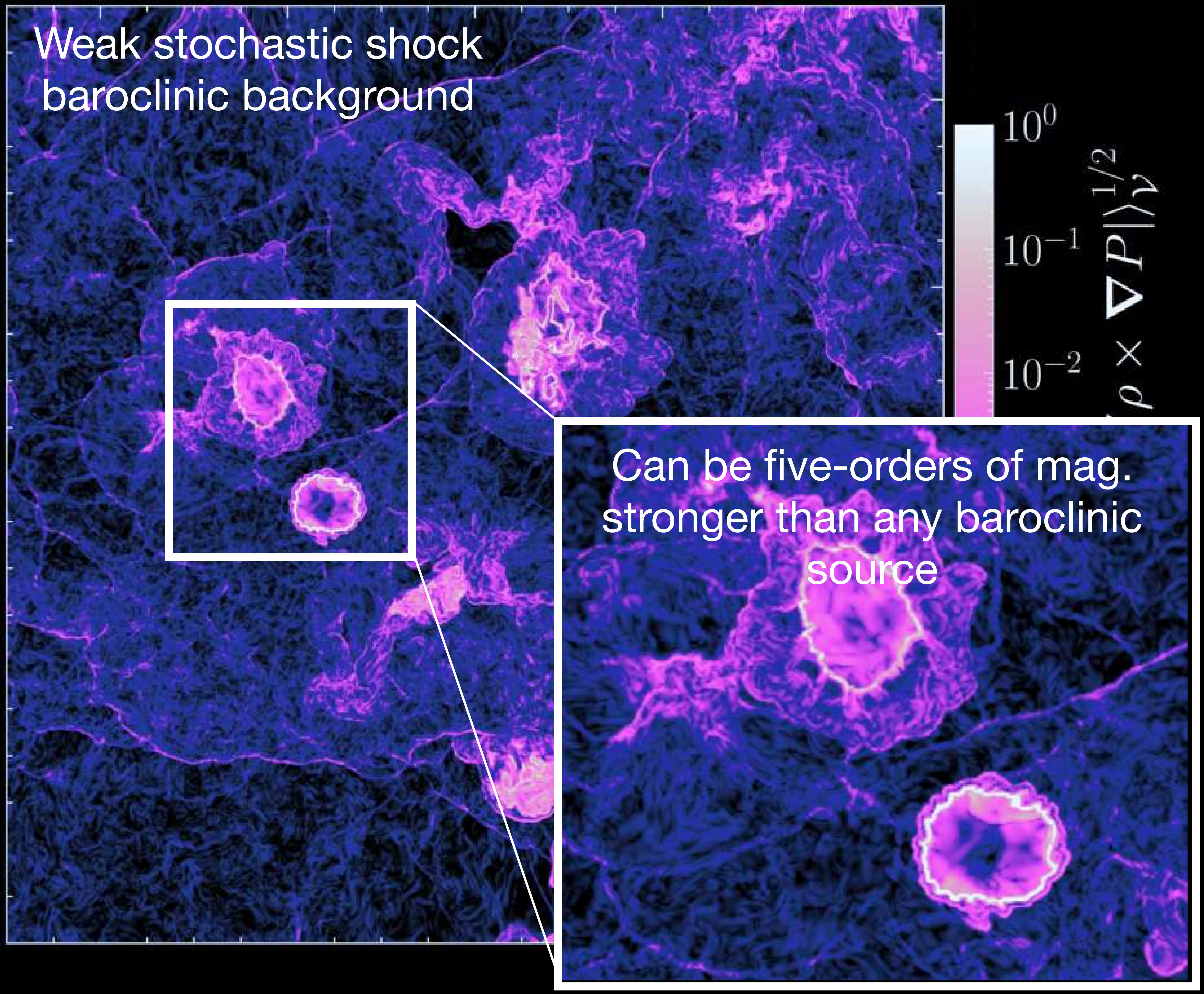

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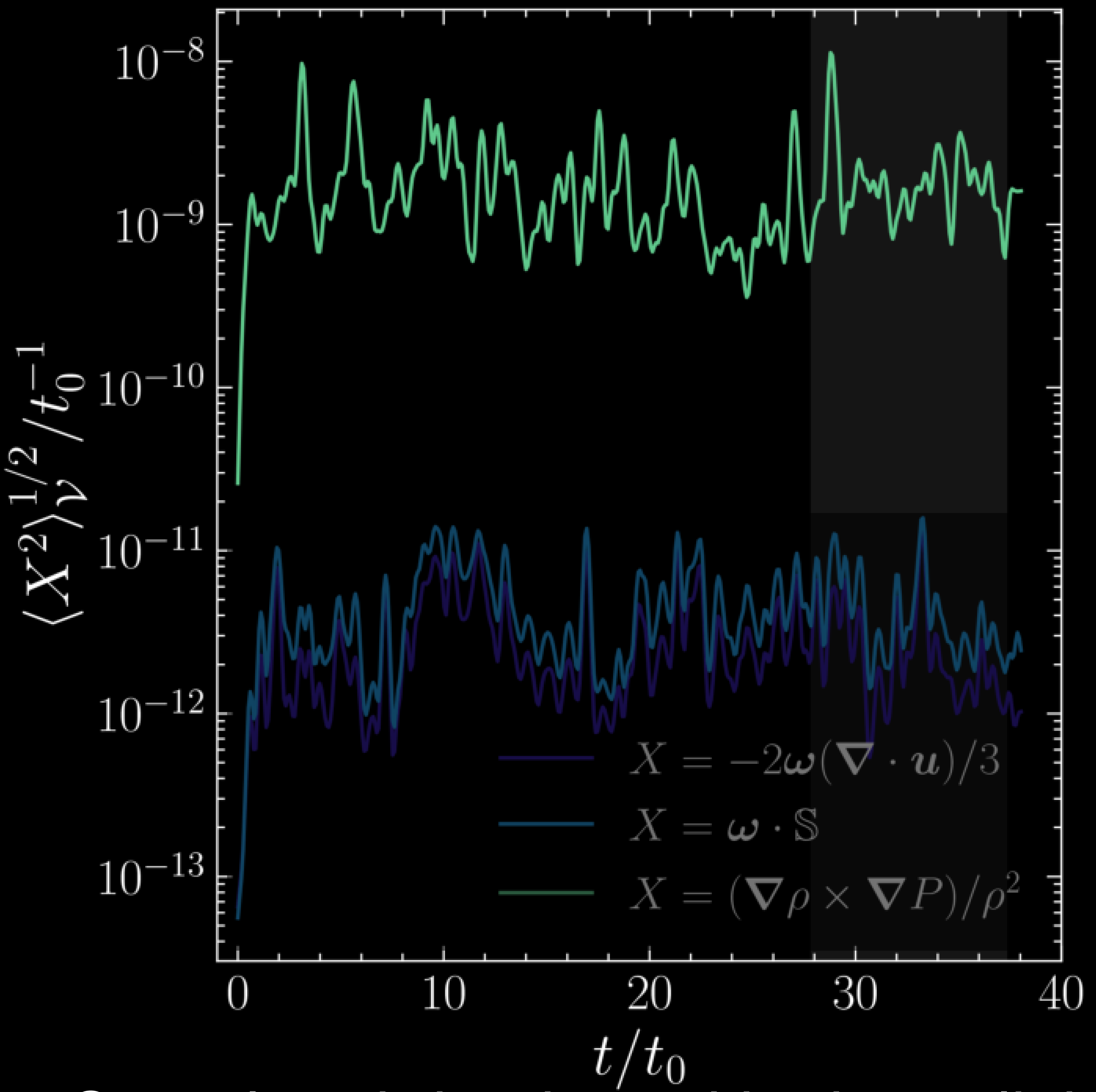

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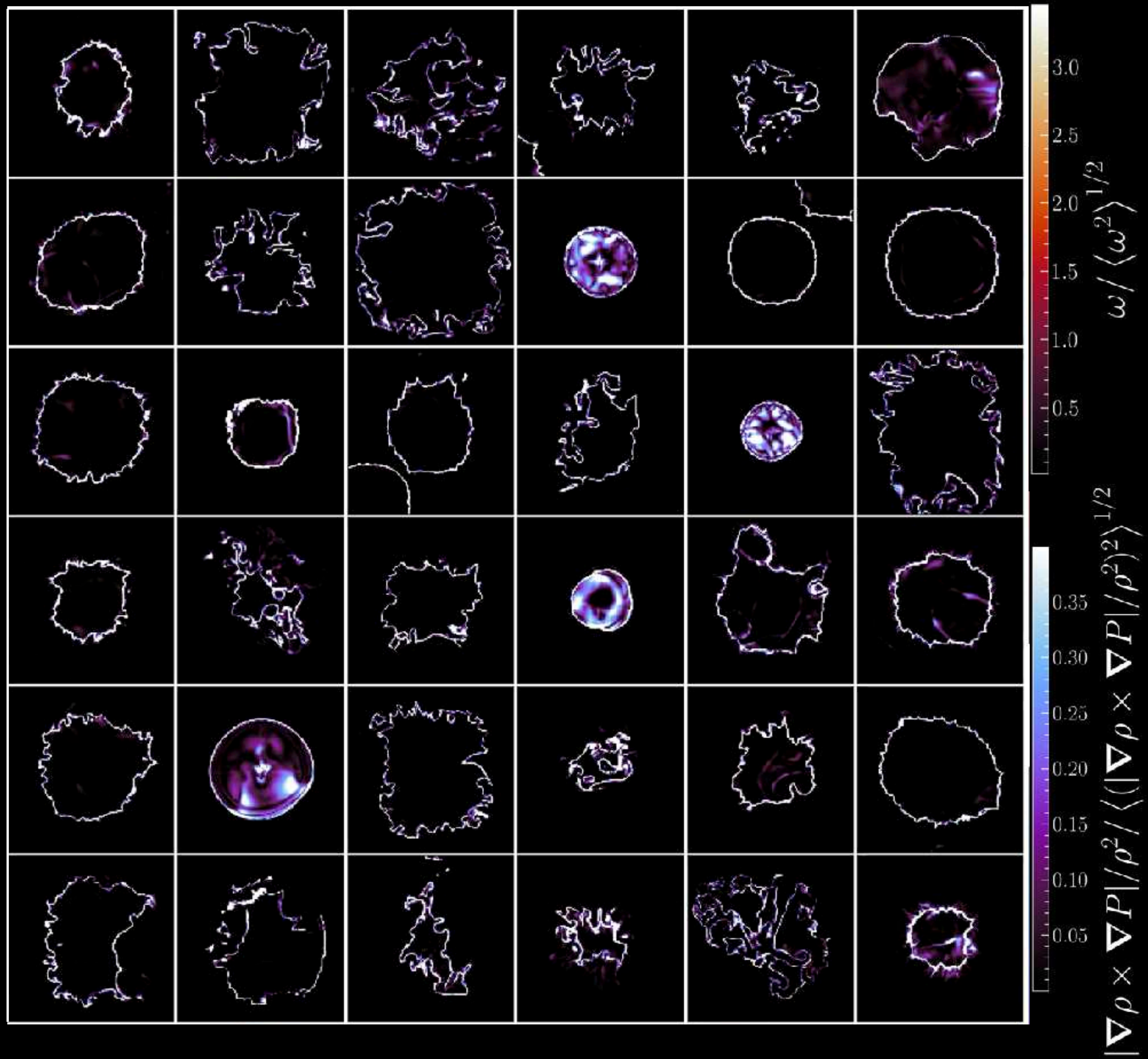
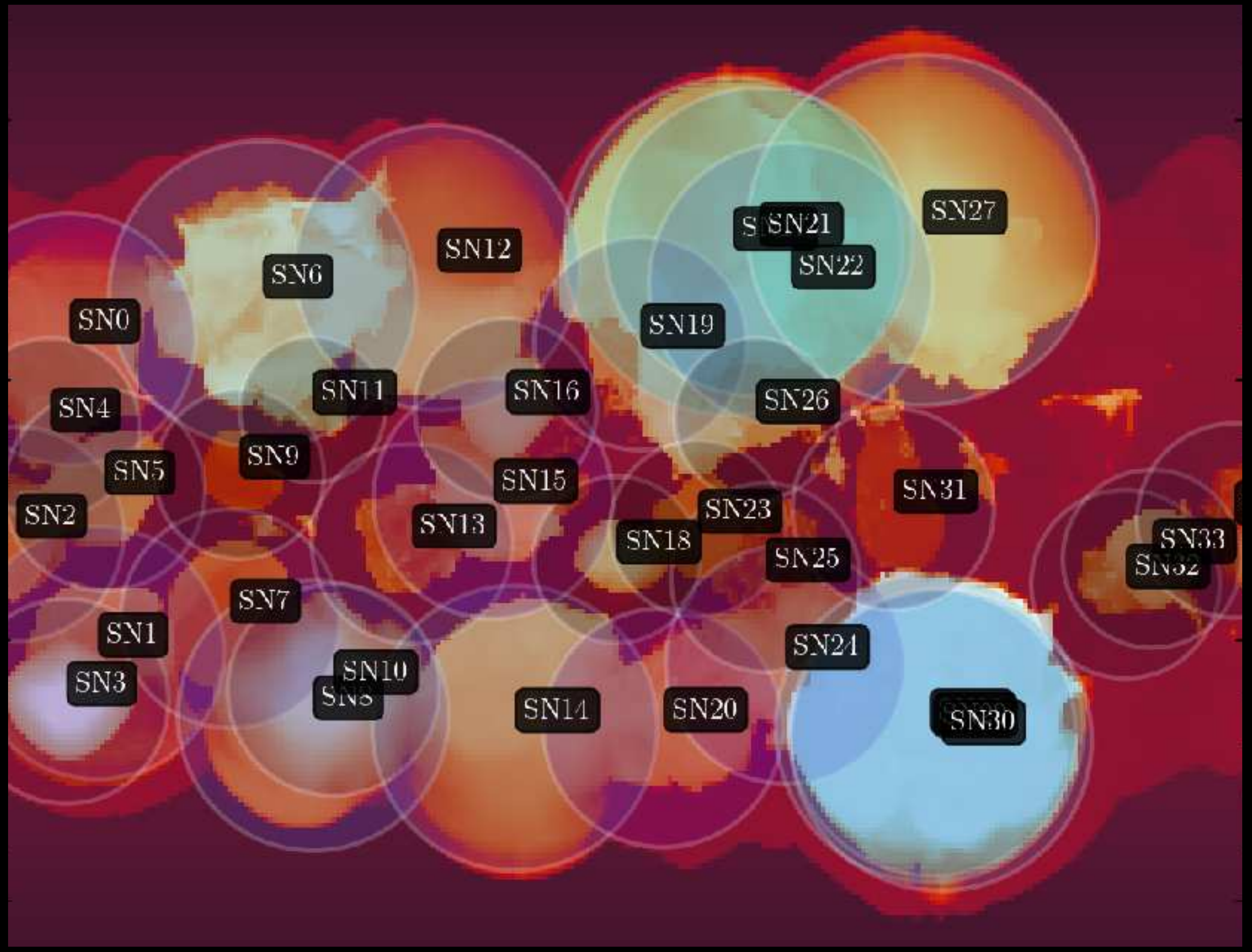
Where does the incompressible turbulence come from?

Beattie (2026; submitted ApJL) Supernovae drive large-scale incompressible turbulence from small-scale instabilities



Cluster all SNR in 3D to extract local statistics



Assuming isotropic, stationary, high-Re

$$\mathscr{P}_{\omega \mathrm{B}}(k) \approx -\frac{d\,\Pi_\omega(k)}{dk}$$

where

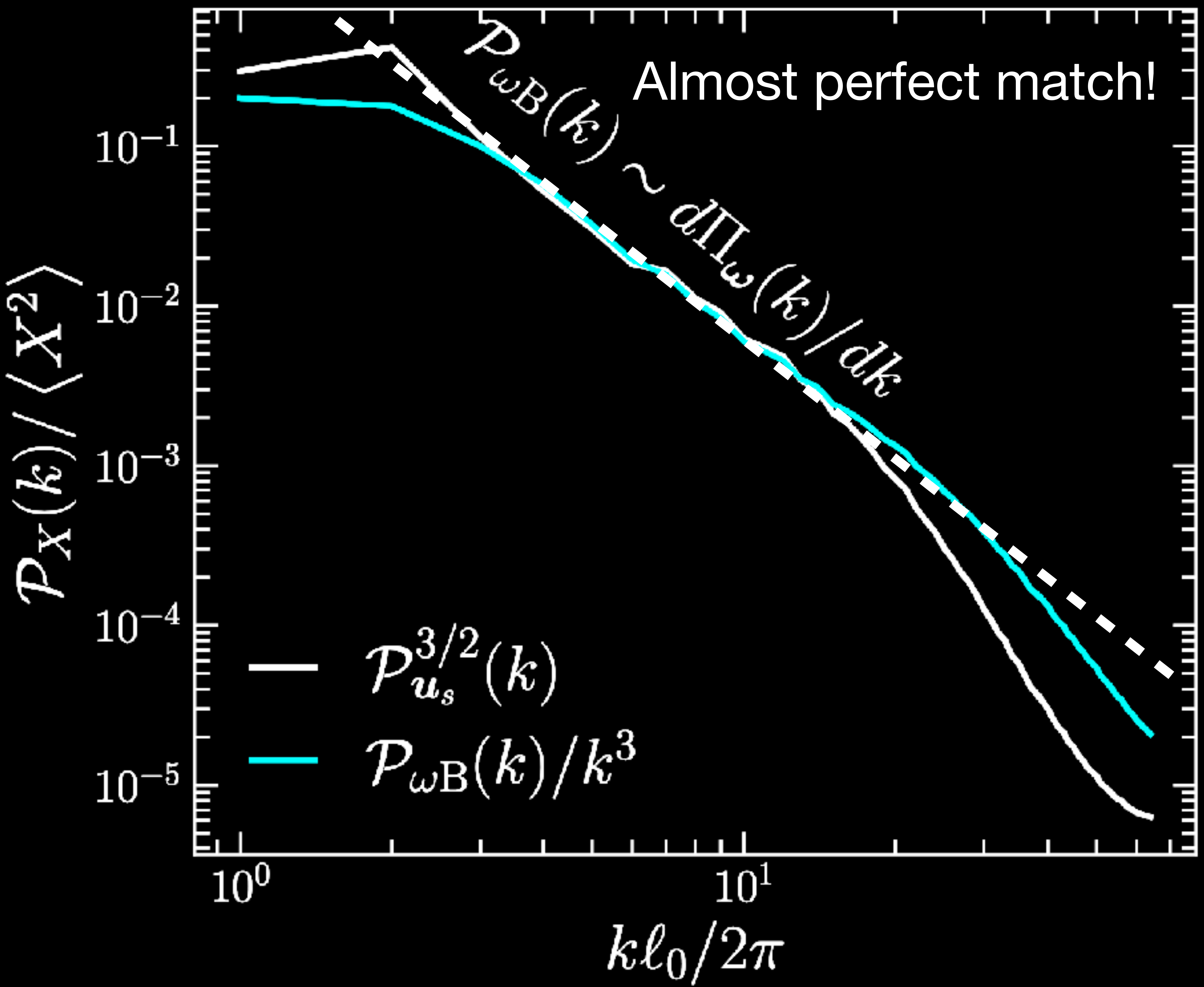
$$\frac{d\,\Pi_\omega(k)}{dk} \sim \frac{1}{t_{\mathrm{turb}}}\int d\Omega_k\, k^2\, \omega(k)\cdot\omega^\dagger(k)$$

$$\sim \frac{d\,\Pi_\omega(k)}{dk} \sim \frac{k^2 \mathscr{P}_{u_s}(k)}{t_{\mathrm{turb}}}$$

relation between incompressible mode spectrum and baroclinic source spectrum

$$\mathscr{P}_{\omega \mathrm{B}}(k) \sim k^3 \mathscr{P}_{u_s}^{3/2}(k)$$

Where does the incompressible turbulence come from?



$k\ell_{\mathrm{SNe}}/2\pi$

Beattie (2026; submitted ApJL) Supernovae drive large-scale incompressible turbulence from small-scale instabilities

Modified schematic from Lancaster+2021

Incompressible modes generated on SNR scale imprinted on all scales

(scales larger than gaseous scale height!!!)

Can we predict the spectrum?

Baroclinicity is born from thin shell instabilities (RT & overstability)

$$\gamma \sim \sqrt{k}$$

Linear RT
Vishniac (1983)

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Baroclinicity is born from thin shell instabilities (RT & overstability)

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Linear RT
Vishniac (1983)

$$t_{\rm nl} \sim \gamma^{-1} \sim k^{-1/2}$$

Motivated by reconnection-mediated turbulence models (e.g., Boldyrev+2017; Dong+2022)

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$$u_\ell^2 / t_{\rm nl} = \varepsilon = \text{const}\,.$$

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Linear RT
Vishniac (1983)

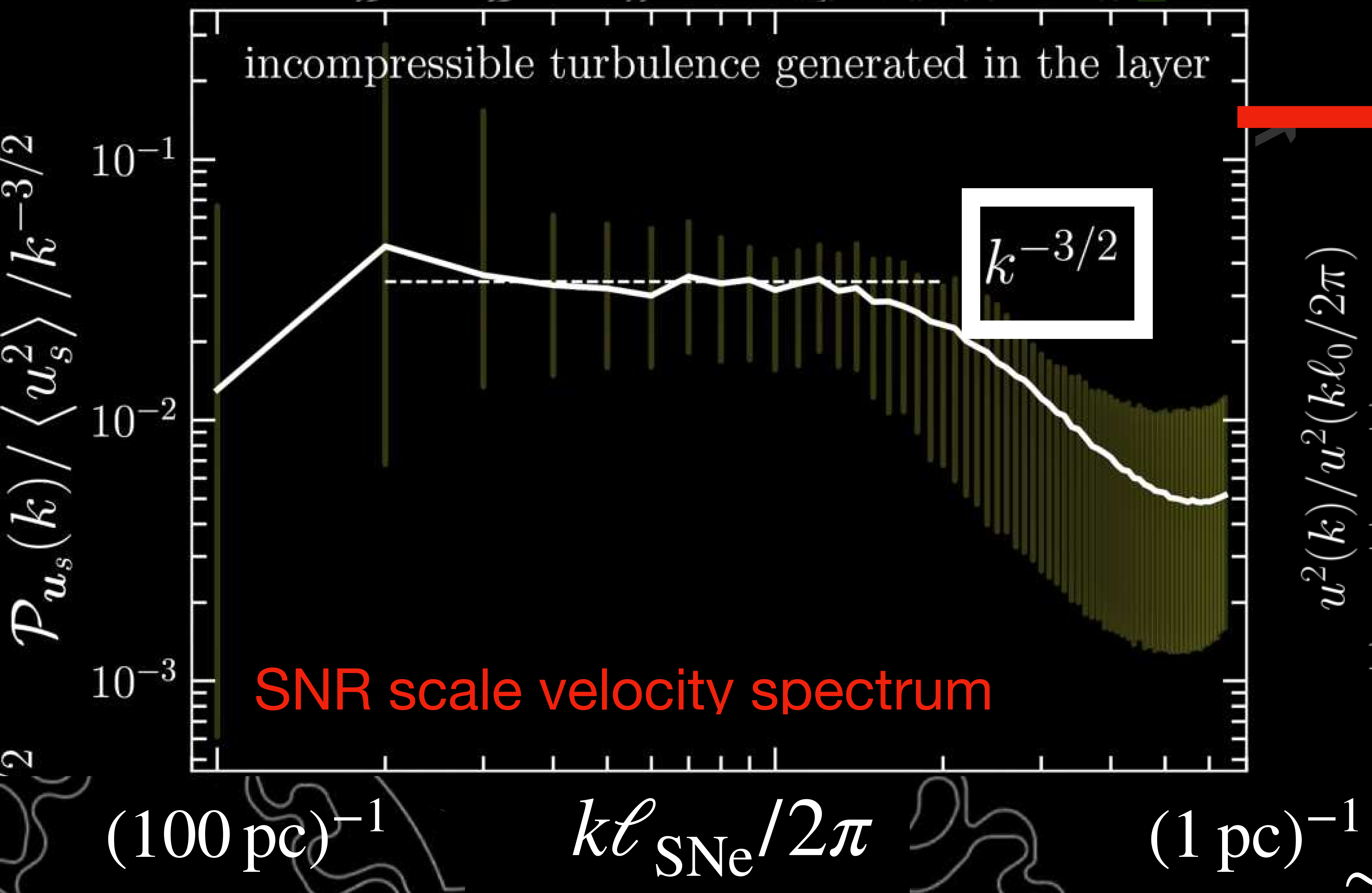
$$t_{\rm nl} \sim \gamma^{-1} \sim k^{-1/2}$$

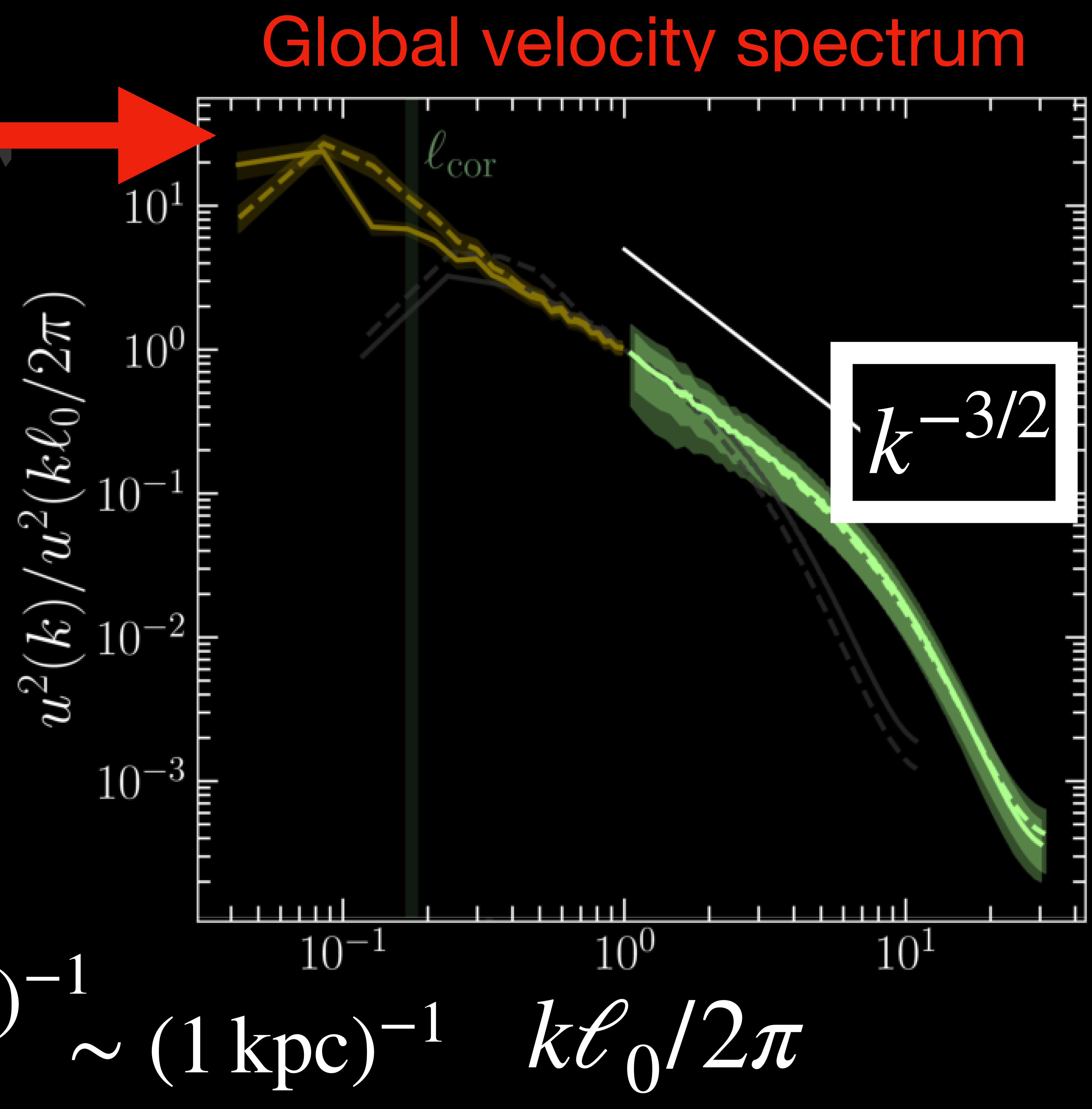
$$u_\ell^2 / t_{\rm nl} = \varepsilon = {\rm const}\,.$$

$$u_\ell \sim \ell^{1/4} \iff u^2(k) \sim k^{-3/2} dk$$

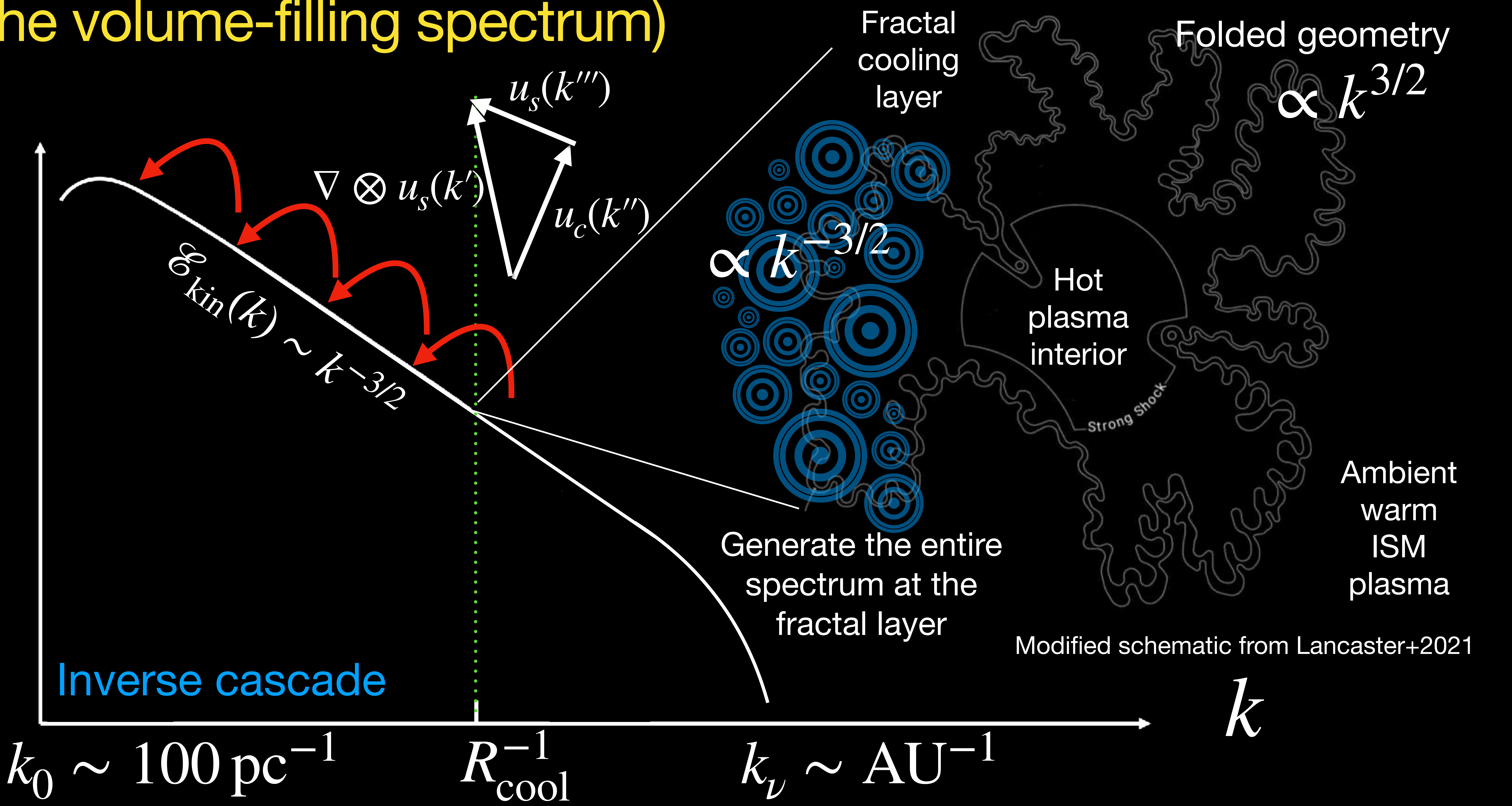
Modified schematic from Lancaster+2021

Shell instabilities drive and imprint themselves on the spectrum!





The SN-driven warm ionized medium spectrum (the volume-filling spectrum)



Summary for this seminar

1. Discuss K41 turbulence in a pedagogical manner.

2. Turbulence in our galaxy.

3. Supernova-driven turbulence — how to fit them into the cascade? Simulations.
   Fit into the small wavelengths, but energies turbulence past the gaseous scale-height.

4. Spectra, cascade directions.
   Non-Kolmogorov scaling in velocity, $k^{-3/2}$, and has flux components that are inverse cascade

5. Where does the incompressible turbulence come from — shell instabilities and spectra model.
   Strong baroclinicity in the unstable thin shell supply the vorticity, drive the turbulence, control the spectrum

Thanks, questions?

james.beattie@princeton.

@astro_magnetism