A large-scale, vertical field driven dynamo from the Kelvin-Helmholtz instability: implications for merging compact objects and other thoughts

Narayan Group Meeting, BHI, Harvard University
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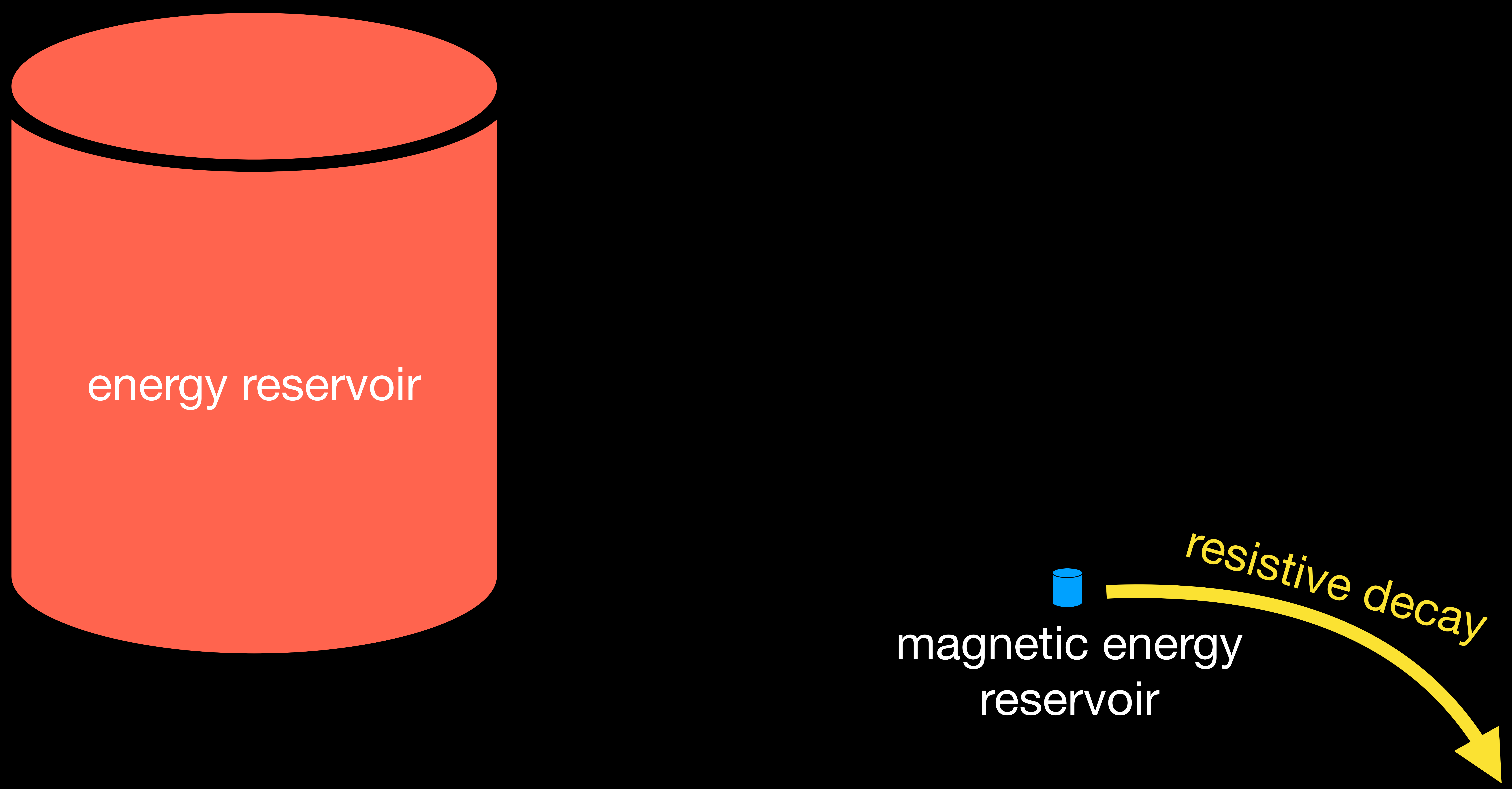
Collaborators for this work: Elias Most (Caltech), Neco Kriel (ANU), Amitava Bhattacharjee (PU), Bart Ripperda (CITA), Sasha Phillippov (UM)

What is a magnetic dynamo?
Starting with a seed magnetic field

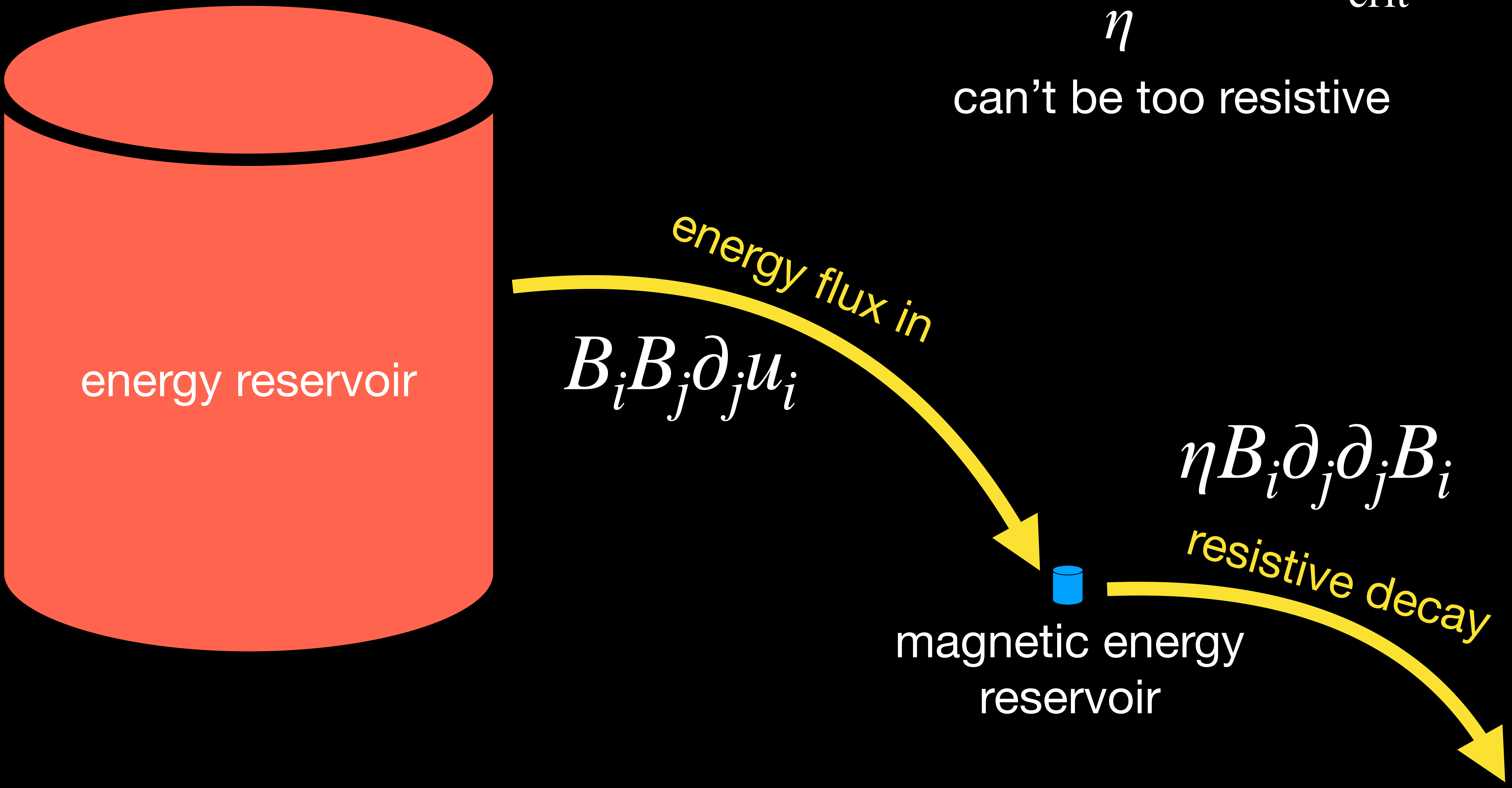


What is a magnetic dynamo?

Growth

$$\mathrm{Rm} \sim \frac{U_0 L}{\eta} > \mathrm{Rm}_{\mathrm{crit}}$$

can't be too resistive



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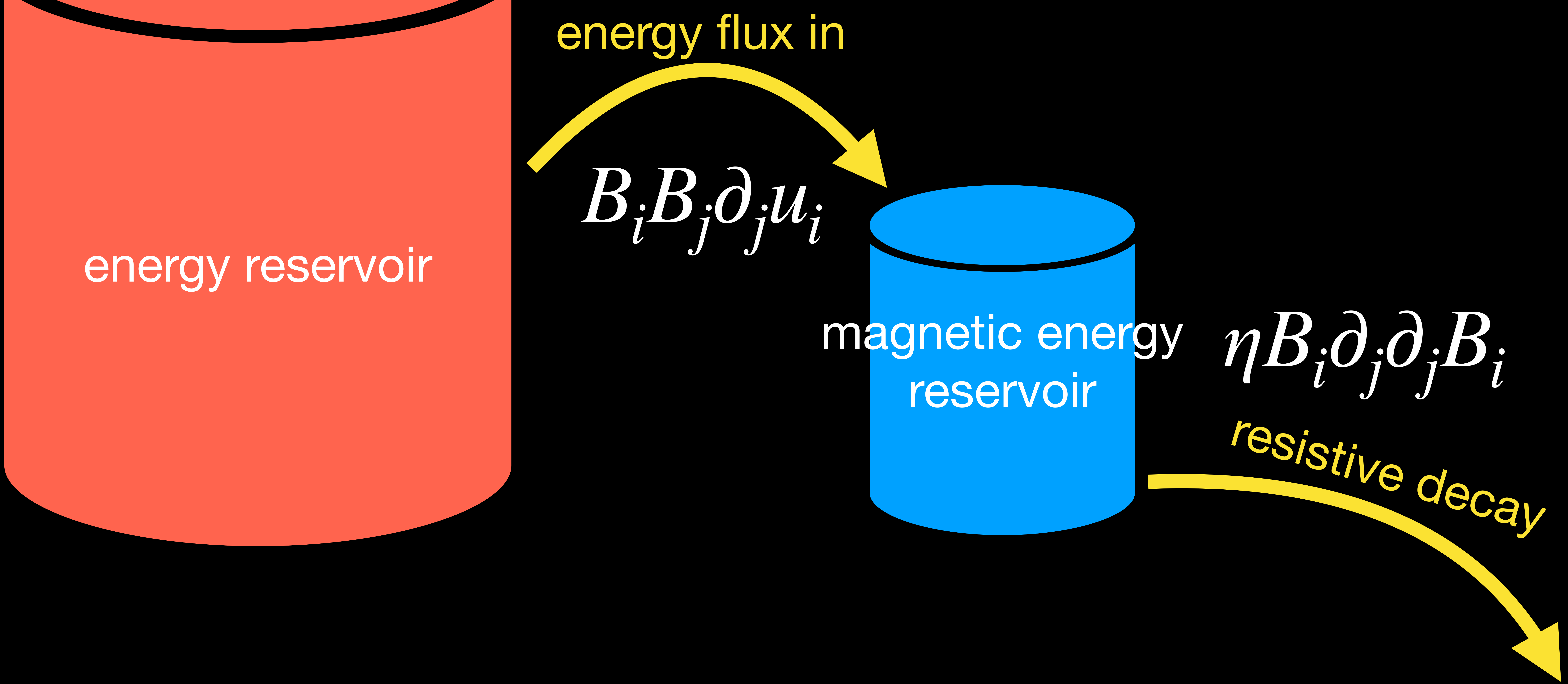
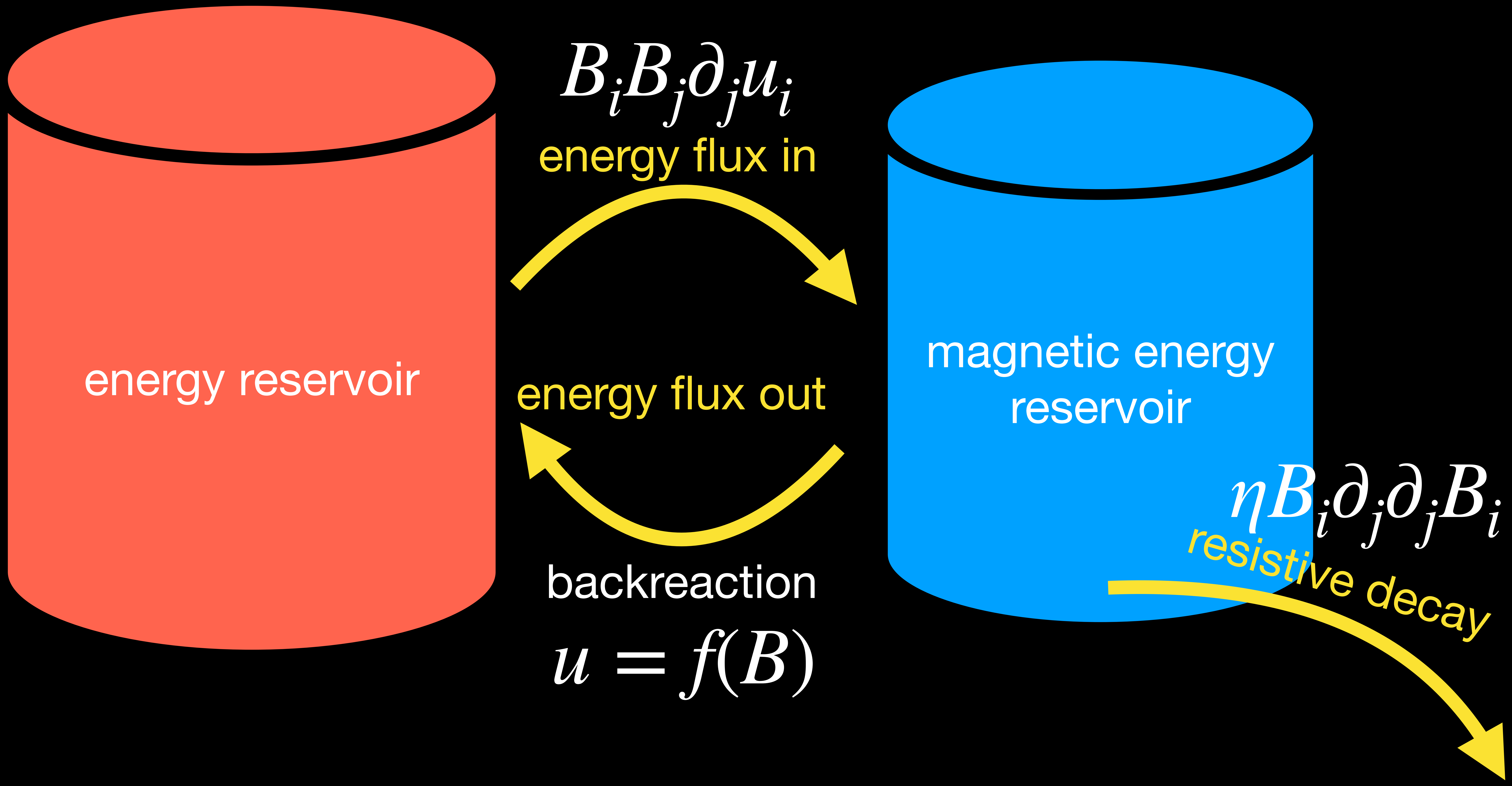


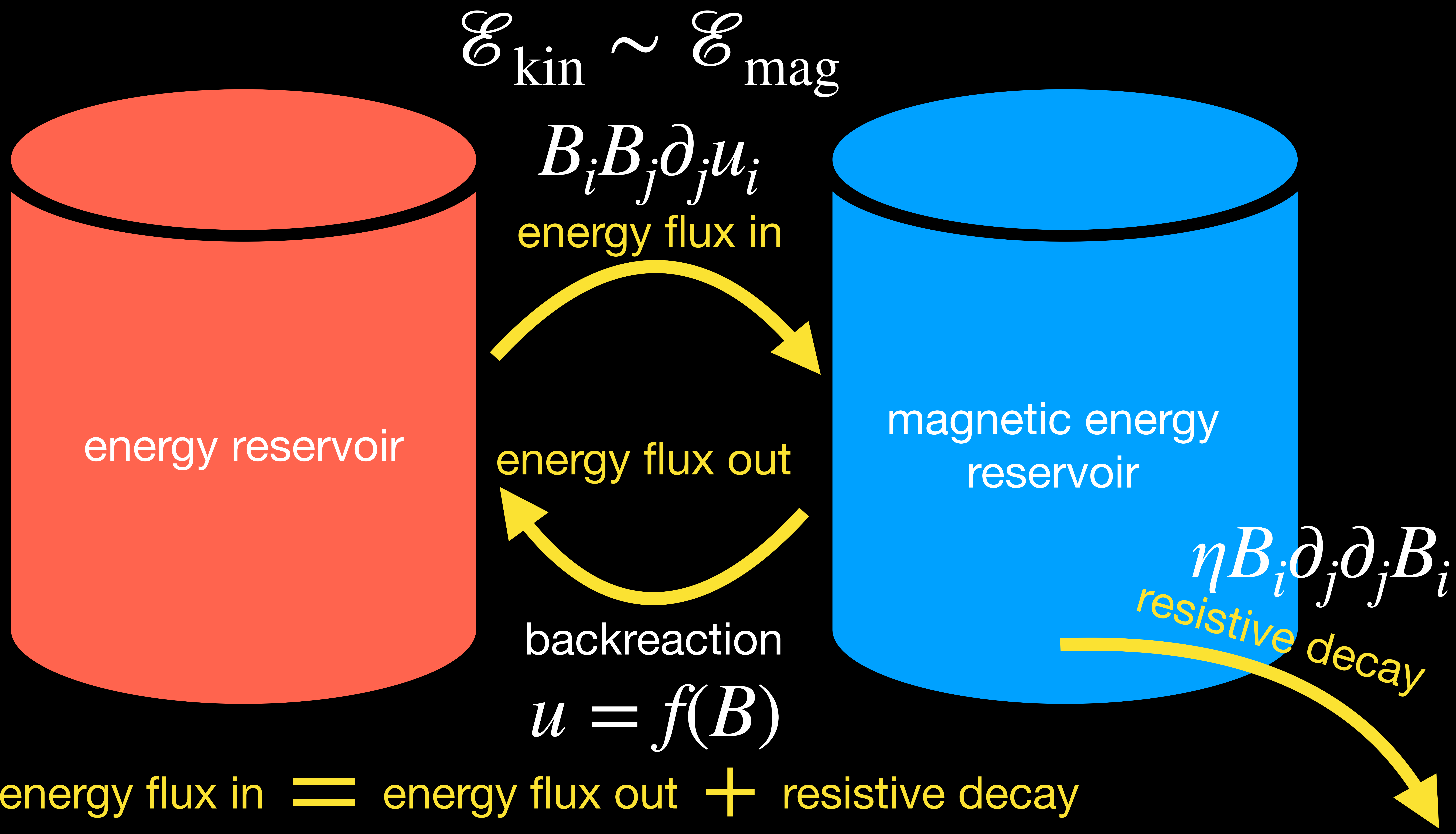
What is a magnetic dynamo?

Nonlinearities and backreaction



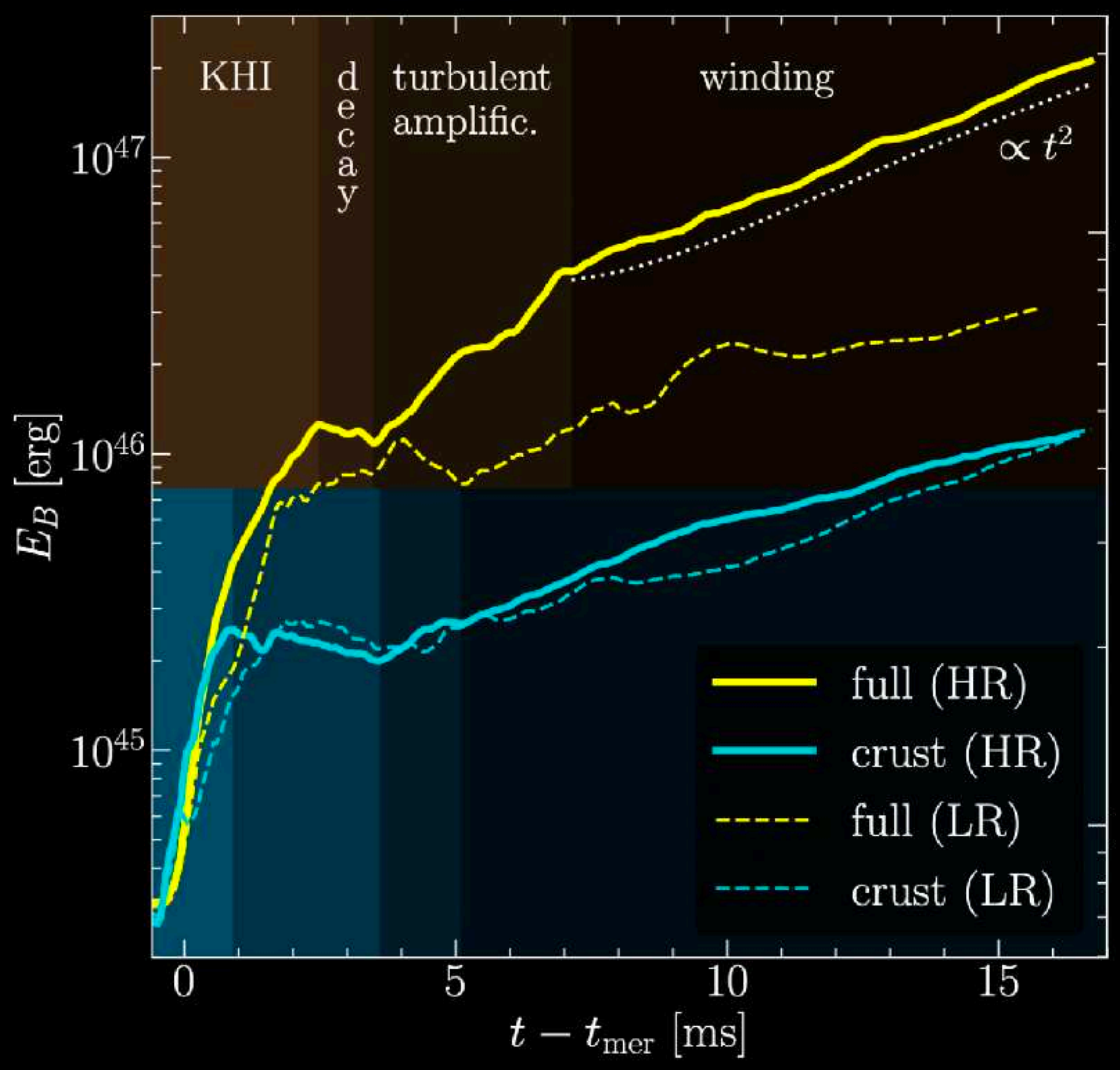
What is a magnetic dynamo?

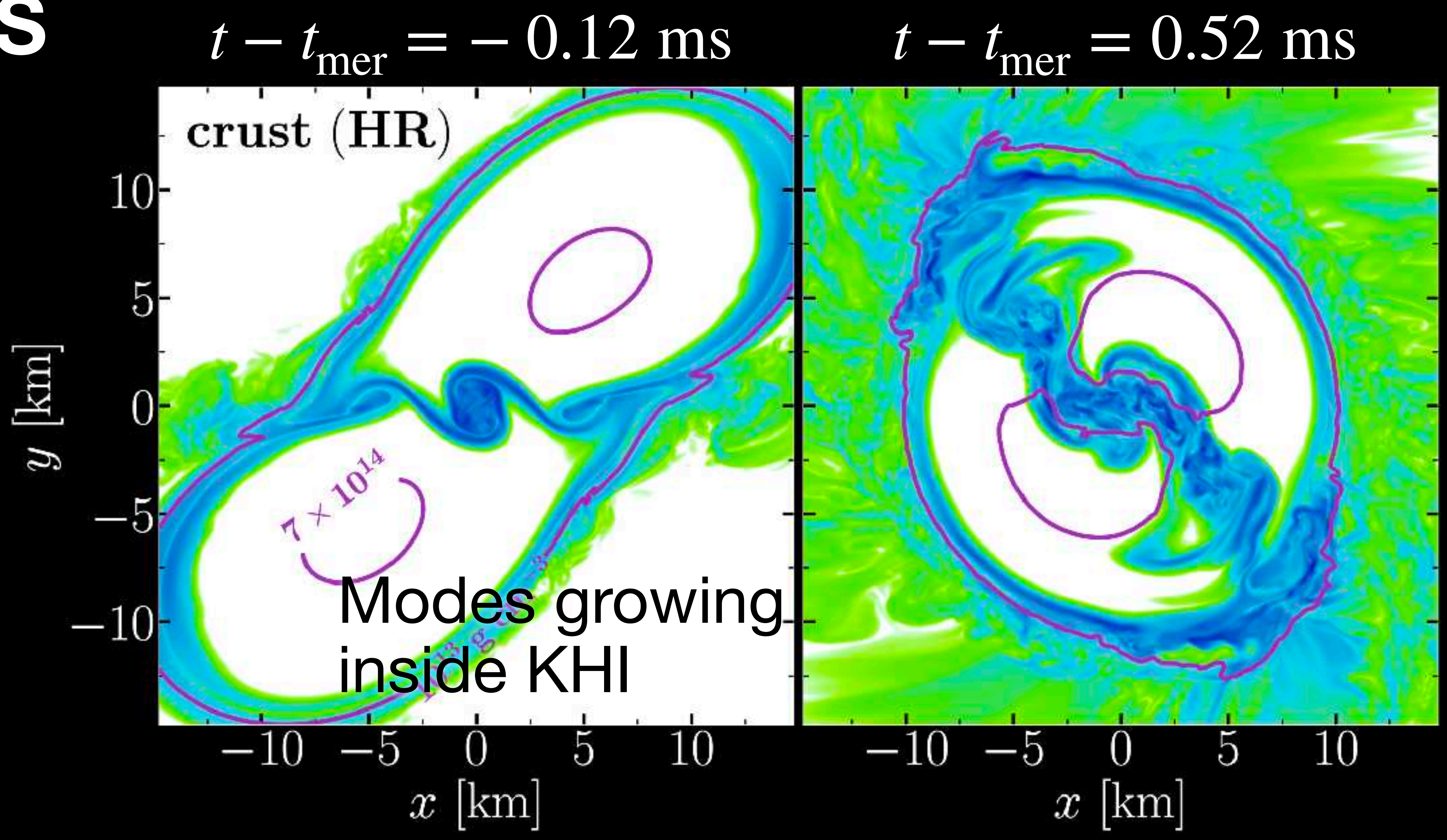
Saturation



Examples of dynamos

KHI instabilities in merging NS





exponential growth → pause

exponential growth ← → LSD

Chabanov+ (2023) *ILES FIL*

Examples of dynamos

Milky Way-type galaxies in cosmological sims

exponential growth

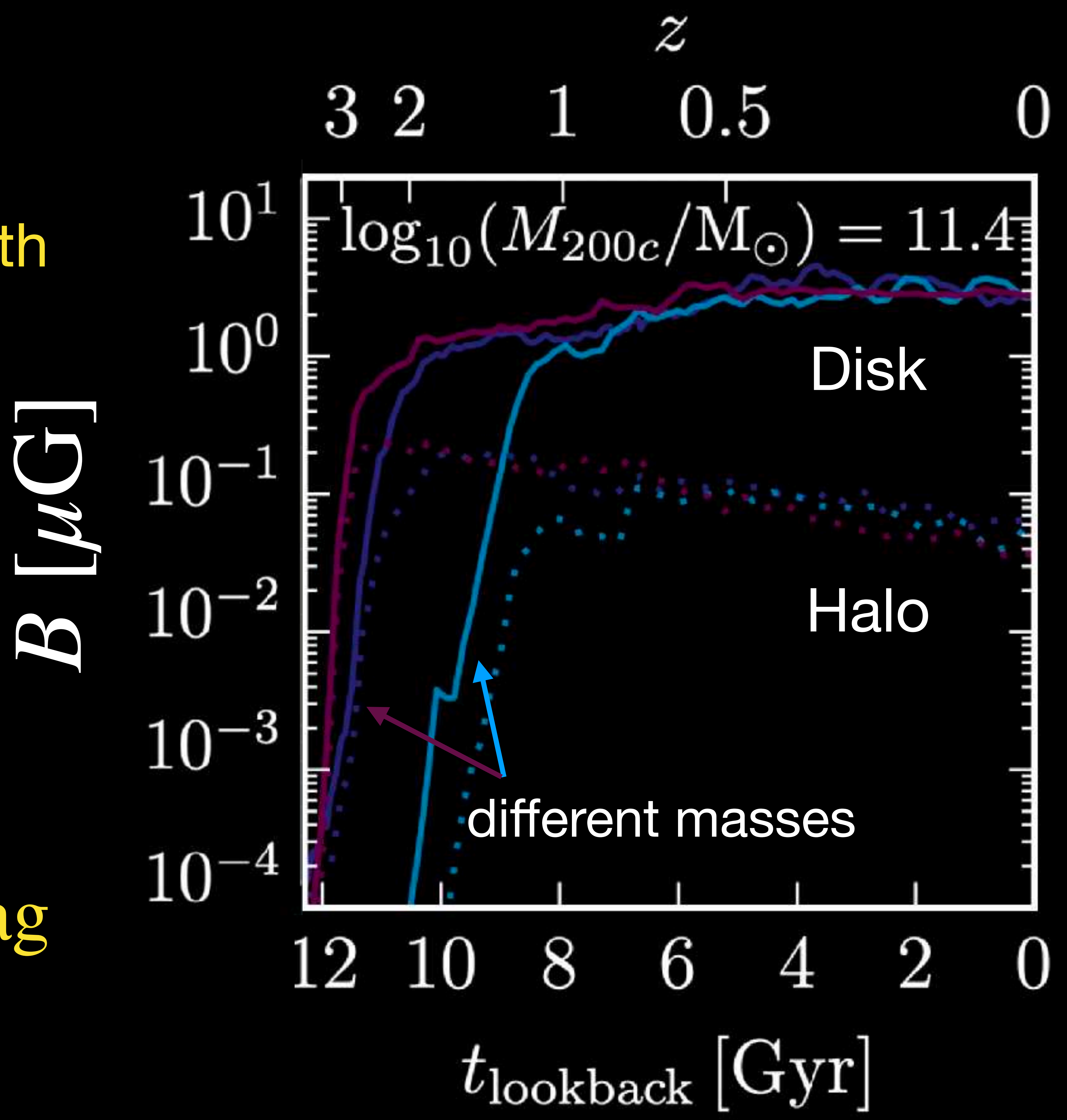
↓

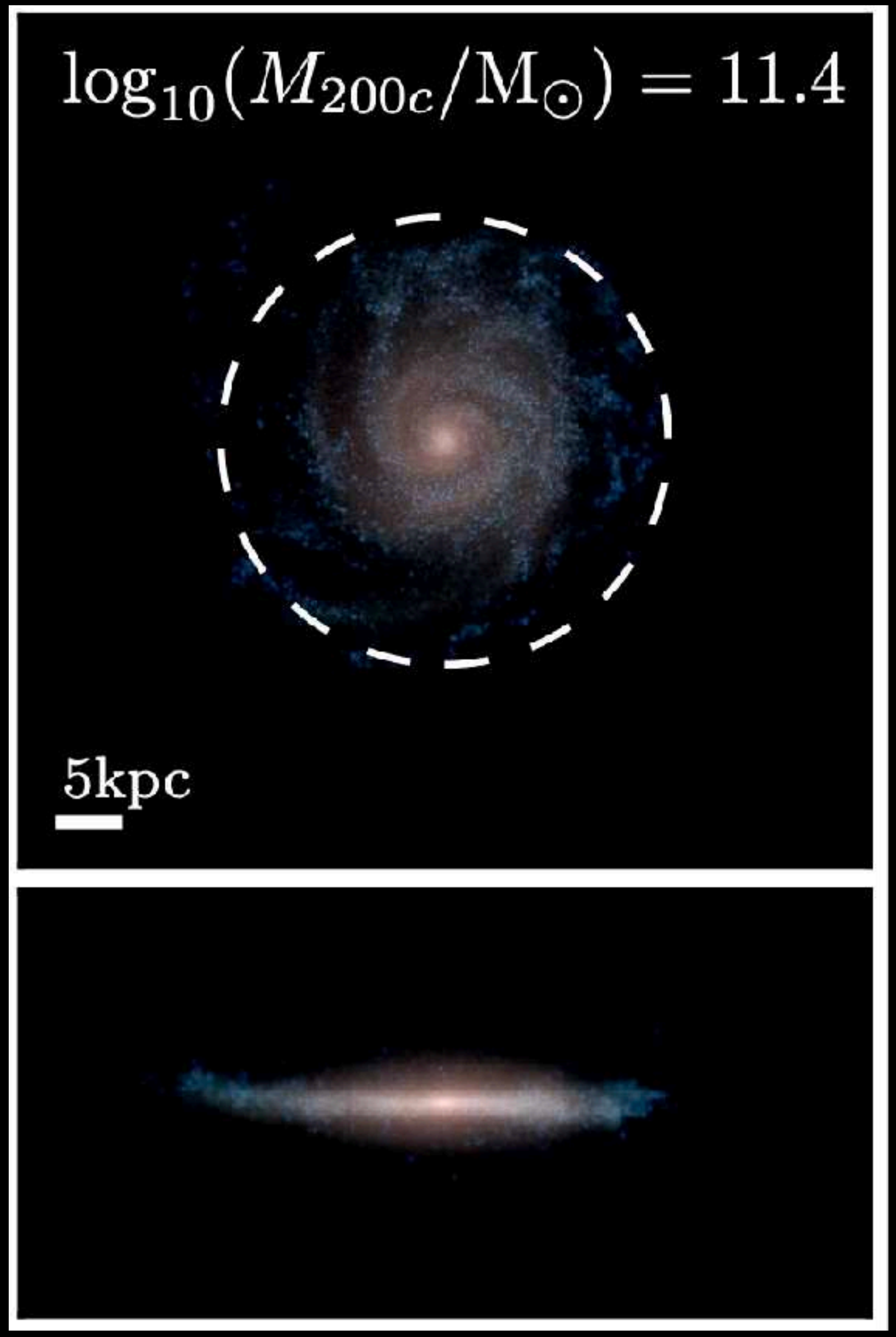
linear growth

↓

saturation

$\mathscr{E}_{\mathrm{kin}} \sim \mathscr{E}_{\mathrm{mag}}$



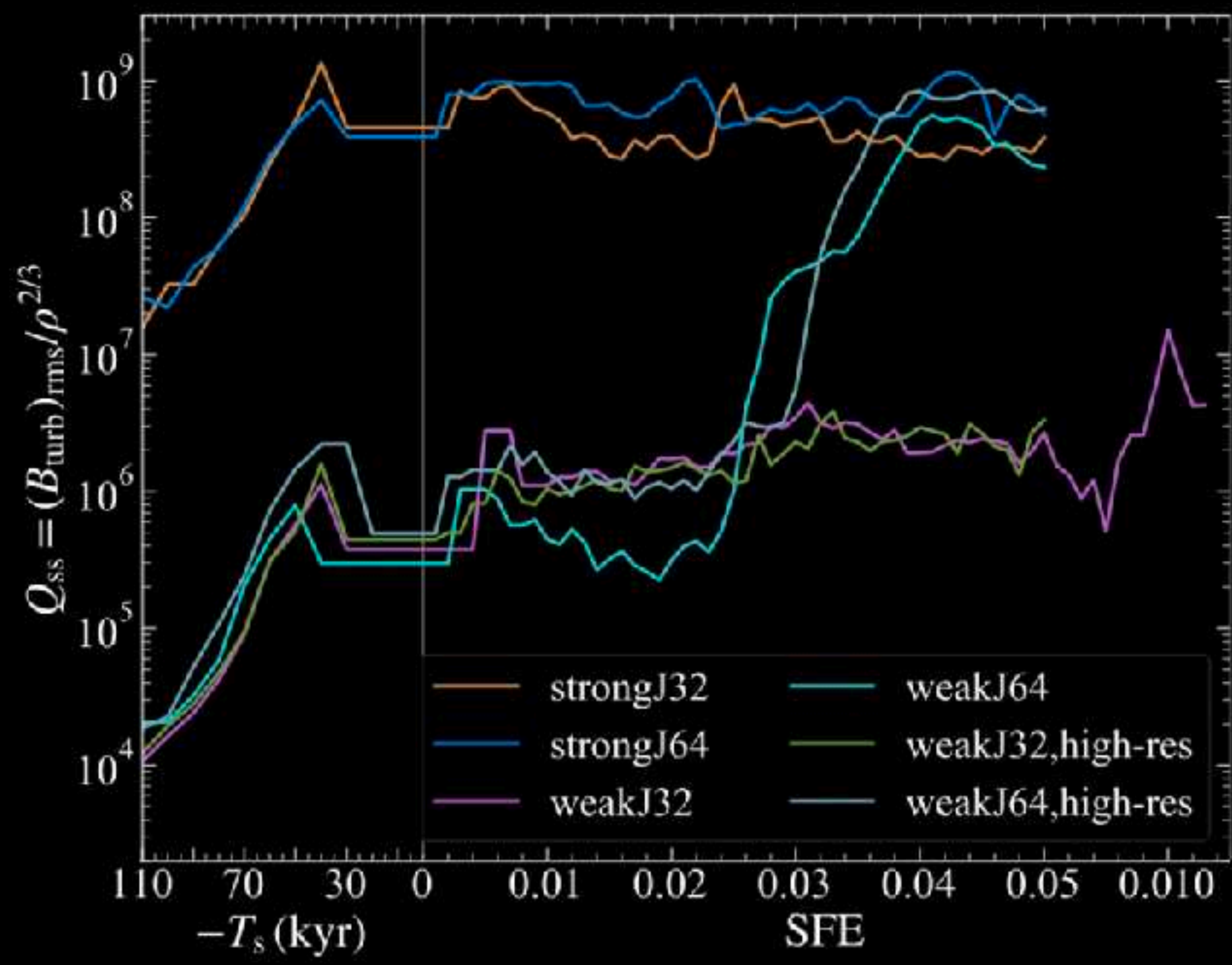


Pakmor+ (2024) *AREPO*

Examples of dynamos

There are many, across all scales (all MHD)

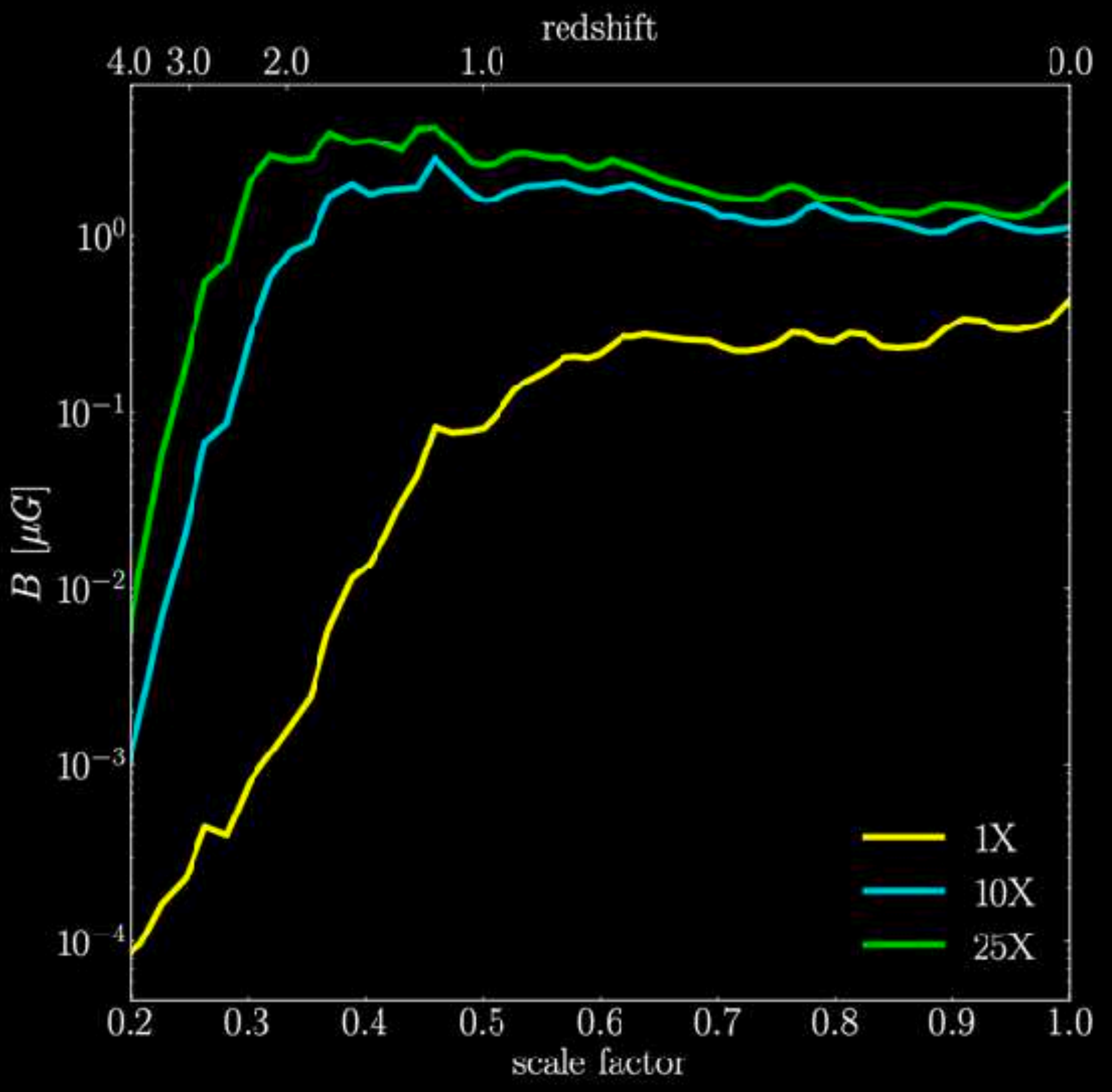
Sharda+2021



Molecular clouds in first generation stars

FLASH

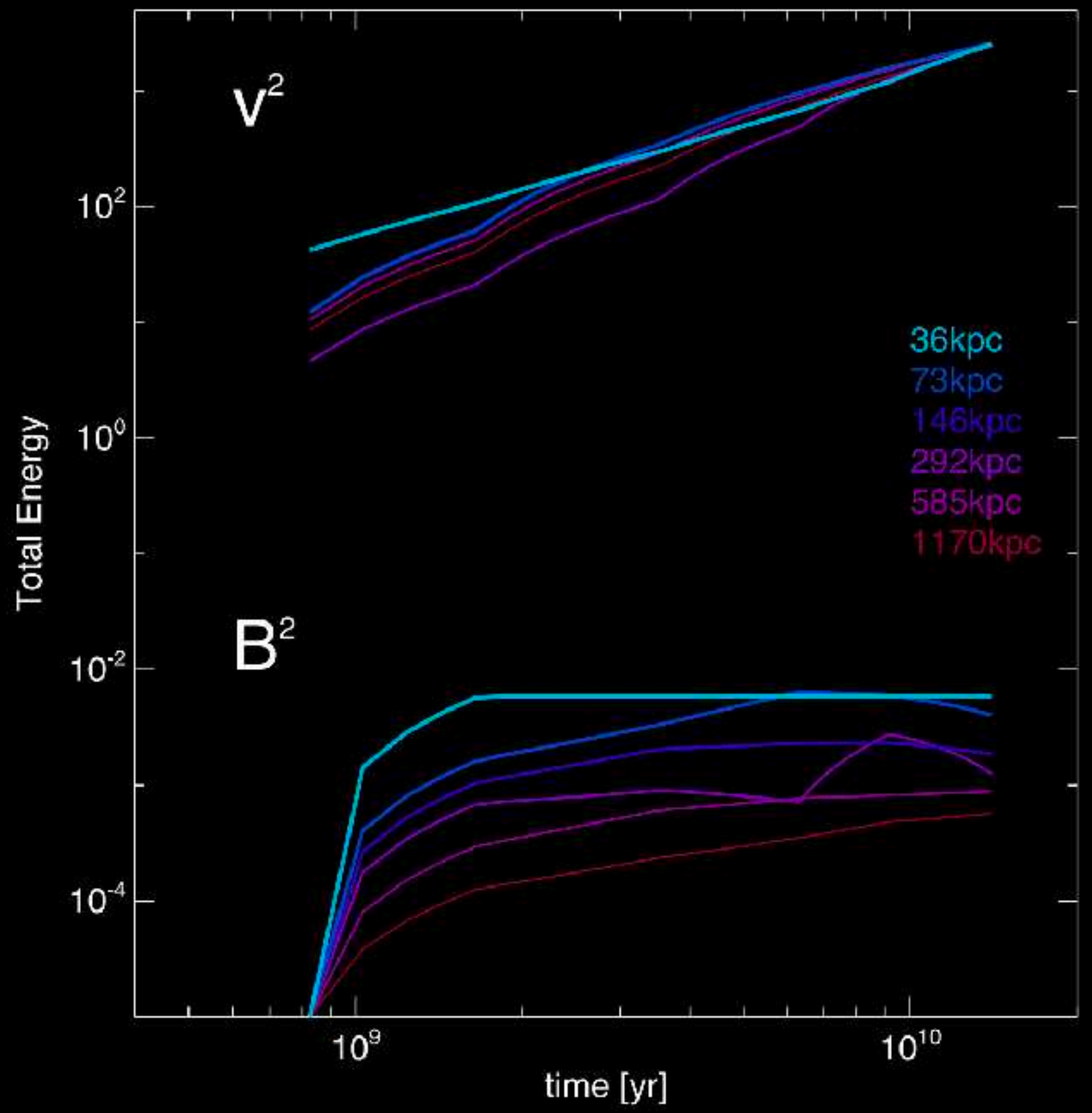
Steinwandel+2021



Intracluster medium

RAMSES

Vazza+2014

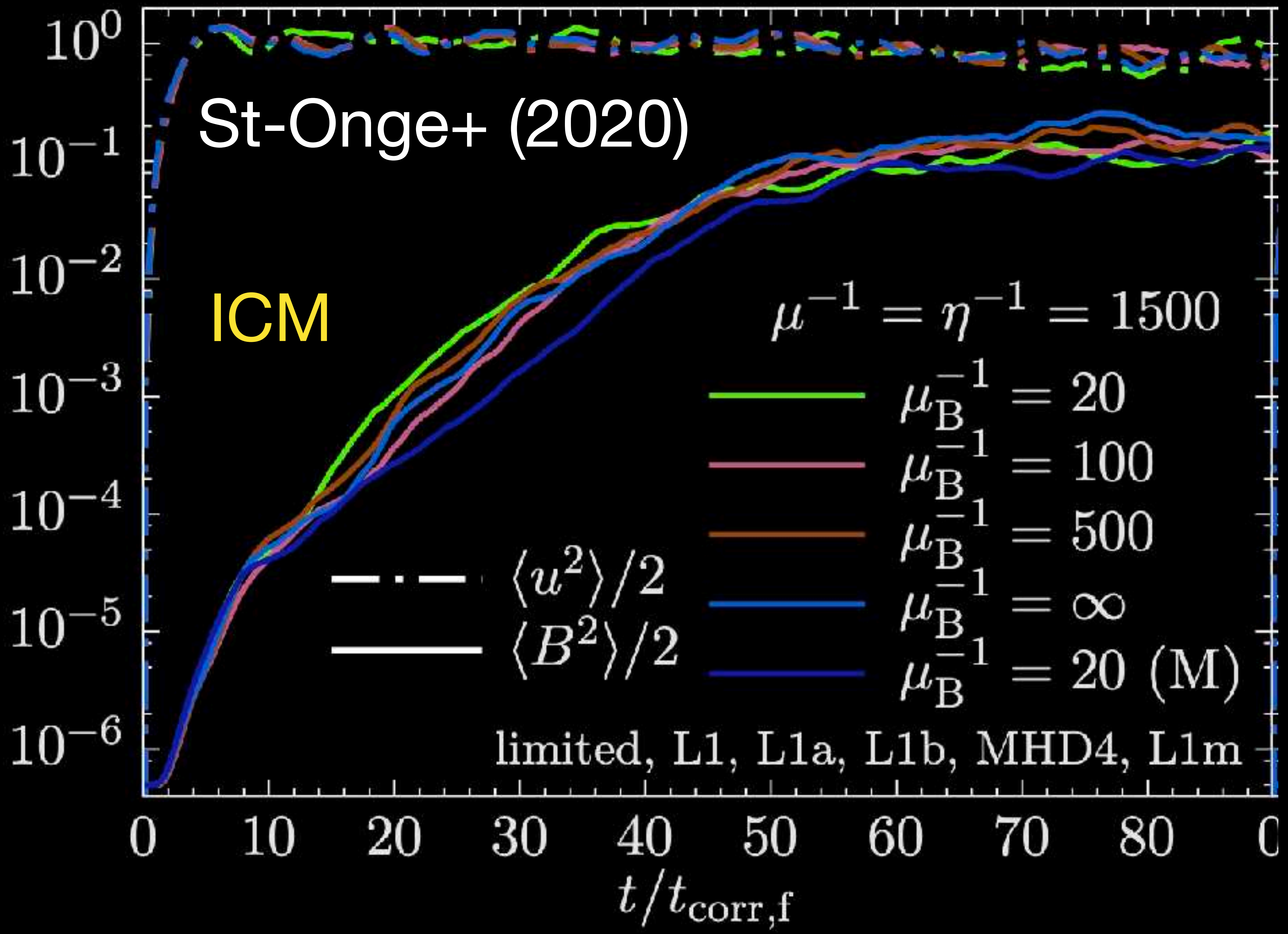


Cosmic filaments

ENZO

Examples of dynamos
in different Knudsen regimes

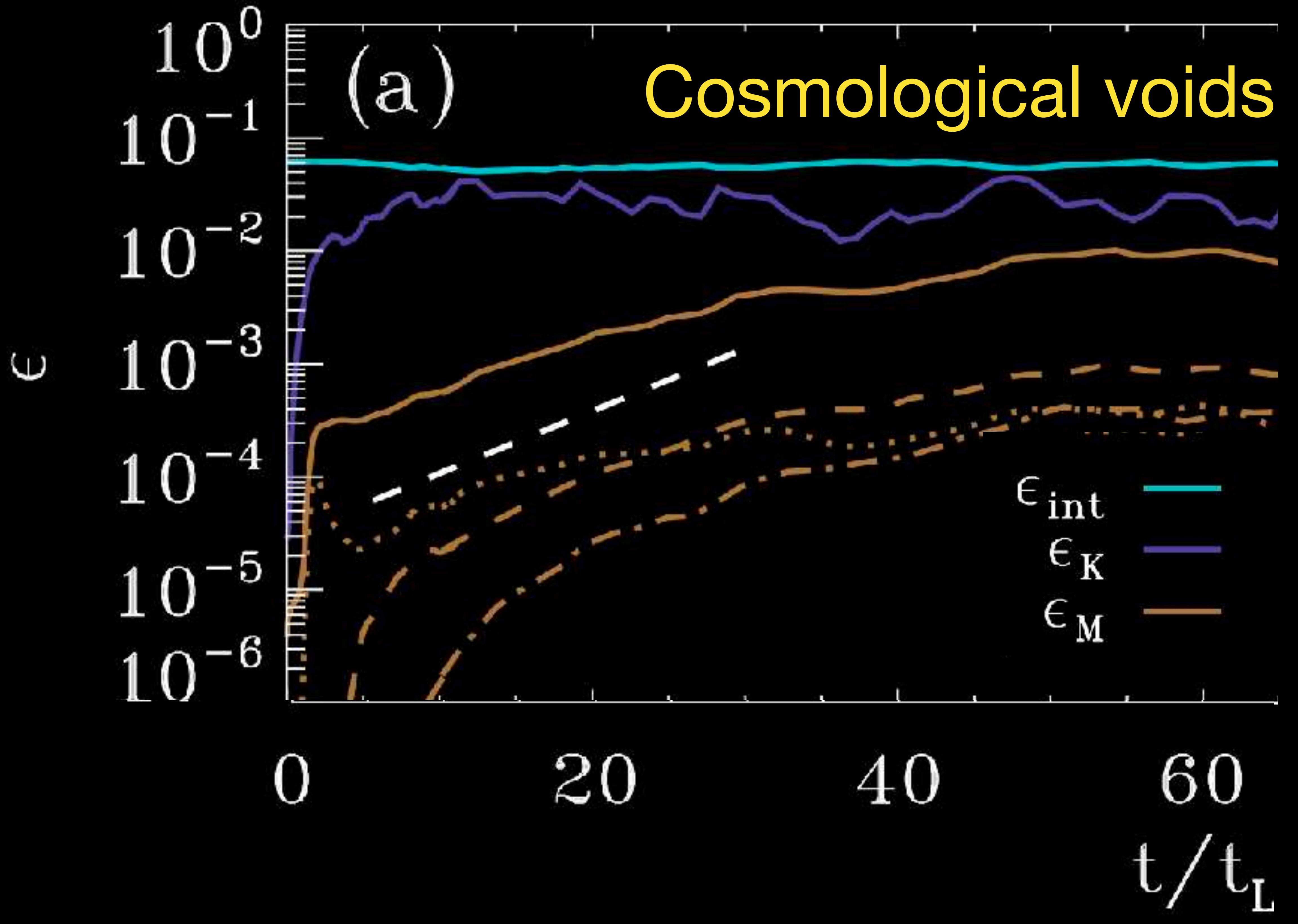
Weakly collisional Braginksii MHD



(added anisotropic viscous Braginskii stress term into MHD)

$\nabla \cdot (\hat{\mathbf{b}} \otimes \hat{\mathbf{b}}(\hat{\mathbf{b}} \otimes \hat{\mathbf{b}} : \nabla \mathbf{v}))$ *Snoopy*

Collisionless plasma magnetogenesis coupled to dynamo



Sironi+2023

(PIC: pair plasma)

TRISTAN—MP

Induction equation is all a dynamo theorists needs… kind of

$$\partial_t \mathbf{B} - \eta \nabla^2 \mathbf{B} = \nabla \times (\mathbf{u} \times \mathbf{B})$$

As long as $\mathbf{u}$ is not a function of $\mathbf{B}$, induction is linear, and the dynamo is kinematic

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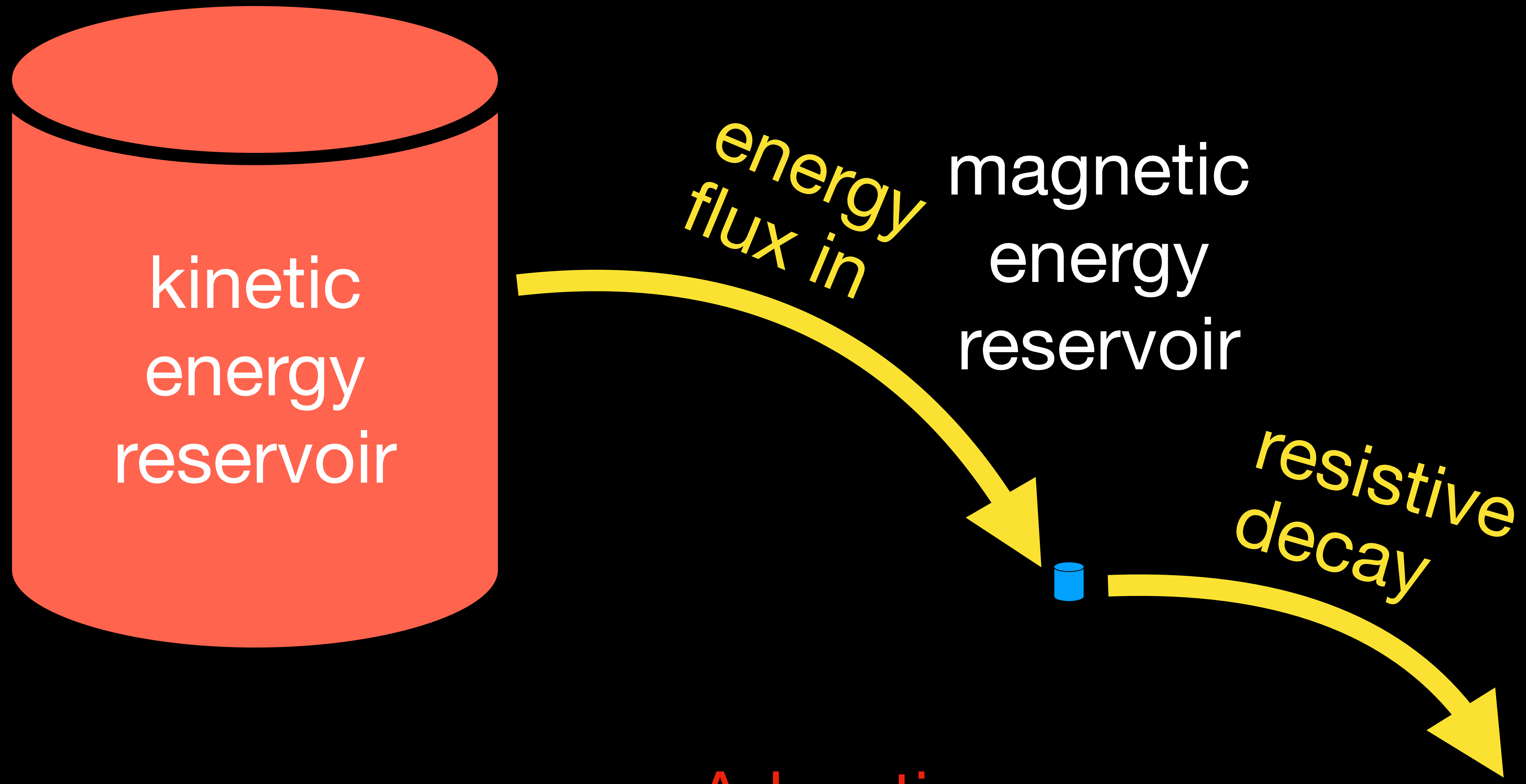
$$\frac{d\langle B^2 \rangle}{dt} = \langle B^2 (\hat{\mathbf{B}} \otimes \hat{\mathbf{B}} : \nabla \otimes \mathbf{u}) \rangle$$

Dynamo theory, in this context, is really a theory for the eigen values of $\partial_i u_j$

Technical note…

Again more quantitative: What is a magnetic dynamo?

Flux terms



$$\mathrm{Rm} = \frac{|\nabla \cdot \mathbb{F}_{\mathbf{b}}|}{|\mathbb{D}_{\eta}(\mathbf{b})|} > \mathrm{Rm}_{\mathrm{crit}}$$

can't be too resistive

$$\mathbf{u} \otimes \mathbf{u} : \nabla \otimes \mathbf{u} = u_j u_i \partial_i u_j$$

Advection

compression

$$\mathbf{b} \cdot \nabla \cdot \mathbb{F}_{\mathbf{b}} = \mathbf{b} \otimes \mathbf{u} : \nabla \otimes \mathbf{b} + \mathbf{b} \otimes \mathbf{b} : \nabla \otimes \mathbf{u} - \frac{1}{2}\mathbf{b} \otimes \mathbf{b} : (\nabla \cdot \mathbf{u})\mathbb{I}$$

coupling to velocity gradients

Again more quantitative: What is a magnetic dynamo?

Gradient coupling

$$\mathbf{b}\cdot\nabla\cdot\mathbb{F}_{\mathbf{b}} = \overbrace{\mathbf{b}\otimes\mathbf{u}:\nabla\otimes\mathbf{b}}^{\text{advection}} - \underbrace{\mathbf{b}\otimes\mathbf{b}:\nabla\otimes\mathbf{u}}_{\text{coupling to velocity gradients}} + \frac{1}{2}\overbrace{\mathbf{b}\otimes\mathbf{b}:(\nabla\cdot\mathbf{u})\mathbb{I}}^{\text{compression}}$$

$$\nabla\otimes\mathbf{u} = \mathbb{A} + \mathbb{S} + \mathbb{B}$$

$$\mathbb{S} = \frac{1}{2}\left(\partial_i u_j + \partial_j u_i\right) - \frac{1}{3}\delta_{ij}\partial_k u_k$$

volume preserving

$$\mathbb{A}_{ij} = \frac{1}{2}\left(\partial_i u_j - \partial_j u_i\right) = -\frac{1}{2}\epsilon_{ijk}\omega_k$$

volume preserving

$$\mathbb{B}_{ij} = \frac{1}{3}\delta_{ij}\partial_k u_k$$

volume changing

Again more quantitative: What is a magnetic dynamo?

Gradient tensor decomp.

$$\mathbf{b}\cdot\nabla\cdot\mathbb{F}_{\mathbf{b}} = \overbrace{\mathbf{b}\otimes\mathbf{u}:\nabla\otimes\mathbf{b}}^{\text{advection}}$$

$$\underbrace{-\mathbf{b}\otimes\mathbf{b}:\mathbb{S}(\mathbf{u})}_{\text{stretching}} - \underbrace{\mathbf{b}\otimes\mathbf{b}:\mathbb{A}(\mathbf{u})}_{\text{rotation}} + \underbrace{\frac{1}{6}\mathbf{b}\otimes\mathbf{b}:(\nabla\cdot\mathbf{u})\mathbb{I}}_{\text{compression}}$$

Again more quantitative: What is a magnetic dynamo?

Gradient tensor decomp.

$$\mathbf{b}\cdot\nabla\cdot\mathbb{F}_{\mathbf{b}} = \mathbf{b}\otimes\mathbf{u}:\nabla\otimes\mathbf{b}$$

$$-\mathbf{b}\otimes\mathbf{b}:\mathbb{S}(\mathbf{u}) - \underbrace{\mathbf{b}\otimes\mathbf{b}:\mathbb{A}(\mathbf{u})} + \frac{1}{6}\mathbf{b}\otimes\mathbf{b}:(\nabla\cdot\mathbf{u})\mathbb{I}$$

rotation ($\mathbb{A}$ is actually a representation of $\mathfrak{SO}(3)$)

symmetric antisymmetric

$$\mathbf{b}\otimes\mathbf{b}:\mathbb{A} = 0$$

Always exactly orthogonal! <u>You can never grow magnetic field flux with rotation operator!</u>

Again more quantitative: What is a magnetic dynamo?
Energy flux

Remaining terms

$$\mathbf{b} \cdot \nabla \cdot \mathbb{F}_{\mathbf{b}} = \mathbf{b} \otimes \mathbf{u} : \nabla \otimes \mathbf{b} - \mathbf{b} \otimes \mathbf{b} : \mathbb{S}(\mathbf{u}) + \frac{1}{6}\mathbf{b} \otimes \mathbf{b} : (\nabla \cdot \mathbf{u})\mathbb{I}$$

Each term could potentially describe an interaction between three difference modes (triad interactions)…

e.g., $\mathbf{b}(\mathbf{k}'), \quad \mathbf{b}(\mathbf{k}''), \quad \mathbf{b}(\mathbf{k}'''), \quad \mathbf{u}(\mathbf{k}'), \quad \ldots$

$$[\mathbf{b} \cdot \nabla \cdot \mathbb{F}_{\mathbf{b}}] \sim U^3/L \quad \text{energy flux density}$$

Again more quantitative: What is a magnetic dynamo?

Cascade versus dynamo

Momentum conservation:

doner receiver

$$\mathbf{k}' + \mathbf{k}'' + \mathbf{k}''' = 0$$

mediator

or

antisymmetry property:
(giveth = - taketh)

doner receiver

$$\mathbf{k}' \xrightarrow{\mathbf{k}''} \mathbf{k}''' = -\mathbf{k}''' \xrightarrow{\mathbf{k}''} \mathbf{k}'$$

mediator

Can extract these interactions directly from stochastic magnetic fields by constructing filtered vector fields

$$\mathbf{b}' = \mathbf{b}(\mathbf{r}') = \int \delta^3(\mathbf{k} - \mathbf{k}')\mathbf{b}(\mathbf{k})\exp\{2\pi i\mathbf{k}\cdot\mathbf{r}\}$$

Again more quantitative: What is a magnetic dynamo?

Cascade versus dynamo

$$\mathbf{k}' \xrightarrow{\mathbf{k}''} \mathbf{k}'''$$

Rewrite magnetic energy equation in terms of triad interactions:

$$\mathbf{b}''' \cdot \partial_t \mathbf{b}' = \partial_t \mathscr{E}_{\mathrm{mag}} = -\,\mathbf{b}''' \cdot \nabla \cdot \mathbb{F}_{\mathbf{b}'} + \frac{1}{\mathrm{Rm}} \mathbf{b}''' \cdot \mathbb{D}_\eta(\mathbf{b}')$$

where

$$\mathbf{b}''' \cdot \nabla \cdot \mathbb{F}_{\mathbf{b}'} = \mathbf{b}''' \otimes \mathbf{u}'' : \nabla \otimes \mathbf{b}'$$

$$-\mathbf{b}''' \otimes \mathbf{b}'' : \nabla \otimes \mathbf{u}' + \frac{1}{2}\mathbf{b}' \otimes \mathbf{b}''' : (\nabla \cdot \mathbf{u}'')\mathbb{I}$$

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$$- \underbrace{\mathbf{b}''' \otimes \mathbf{b}'' : \nabla \otimes \mathbf{u}'}_{\text{kinetic to magnetic energy transfer}}$$

looks like flux generation via compression… it's not

Again more quantitative: What is a magnetic dynamo?

Growth

$$\mathbf{k}' \xrightarrow{\mathbf{k}''} \mathbf{k}'''$$

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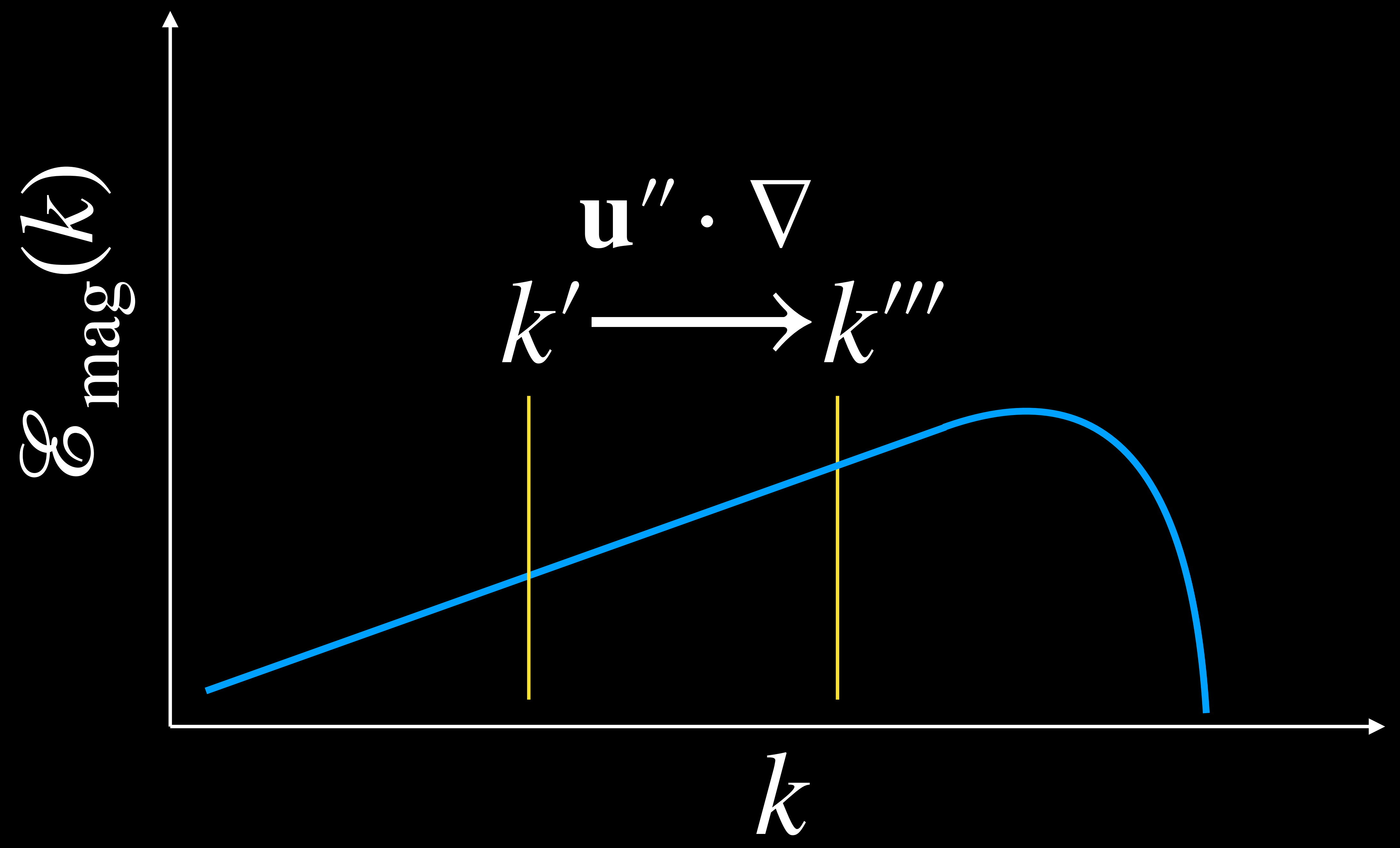

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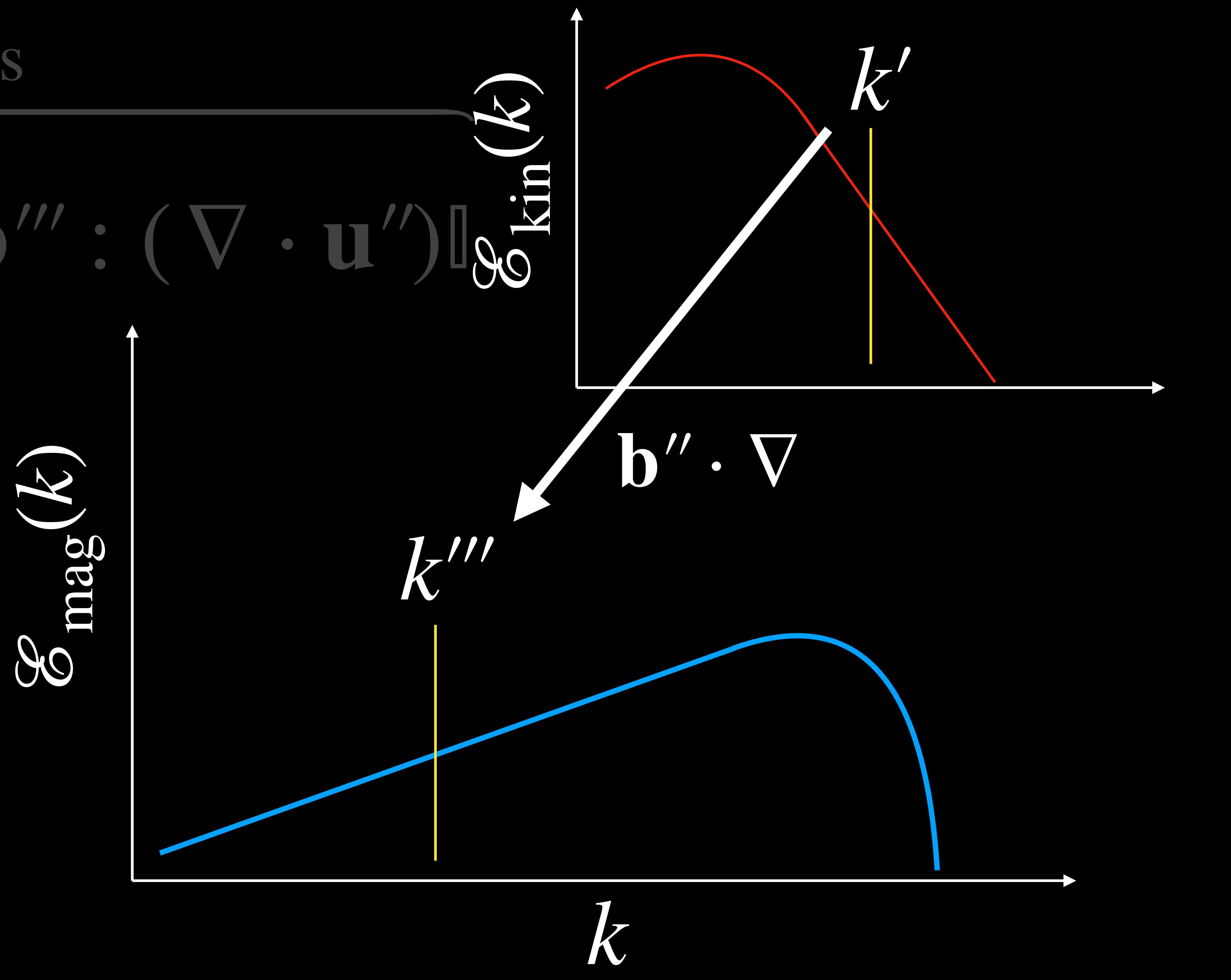

End technical note…

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Dynamo theory, in this context, is really a theory for the eigen values of $\partial_i u_j$

Small-scale dynamo

The engine of the kinematic dynamo

Growth rate dominated by the smallest possible scales of the flow gradients

$$\gamma = \langle \hat{\mathbf{B}} \otimes \hat{\mathbf{B}} : \nabla \otimes \mathbf{u} \rangle \sim u_\nu / \ell_\nu \sim 1/t_\nu,$$

$$u_\ell / \ell \sim \varepsilon^{1/3} \ell^{-2/3}, \quad t_\ell \sim \ell^{2/3}.$$

put in units of outer scale turnover time $t_0 = \ell_0 / u_0$

$$t_0 \gamma \sim t_0 / t_\nu,$$

K41 spectrum (incompresible)

$$t_0 \gamma \sim (\ell_0 / \ell_\nu)^{2/3} \sim \left(\mathrm{Re}^{3/4}\right)^{2/3} \sim \mathrm{Re}^{1/2},$$

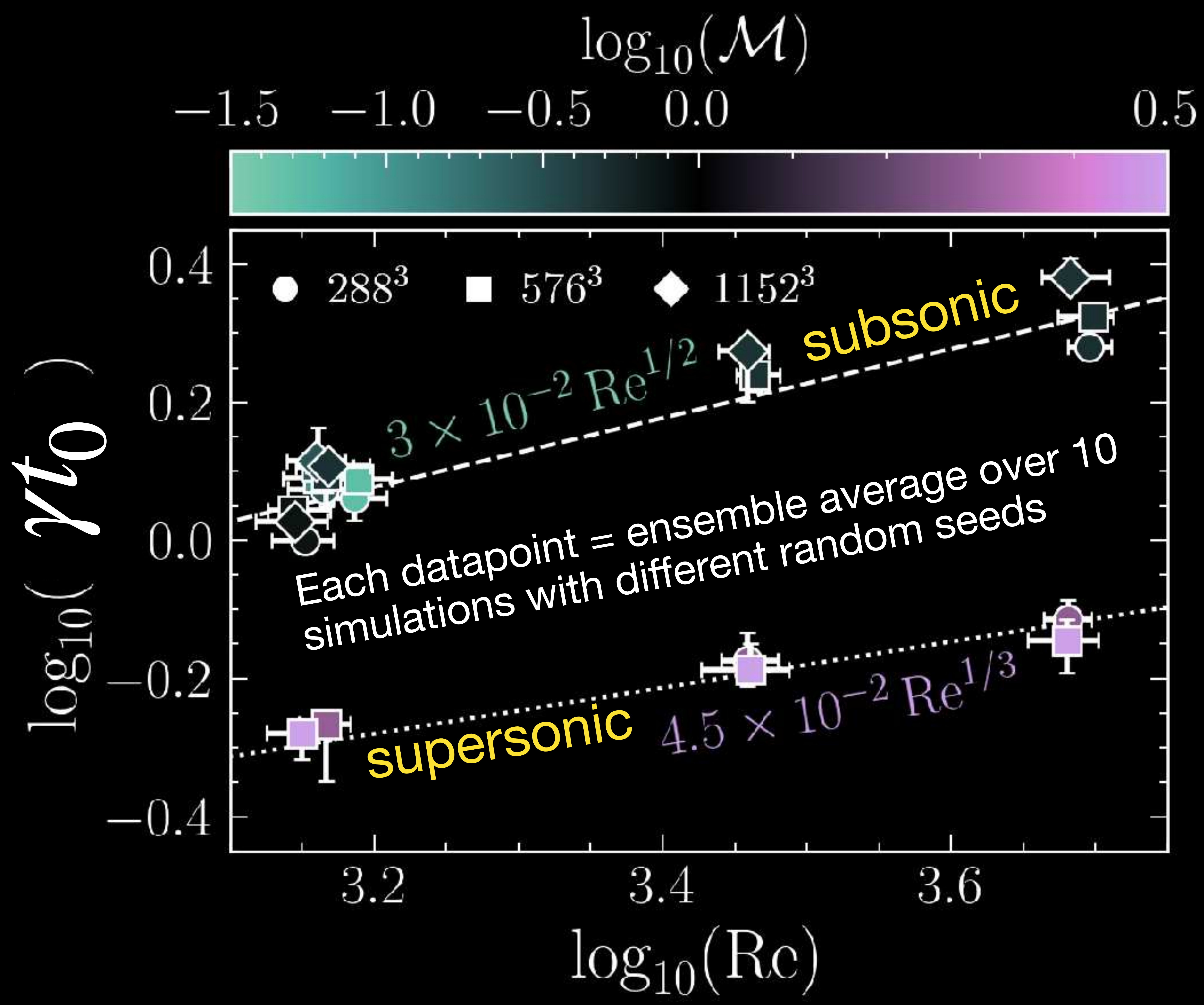
and to summarise,

Burgers spectrum (supersonic)

$$t_0 \gamma \sim (\ell_0 / \ell_\nu)^{1/2} \sim \left(\mathrm{Re}^{2/3}\right)^{1/2} \sim \mathrm{Re}^{1/3}.$$

Small-scale dynamo

Confronting γ with data



The engine of the kinematic turbulent dynamo is k_ν

K41 spectrum (incompressible)

$$t_0\gamma \sim \mathrm{Re}^{1/2},$$ ✓

Burgers spectrum (supersonic)

$$t_0\gamma \sim \mathrm{Re}^{1/3}.$$ ✓

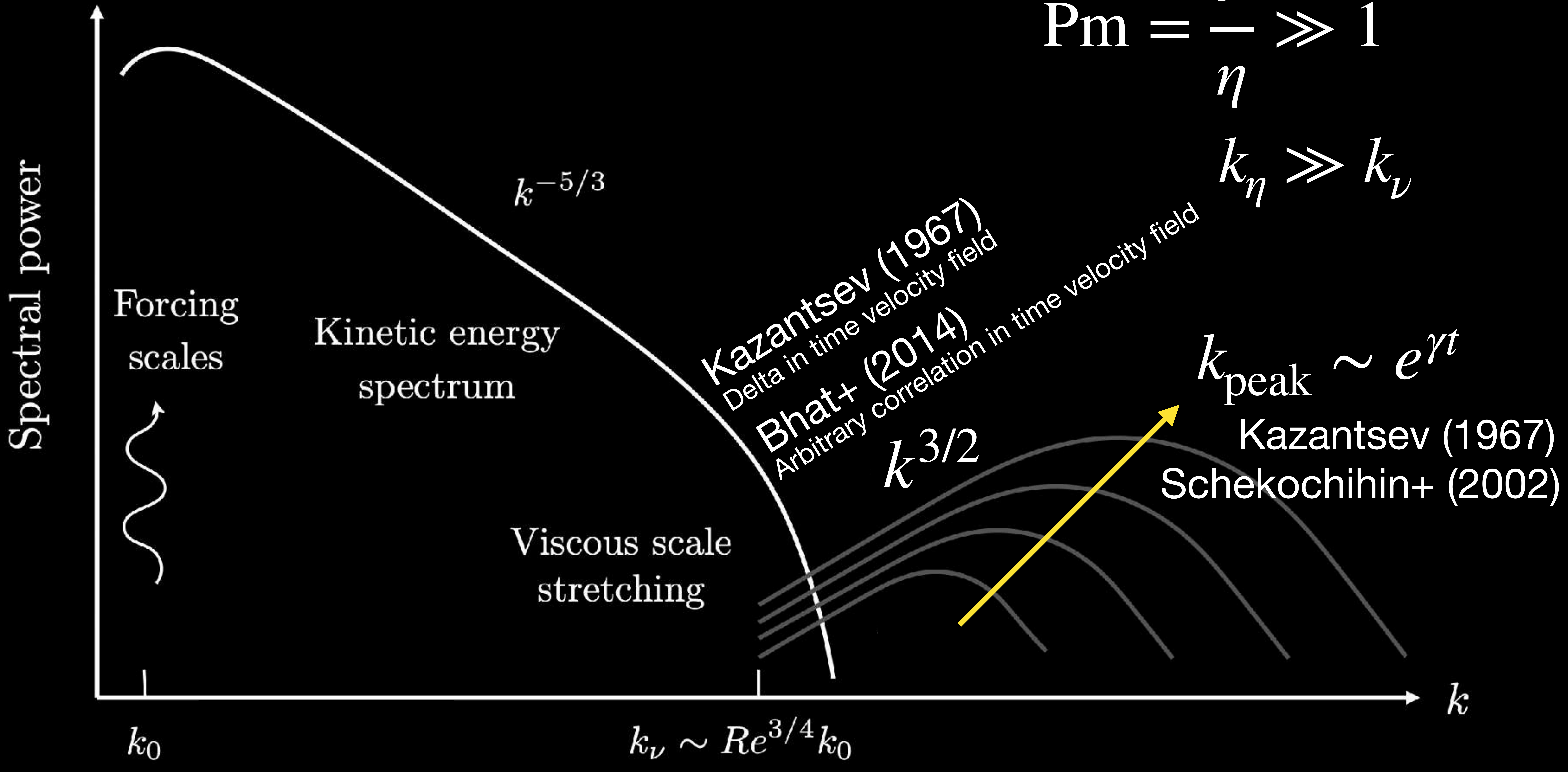
Kriel, **Beattie**+ (accepted PRD.). *The growth of magnetic energy during the nonlinear phase of the subsonic and supersonic small-scale dynamo*

Kinematic Small-scale dynamo

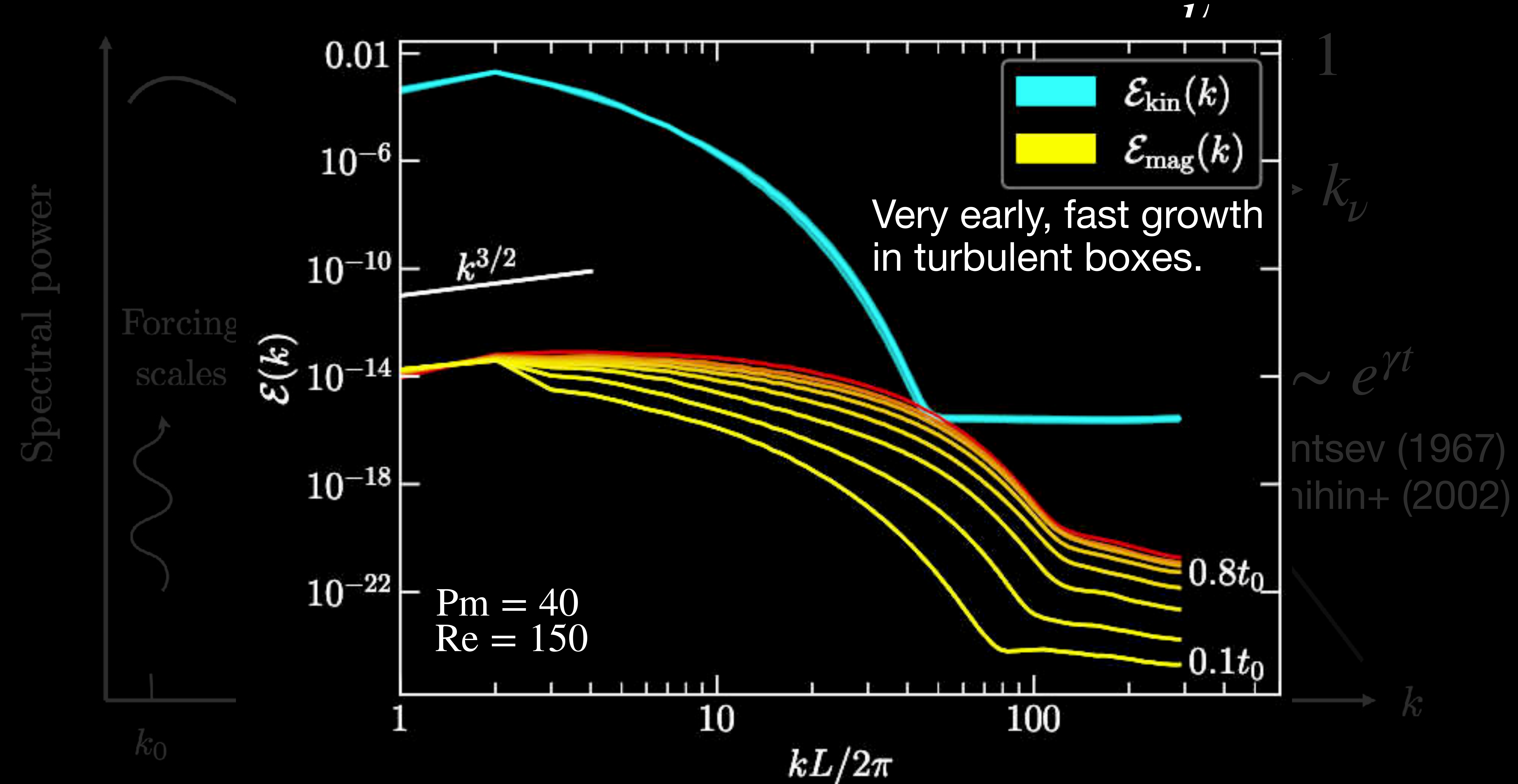
Modified from Rincon (2019)

$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

$$k_\eta \gg k_\nu$$



Kinematic Small-scale dynamo



Very early, fast growth in turbulent boxes.

Turbulent Dynamo Action in Binary Neutron Star Mergers

Eduardo M. Gutiérrez,[1,2] David Radice,[1,2,3] Jacob Fields,[4] and James M. Stone[4]

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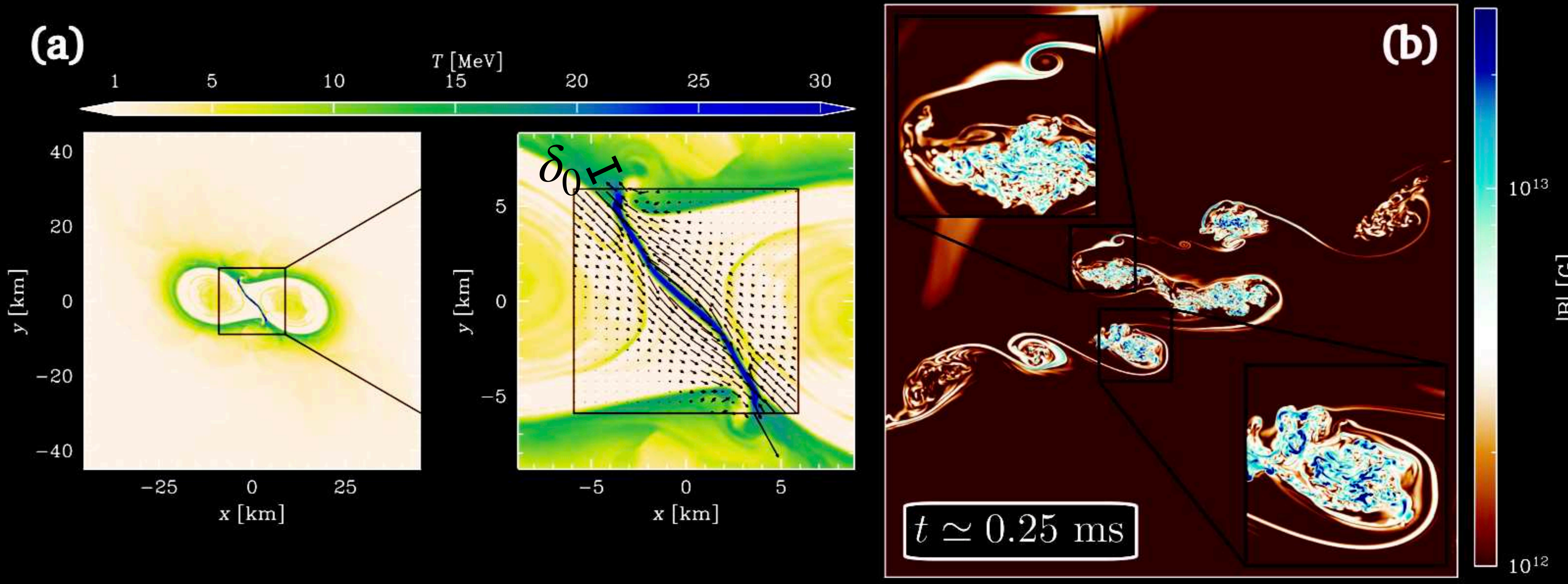
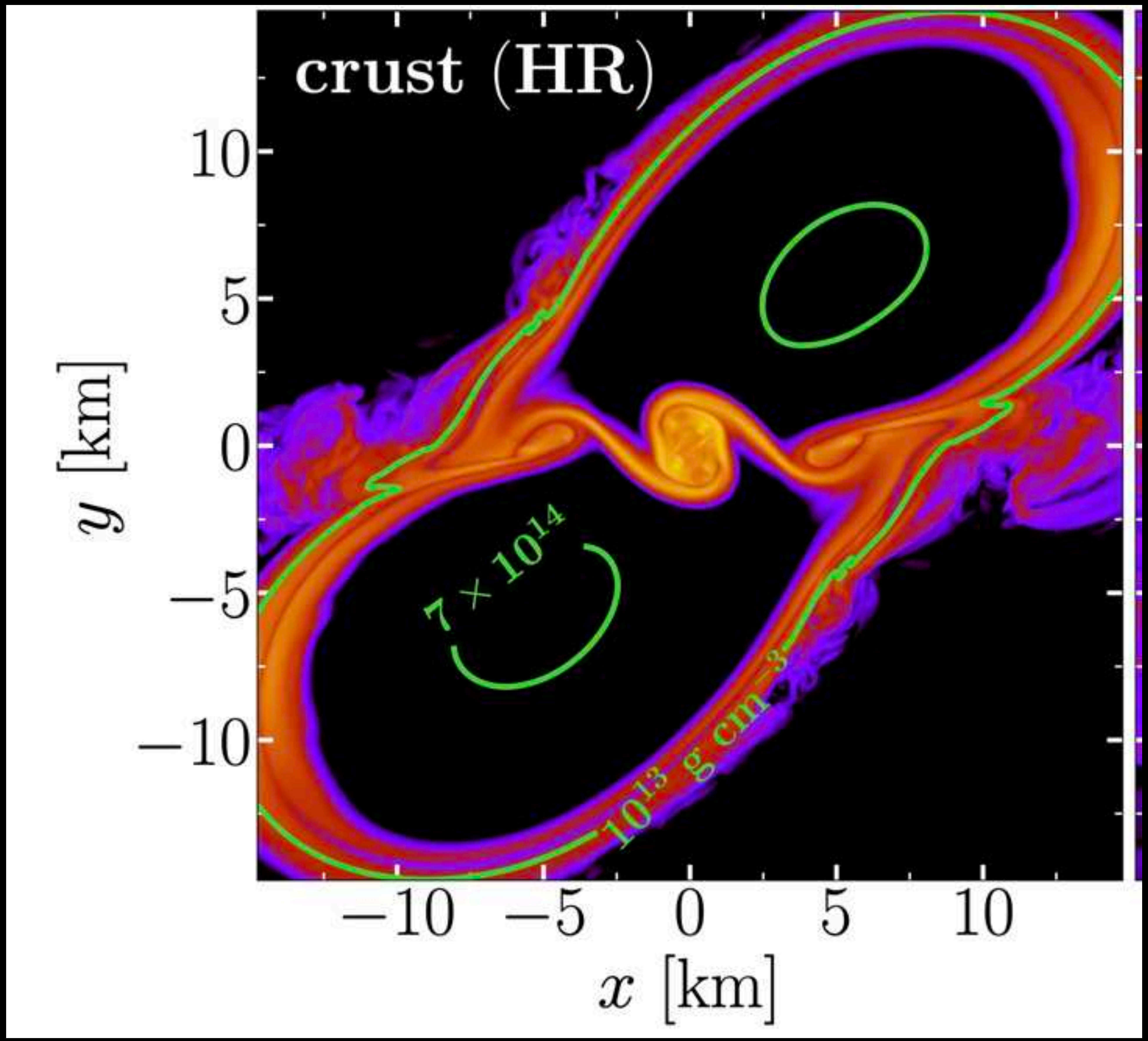
Abstract

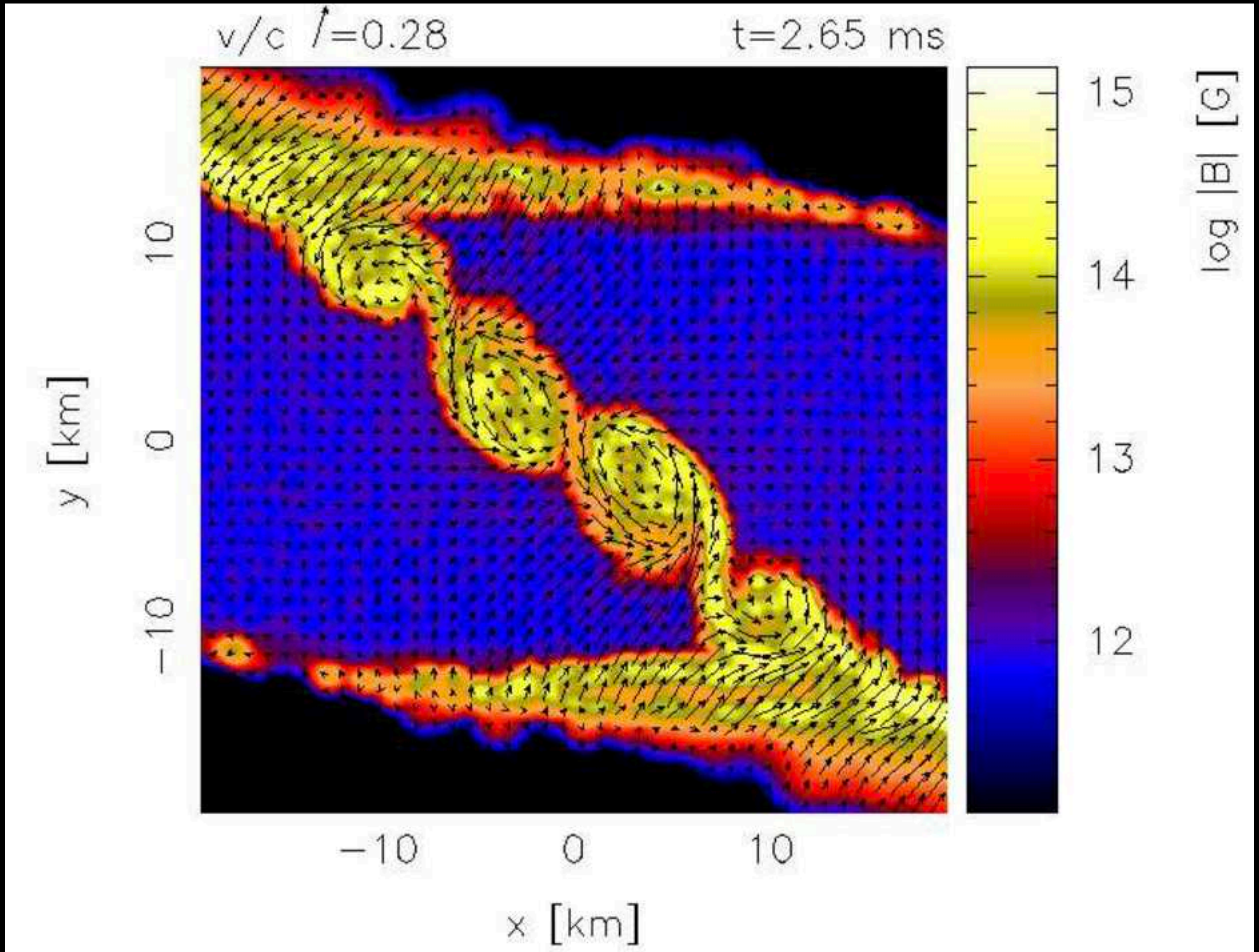
Binary neutron star mergers are expected to generate intense magnetic fields that power relativistic and non-relativistic outflows and shape their multimessenger signatures. These fields likely arise from the turbulent amplification of initially weak magnetic fields during the merger, particularly via the Kelvin–Helmholtz instability at the collisional interface between the stars. While previous studies have shown efficient amplification to magnetar-level strengths, the degree of large-scale coherence of the resulting field remains uncertain. We present general-relativistic, dynamical spacetime, magnetohydrodynamic simulations following the evolution of initially weak, pulsar-like magnetic fields in a binary neutron star merger. We find rapid magnetic field growth at small scales with clear signatures of small-scale turbulent dynamo action. At the highest resolutions, we additionally observe the emergence of coherent magnetic structures on larger scales. Our results imply that strong, ordered magnetic fields may be present immediately after merger, with important implications for the subsequent evolution of the remnant and its observable electromagnetic and gravitational-wave signals.

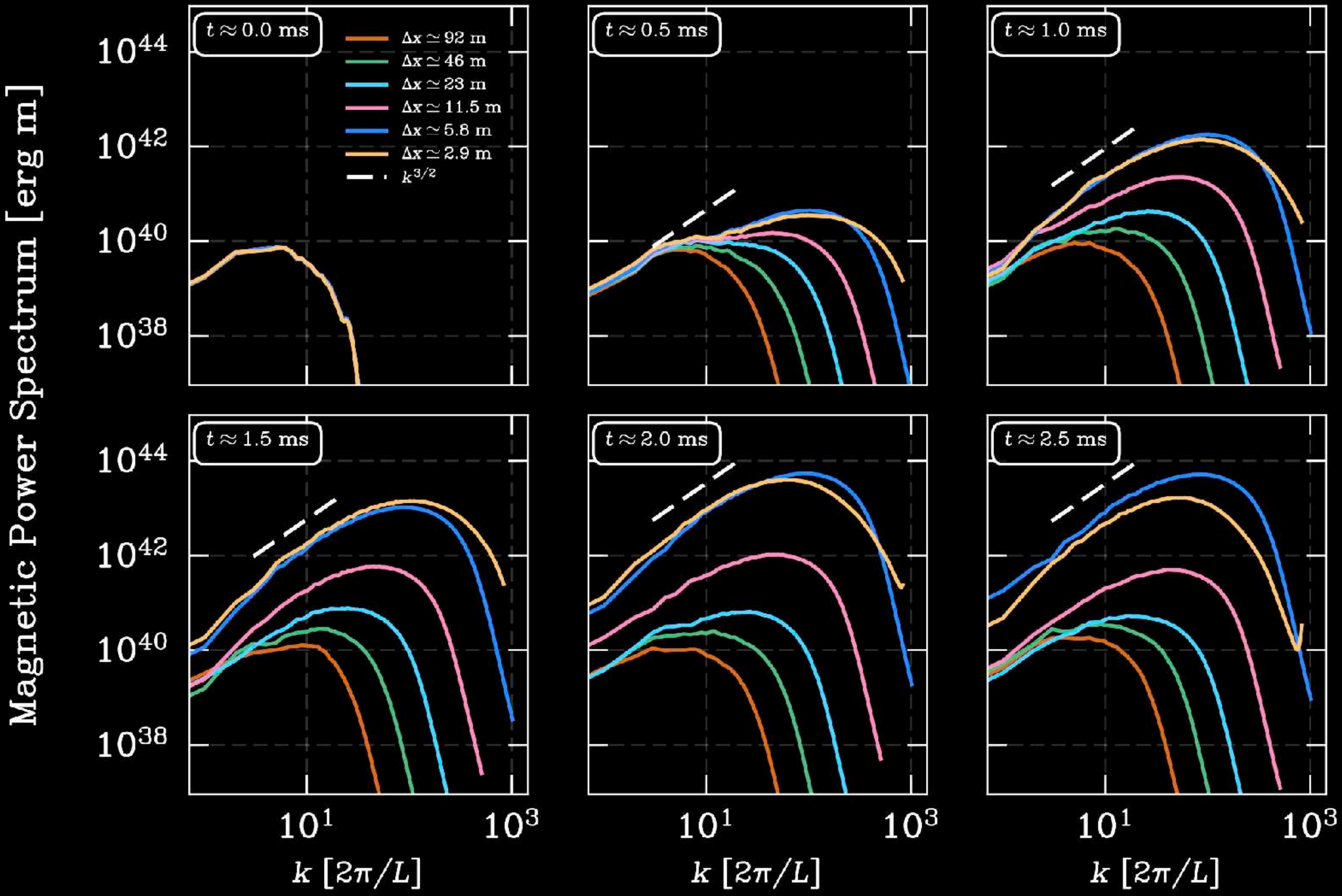


$$t_\Delta = \frac{\delta_0}{\Delta U} \sim \frac{65\,\mathrm{m}}{0.2c} = \mathcal{O}(10^{-6}\,\mathrm{sec}) = 10^{-3}\mathrm{t}_{\mathrm{merge}}$$



Price & Rosswog 2006

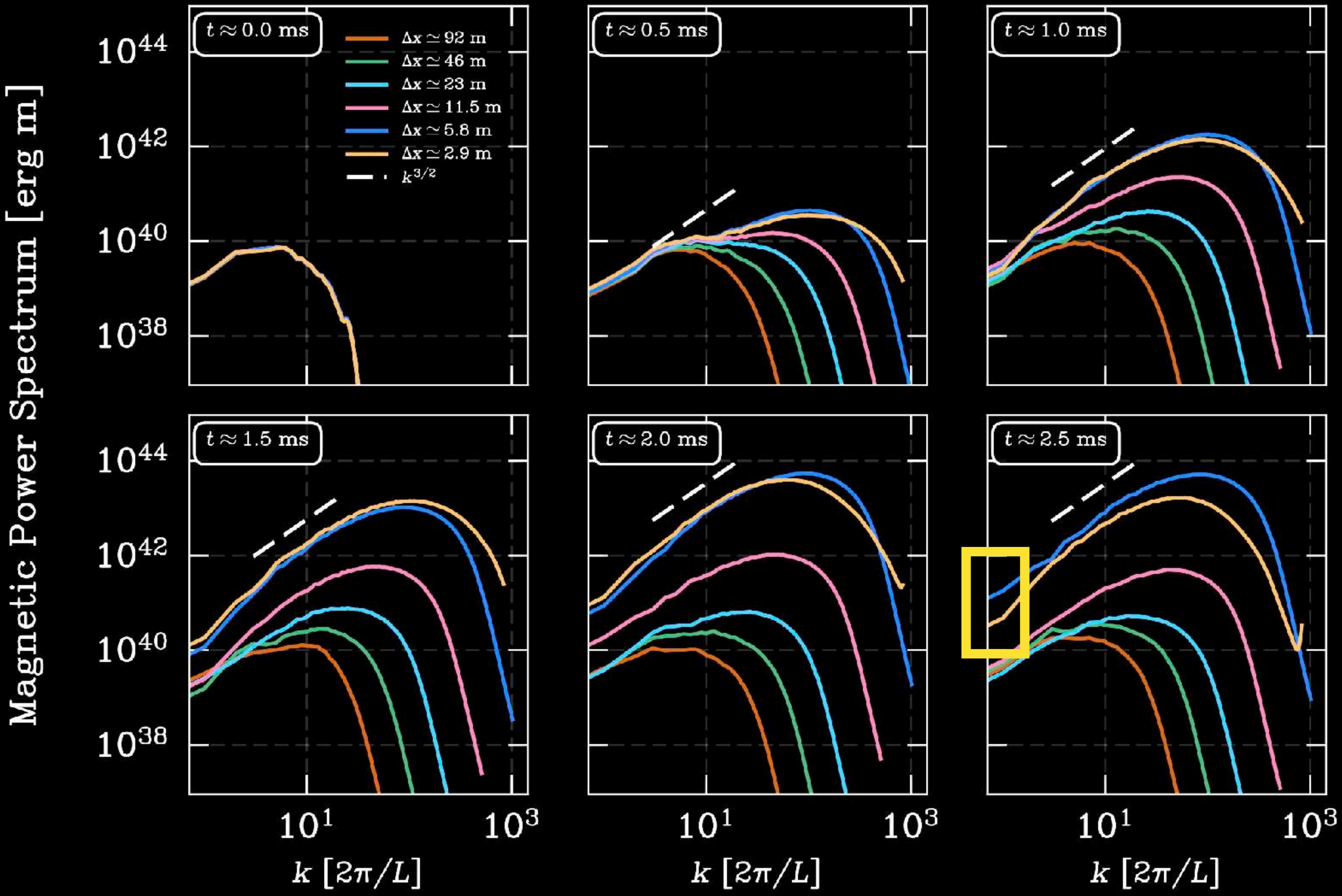




Magnetic Power Spectrum [erg m]
$t \approx 0.0$ ms
$t \approx 0.5$ ms
$t \approx 1.0$ ms
$t \approx 1.5$ ms
$t \approx 2.0$ ms
$t \approx 2.5$ ms
$\Delta x \simeq 92$ m
$\Delta x \simeq 46$ m
$\Delta x \simeq 23$ m
$\Delta x \simeq 11.5$ m
$\Delta x \simeq 5.8$ m
$\Delta x \simeq 2.9$ m
$k^{3/2}$
10^{44}
10^{42}
10^{40}
10^{38}
10^{1}
10^{3}
$k\ [2\pi/L]$

Abstract

Binary neutron star mergers are expected to generate intense magnetic fields that power relativistic and non-relativistic outflows and shape their multimessenger signatures. These fields likely arise from the turbulent amplification of initially weak magnetic fields during the merger, particularly via the Kelvin–Helmholtz instability at the collisional interface between the stars. While previous studies have shown efficient amplification to magnetar-level strengths, the degree of large-scale coherence of the resulting field remains uncertain. We present general-relativistic, dynamical spacetime, magnetohydrodynamic simulations following the evolution of initially weak, pulsar-like magnetic fields in a binary neutron star merger. We find rapid magnetic field growth at small scales with clear signatures of small-scale turbulent dynamo action. At the highest resolutions, we additionally observe the emergence of coherent magnetic structures on larger scales. Our results imply that strong, ordered magnetic fields may be present immediately after merger, with important implications for the subsequent evolution of the remnant and its observable electromagnetic and gravitational-wave signals.



Magnetic Power Spectrum [erg m]
k [2π/L]
t ≈ 0.0 ms
t ≈ 0.5 ms
t ≈ 1.0 ms
t ≈ 1.5 ms
t ≈ 2.0 ms
t ≈ 2.5 ms
Δx ≃ 92 m
Δx ≃ 46 m
Δx ≃ 23 m
Δx ≃ 11.5 m
Δx ≃ 5.8 m
Δx ≃ 2.9 m
k^{3/2}
10^44
10^42
10^40
10^38
10^1
10^3

But how to do dynamo of just the coherent field?

Always start with induction

$$\partial_t \mathbf{B} - \eta \nabla^2 \mathbf{B} = \nabla \times (\mathbf{u} \times \mathbf{B}),$$

Then take mean, with some yet-to-be specified kernel $\mathbf{B} = \langle \mathbf{B} \rangle + \delta \mathbf{b}$

$$\partial_t \overline{\mathbf{B}} - \eta \nabla^2 \overline{\mathbf{B}} = \nabla \times (\overline{\delta \mathbf{u} \times \delta \mathbf{b}}),$$

We call the trouble the EMF (electromotive force):

$$\nabla \times (\overline{\delta \mathbf{u} \times \delta \mathbf{b}}) = \nabla \times \overline{\mathscr{E}}$$

The main goal in large-scale dynamo theory is to write down $\overline{\mathscr{E}}$ in terms of mean-field quantities. It's a closure problem for the $\varepsilon_{ijk}\overline{\delta b_j \delta u_k}$ correlator.

But how to do dynamo of just the coherent field?

Construct fluctuating field induction (assuming homogenous velocity field)

$$\frac{\partial \delta \mathbf{b}}{\partial t} - \eta \nabla^2 \delta \mathbf{b} = \nabla \times \left(\delta \mathbf{u} \times \overline{\mathbf{B}}\right),$$

Admits to solution via Green's function

$$\delta b_k(\mathbf{x}, t) = \int dt' \int d^3x' \, G_{km}(\mathbf{x}, t; \mathbf{x}', t') \left[\partial'_s(\delta u_m \overline{B}_s) - \partial'_r(\delta u_r \overline{B}_m)\right]_{(\mathbf{x}', t')} .$$

Assuming isotropy and homogeneity of the turbulent transport tensor, and that $\delta b_i(B_i)$

$$\overline{\mathcal{E}}_i(\mathbf{x}, t) = \int d\tau \int d^3r \, K_{ij}(r, \tau) \, \overline{B}_j(\mathbf{x}', t') .$$

$$r = x - x'$$
$$\tau = t - t'$$

But how to do dynamo of just the coherent field?

$$\overline{\mathcal{E}}_i(\mathbf{x}, t) = \int d\tau \int d^3r \; K_{ij}(r, \tau) \; \overline{B}_j(\mathbf{x}', t') \, .$$

Now Taylor expanding

$$\overline{B}_j(\mathbf{x}', t') = \overline{B}_j - r_\ell \, \partial_\ell \overline{B}_j - \tau \, \partial_t \overline{B}_j + \frac{1}{2} r_\ell r_m \, \partial_\ell \partial_m \overline{B}_j + \cdots$$

Substituting back into the EMF

$$\overline{\mathcal{E}}_i = a_{ij} \, \overline{B}_j + b_{ij\ell} \, \partial_\ell \overline{B}_j + c_{ij} \, \partial_t \overline{B}_j + d_{ij\ell m} \, \partial_\ell \partial_m \overline{B}_j + \cdots$$

But how to do dynamo of just the coherent field?

Utilizing the same homogeneity and isotropy assumptions, as well as slowly varying $\overline{B}$, we can write out a scalar theory for large-scale dynamo EMF, e.g.,

$$\overline{\mathscr{E}}_i = a_{ij}\overline{B}_j + b_{ij\ell}\,\partial_\ell \overline{B}_j + c_{ij}\,\partial_t \overline{B}_j + d_{ij\ell m}\,\partial_\ell \partial_m \overline{B}_j + \cdots$$

becomes

$$\overline{\mathscr{E}}_i = \alpha \overline{B}_j - \beta \varepsilon_{ijk} \partial_j \overline{B}_k + \cdots$$

alpha term   beta term

$$\alpha = t_{\rm nl}\overline{\delta \mathbf{u} \cdot \nabla \times \delta \mathbf{u}}$$

$$\beta = t_{\rm nl}\overline{\delta u^2}$$

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Nobody has done the relativistic version of this theory…

But how to do dynamo of just the coherent field?

But something very wrong for the KHI dynamo… the velocity is not homogenous!
That means we cannot Galilean transfer away $\overline{u}$ in

$$\frac{\partial \delta \mathbf{b}}{\partial t} - \eta \nabla^2 \delta \mathbf{b} = \nabla \times \left(\delta \mathbf{u} \times \overline{\mathbf{B}} \right),$$

hence one needs,

$$\frac{\partial \delta \mathbf{b}}{\partial t} - \eta \nabla^2 \delta \mathbf{b} = \nabla \times \left(\delta \mathbf{u} \times \overline{\mathbf{B}} \right) + \nabla \times (\overline{\mathbf{u}} \times \delta \mathbf{b}),$$

First realized by Tripathi + (2026) this year!

But how to do dynamo of just the coherent field?

Expanding now in $\overline{u}$ and $\overline{B}$, to leading order (making same time assumptions):

$$\overline{\mathcal{E}}_i = \alpha \overline{B}_j - \beta \varepsilon_{ijk} \partial_j \overline{B}_k + \Upsilon \varepsilon_{ijk} \partial_j \overline{u}_k$$

alpha term beta term upsilon term

$$\alpha = t_{\mathrm{nl}} \overline{\delta \mathbf{u} \cdot \nabla \times \delta \mathbf{u}}$$

$$\beta = t_{\mathrm{nl}} \overline{\delta u^2}$$

$$\Upsilon = t_{\mathrm{nl}} \overline{\delta u \cdot \delta b}$$

Now we have all the theory we need to do our unstable shear layer large-scale dynamo!

Numerical Setup:

3D, visco-resistive, compressible MHD equation, with KHI ICs:

$$\frac{u_y(x,t_0)}{U_y} = \Delta(x) - 1, \qquad \Delta U = 2U_y, \qquad \text{shear profile}$$

$$\frac{\rho(x,t_0)}{\rho_0} = 1 + \frac{\delta\rho}{2\rho_0}\Delta(x), \qquad \text{stratification}$$

$$\Delta(x) = \tanh\left(\frac{x-x_1}{\delta_0}\right) - \tanh\left(\frac{x-x_2}{\delta_0}\right),$$

$$\frac{u_x(x,y,t_0)}{U_x} = \frac{1}{\sqrt{n_{\max}+1}} \sum_n^{n_{\max}} \sin\left(\frac{2n\pi}{L}y + \phi_n\right) \times \qquad \text{perturbation}$$

$$\left(\exp\left\{-\frac{(x-x_1)^2}{\sigma^2}\right\} + \exp\left\{-\frac{(x-x_2)^2}{\sigma^2}\right\}\right),$$

with initial random magnetic field that satisfies:

$$\widetilde{B}_i(\boldsymbol{k}) = \widetilde{A}(k)(\delta_{ij} - k_i k_j/k^2)\widetilde{g}_j(\boldsymbol{k}) \iff \boldsymbol{k}\cdot\widetilde{\boldsymbol{B}}(\boldsymbol{k}) = 0, \quad k \neq 0,$$

$$\langle \widetilde{g}_i(\boldsymbol{k})\rangle = 0, \qquad \langle \widetilde{g}_i(\boldsymbol{k})\widetilde{g}_j^\dagger(\boldsymbol{k}')\rangle = \delta_{ij}\,\delta_{\boldsymbol{k}\boldsymbol{k}'}.$$

magnetic field breaks z symmetry

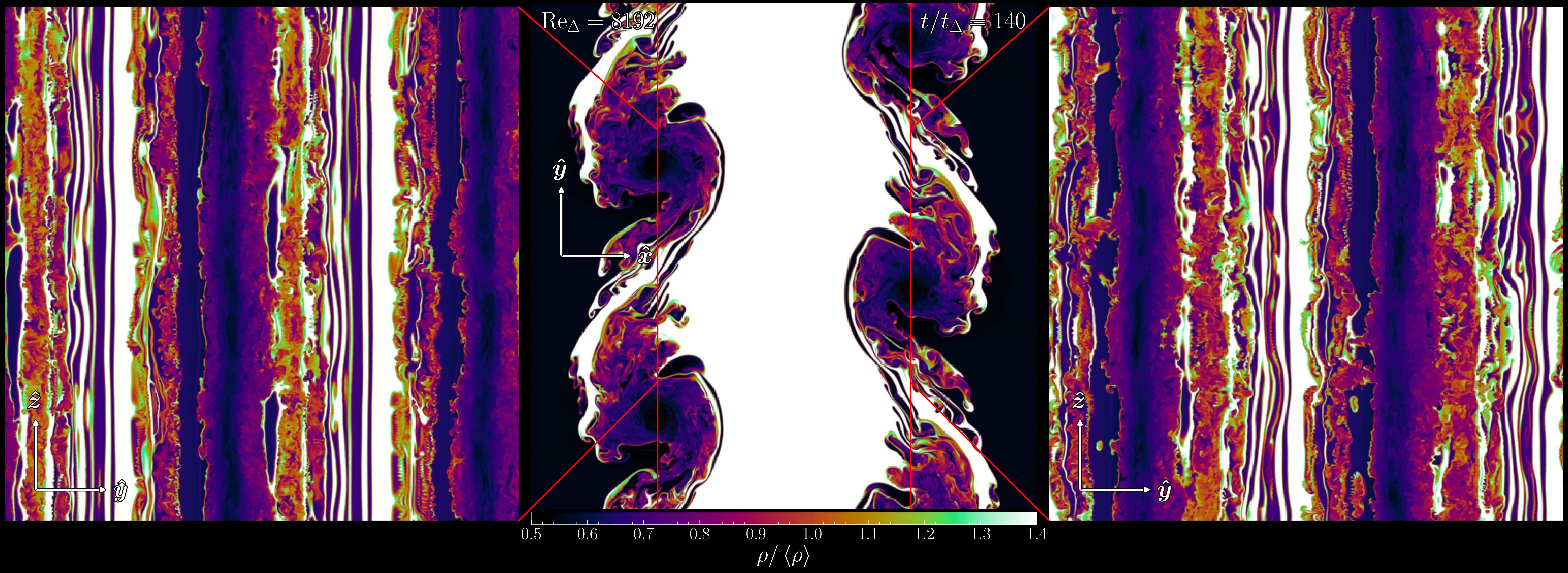
discretized on uniform grids: $288^3 \leq N_{\text{grid}}^3 \leq 4{,}608^3$

$$\text{Re} = \frac{\Delta U \delta_0}{\nu}$$

$$\text{Rm} = \frac{\Delta U \delta_0}{\eta}$$

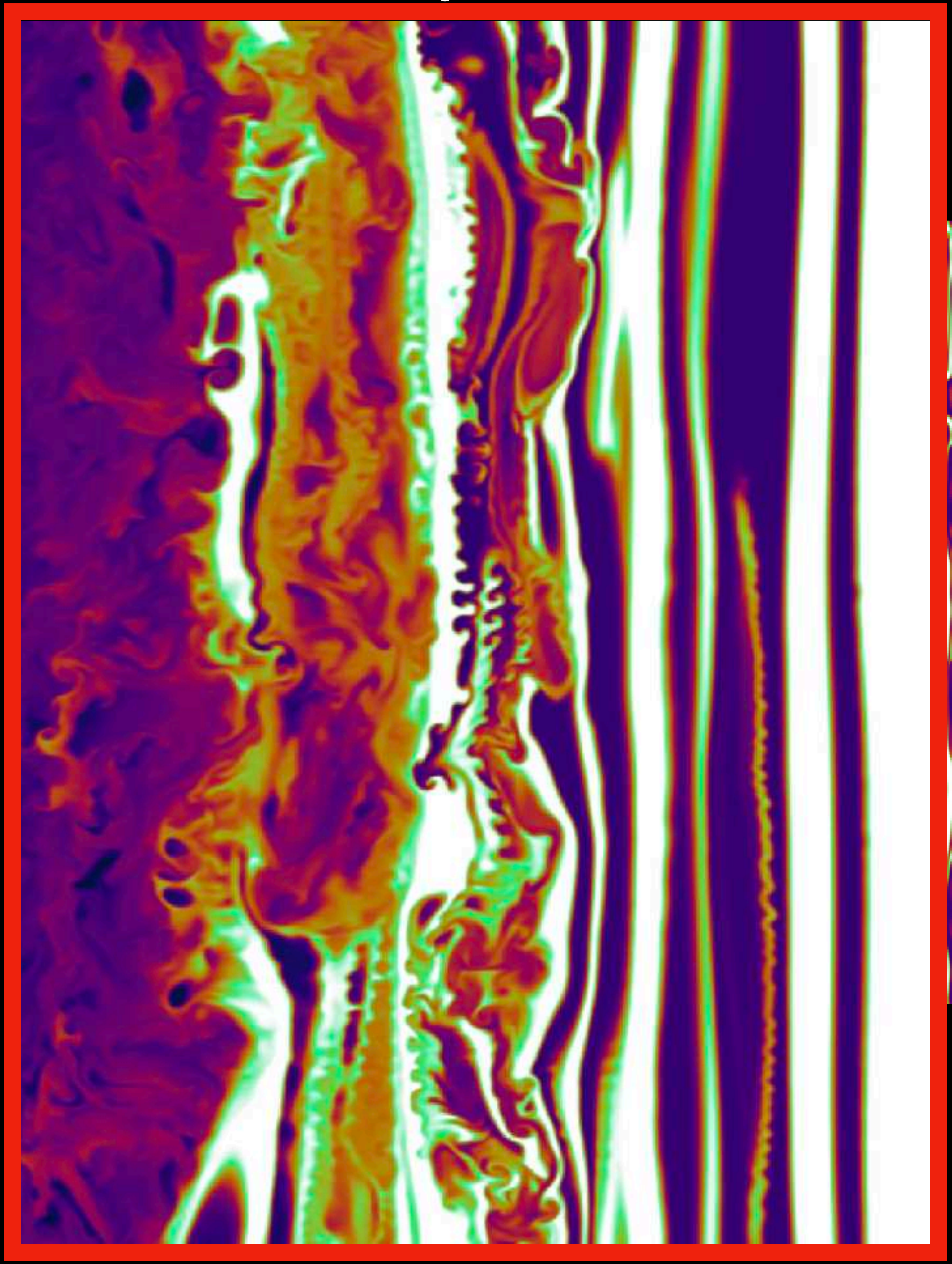
$$\text{Pm} = \frac{\nu}{\eta} = 1$$

Cat's eye / billows



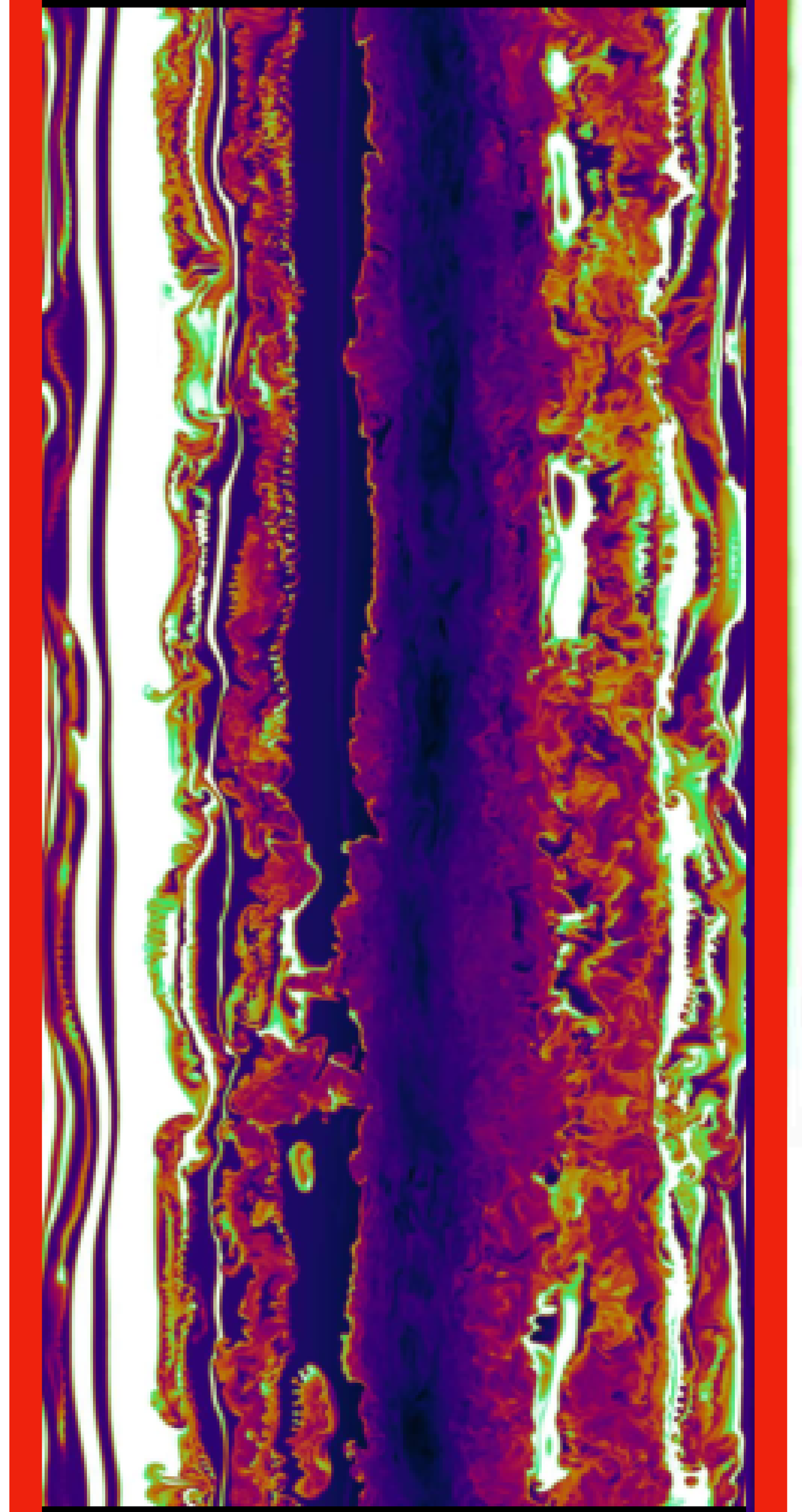
billows in 3D form columns
that go 3D unstable starting at high-k

Secondary KH modes

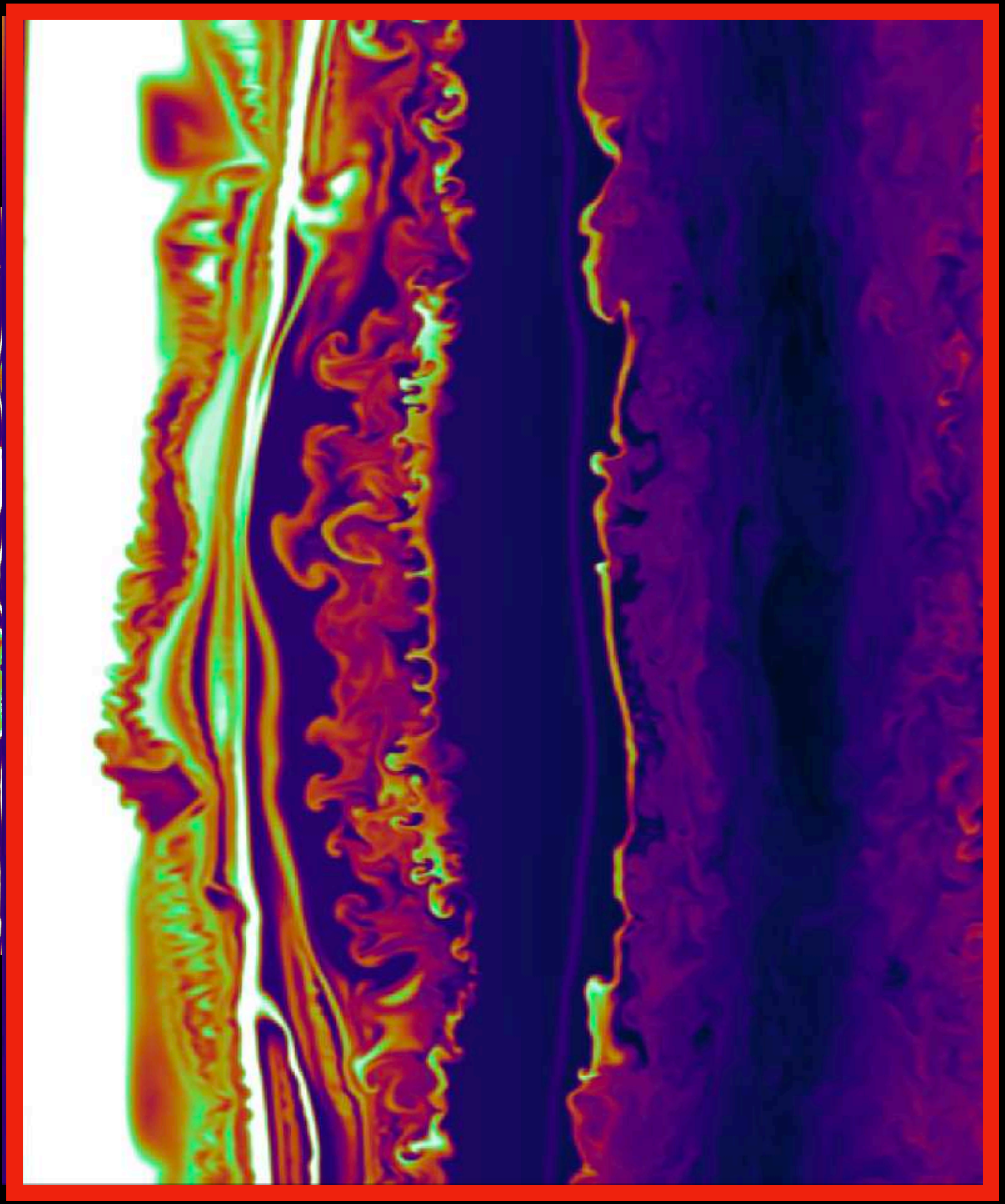


Zoo of 3D instabilities

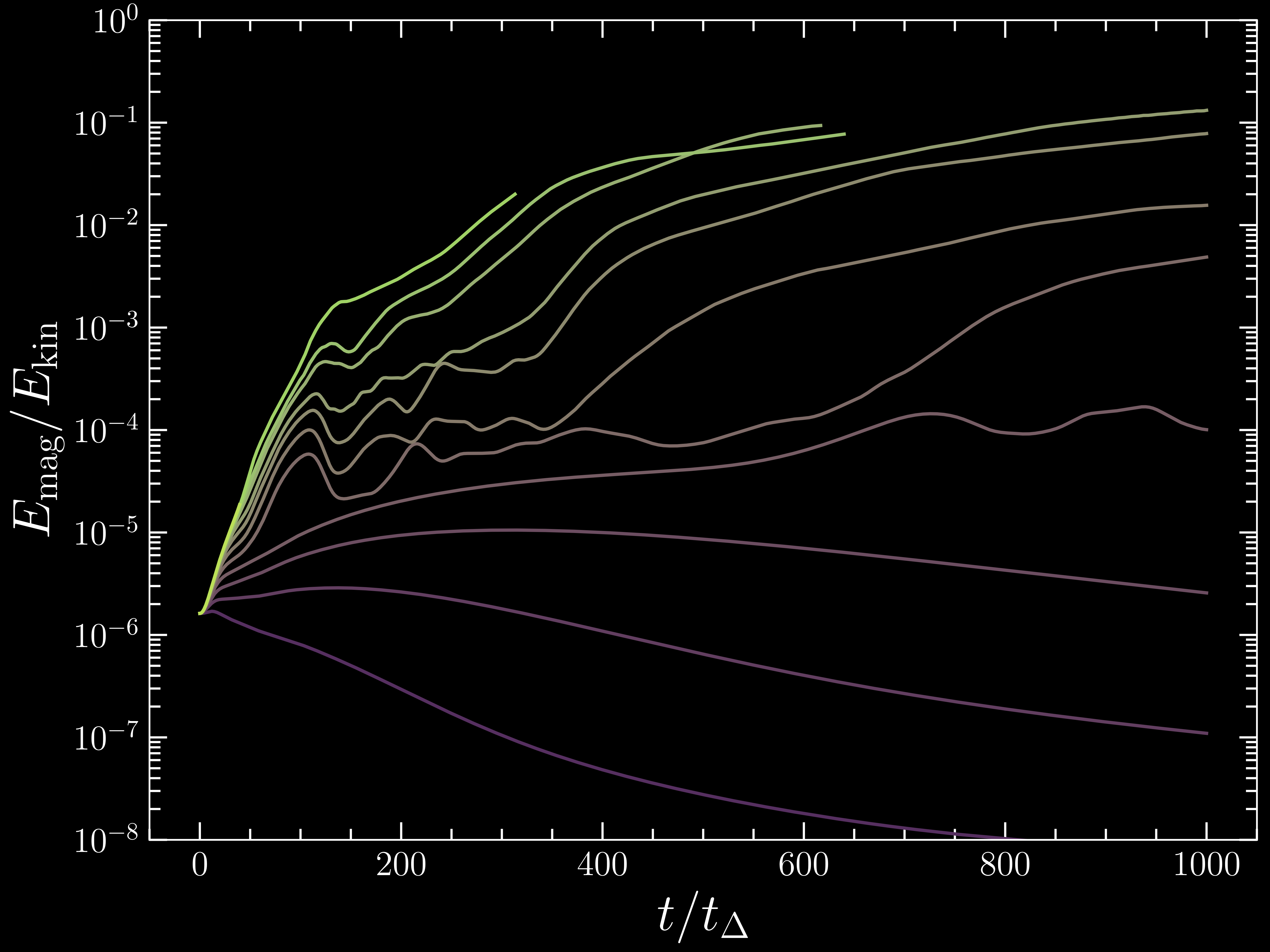
Mashayek+(2012)



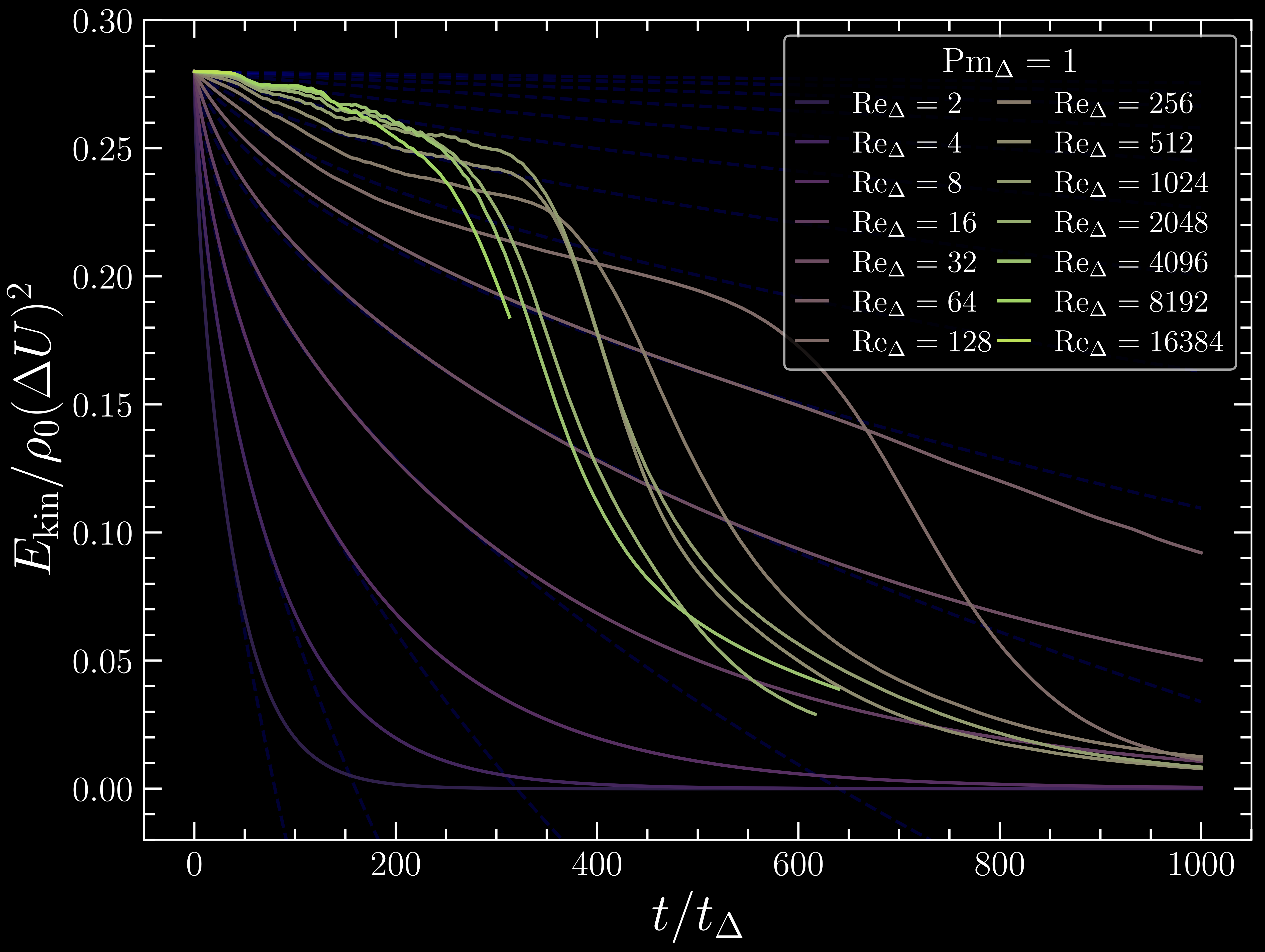
RT modes



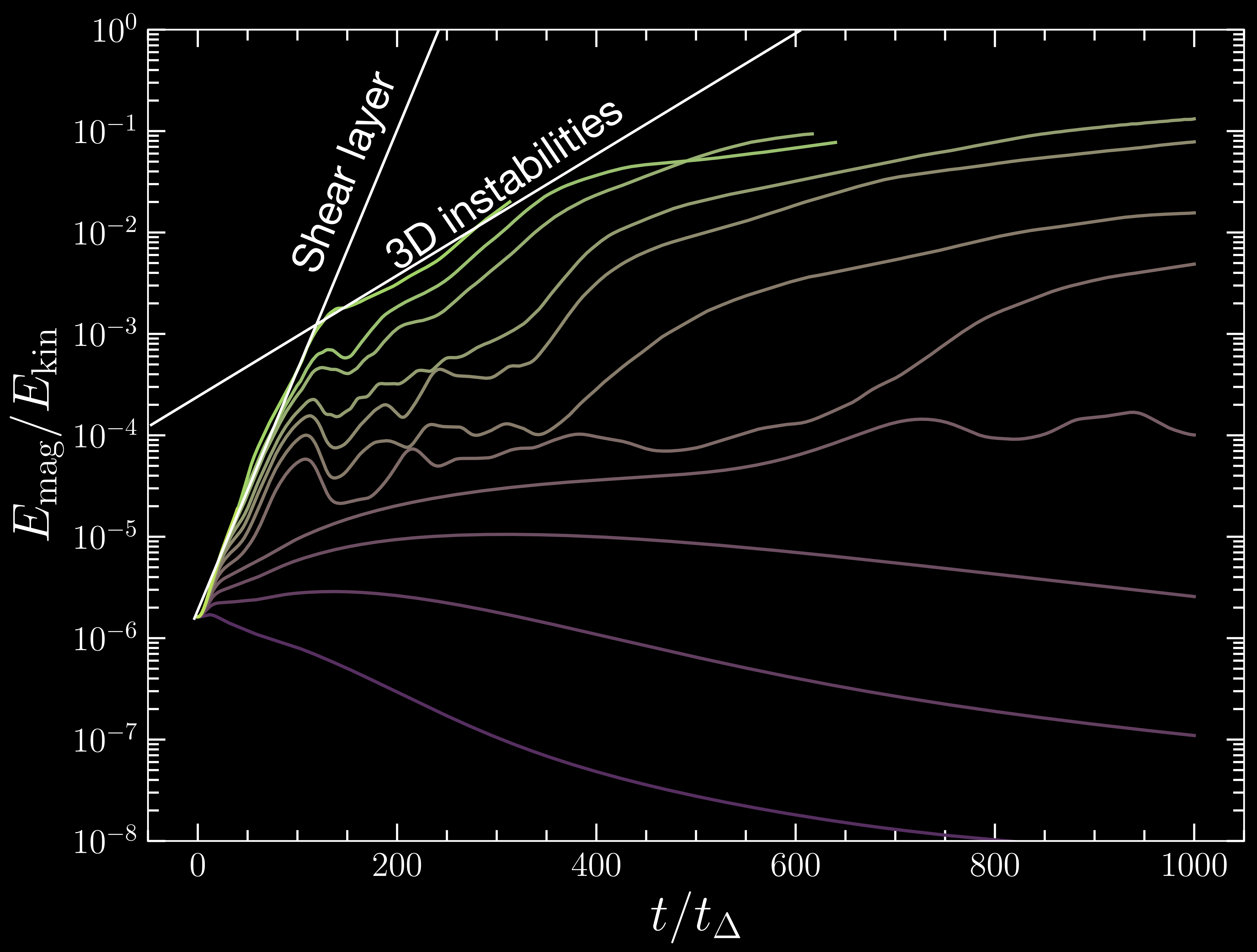
Integral magnetic energy

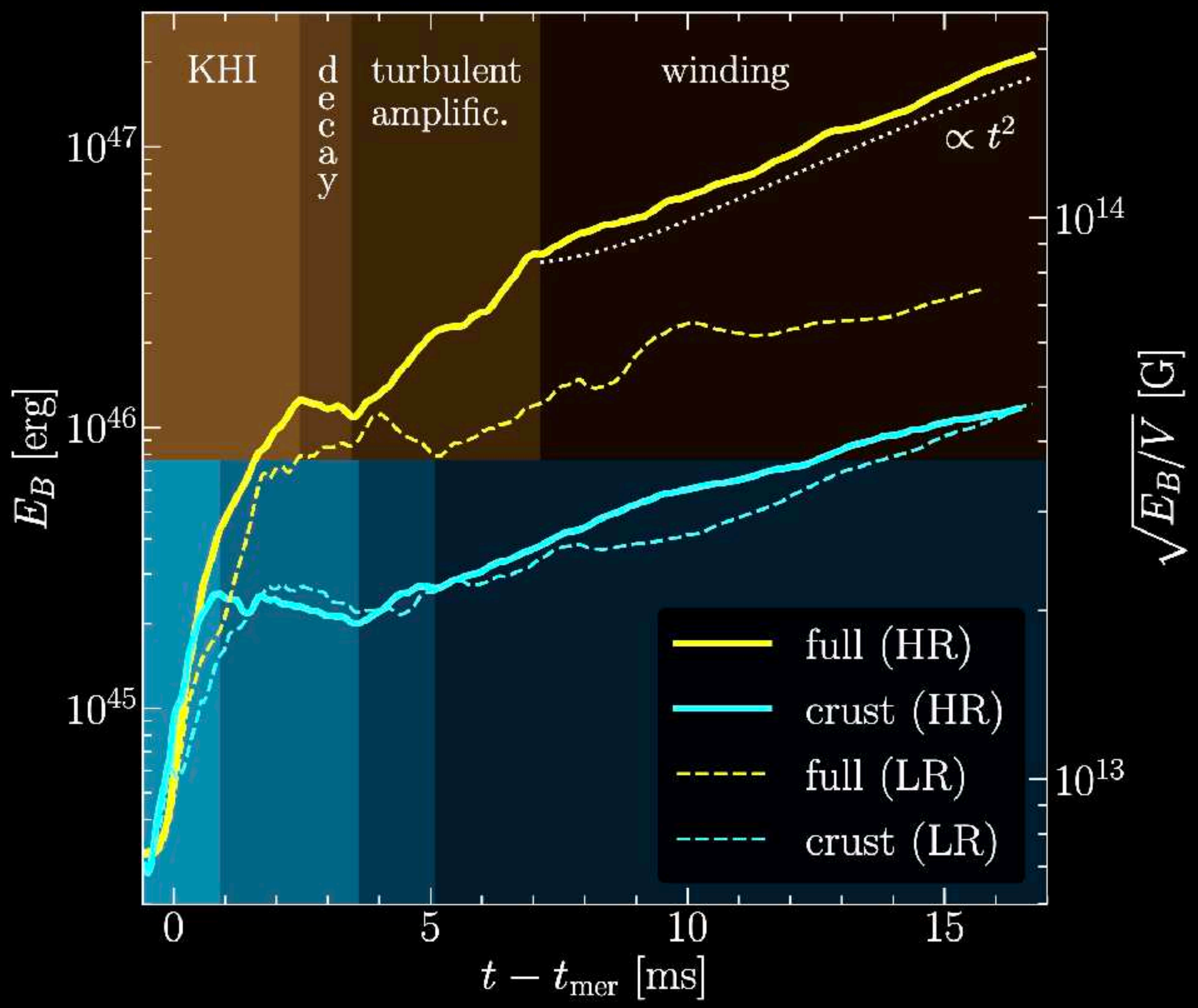


Integral kinetic energy



Integral energy growths look remarkably similar to global simulations!





Vertical field dynamo!! It’s possible!