

Luncheon talk:

**Beattie**, Federrath, Klessen, Cielo, Bhattacharjee, 2025 (*Nature Astronomy*). The spectrum of magnetized turbulence in the interstellar medium

## Supernova remnants as engines of non-Kolmogorov galactic turbulence (and why we need to throw away old ideas applied to ISM turbulence)

ITC Colloquium, Harvard University

James Beattie

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In collaboration: Isabelle Connor (UCSC), Anne Noer Kolborg (UCSC), Enrico Ramirez Ruiz (UCSC), Amitava Bhattacharjee (Princeton), Christoph Federrath (ANU), Ralf Klessen (UH), Salvatore Cielo (LRZ)

**Beattie** + (2025; ApJ) So long Kolmogorov: the forward and backwards cascades in a supernovae-driven, multiphase ISM

Connor, **Beattie** + (2025, ApJ) Cascading from the winds to the disk: universality of supernovae-driven turbulence in different galactic ISMs

**Beattie** (2026; submitted ApJL) Supernovae drive large-scale incompressible turbulence from small-scale instabilities

# Roadmap for this colloquium

1. Discuss vanilla K41 turbulence in a pedagogical manner & turbulence in our galaxy.
2. Supernova-driven turbulence — how to fit them into the cascade? Simulations.
3. Spectra, cascade directions.
4. Where does the incompressible turbulence come from — shell instabilities and spectra model.

# The ISM: A laboratory for interesting nonlinear physics

- Multiphase trans-to-supersonic compressible plasma
- $Re = UL/\nu \gg 1 \implies$  highly-turbulent
- Turbulence driven by a zoo of sources
  - Gravitational instabilities, SN explosions, stellar feedback...
- Pervaded by ultra-relativistic particles with non-negligible energy densities (peaked number density in GeV – TeV range)
- Non-Gaussian gas-density structures related to star formation (multi-scale filaments, ribbons, striations, sheets)
- Dynamically important, maintained magnetic fields
- $Rm = UL/\eta \gg 1 \implies$  highly-turbulent magnetic fields
- ....

Krumholz & McKee (2005); Federrath & Klessen (2012); Tritsis+(2016); Xu & Lazarian (2016); Soler & Hennebelle (2017); Squire & Hopkins (2017); Mocz & Burkhart (2018), Burkhart & Mocz (2018); Heyer+(2012); Tritsis+(2018); Hu+(2019); Heyer+(2020); Krumholz+(2020); Beattie & Federrath (2020); Beattie+(2020); Körtgen & Soler (2020); Skalidis & Tassis (2020); Federrath+(2021); Burkhart (2021); Barreto-Mota+(2021); Hopkins+(2022); Beattie+(2022); Fielding+(2022); Sampson+ (2022); Kriel+(2022); Galishnikova+(2022); Beattie+(2023)

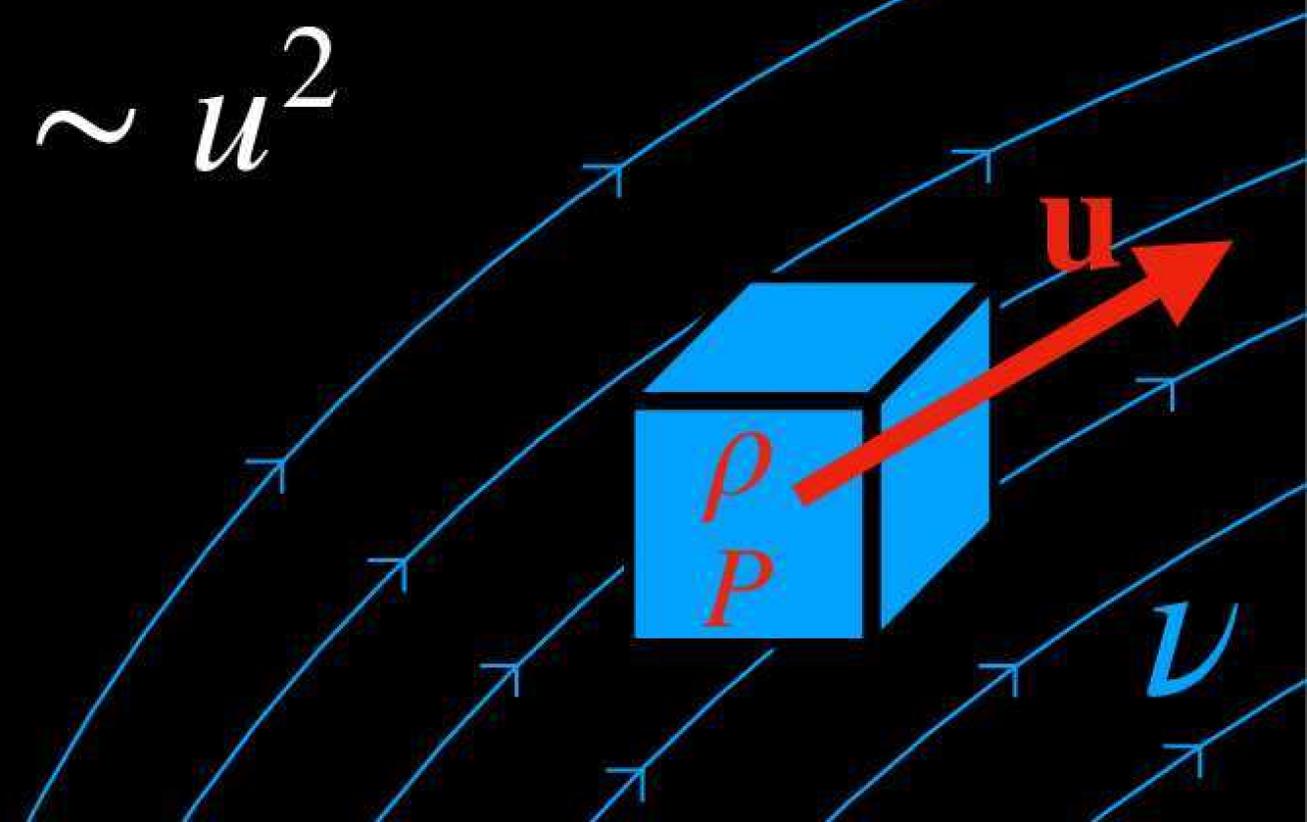
# Turbulence

What is it (hand wavy)?

Momentum conservation for a hydrodynamical fluid element  
(first moment of Boltzmann eq. for monoatomic plasma slightly out of LTE)

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) = -\nabla \cdot P \mathbb{I} + 2\nu \nabla \cdot (\rho \mathcal{S})$$
$$\mathbf{u} \otimes \mathbf{u} = u_i u_j \sim u^2$$

$$\lambda_{\text{mfp}}/L \ll 1$$



# Turbulence

What is it (hand wavy)?

Reynolds stress



$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) = -\nabla \cdot P \mathbb{I} + 2\nu \nabla \cdot (\rho \mathcal{S})$$

$$\mathbf{u} \otimes \mathbf{u} = u_i u_j \sim u^2$$



Viscous stress

# Turbulence

What is it (hand wavey)?

creating nonlinearity



$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) = -\nabla \cdot P \mathbb{I} + 2\nu \nabla \cdot (\rho \mathcal{S})$$

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Smooths out nonlinear things in the fluid

# Turbulence

What is it (hand wavy)?

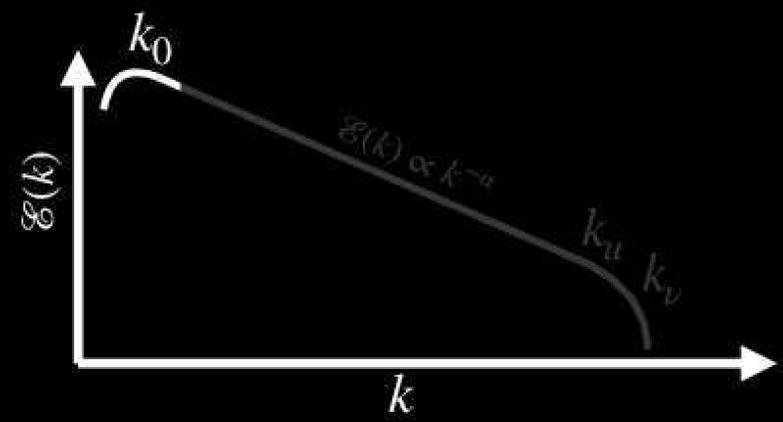
$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) = -\nabla \cdot P\mathbb{I} + 2\nu \nabla \cdot (\rho \mathcal{S})$$

Creating nonlinear things in the fluid

$$\text{Re} = \frac{|\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})|}{|2\nu \nabla \cdot (\rho \mathcal{S})|} \sim \frac{UL}{\nu}$$

Smoothing out nonlinear things in the fluid

# Turbulence Cascade



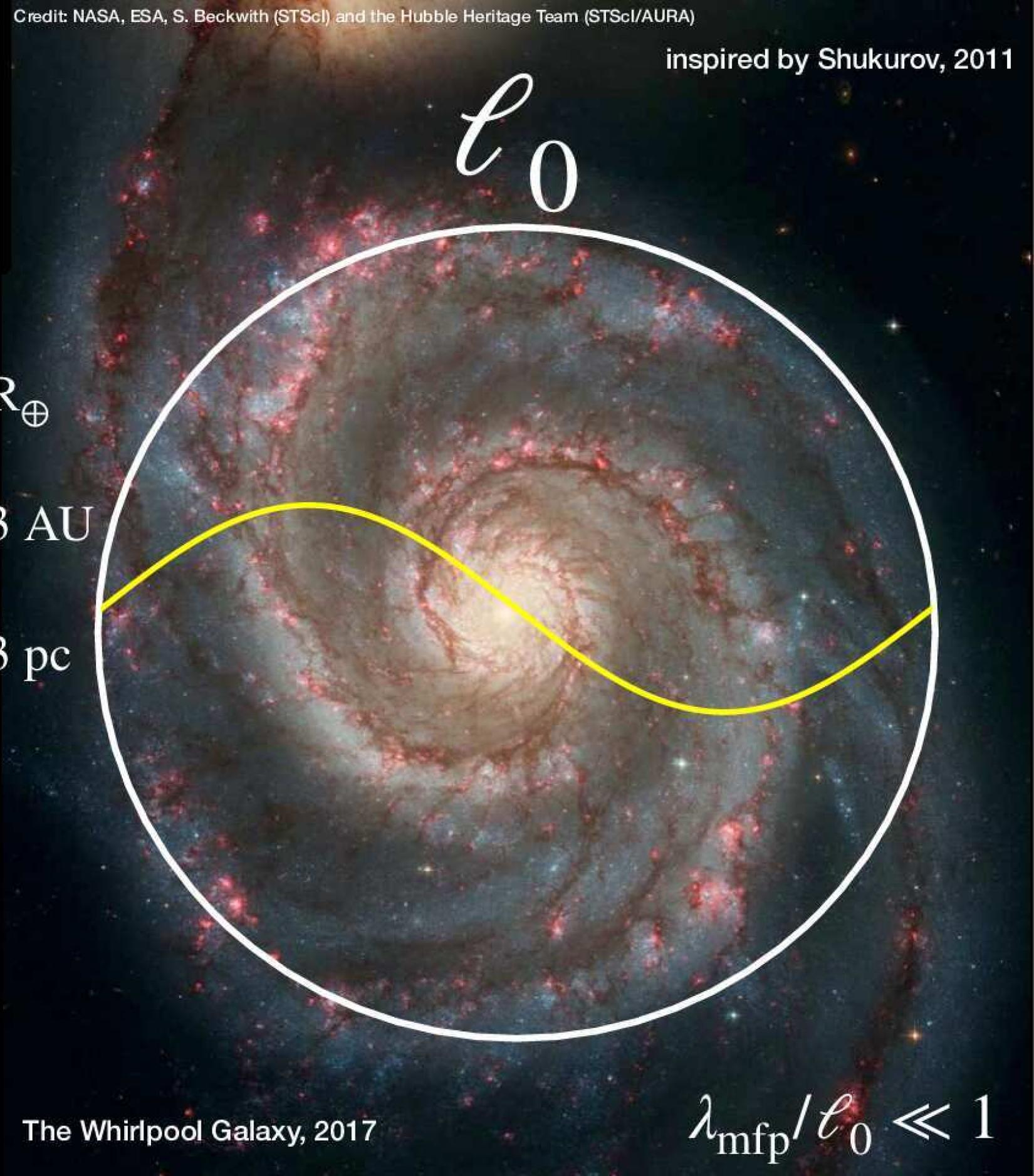
- WIM:  $Re \sim 10^7$      $\lambda_{mfp} \sim 5 R_{\oplus}$
- WNM:  $Re \sim 10^7$      $\lambda_{mfp} \sim 1.3 \text{ AU}$
- CNM:  $Re \sim 10^{10}$      $\lambda_{mfp} \sim 0.3 \text{ pc}$

Ferrière, 2020;

The quadratic nonlinear term

$$|\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})|$$

dominates on large scales,  $\ell$



The Whirlpool Galaxy, 2017

$$\lambda_{mfp} / \ell_0 \ll 1$$

# Turbulence

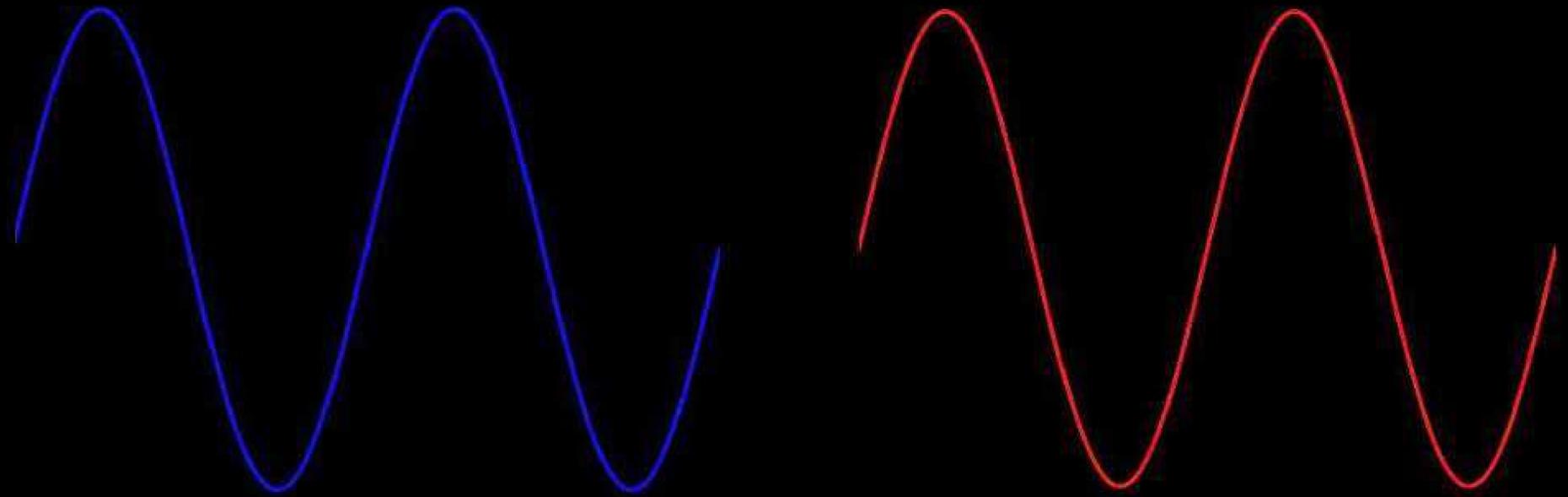
What does  $\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})$  want to do?

$$k_1 = k_2$$

Consider two waves

$$u_1(x) = \sin(k_1 x)$$

$$u_2(x) = \sin(k_2 x)$$



The nonlinear term creates a new wave

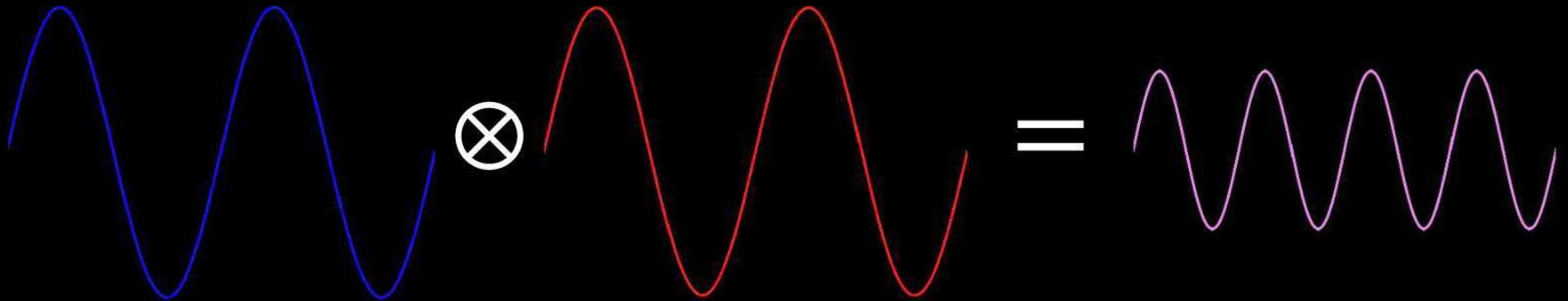
$$\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) \sim u_1 \partial_x u_2 = k_2 \sin(k_1 x) \cos(k_2 x) \propto \sin(k_3 x)$$

$$k_1 + k_2 = k_3 \quad (\text{momentum conservation})$$

# Turbulence

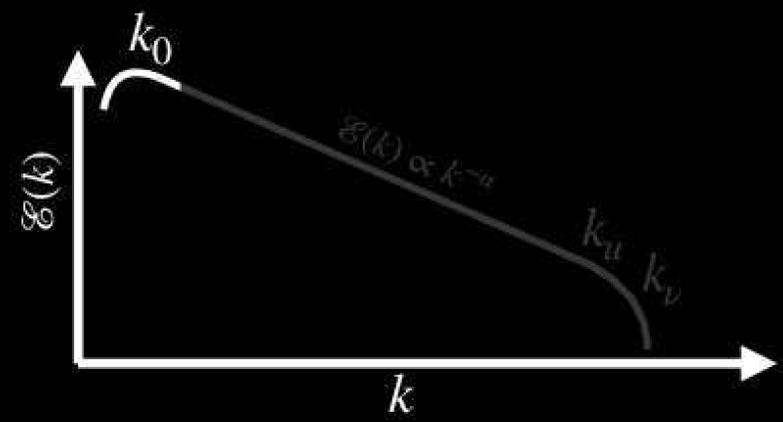
What does  $\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})$  want to do — cascade!!!

$$\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})$$



$$k_1 + k_2 = k_3$$

# Turbulence Cascade



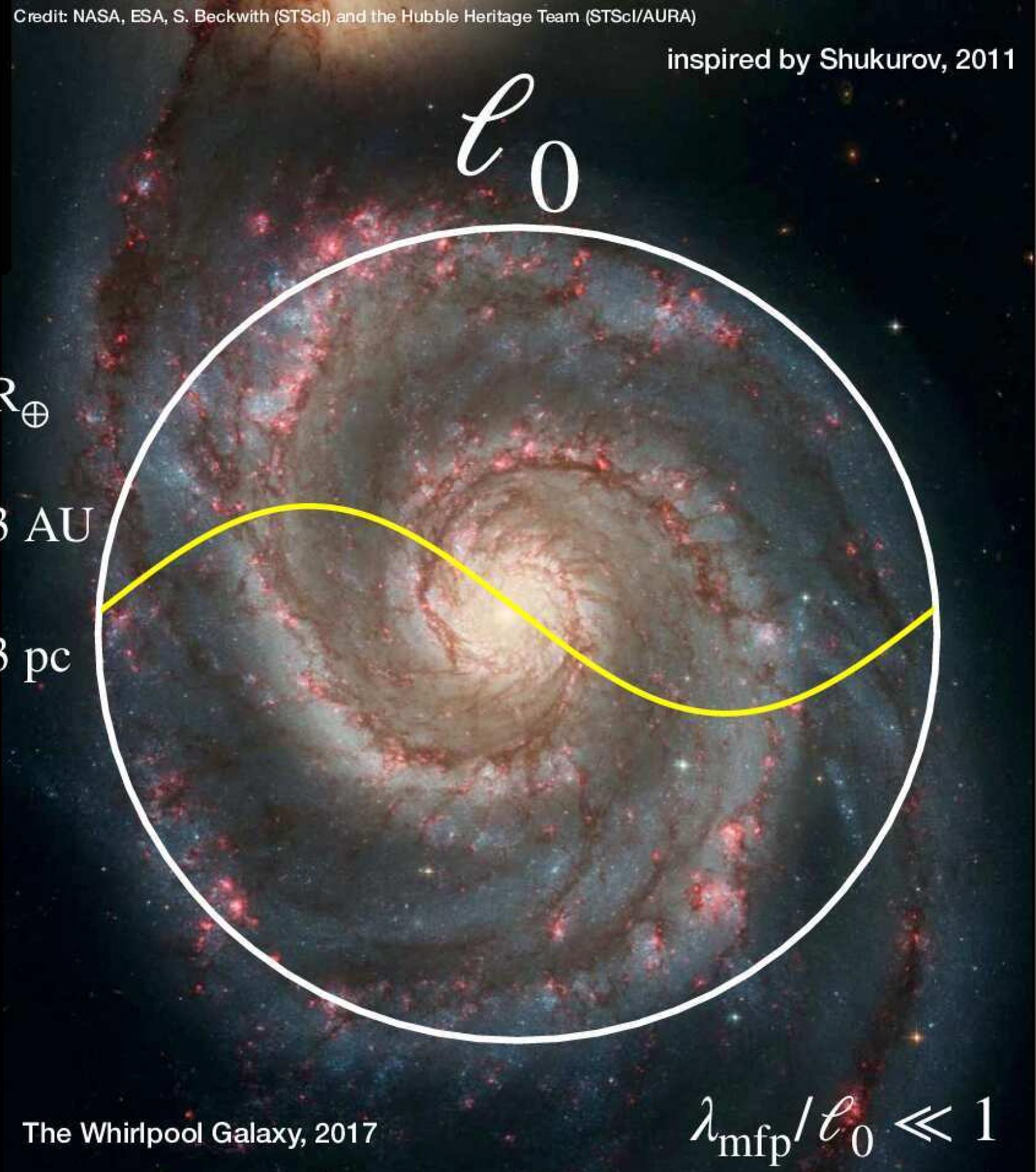
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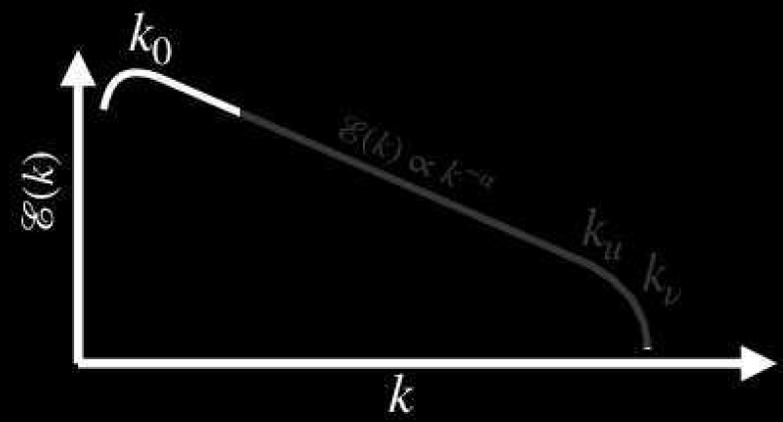
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The Whirlpool Galaxy, 2017

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# Turbulence Cascade



The quadratic nonlinear term

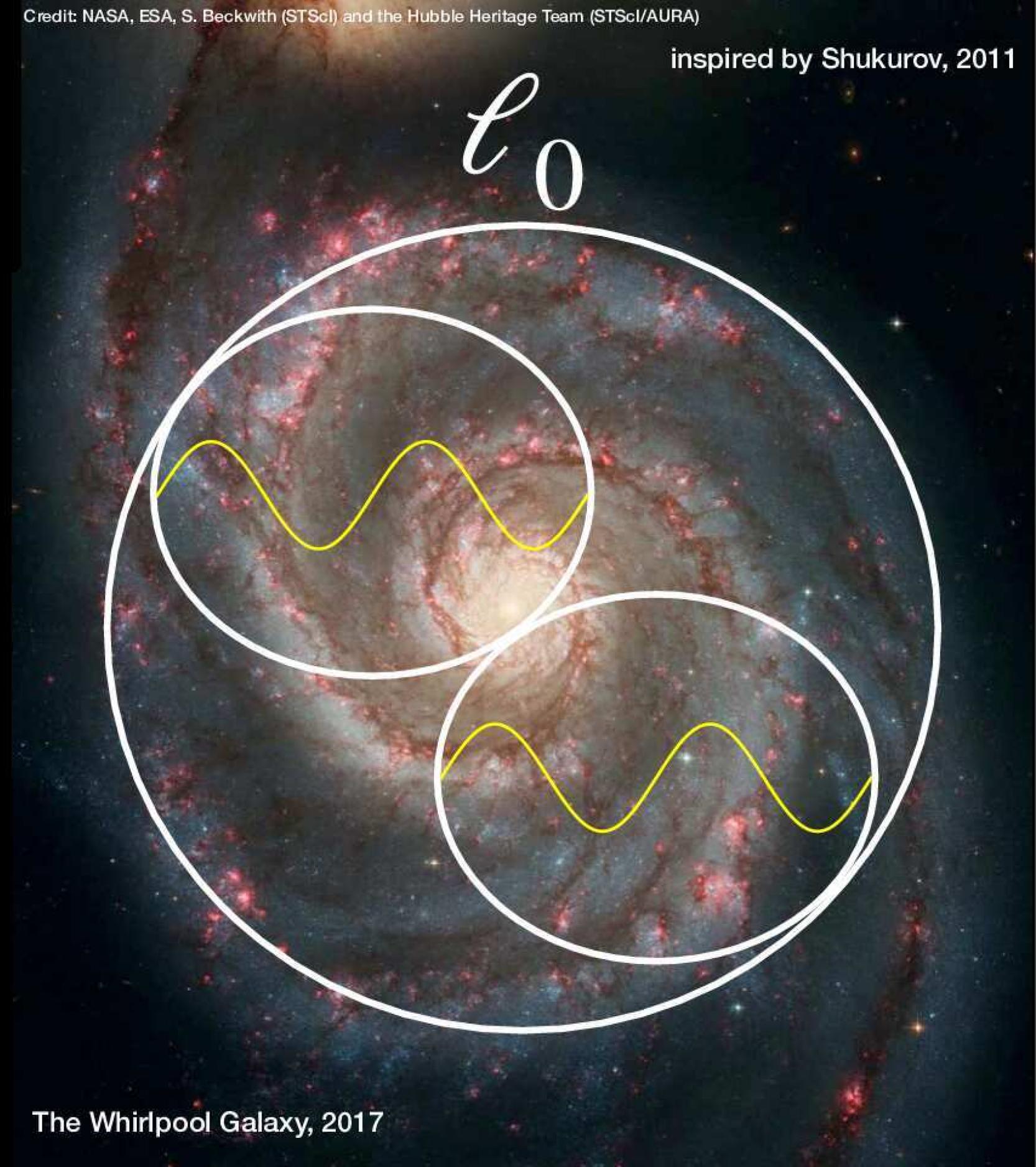
$$|\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})|$$

dominates on large scales,  $\ell$

Creates new modes on

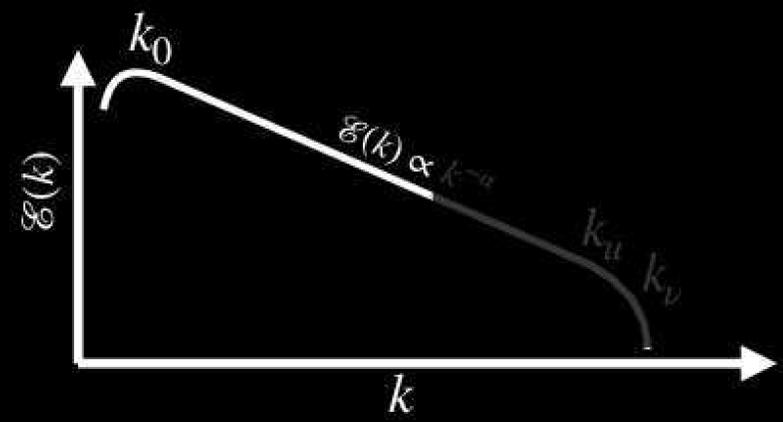
$$t_{nl} \sim L/U \sim \mathcal{O}(\text{Gyr})$$

$$\dot{\epsilon} \sim u_0^3 / \ell_0$$

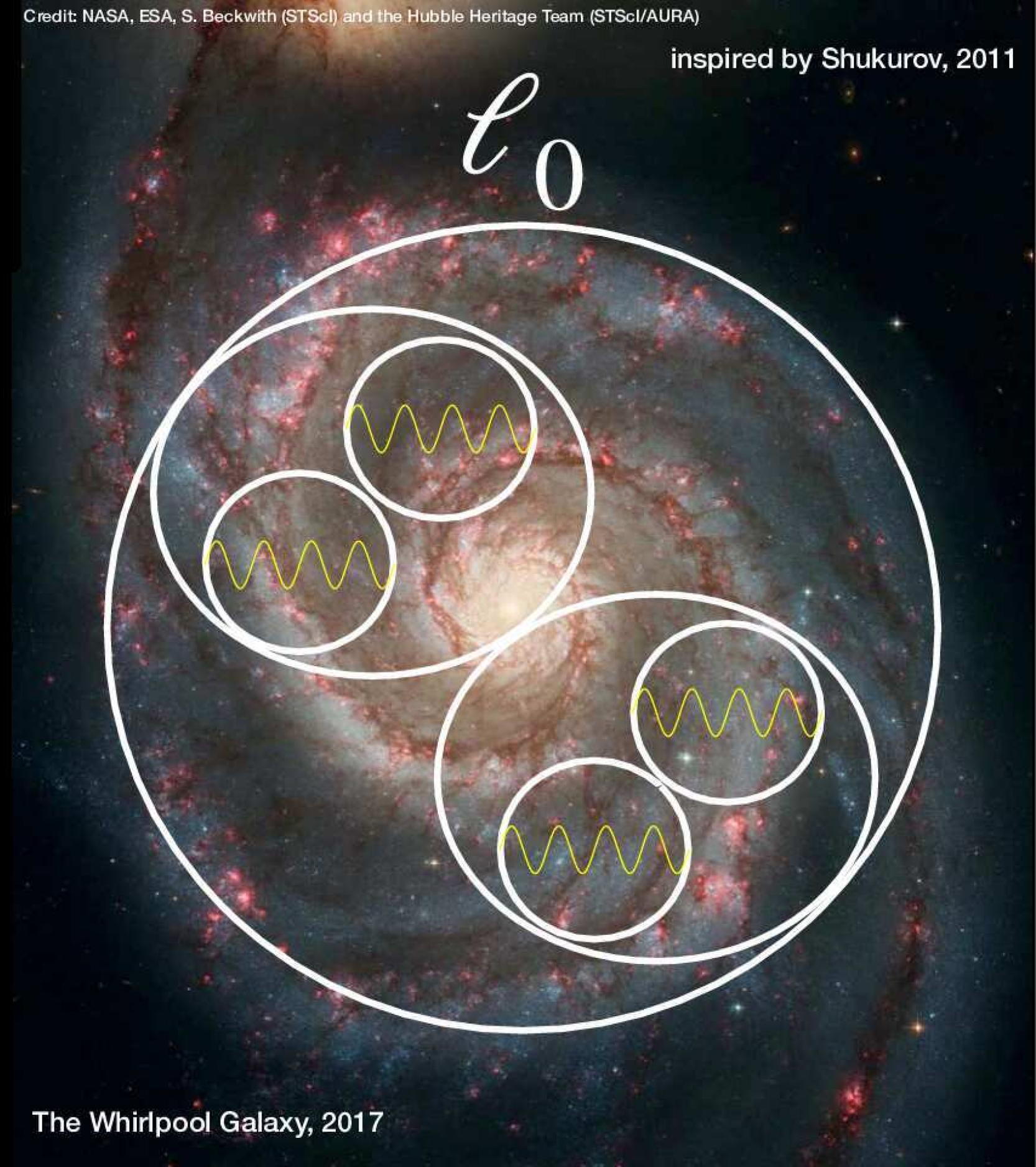


The Whirlpool Galaxy, 2017

# Turbulence Cascade

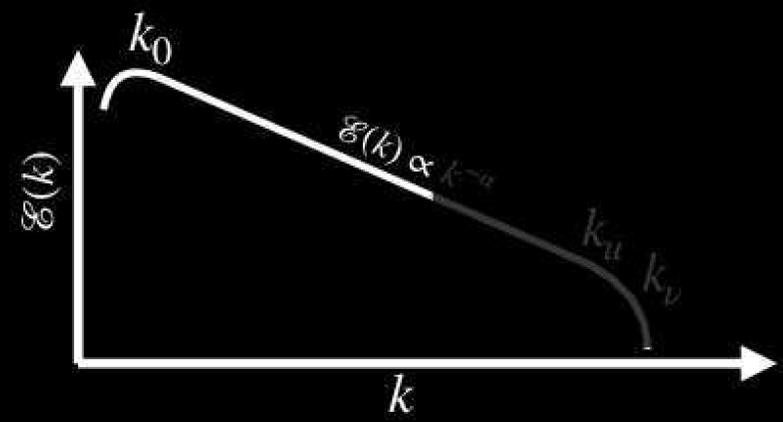


inside of the cascade



The Whirlpool Galaxy, 2017

# Turbulence Cascade

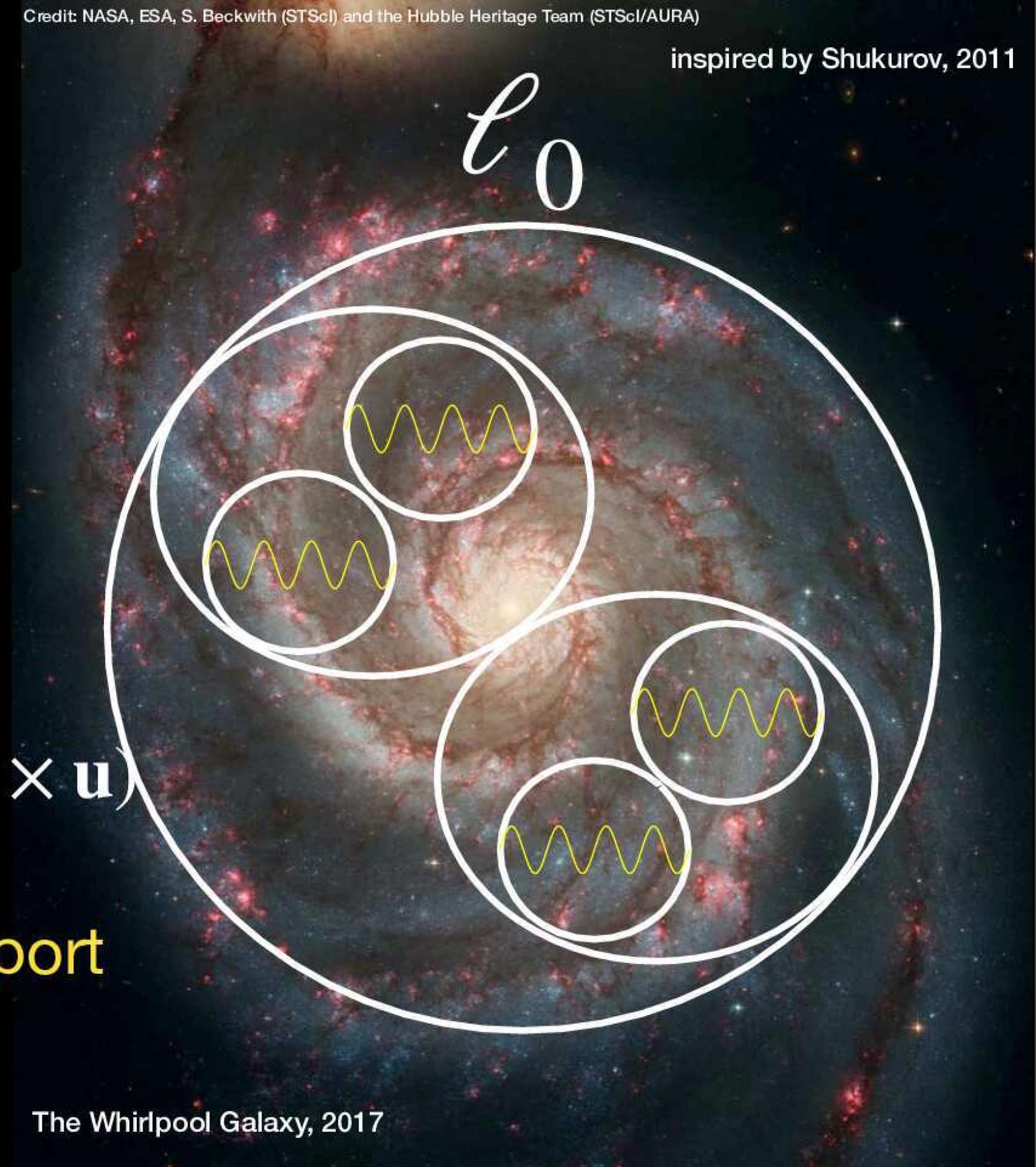


inside of the cascade

The quadratic nonlinear term

$$\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) = \underbrace{1/2 \nabla u^2}_{\text{turbulent pressure}} - \mathbf{u} \times (\nabla \times \mathbf{u})$$

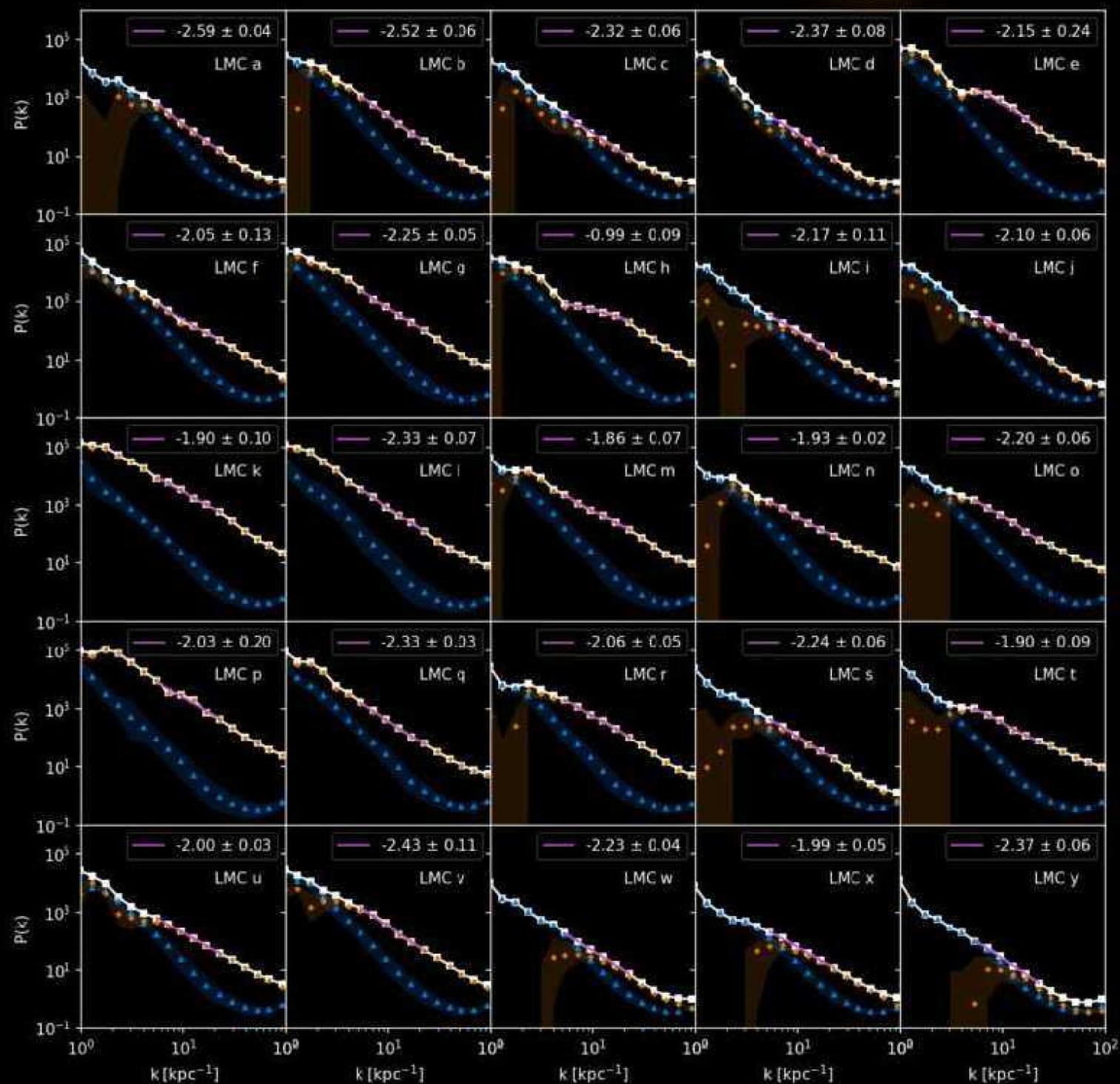
Turbulent pressure = turbulent support



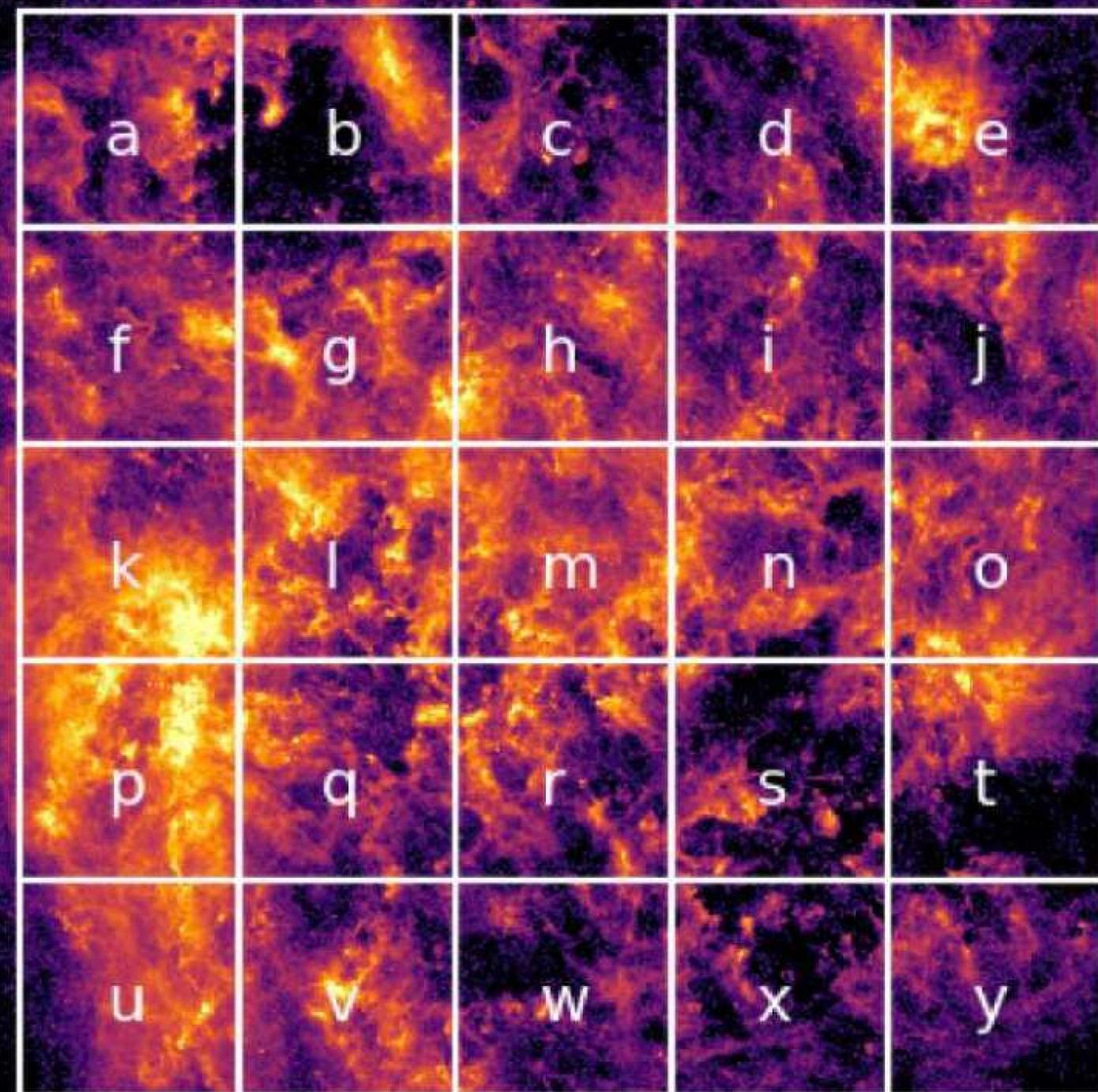
# Turbulence



LMC:  $500\mu\text{m}$ , *Herschel* (processing Gordon et al. 2014)



1 kpc

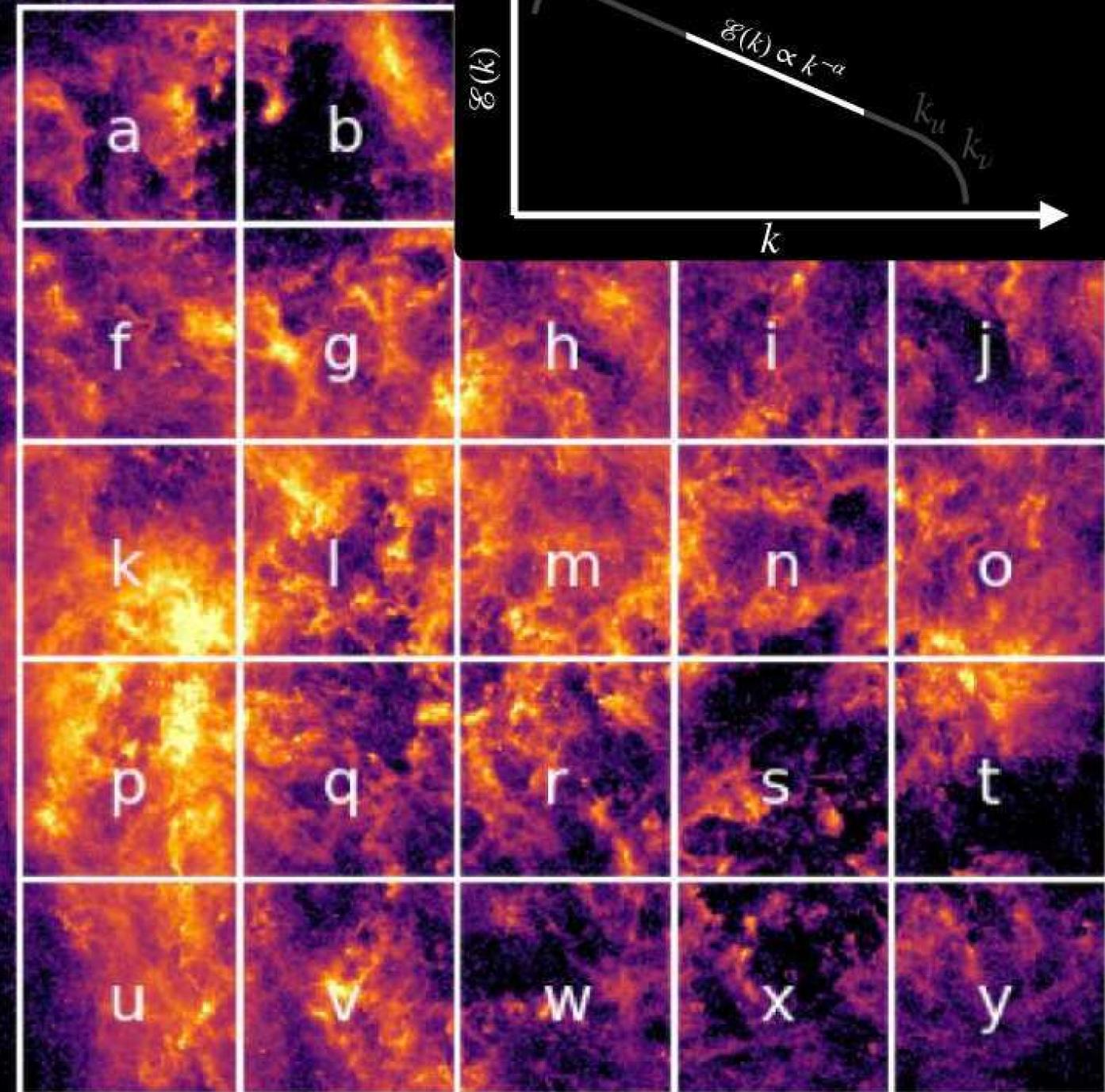
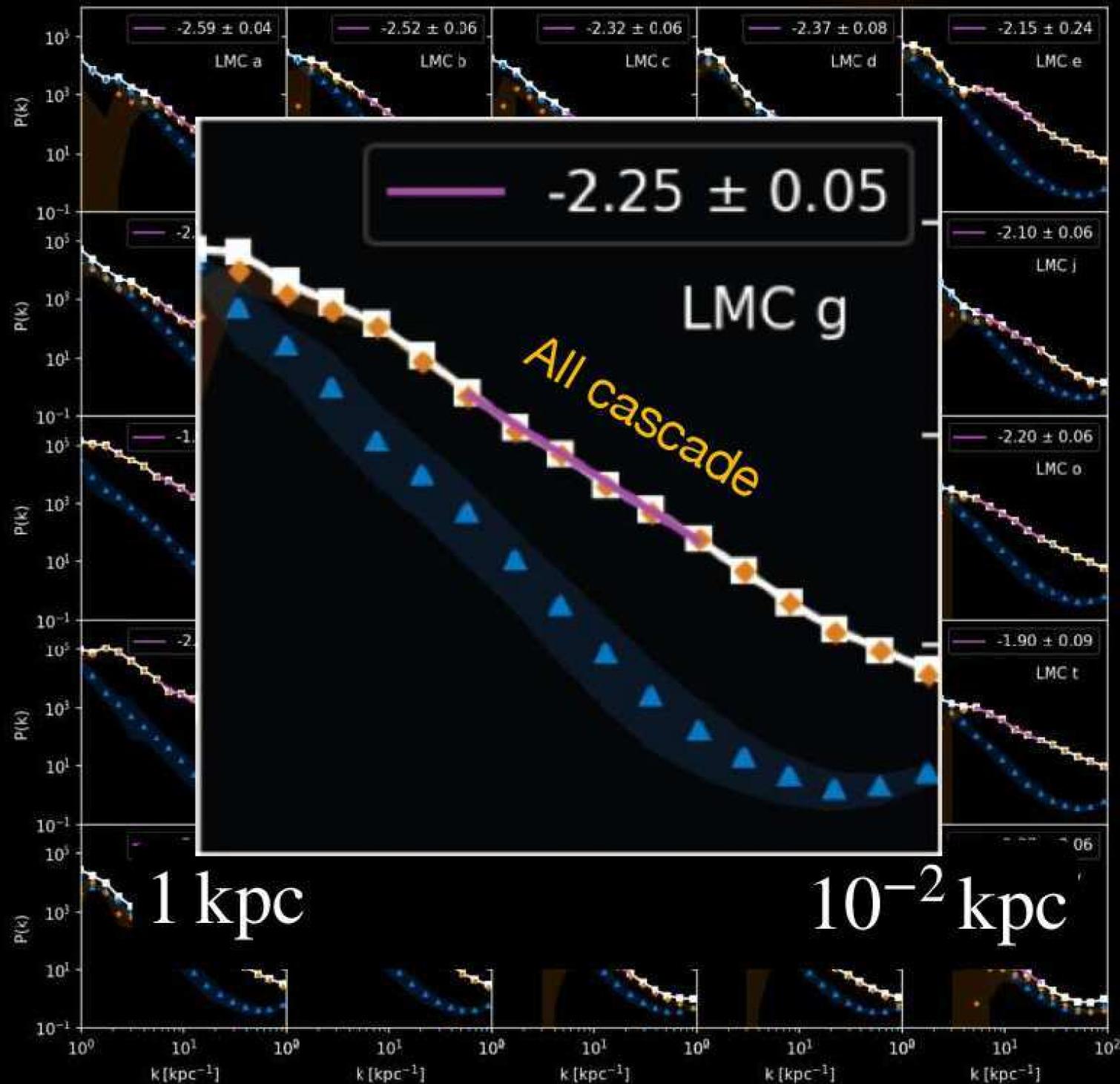


Colman, et al., 2022, MNRAS

# Turbulence

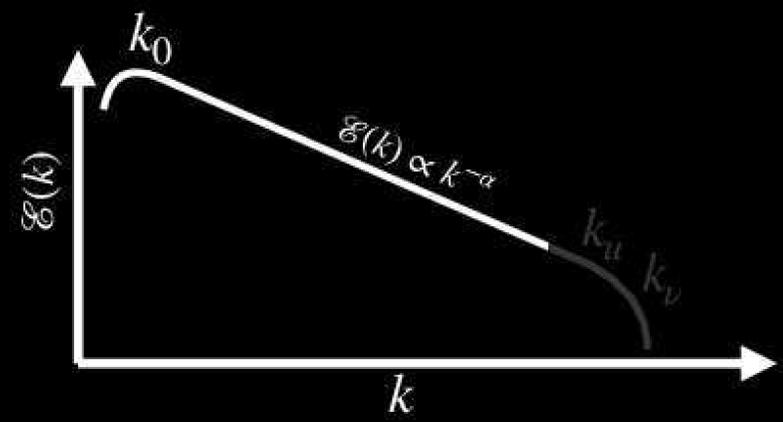


LMC: 500 $\mu$ m, *Herschel* (processing Gordon et al. 2014)



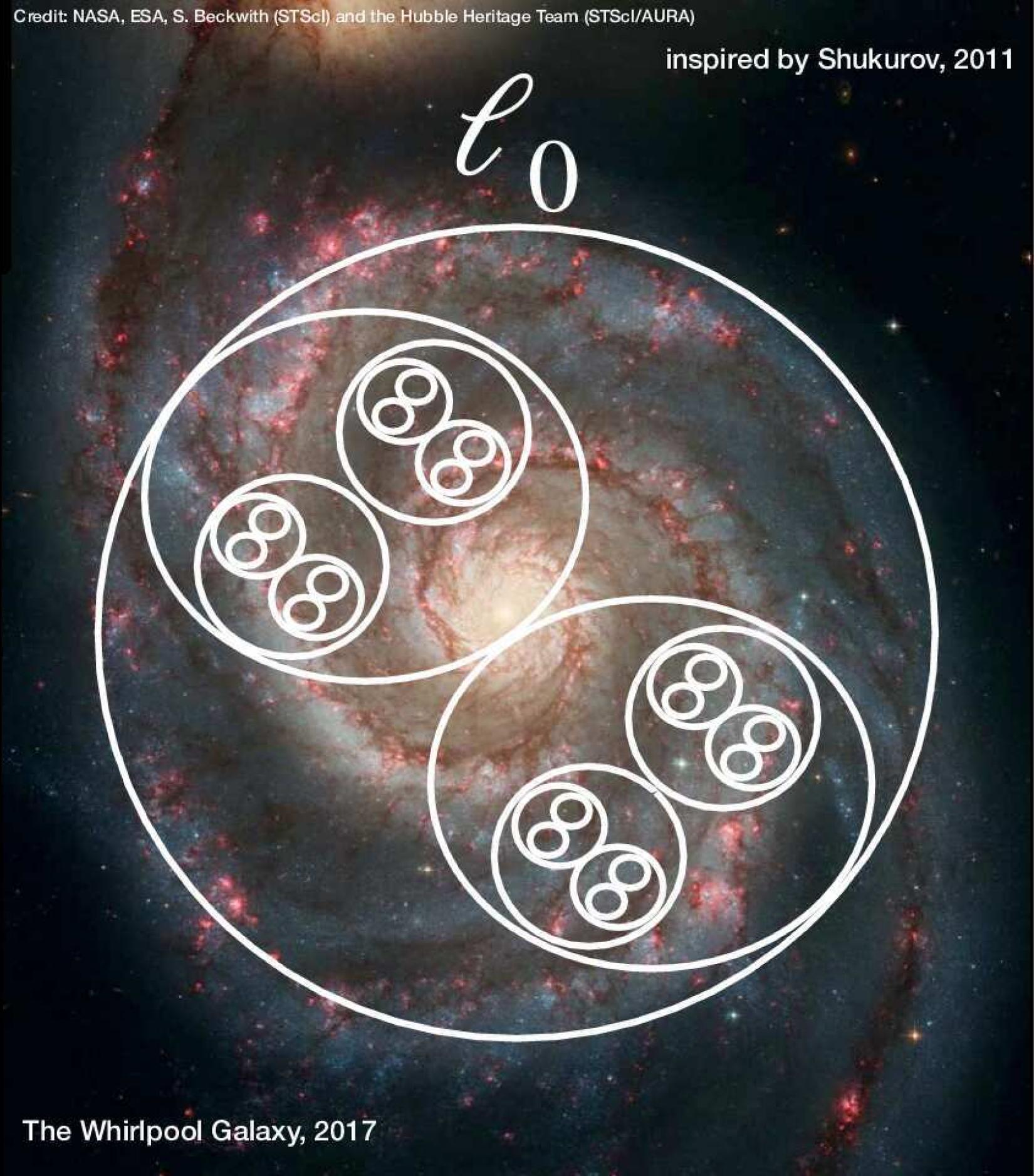
Colman, et al., 2022, MNRAS

# Turbulence Cascade



Reynolds number is shrinking

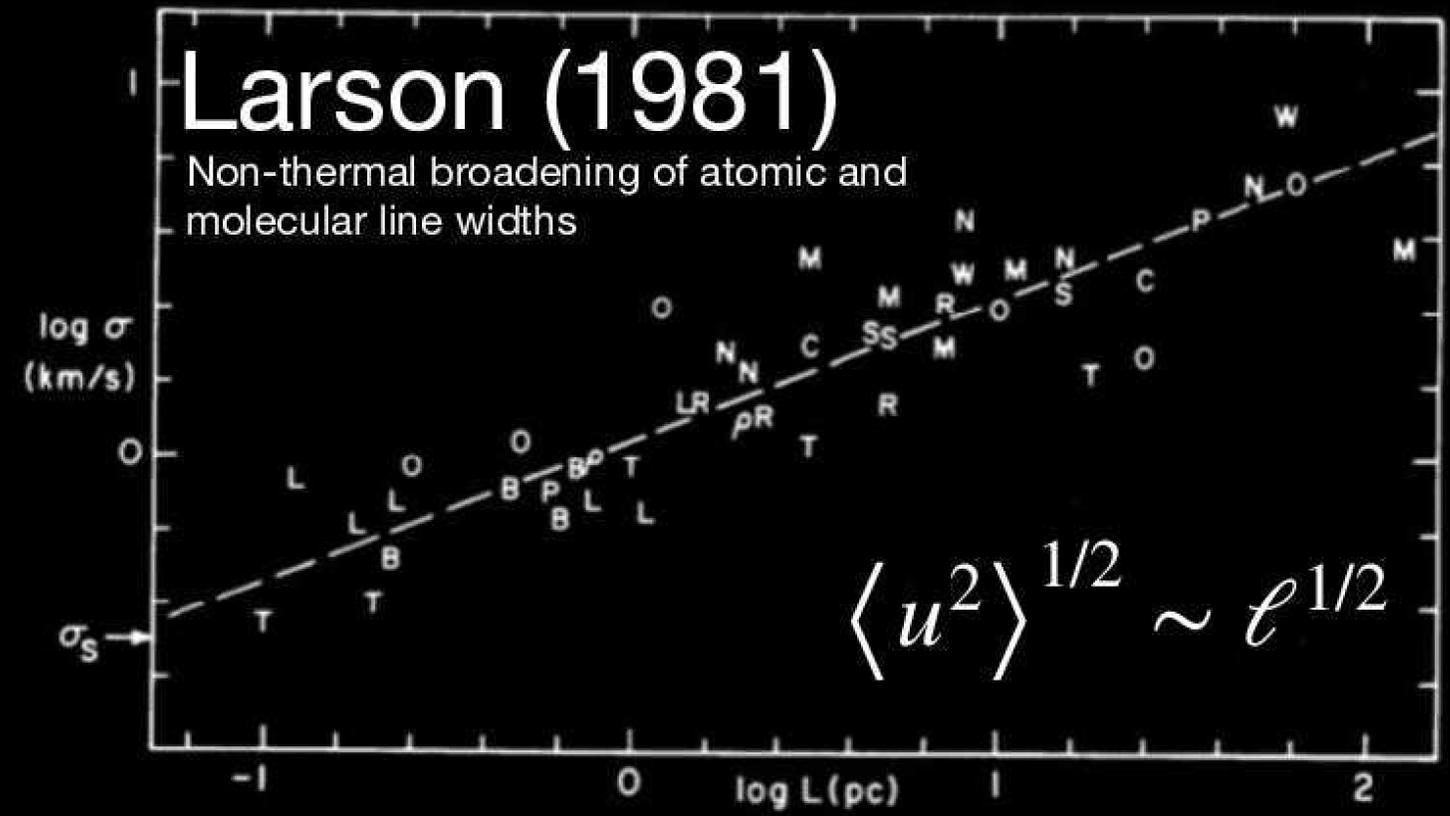
$$Re \sim \langle u^2 \rangle^{1/2} \ell / \nu \sim \ell^{3/2} / \nu$$



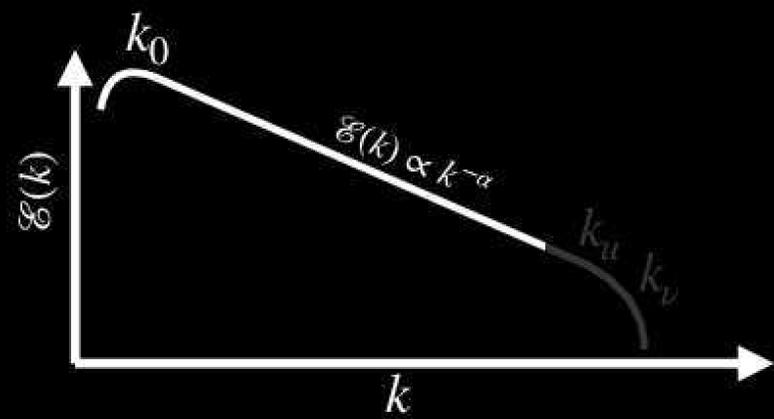
The Whirlpool Galaxy, 2017

## Larson (1981)

Non-thermal broadening of atomic and molecular line widths

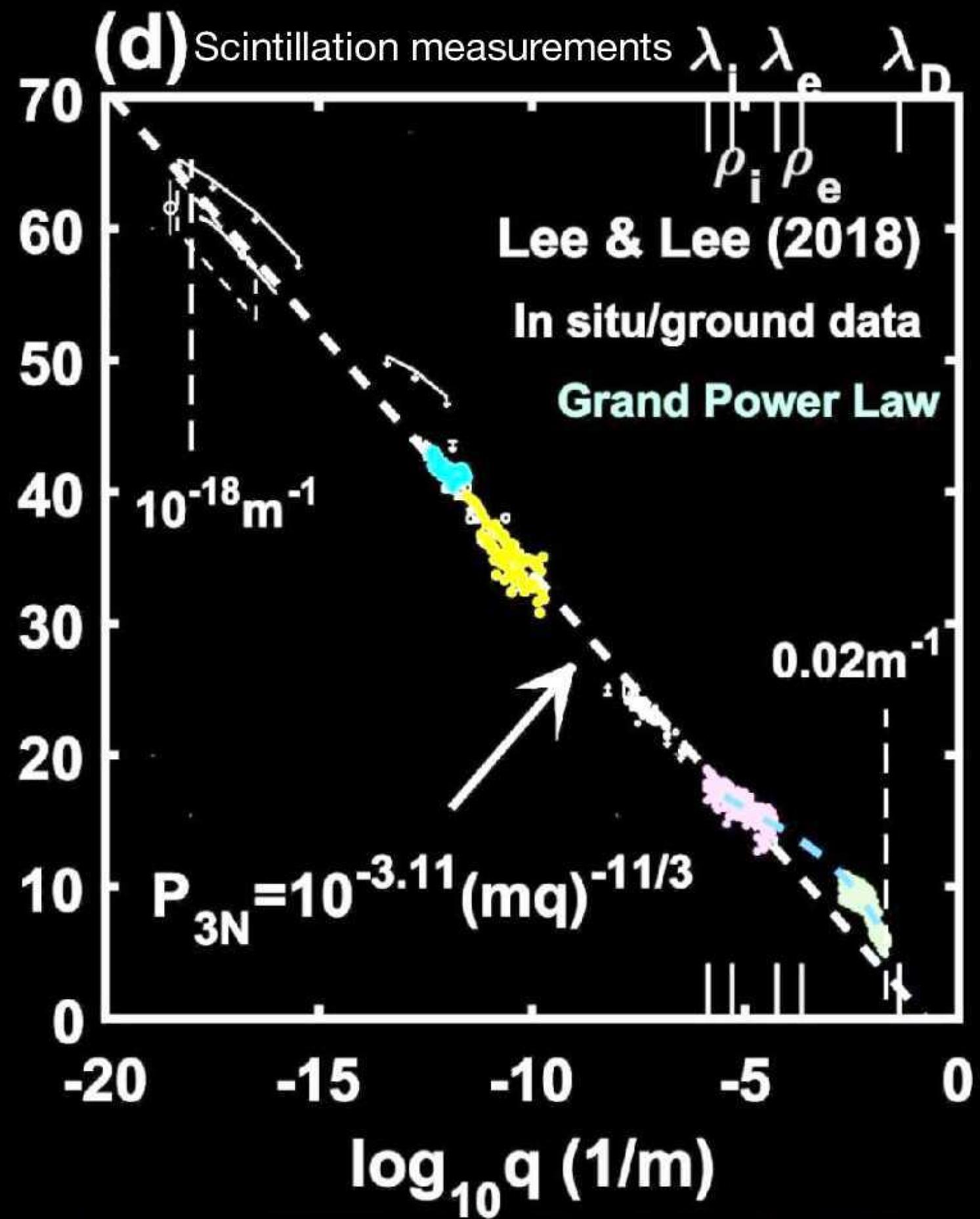
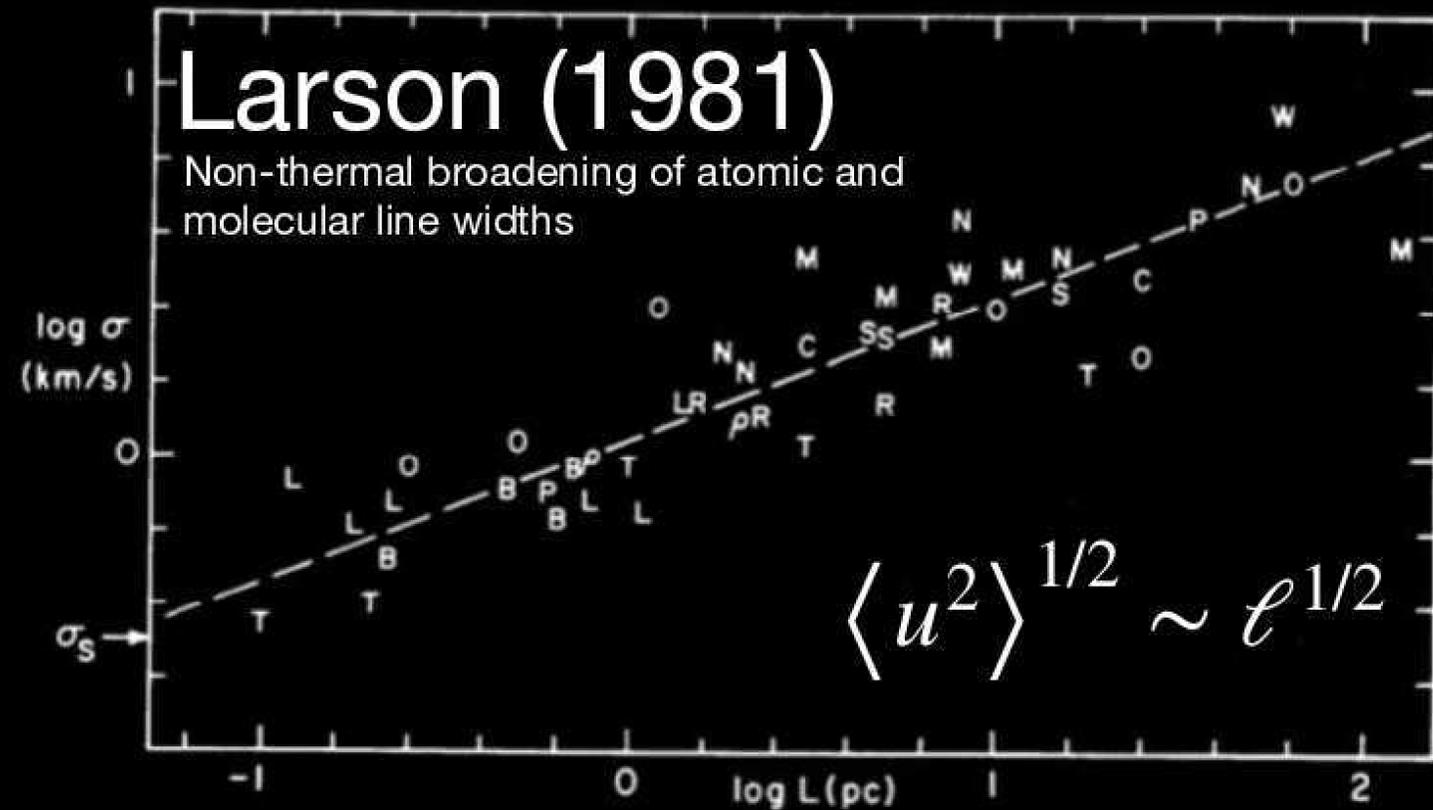


# Turbulence Cascade



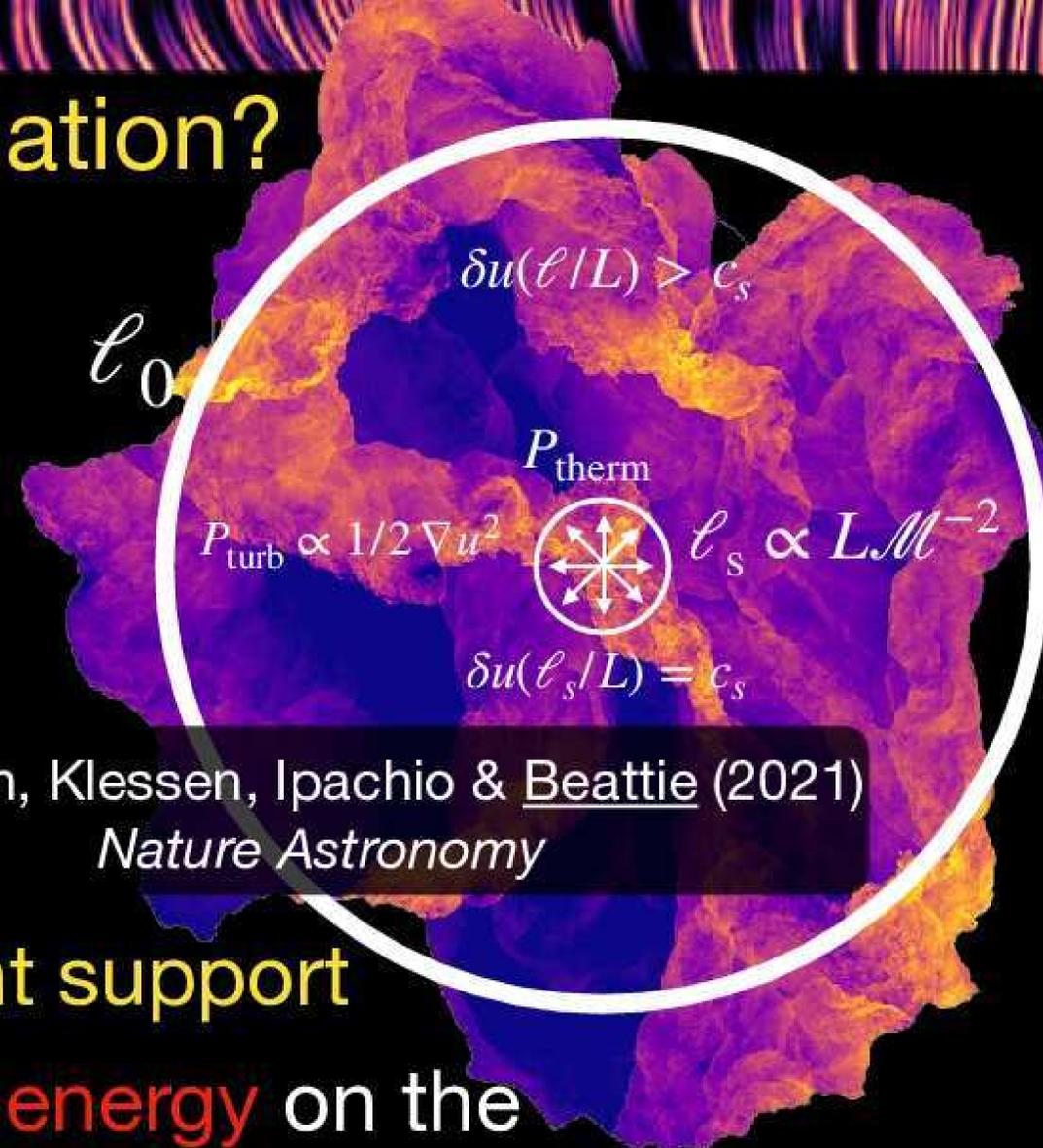
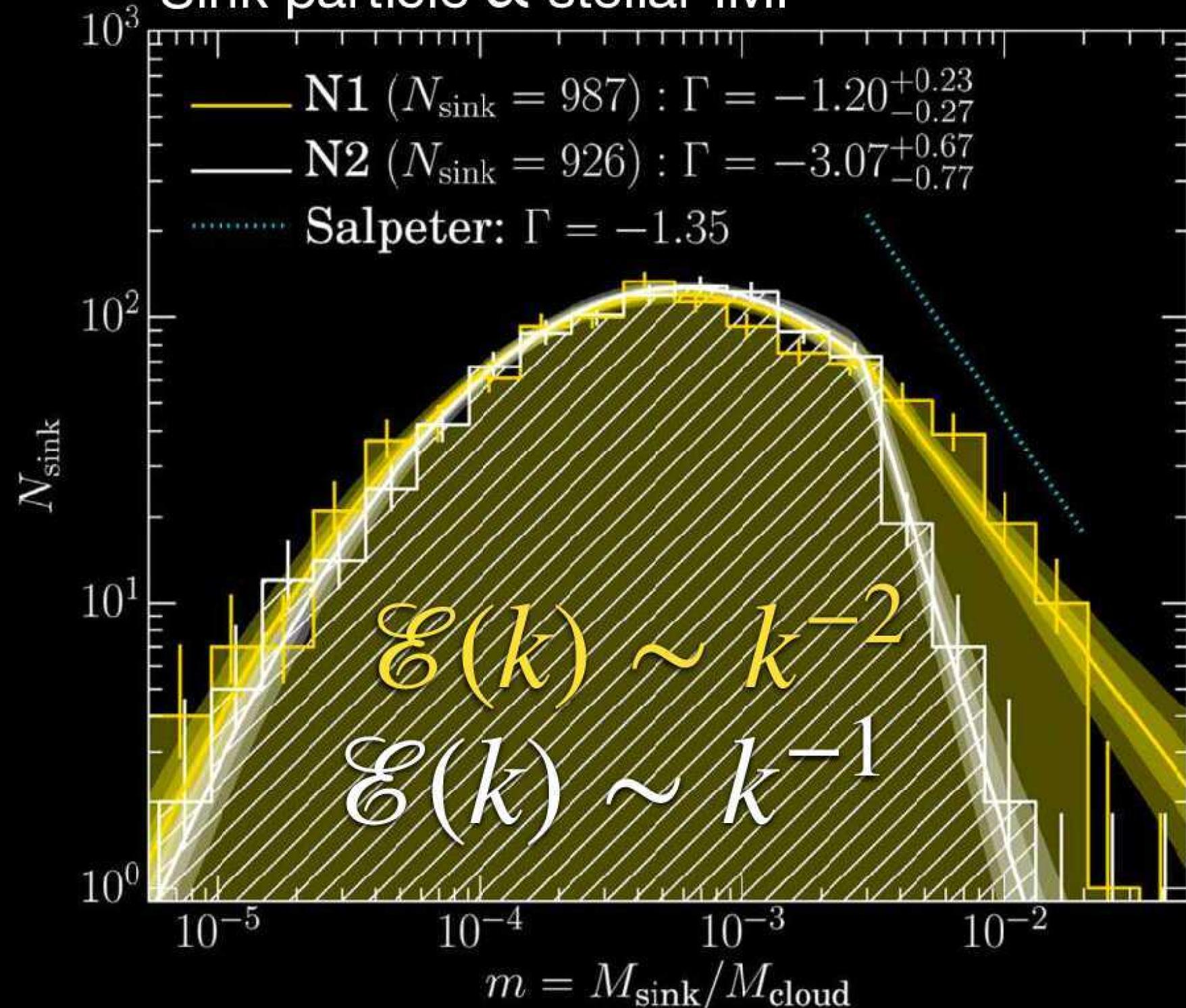
Reynolds number is shrinking

$$Re \sim \langle u^2 \rangle^{1/2} \ell / \nu \sim \ell^{3/2} / \nu$$



# Why should astrophysicists care — star formation?

Sink particle  $\propto$  stellar IMF



Federrath, Klessen, Ipachio & Beattie (2021)  
*Nature Astronomy*

## Turbulent support

More energy on the small-scales (more small-scale support) implies less high-mass stars formation

$$\mathbf{u} \cdot \nabla \otimes \mathbf{u} = \frac{1}{2} \nabla u^2 - \mathbf{u} \times \nabla \times \mathbf{u}$$

# Why should astrophysicists care — cosmic ray transport?

## Testing Physical Models for Cosmic Ray Transport Coefficients on Galactic Scales: **Self-Confinement** and **Extrinsic Turbulence** at $\sim$ GeV Energies

Philip F. Hopkins<sup>1</sup>, Jonathan Squire<sup>2</sup>, T. K. Chan<sup>3,4</sup>, Eliot Quataert<sup>5</sup>,  
Suoqing Ji<sup>1</sup>, Dušan Kereš<sup>2</sup>, Claude-André Faucher-Giguère<sup>6</sup>

Sampson, Beattie et al. (2023, MNRAS). *Turbulent diffusion of streaming cosmic rays in compressible, partially ionised plasma*

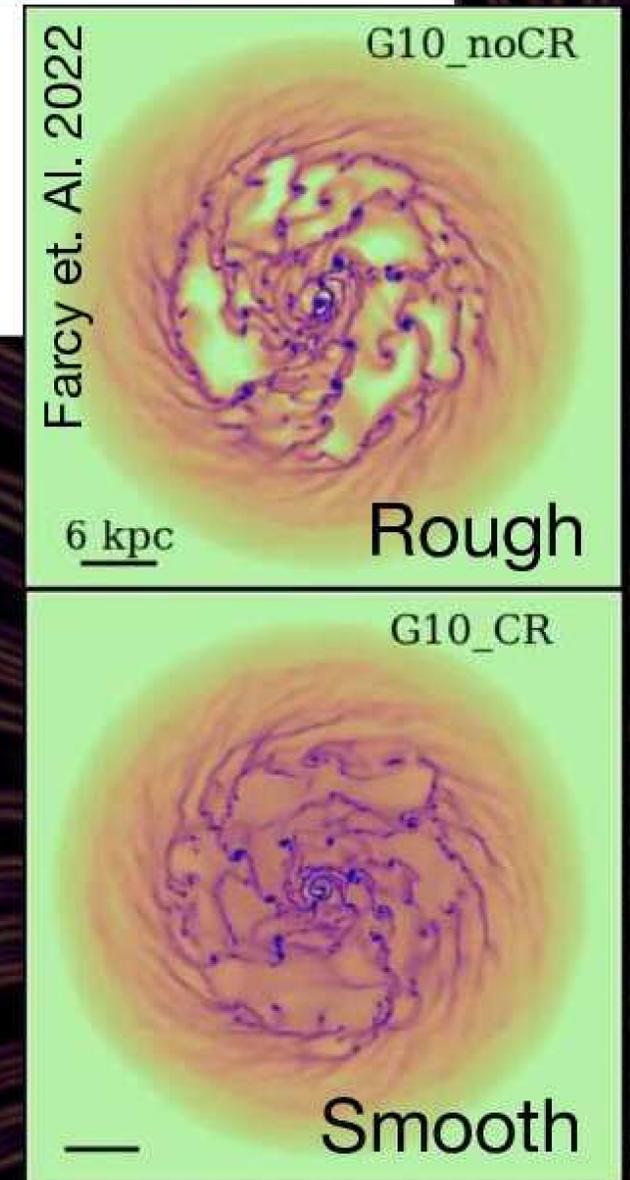
Provides source of diffusion on large-scales

Beattie et al. (2023, Frontiers). *Ion Alfvén velocity fluctuations and implications for the diffusion of streaming cosmic rays*

The energy cascade — fundamental for CR transport

Determines the properties of turbulence

CRs add extra pressure support + smooth plasma gradients



# Two great insights from Kolmogorov

- Constant energy flux between modes
- No magnetic field
- No inhomogeneities
- No density fluctuations
- Isotropic

$$\varepsilon \sim u_\ell^3 / \ell \sim \text{const.}$$

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$$u_\ell \sim (\varepsilon \ell)^{1/3}$$

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$$u_\ell \sim (\varepsilon \ell)^{1/3} \iff u^2(k) \sim k^{-5/3} dk$$

# Two great insights from Kolmogorov

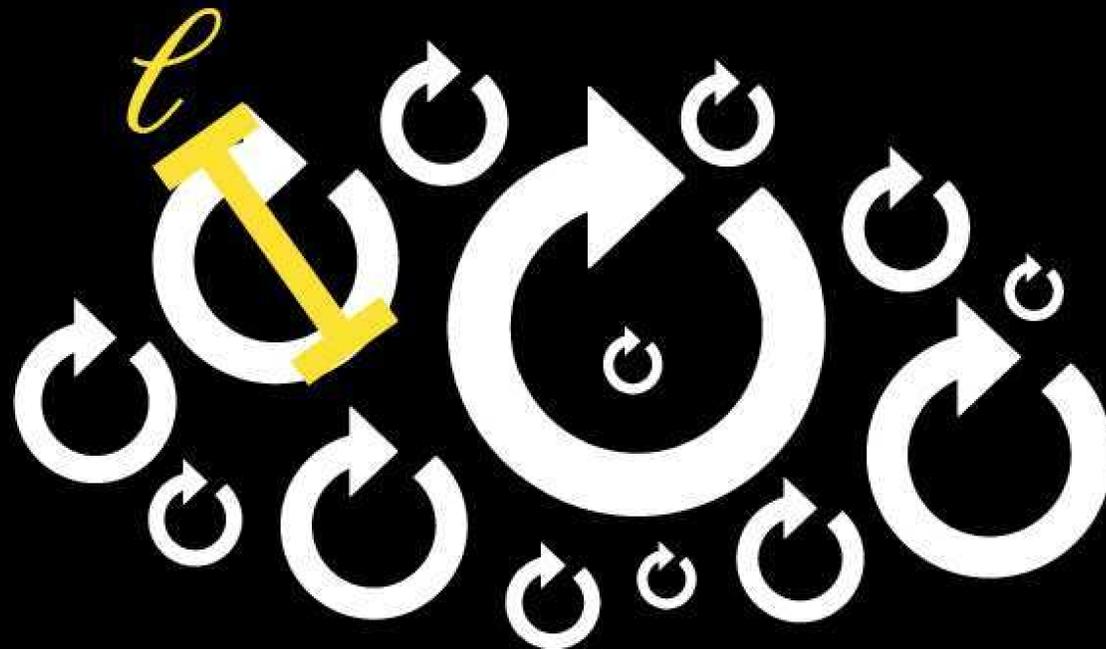
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$$u_\ell \sim (\varepsilon \ell)^{1/3}$$

“structure function”

$$u(\ell) = \langle |u(\mathbf{x}) - u(\mathbf{x} + \ell) \cdot \ell| \rangle_{\mathbf{x}} \sim \ell^{1/3}$$

$\ell$  picks out an eddy



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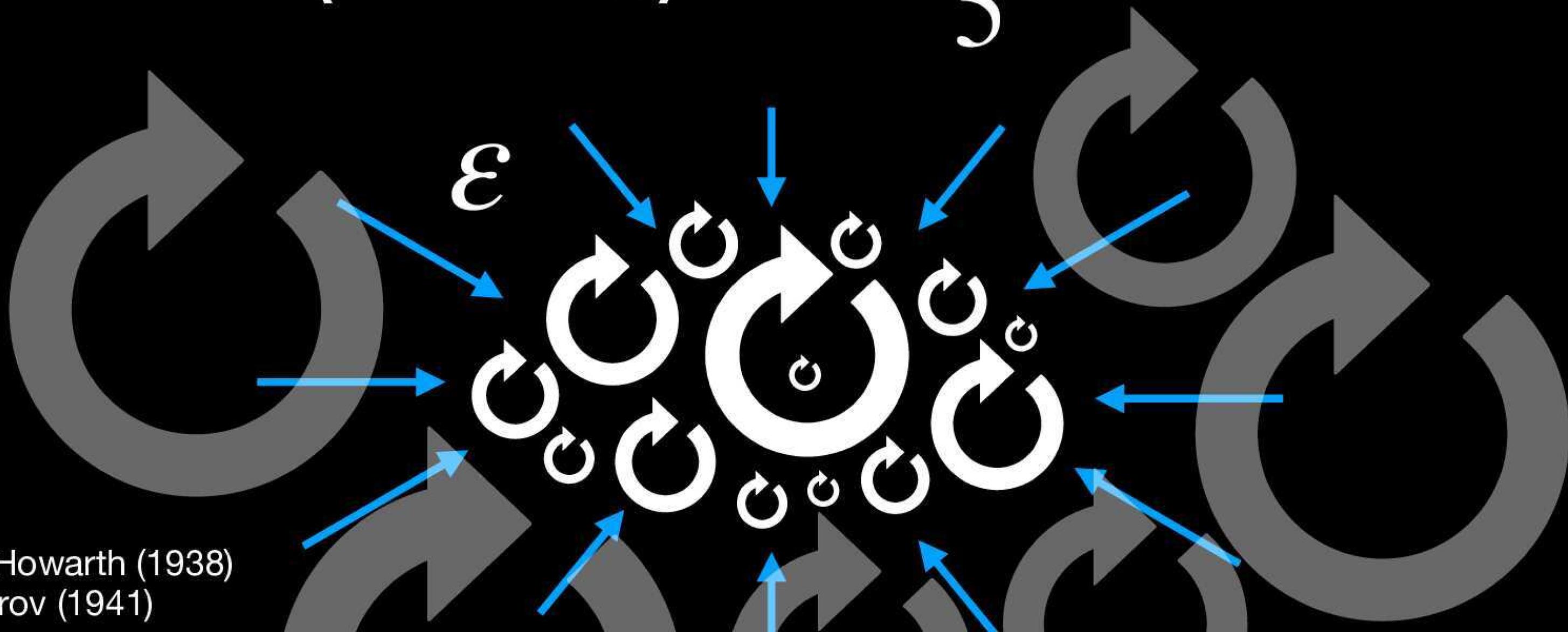
$$u(\ell) = \left\langle |u(\mathbf{x}) - u(\mathbf{x} + \ell) \cdot \ell| \right\rangle_{\mathbf{x}} \sim \ell^{1/3}$$

$$u(\ell) = \left\langle |\delta u_L| \right\rangle_{\mathbf{x}} \sim \ell^{1/3}$$

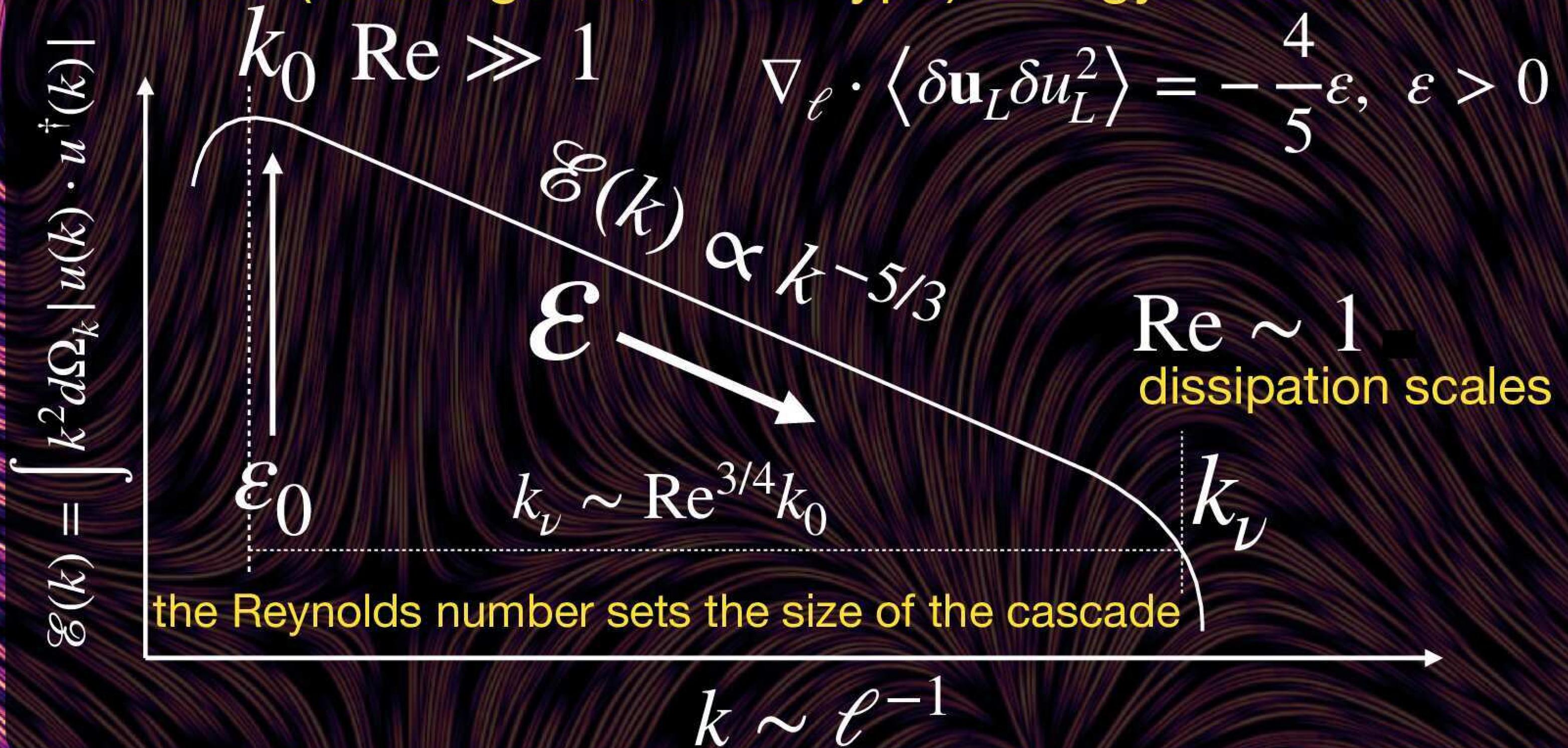
# An exact relation for turbulence

## Kolmogorov's 4/5 law

$$\nabla_{\ell} \cdot \langle \delta \mathbf{u}_L \delta u_L^2 \rangle = -\frac{4}{5} \varepsilon, \quad \varepsilon > 0$$



# The (Kolmogorov, 1941-type) energy cascade



Fact: the galaxy that we reside in is in a state of highly-compressible fluid turbulence

$$E_{\text{SNe}} \approx 10^{51} \text{ erg}$$

$$\gamma_{\text{SNe}} \approx 0.01 - 0.03 \text{ yr}^{-1} \quad \text{Diehl + (2016)}$$

$$\dot{E}_{\text{SNe}} \approx E_{\text{SNe}} \gamma_{\text{SNe}} \approx 10^{41} \text{ erg s}^{-1}$$

$$E_{\text{turb}} \approx 10^{54} \text{ erg}$$

$$t_{\text{turb}} \approx \ell_0 / \langle u^2 \rangle^{1/2} \approx 10^7 \text{ yr} \quad \text{Ferriere + (2020)}$$

$$\dot{E}_{\text{turb}} \approx E_{\text{turb}} / t_{\text{turb}} \approx 10^{39} \text{ erg s}^{-1}$$

$$\dot{E}_{\text{turb}} / \dot{E}_{\text{SNe}} \approx 10^{-2}$$

CC-supernovae alone provide enough energy flux to fuel a continuously driven cascade

Connor, [Beattie](#) + (2025, ApJ) Cascading from the winds to the disk: universality of supernovae-driven turbulence in different galactic ISMs

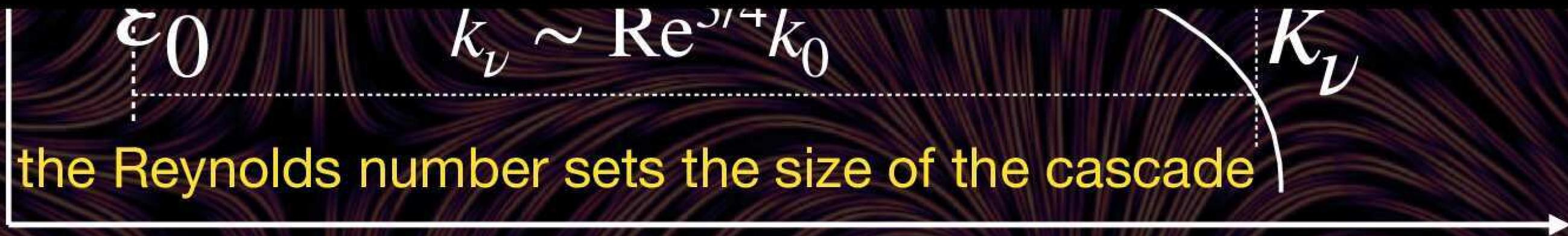
The (Kolmogorov, 1941-type) energy cascade

$$k_0 \text{ Re} \gg 1 \quad \nabla_{\ell} \cdot \langle \delta \mathbf{u}_L \delta u_L^2 \rangle = -\frac{4}{5} \varepsilon, \quad \varepsilon > 0$$

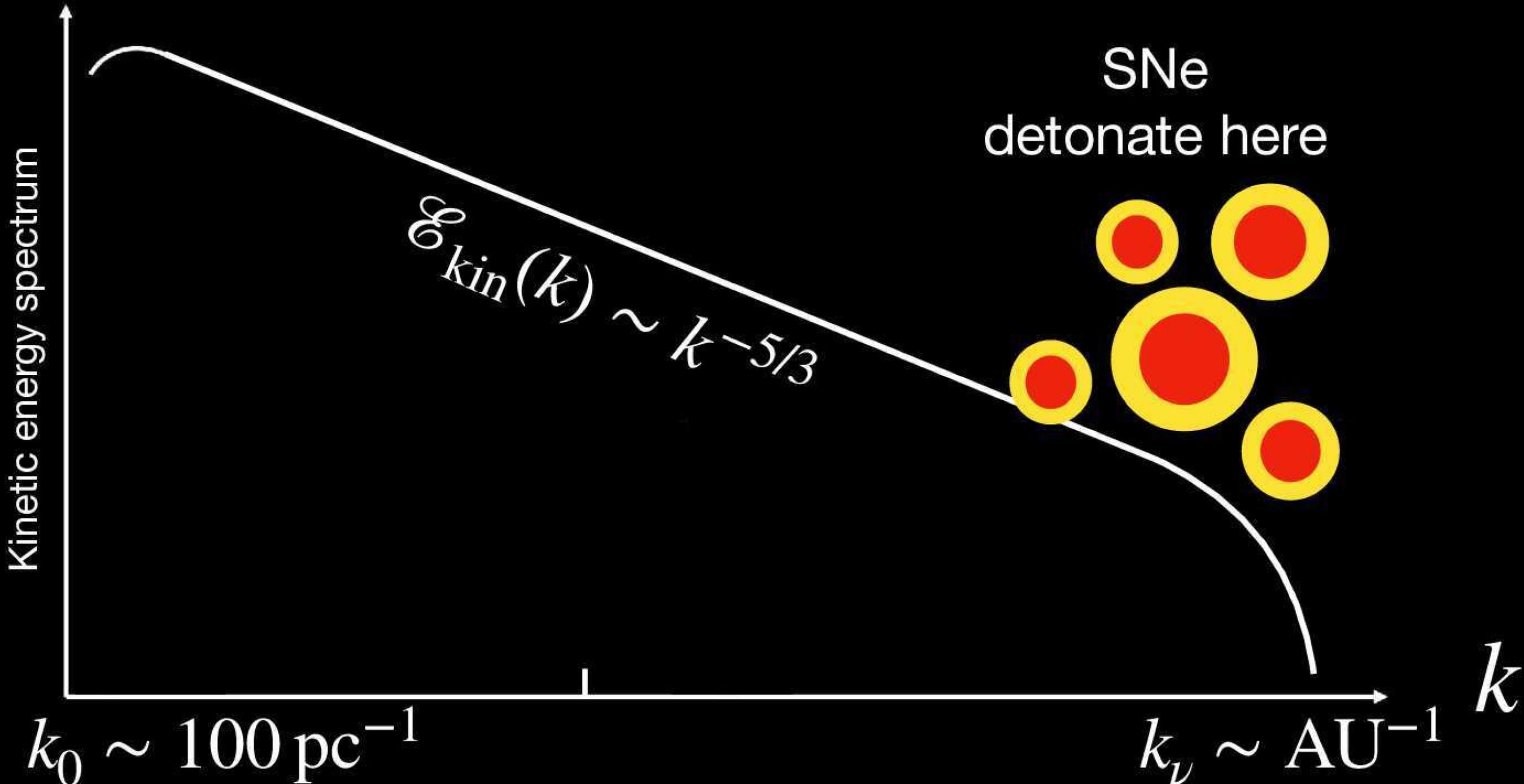
Does it resemble this idea at all?

ales

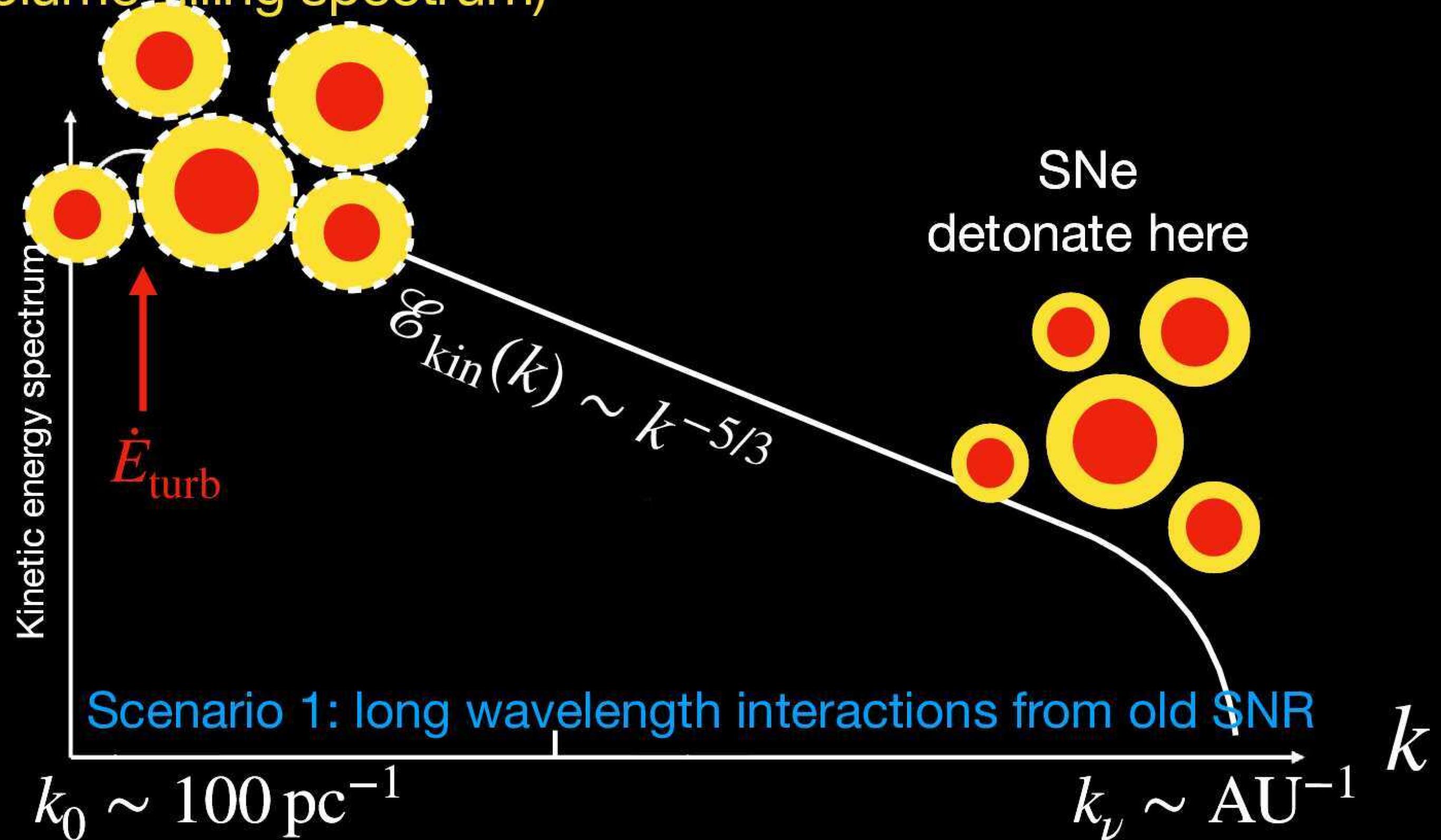
$$\mathcal{E}(k) = \int k^2 d\Omega_k |u(k) \cdot u^*(k)|$$



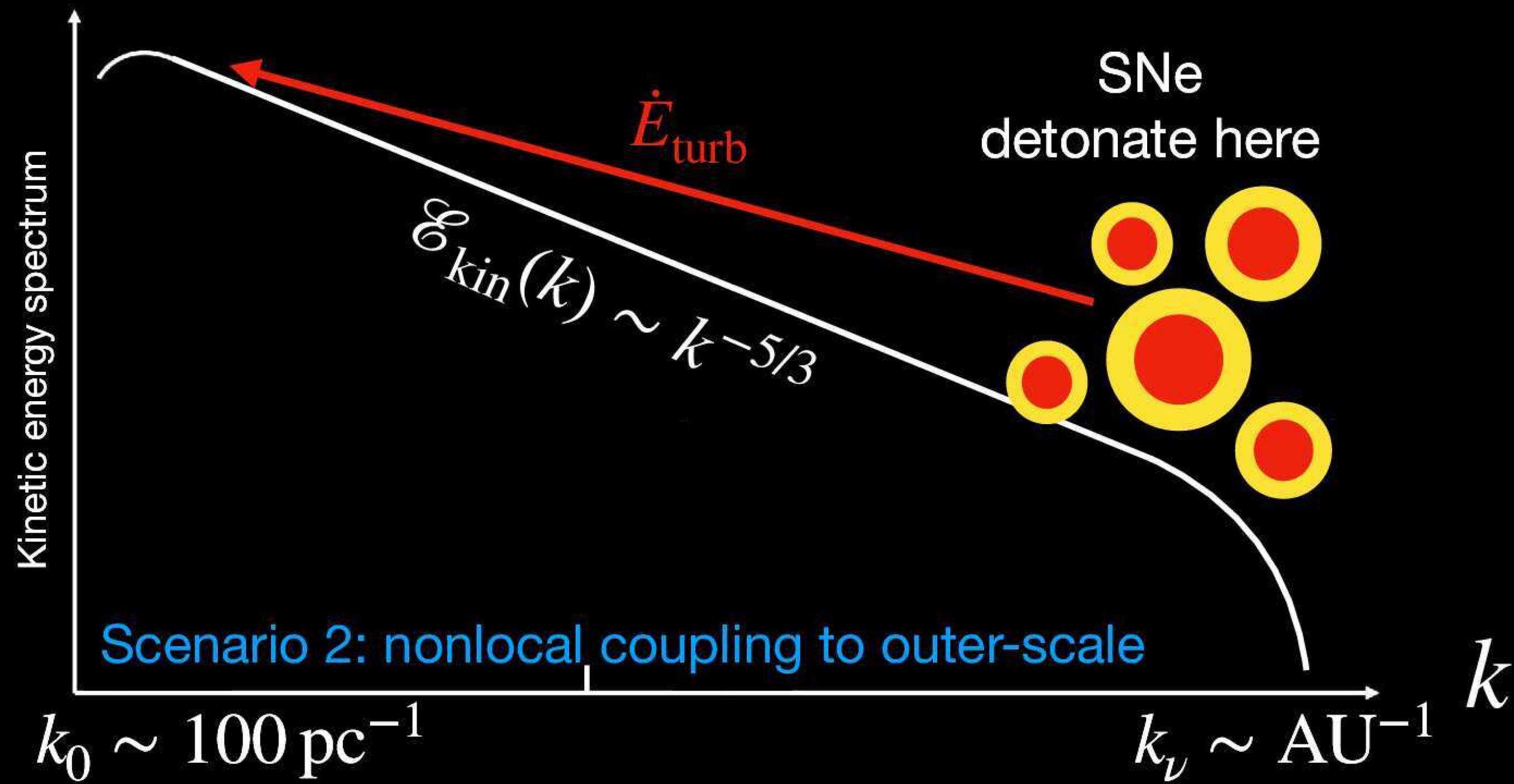
# The naive ( $\mathcal{M} \approx 2$ ) warm ionized medium spectrum (the volume-filling spectrum)



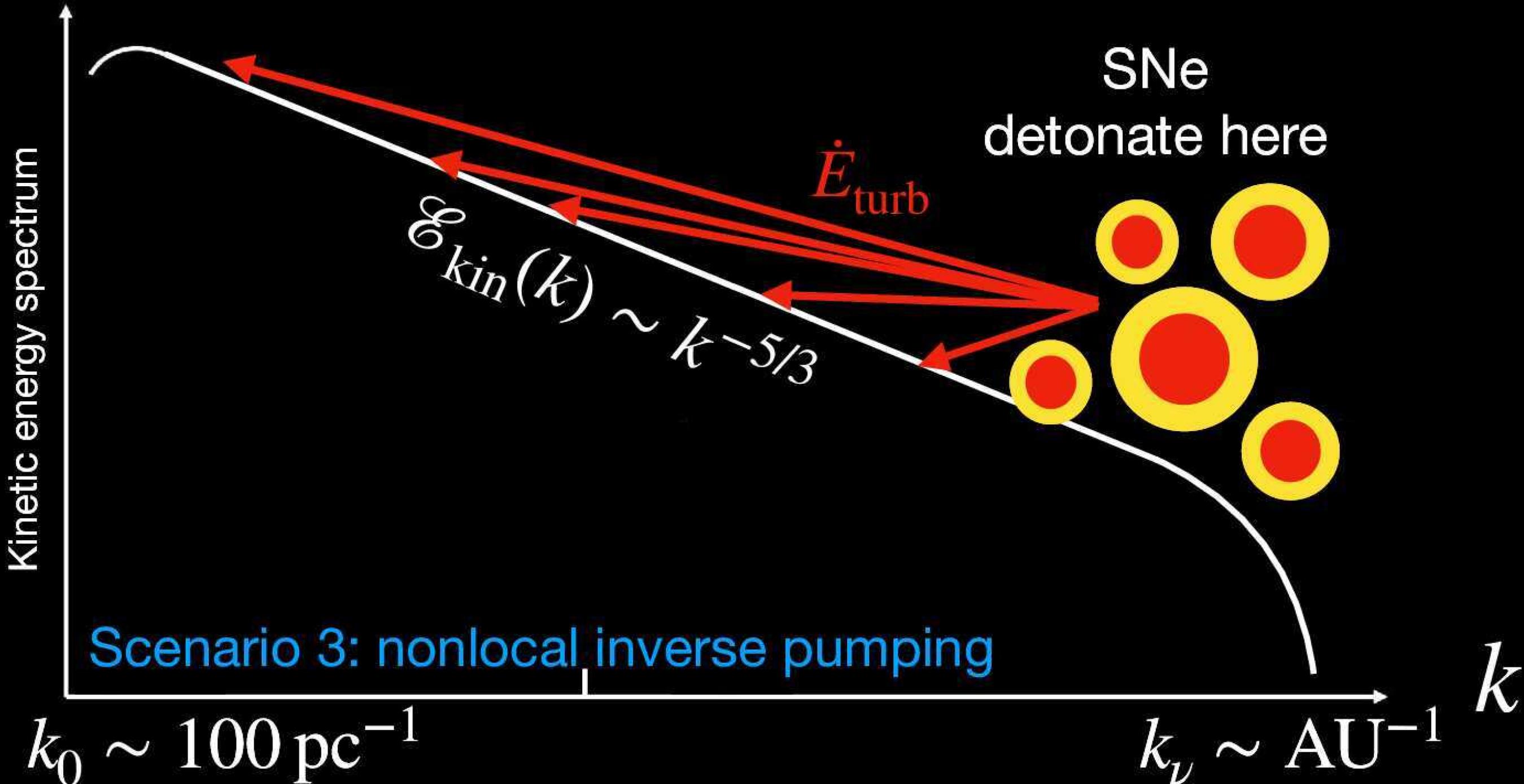
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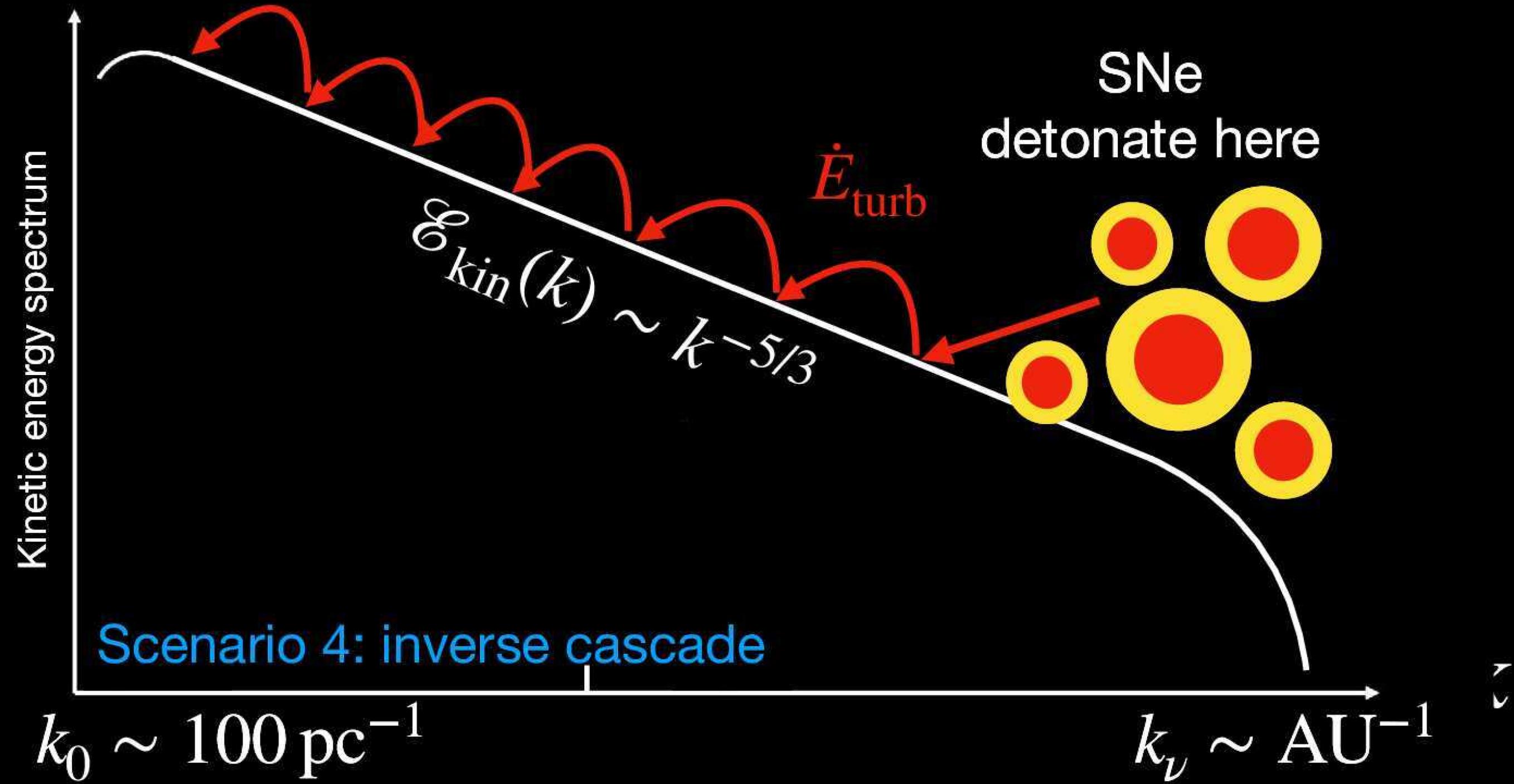
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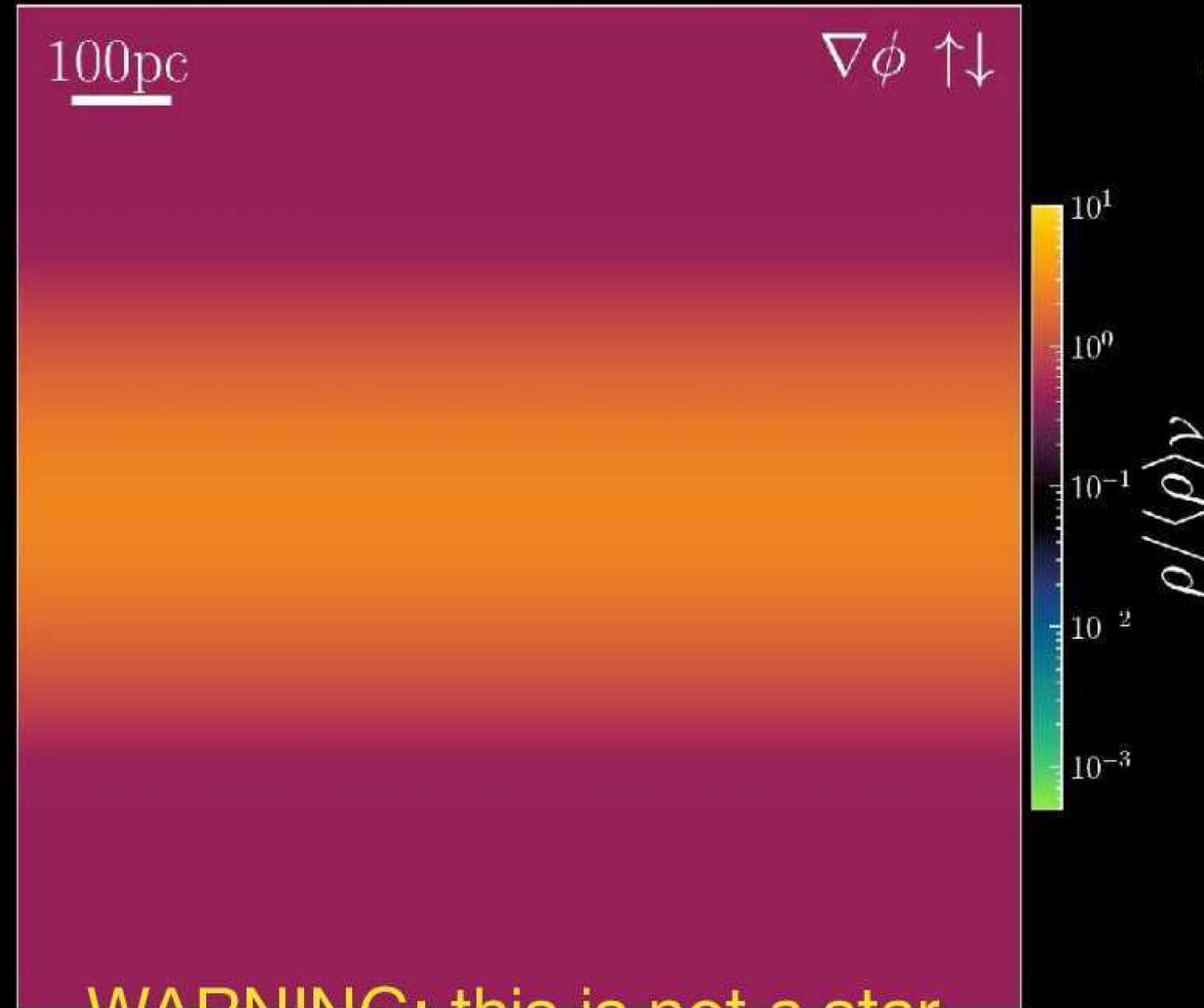
# The simplest SNe-driven simulations possible

$L = 1 \text{ kpc}$

Martizzi+2016

RAMSES (Teyssier 2002)

Supernova driven  
gravito-hydro dynamical model



**WARNING:** this is not a star formation simulation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = \dot{n}_{\text{SNe}} M_{\text{ej}}, \quad (1)$$

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u} + P \mathbb{I}) = -\rho \nabla \phi + \dot{n}_{\text{SNe}} \mathbf{p}_{\text{SNe}}(Z, n_{\text{H}}), \quad (2)$$

$$\frac{\partial \rho e}{\partial t} + \nabla \cdot [\rho (e + P) \mathbf{u}] = -n_{\text{H}}^2 \Lambda - \quad (3)$$

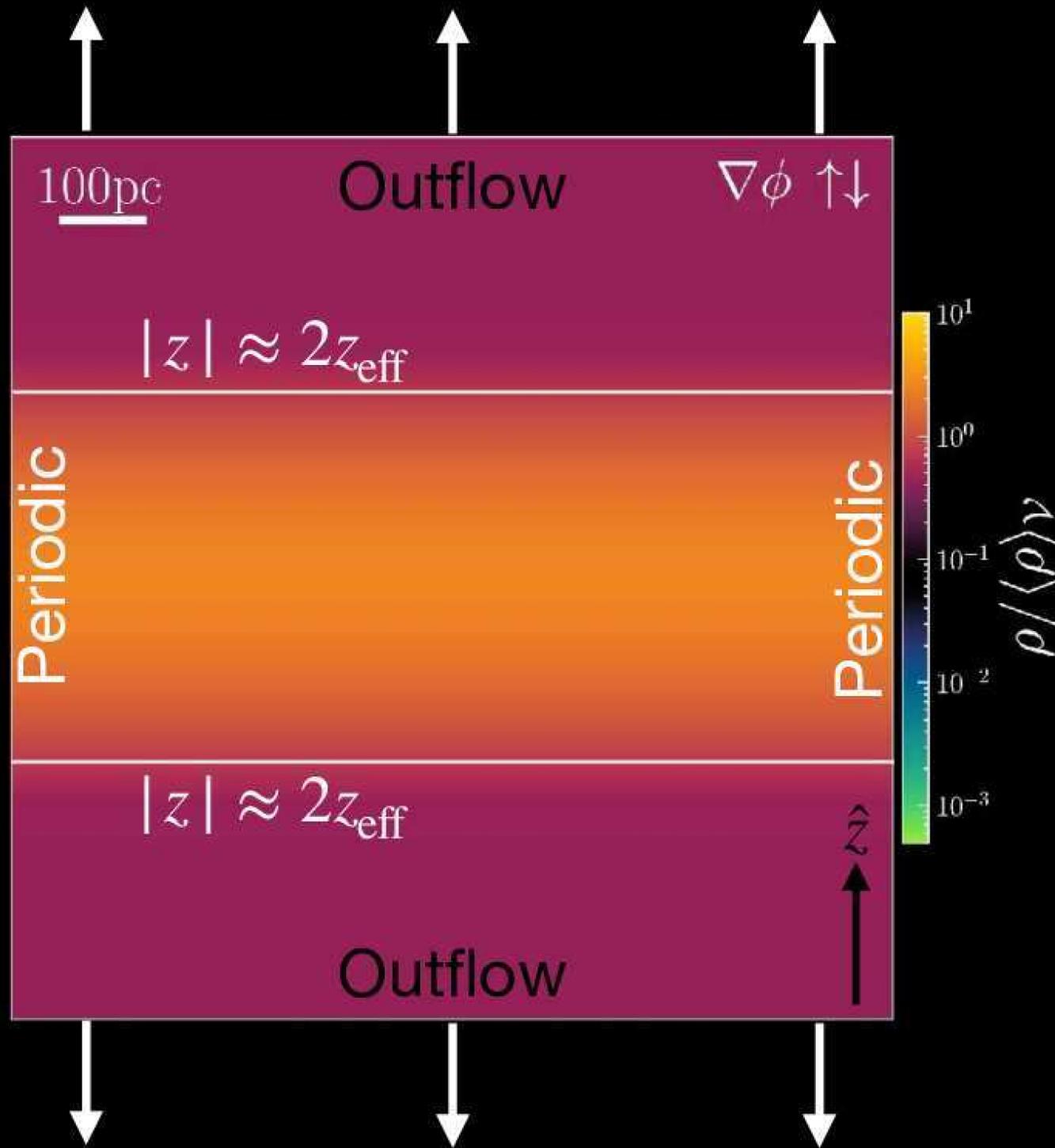
$$\rho \mathbf{u} \cdot \nabla \phi + \dot{n}_{\text{SNe}} \left[ E_{\text{th,SNe}}(Z, n_{\text{H}}) + \frac{\mathbf{p}_{\text{SNe}}(Z, n_{\text{H}})^2}{2(M_{\text{ej}} + M_{\text{swept}})} \right], \quad (4)$$

$$e = \epsilon + \frac{u^2}{2}, \quad P = \frac{2}{3} \rho e, \quad (5)$$

$$N_{\text{grid}}^3 = 1024^3 \implies \Delta x \sim 1 \text{ pc}$$

$$\text{numerical diss.} \implies \Delta x \sim 10 \text{ pc}$$

# The simplest SNe-driven simulations possible



## Static gravitational potential

$$\phi(z) = 2\pi G \Sigma_* \underbrace{\left( \sqrt{z^2 - z_0^2} - z_0 \right)}_{\text{stratified disk}} + \underbrace{\frac{2\pi G \rho_{\text{halo}}}{3} z^2}_{\text{spherical halo}}$$

stratified disk

spherical halo

## Supernova driving prescription

(following 1D evolution models for momentum and energy deposition in Martizzi + 2016)

$$\dot{n}_{\text{SNe}} = \frac{\dot{\Sigma}_*}{2z_{\text{eff}} 100M_{\odot}} \quad \text{1 SN per } 100M_{\odot} \text{ of SF}$$

$$\dot{\Sigma}_* \propto \Sigma^{1.4}$$

KS relation

$$p(z) = \begin{cases} \frac{1}{4z_{\text{eff}}}, & |z| \leq 2z_{\text{eff}} \\ 0, & |z| > 2z_{\text{eff}} \end{cases}$$

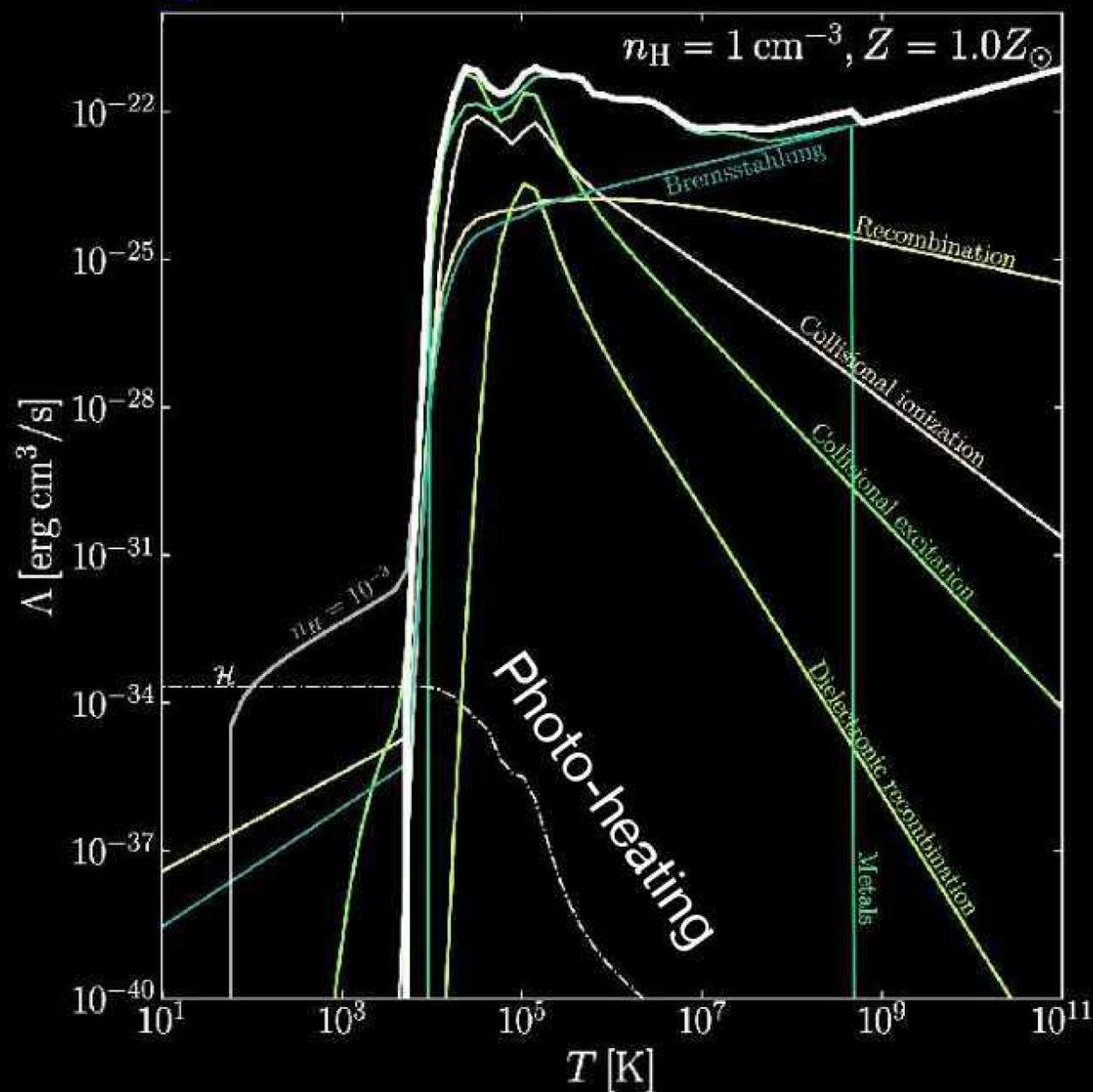
$$p(n | \lambda) = \lambda^n e^{-\lambda} / n!$$

$$\lambda(x, y, z) = \dot{n}_{\text{SNe}} \mathcal{V}_{\text{cell}} \Delta t$$

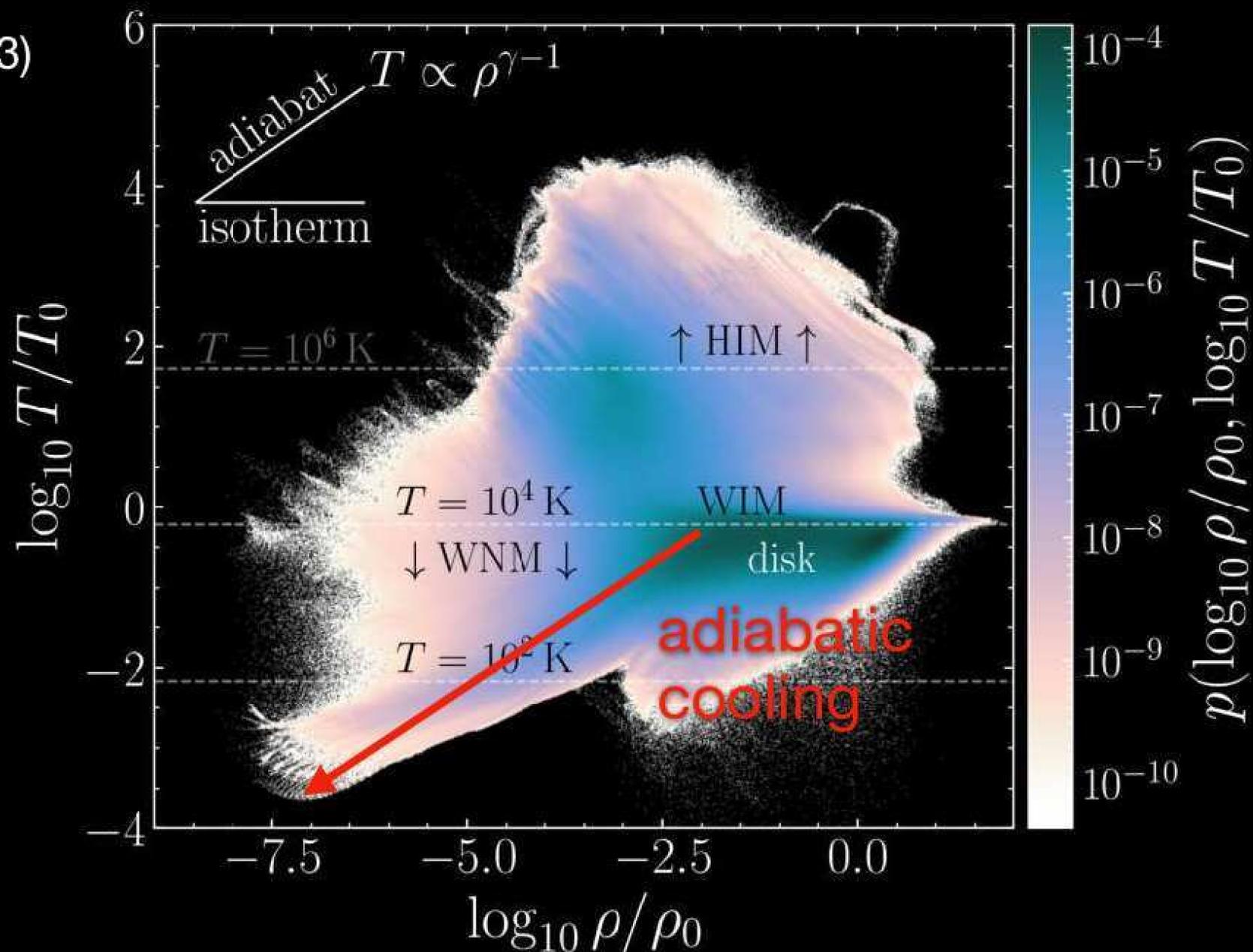
# The simplest SNe-driven simulations possible

Theuns+(1998)  
Sutherland & Dopita(1993)

## Cooling function



HI, HII, HeI, HeII, HeIII and free electrons



$$\rho_0 = 2.1 \times 10^{-24} \text{ g cm}^{-3}$$

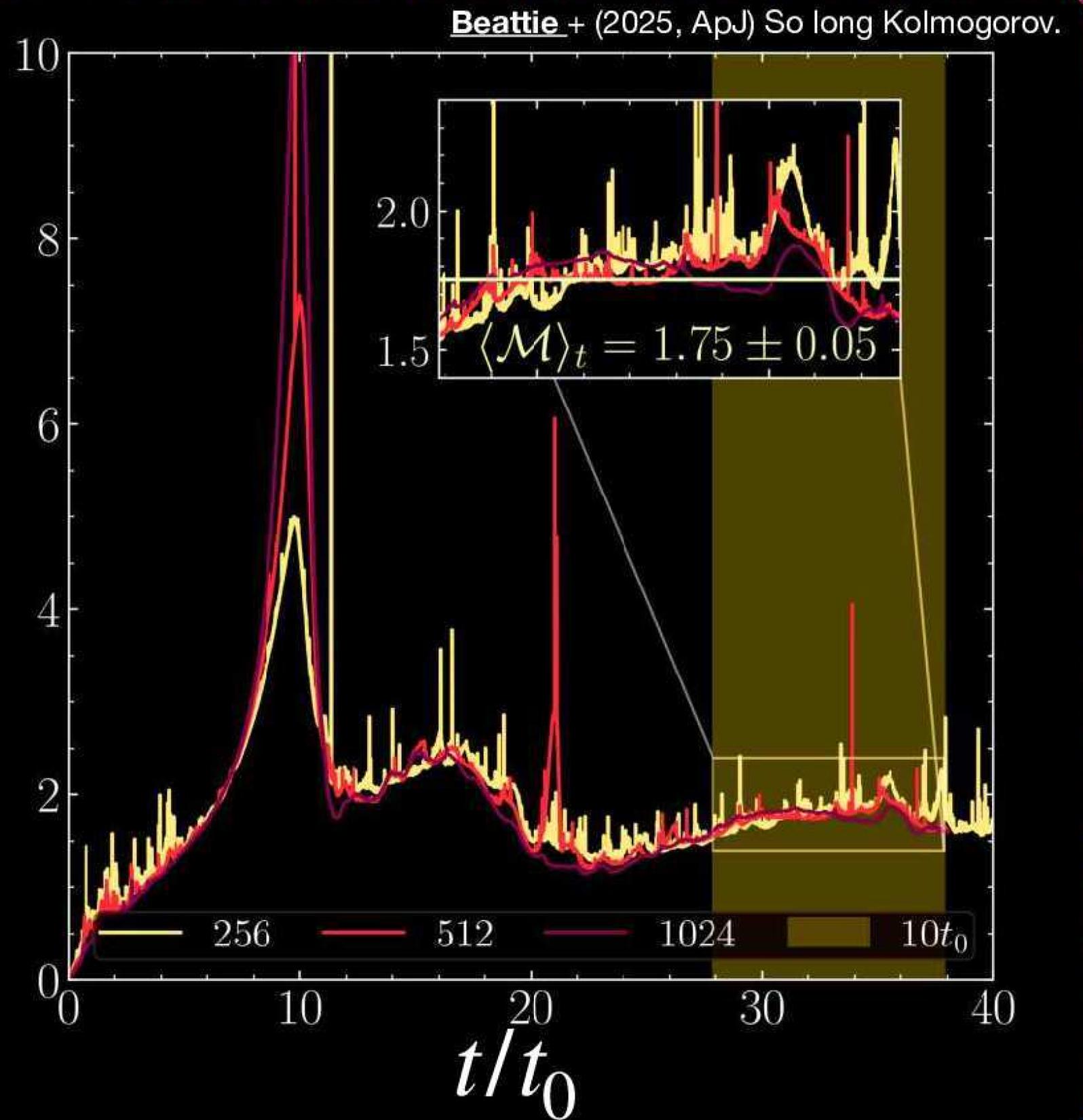
$$P_0 = 2.2 \times 10^{-12} \text{ Ba}$$

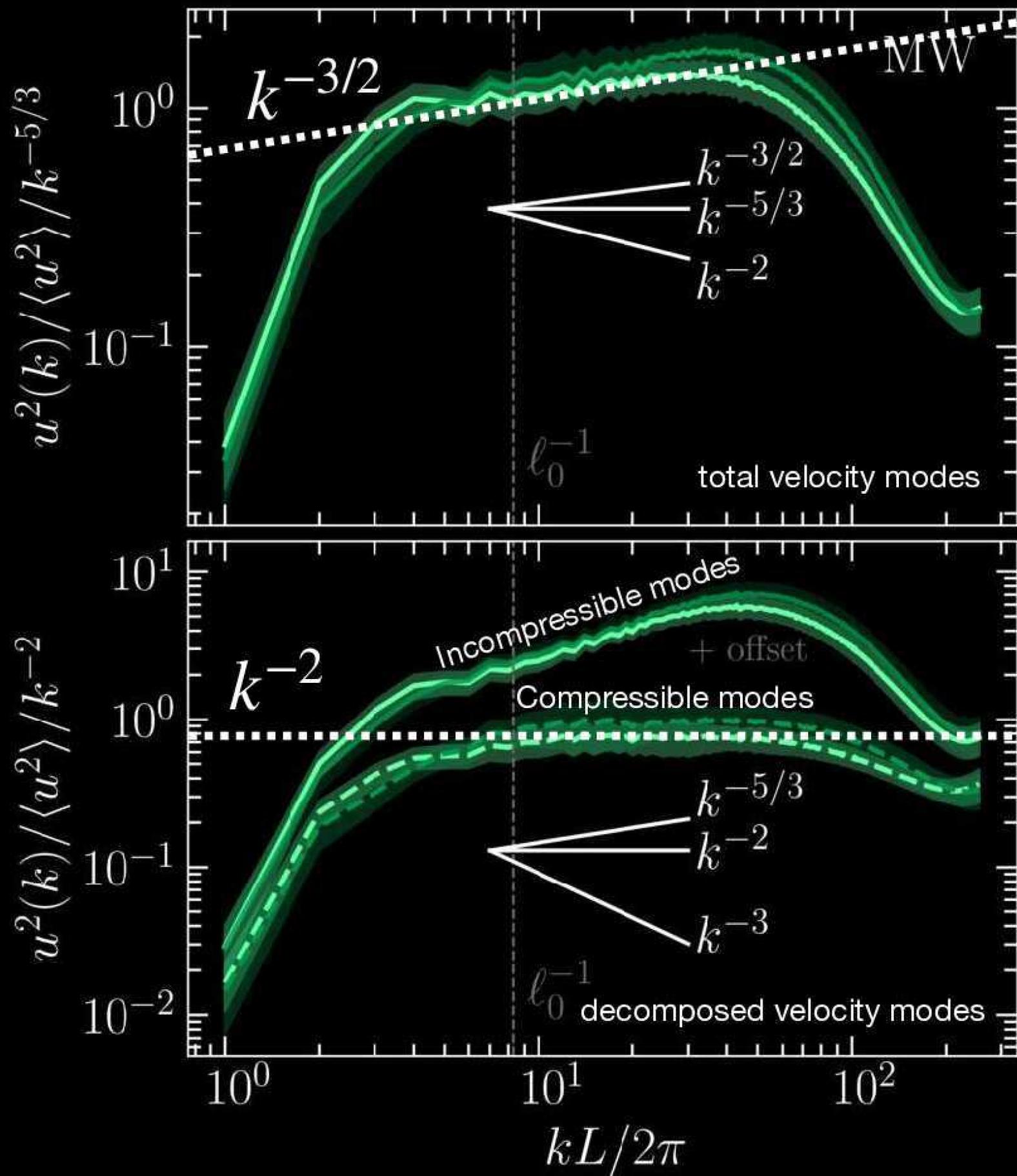
# Stationarity and Mach number

$$\mathcal{M} = \left\langle \left( \frac{u}{c_s} \right)^2 \right\rangle^{1/2} \mathcal{M}$$

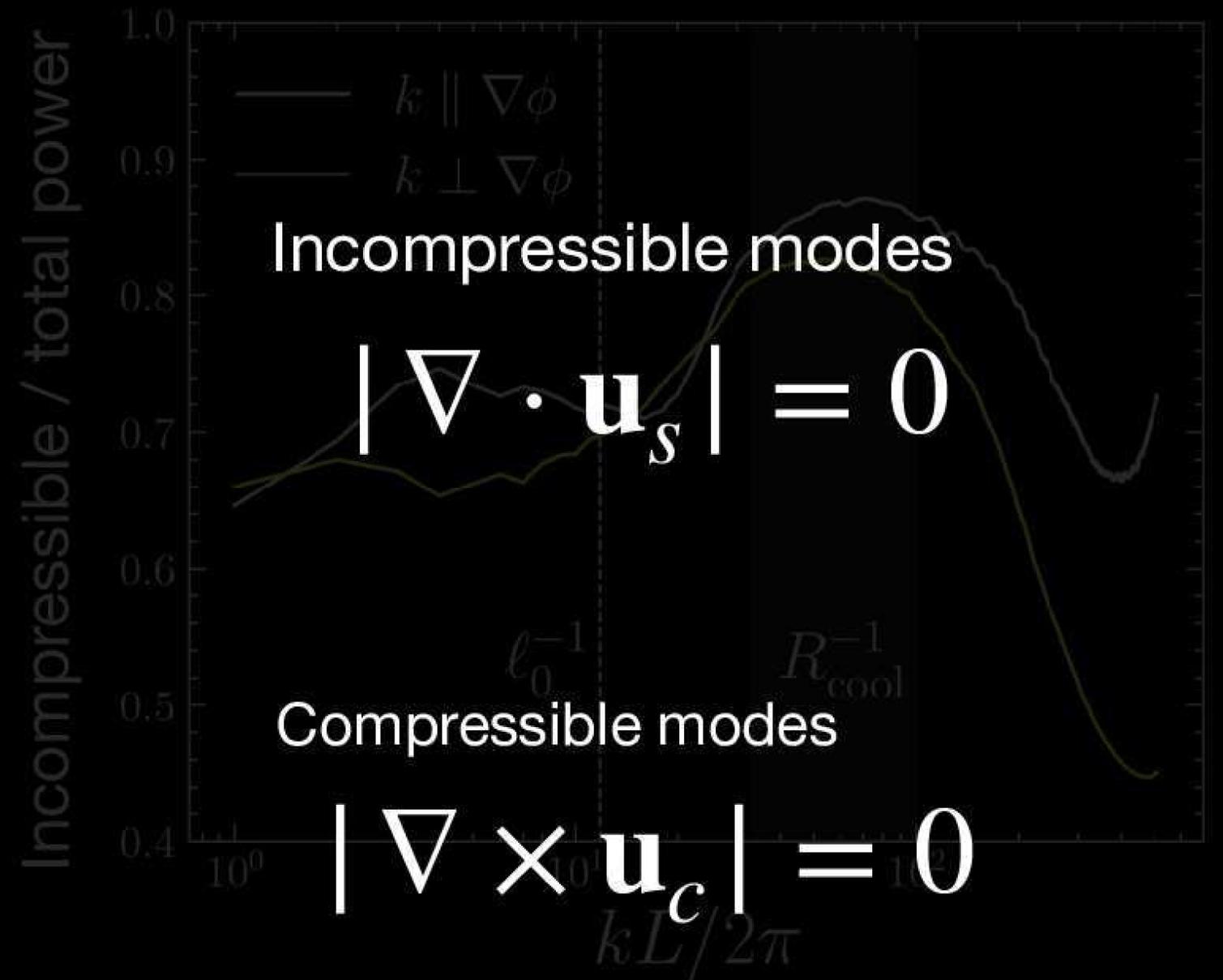
$$t_0 \sim \frac{\ell_0}{\langle u^2 \rangle^{1/2}} \sim 80 \text{ Myr}$$

reaches a quasi stationary state



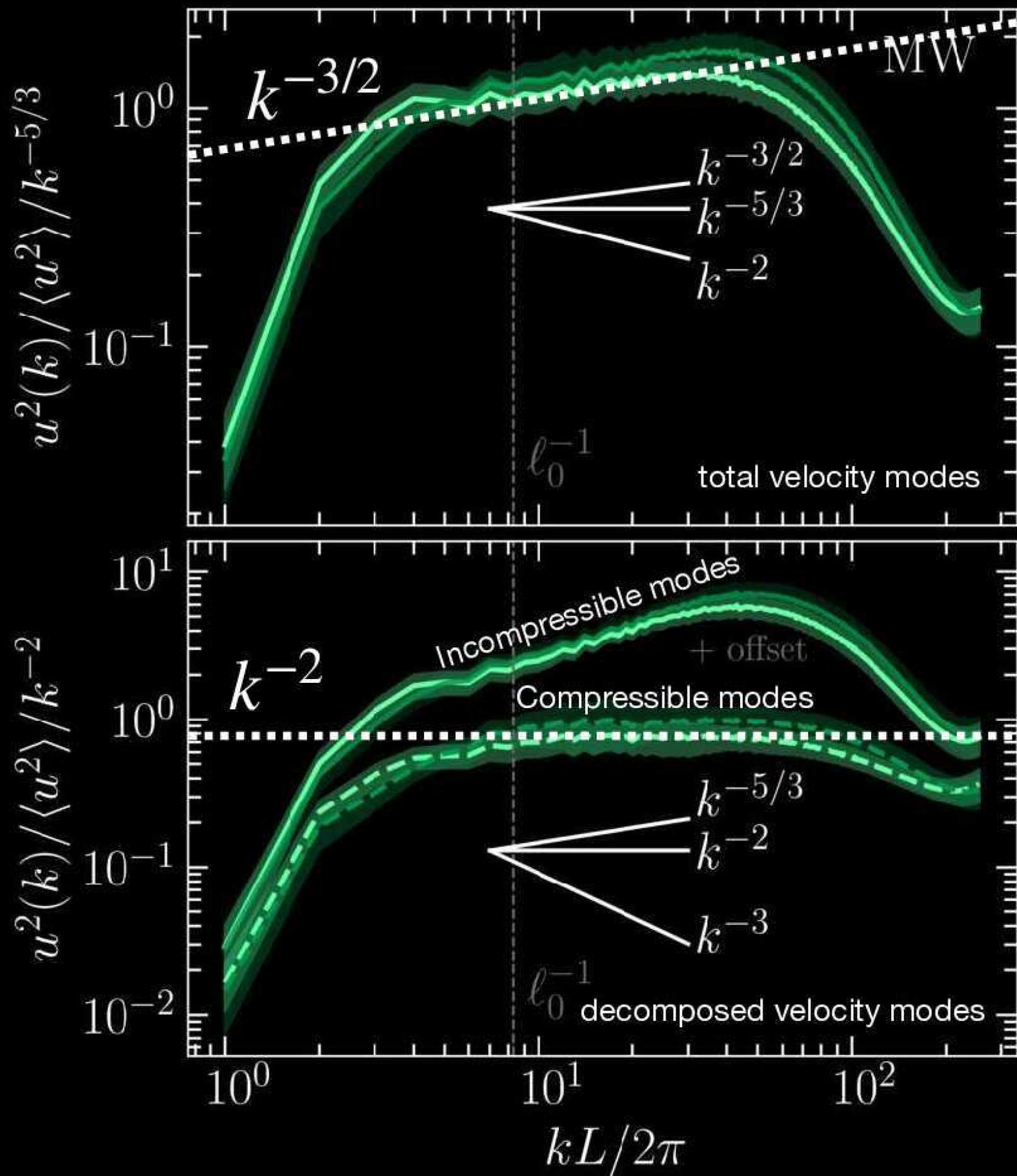


## Basic spectral properties



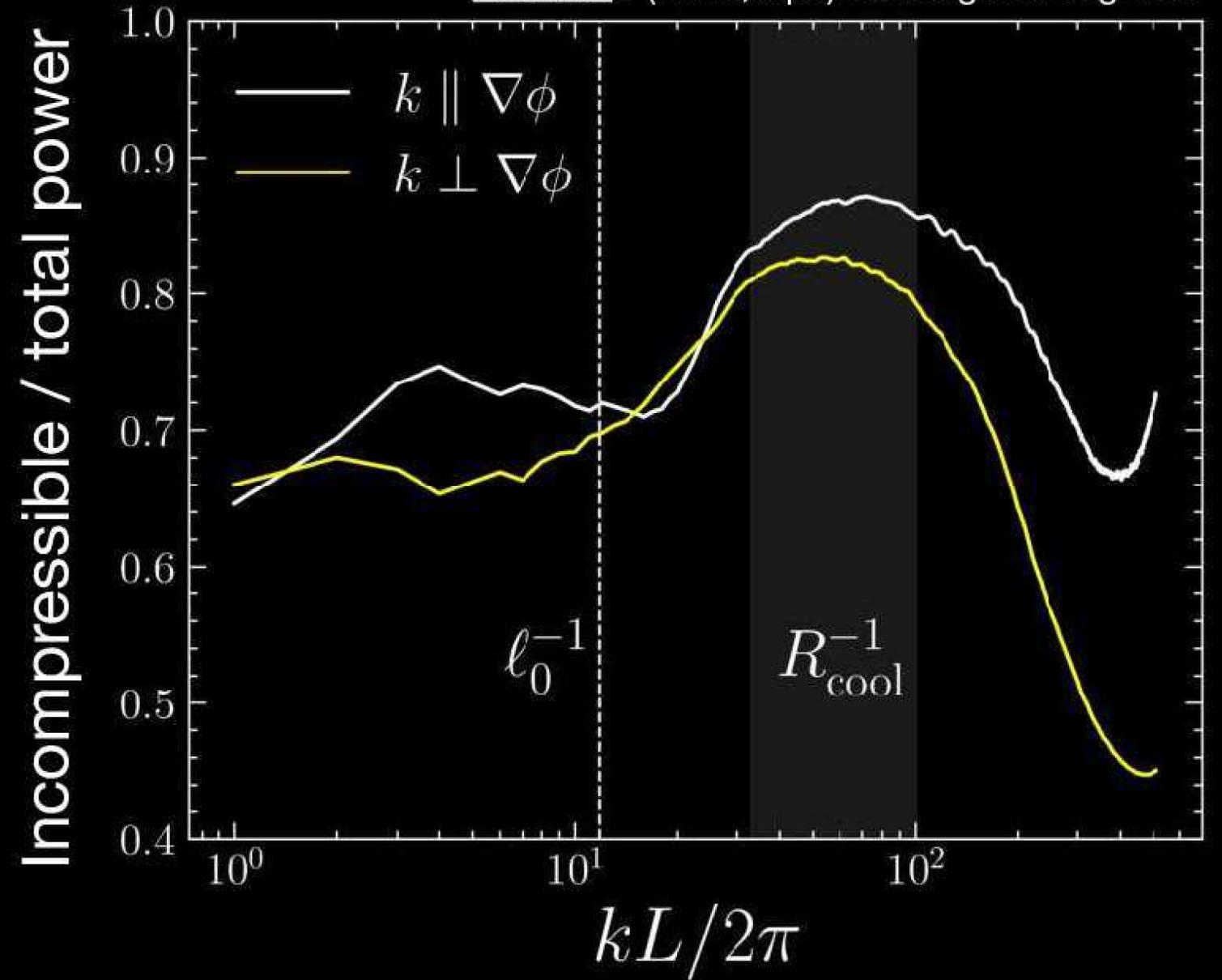
$$R_{cool} \approx 14 \text{ pc} \left( \frac{E_{SN}}{10^{51} \text{ erg}} \right) \left( \frac{0.29}{n_0 \text{ cm}^{-3}} \right)^{-0.43}$$

Cioffi + 1998  
Blondin + 1998



## Basic spectral properties

Beattie + (2025, ApJ) So long Kolmogorov.



$$R_{\text{cool}} \approx 14 \text{ pc} \left( \frac{E_{\text{SN}}}{10^{51} \text{ erg}} \right) \left( \frac{0.29}{n_0 \text{ cm}^{-3}} \right)^{-0.43}$$

Cioffi + 1998  
Blondin + 1998

# Phase fluctuations and thermal response is mostly adiabatic



Izzy Connor  
(undergrad. UCSC)

Consider the adiabatic limit

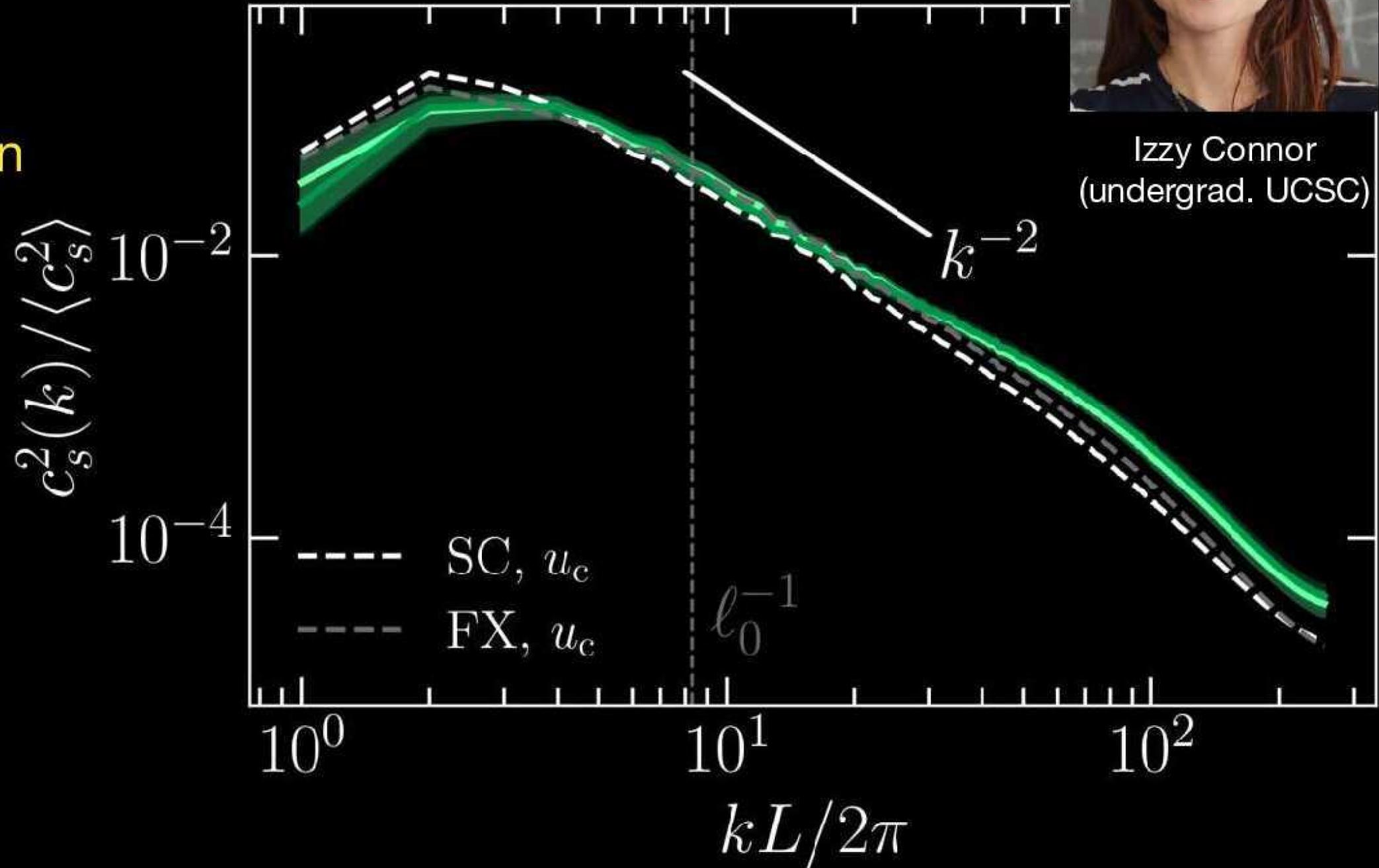
$$t_{\text{turb}} \ll t_{\text{cool}}$$

via linearized continuity equation

$$\delta \tilde{c}_s \sim c_{s,0} \frac{\gamma - 1}{2\omega} \mathbf{k} \cdot \tilde{\mathbf{u}}_c$$

$$\omega \sim c_{s,0} k$$

$$\delta \tilde{c}_s \sim \frac{\gamma - 1}{2} \tilde{u}_c$$

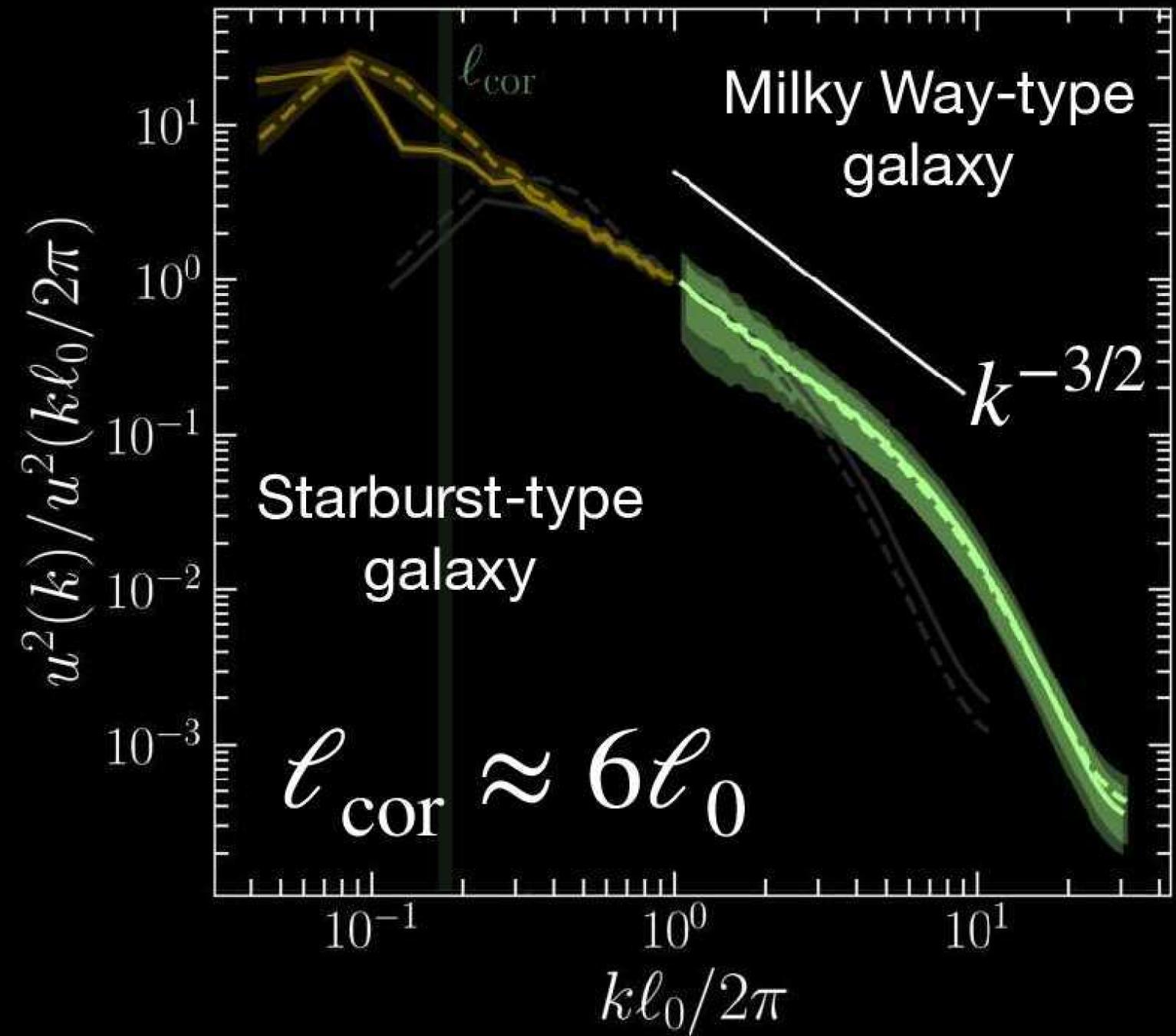


thermal response / phase fluctuations controlled by compressible modes

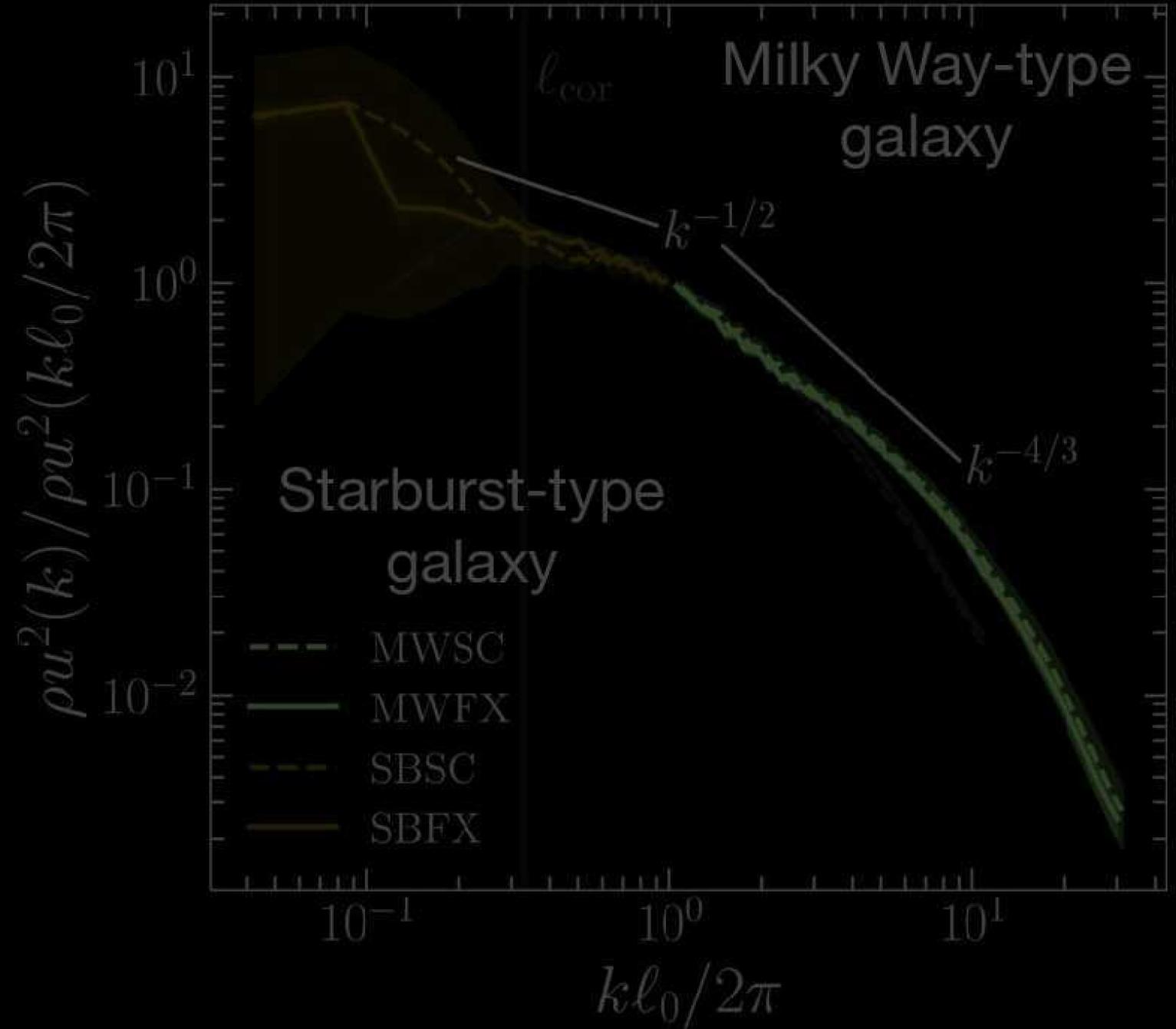
# Universality between galactic models

$$\gamma_{\text{SN}_e, \text{SB}} = 10 \gamma_{\text{SN}_e, \text{MW}}$$

Velocity spectrum



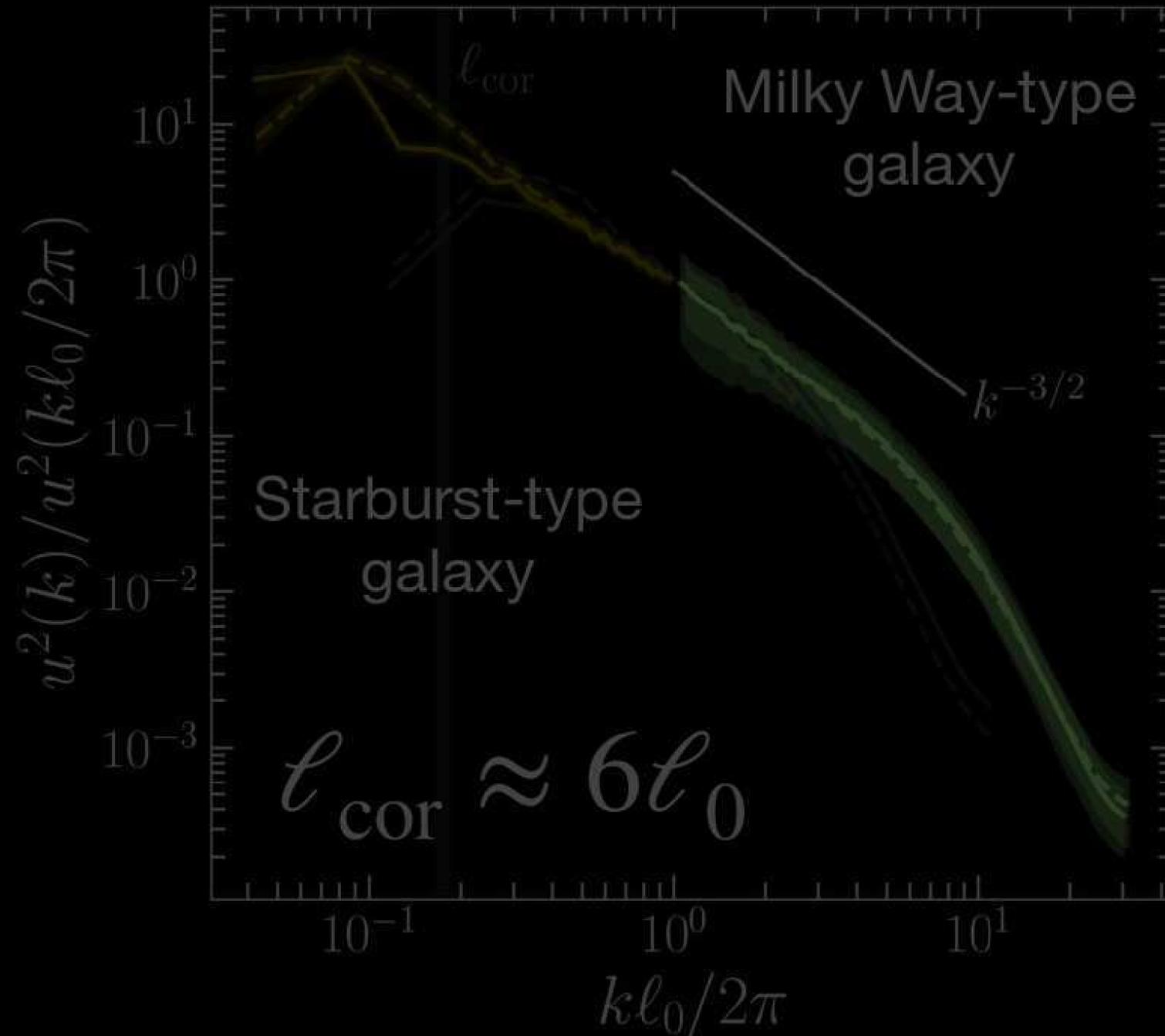
Kinetic energy spectrum



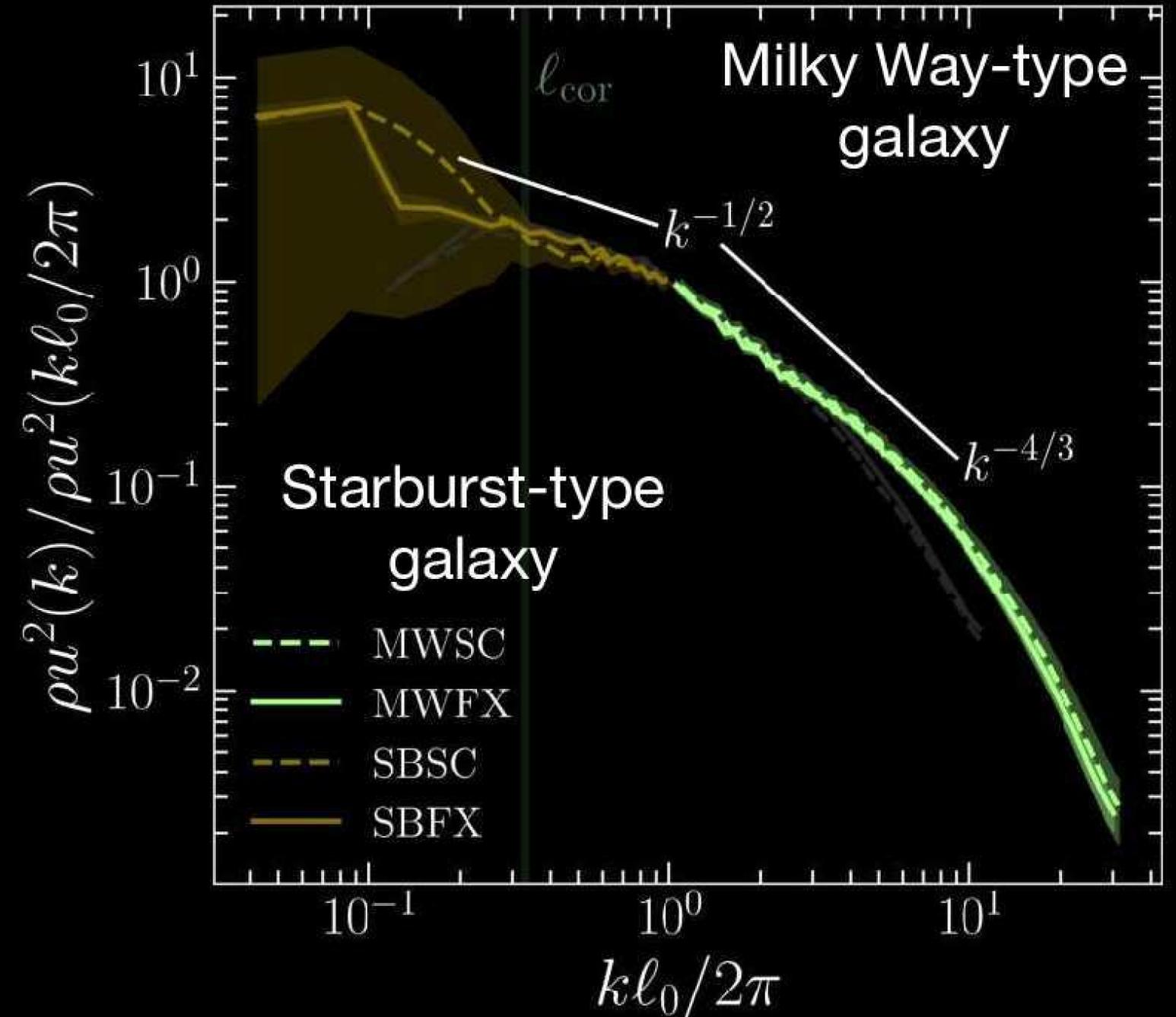
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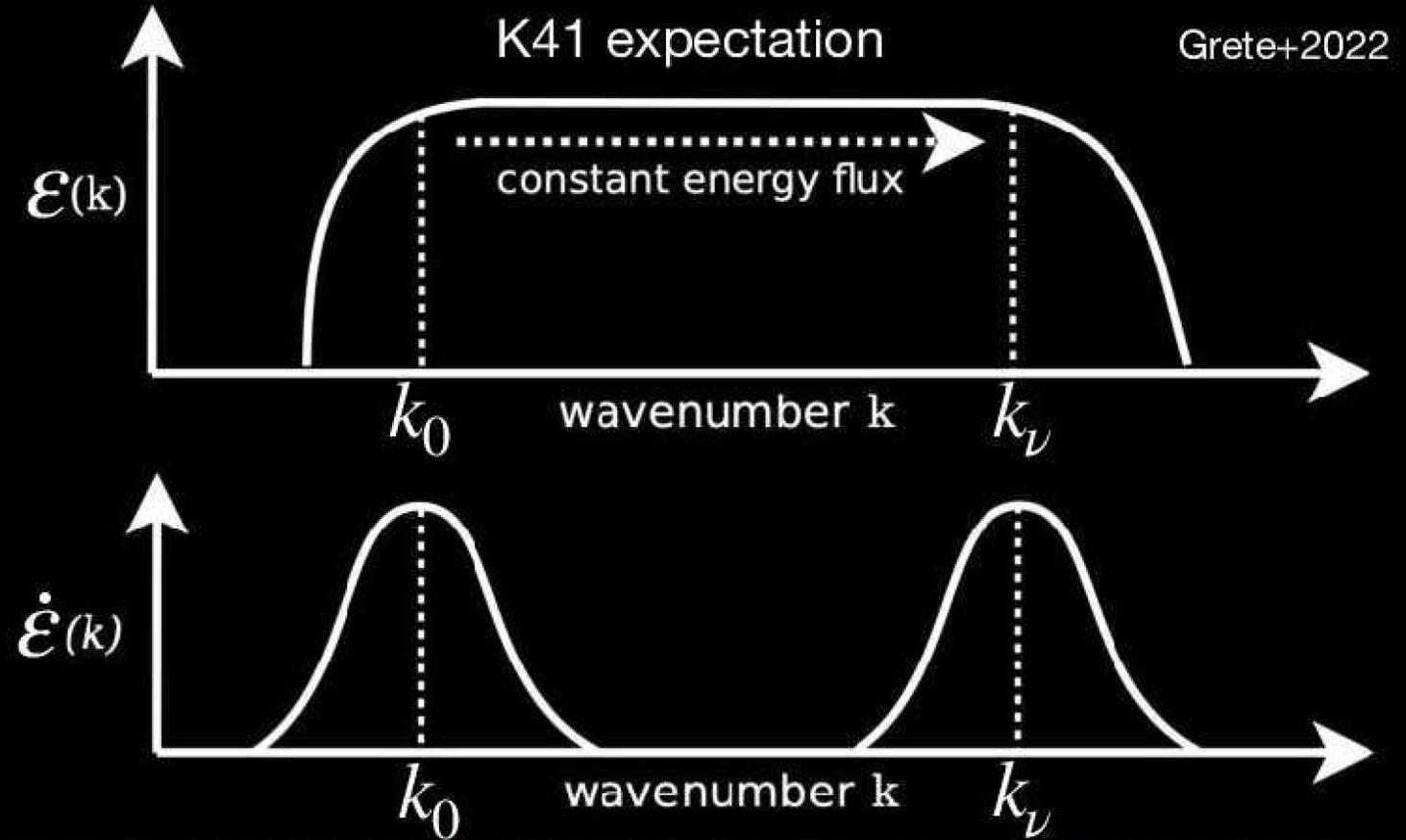
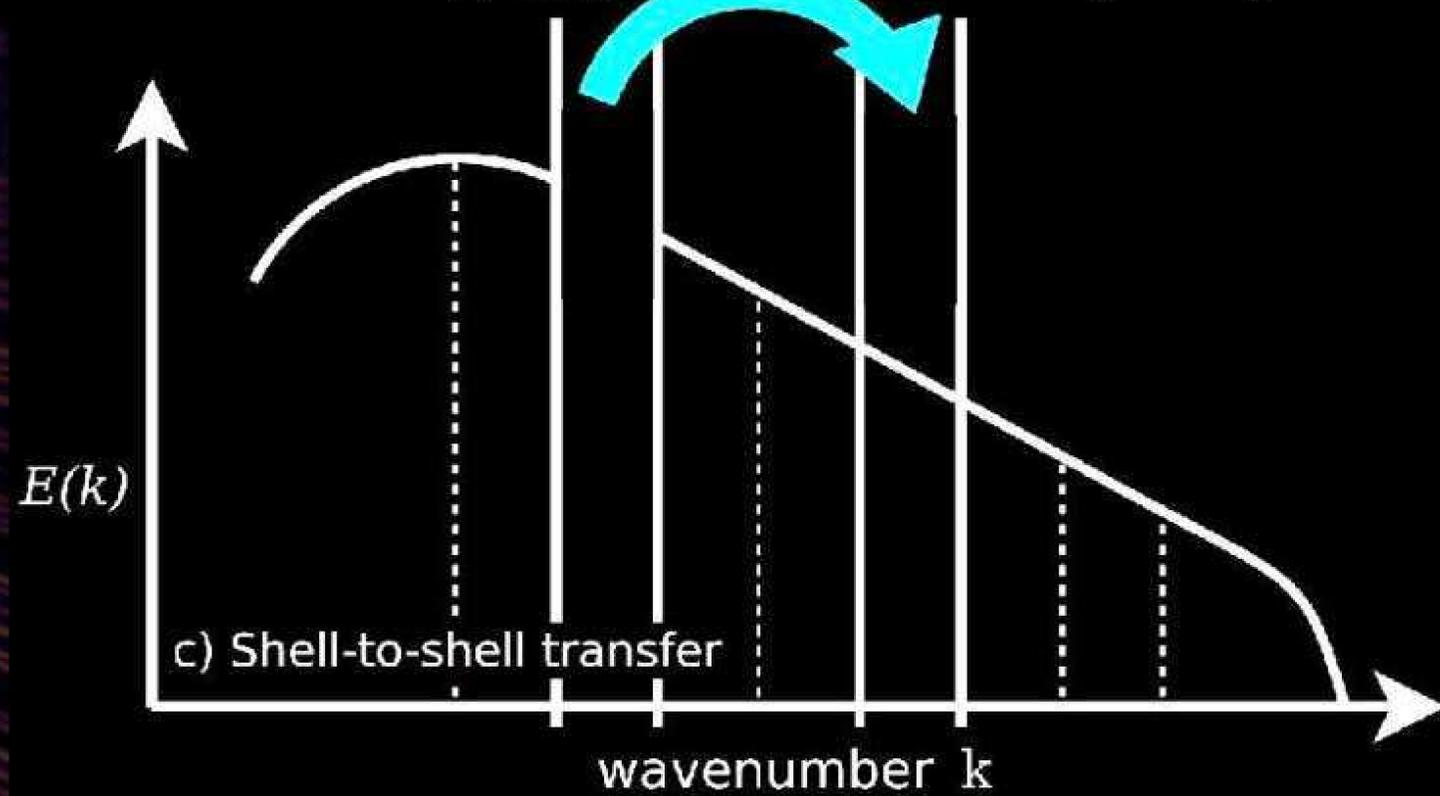


Kinetic energy spectrum



But to understand the turbulence we must understand the flux rates from mode to mode!

$$k''(Q) \quad \varepsilon \quad k'''(K)$$



Grete, O'Shea +2022a,b,23

Dar+2001; Verma+2004; Mininni+2005; Alexakis+2005

But to understand the turbulence we must understand the flux rates from mode to mode!

Momentum conservation:

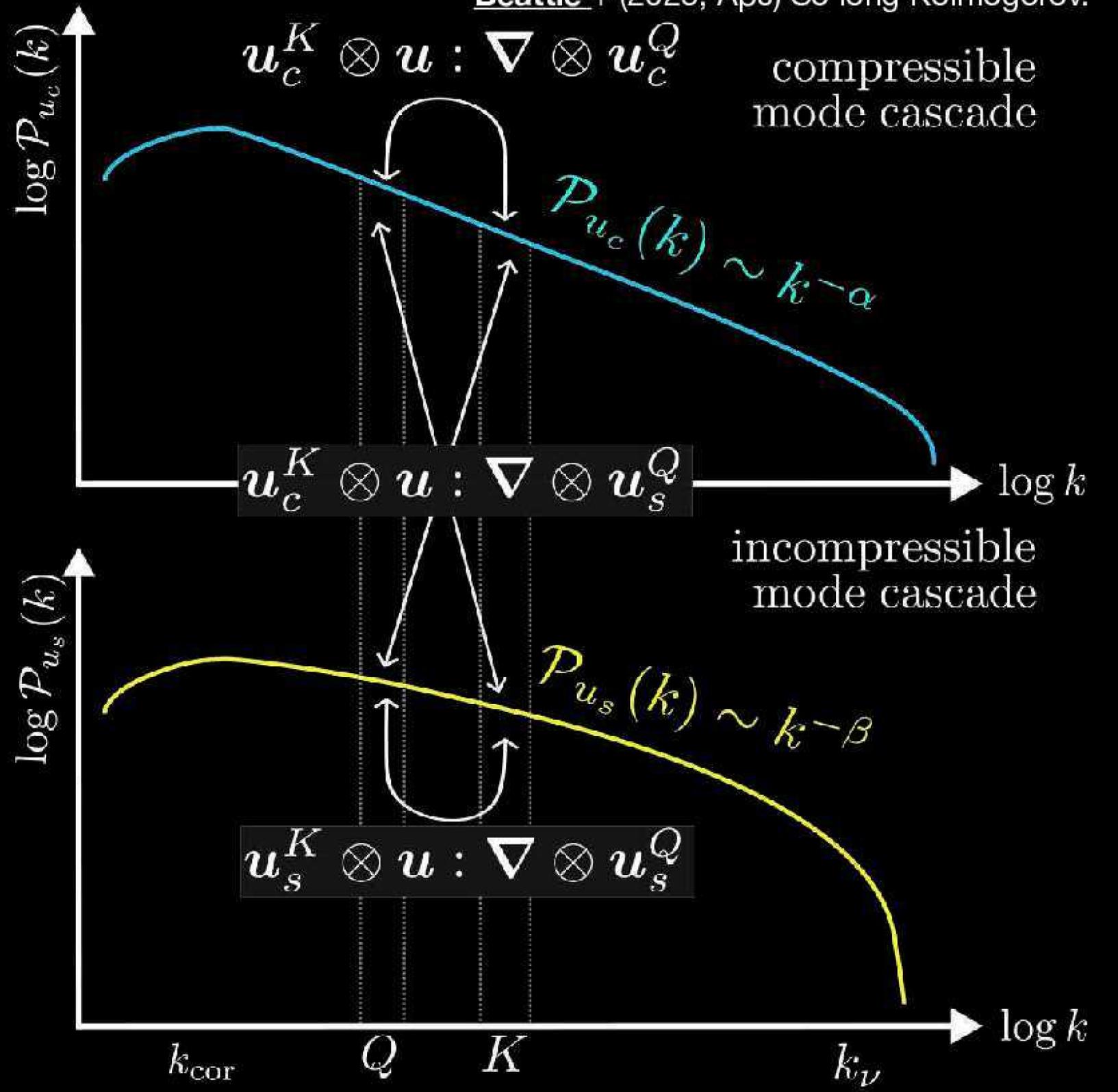
$$\begin{array}{c}
 \text{doner} \\
 Q + P + K = 0 \\
 \text{mediator}
 \end{array}$$

$$\begin{array}{c}
 \text{doner} \quad \text{receiver} \\
 Q \xrightarrow{P} K = -K \xrightarrow{P} Q \\
 \text{mediator}
 \end{array}$$

$$\mathbf{u}^Q = \mathbf{u}(\mathbf{r}^Q) = \int \delta^3(\mathbf{k} - Q) \mathbf{u}(\mathbf{k}) \exp \{ 2\pi i \mathbf{k} \cdot \mathbf{r} \}$$

Compressible modes and solenoidal modes live very different lives!

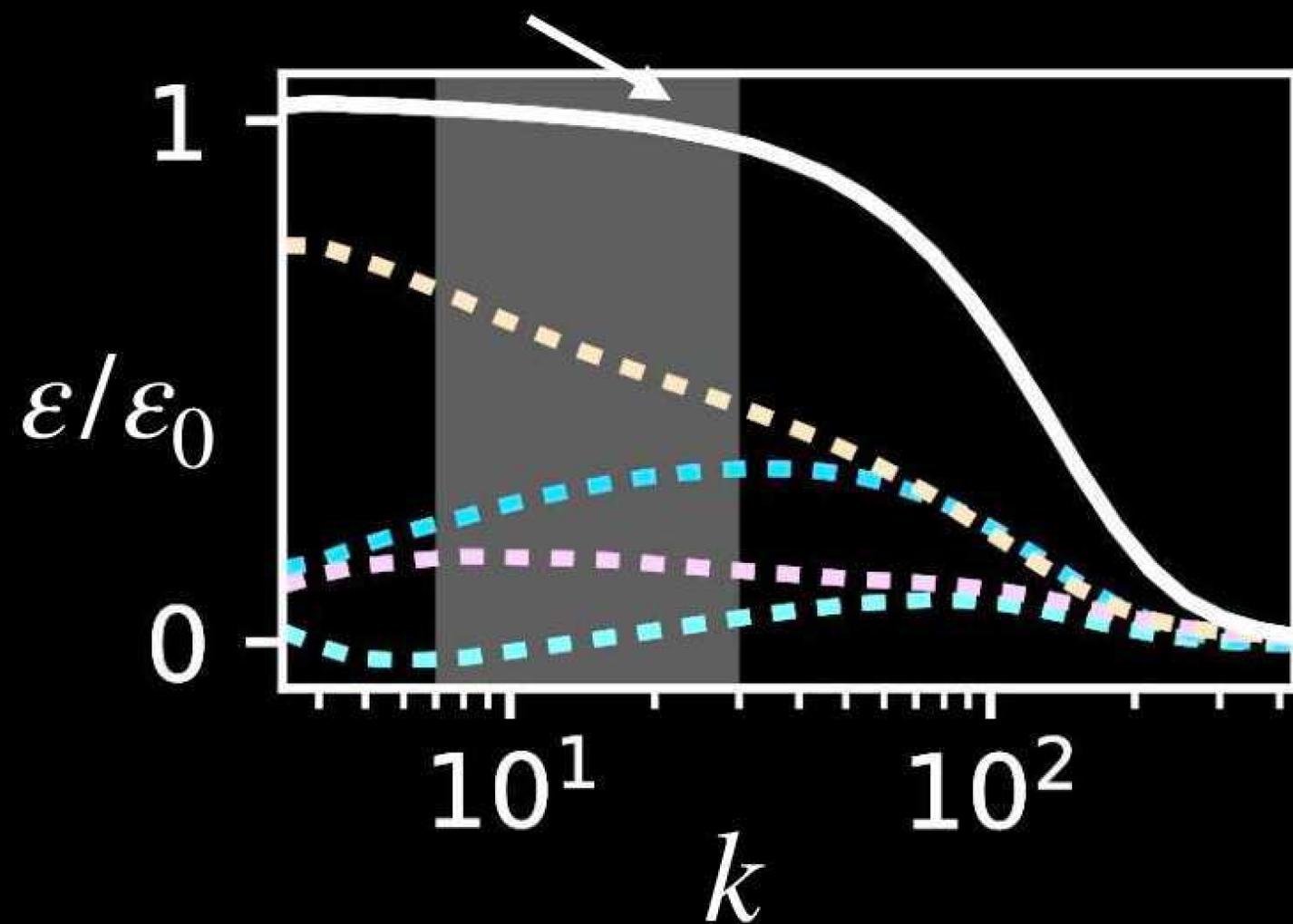
We should study them separately!



# Expectations from compressible MHD turbulence in a box

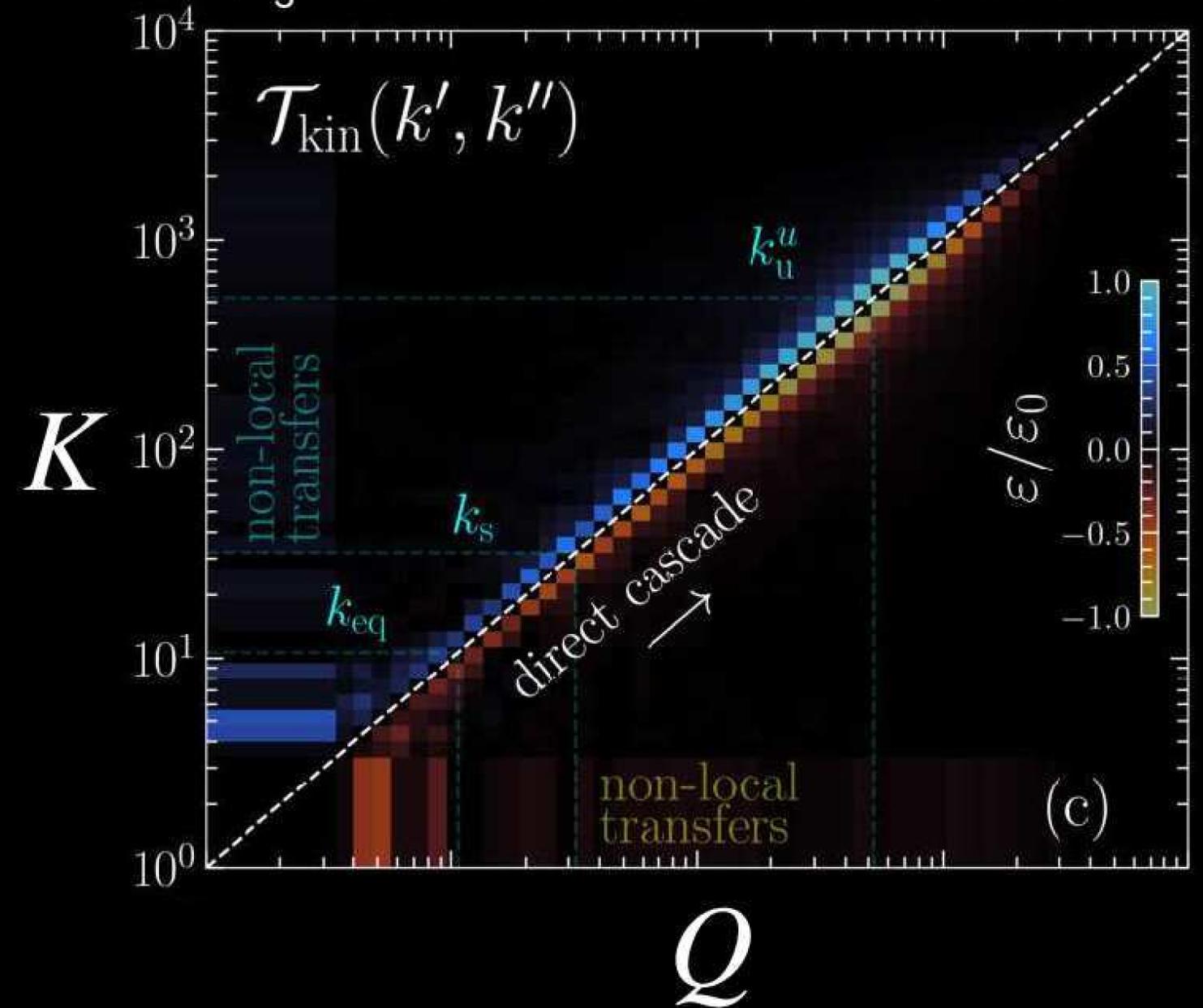
$$Q \xrightarrow{P} K$$

$$\varepsilon \sim u^3/\ell \approx \text{const.}$$

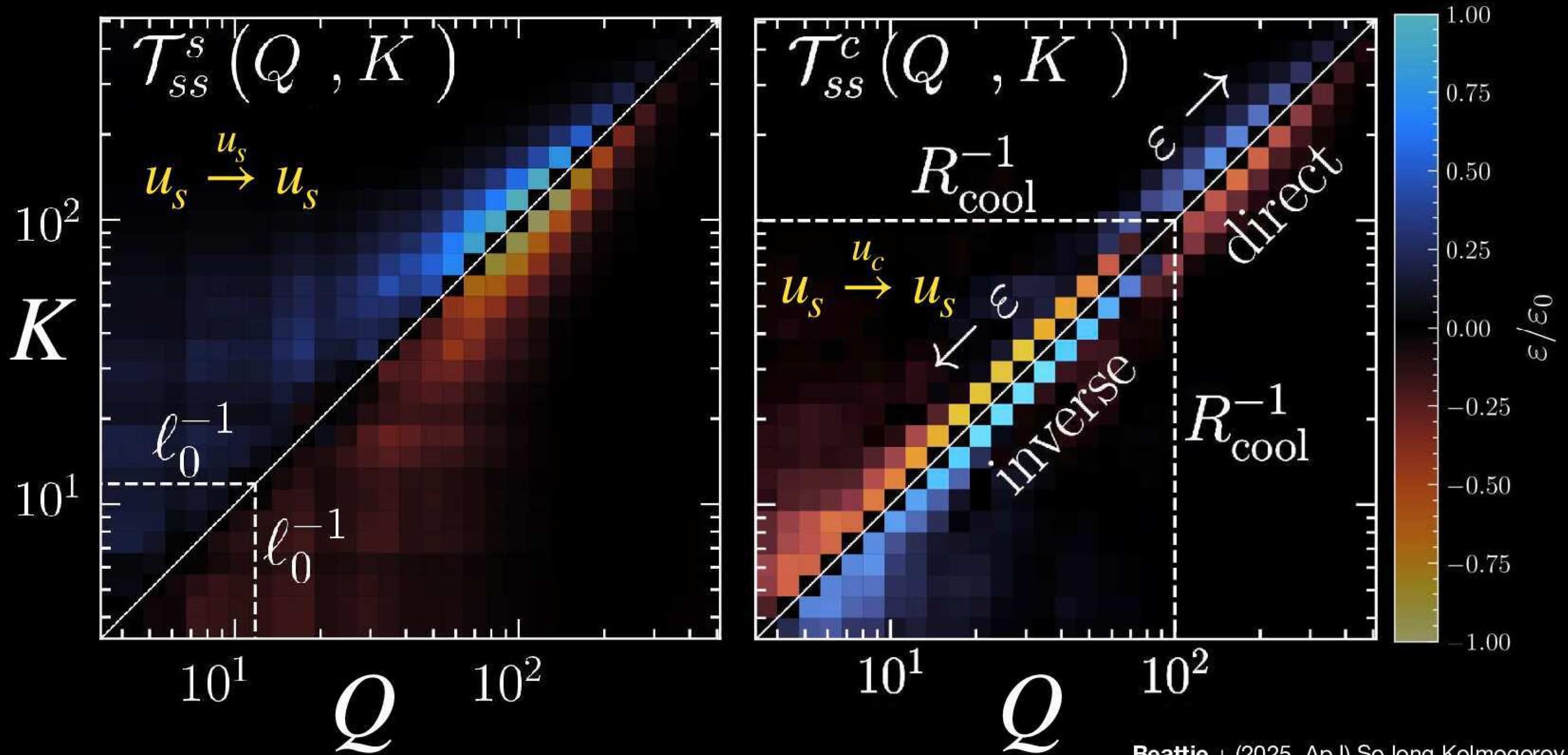


Grete, O'shea + 2022

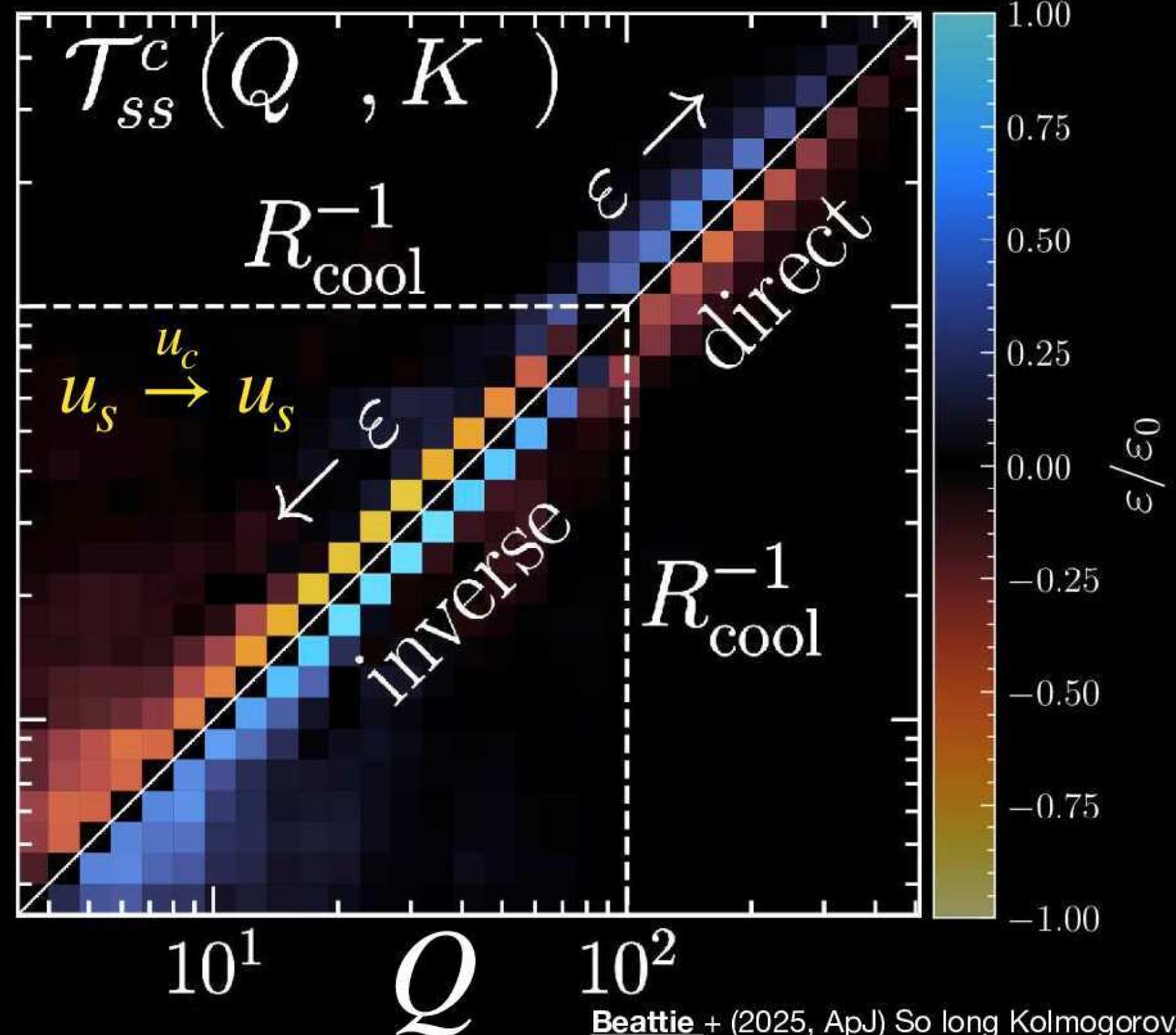
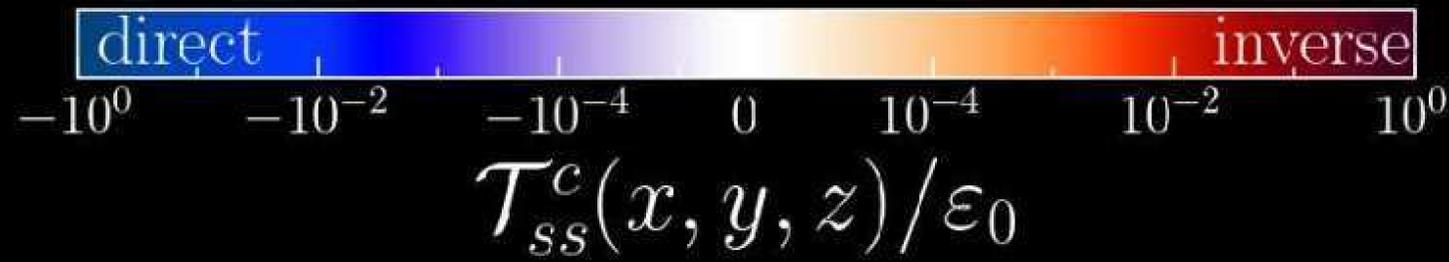
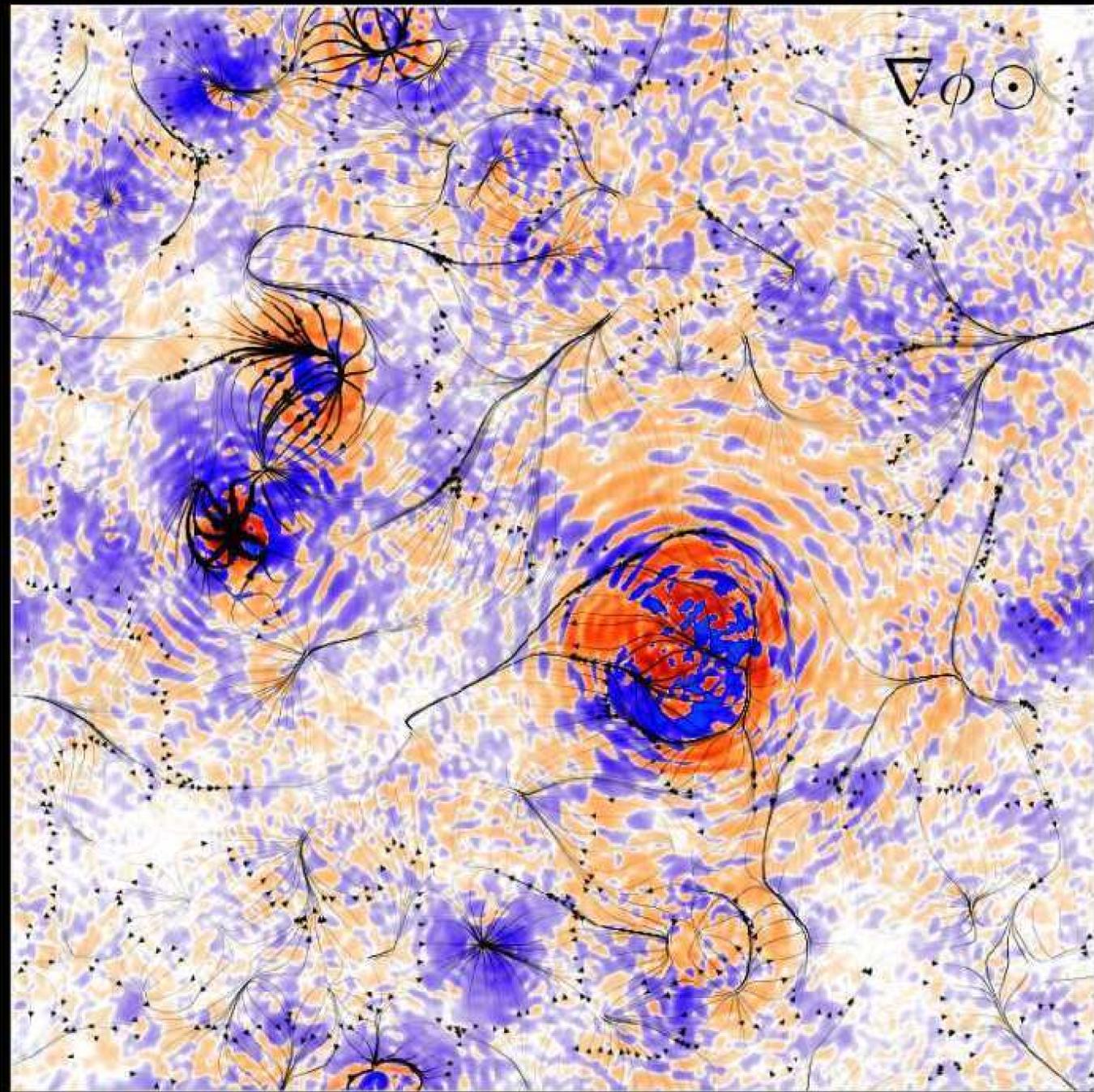
*Beattie+2025 (Nature Astronomy)*. The spectrum of magnetized turbulence in the interstellar medium



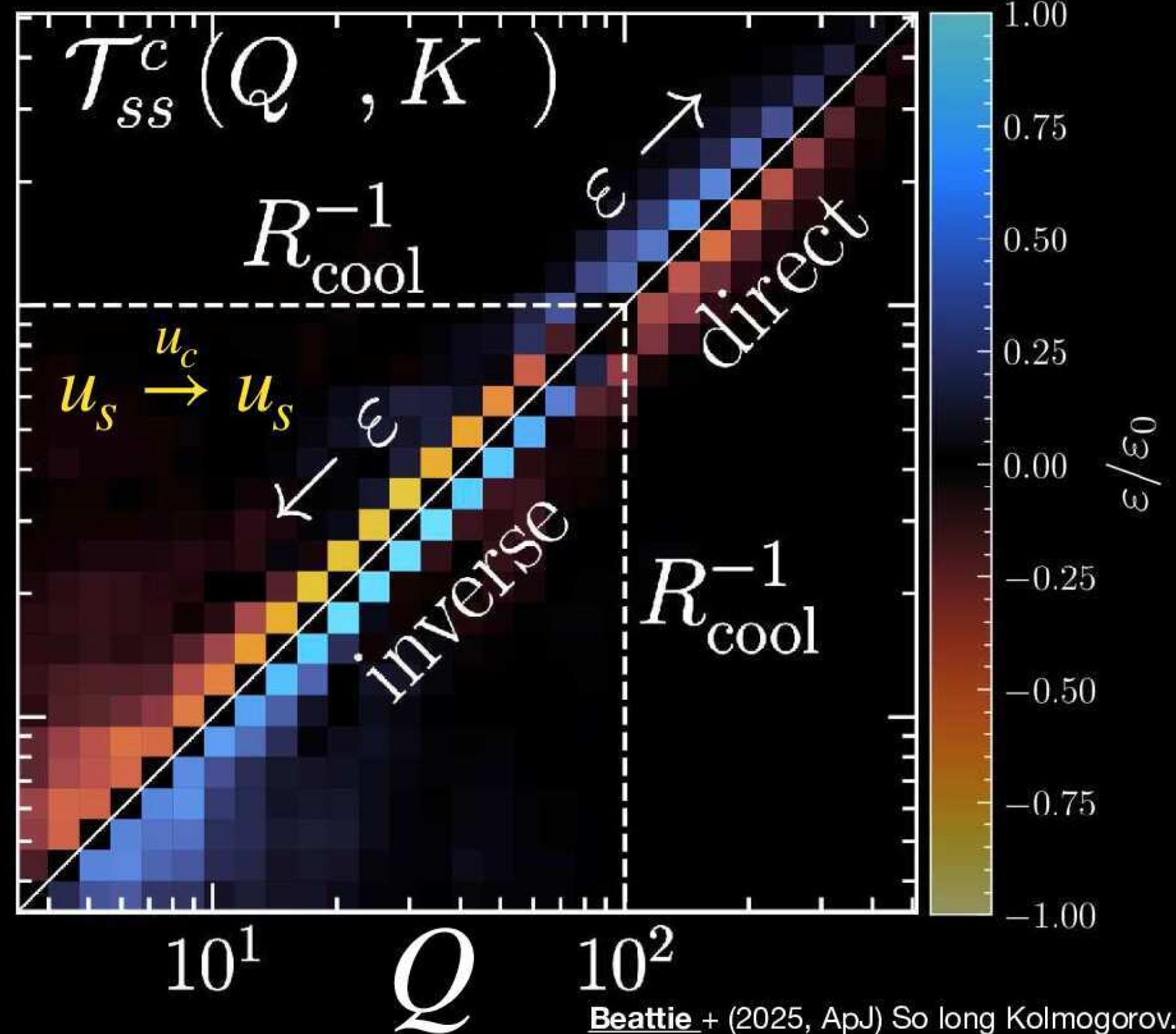
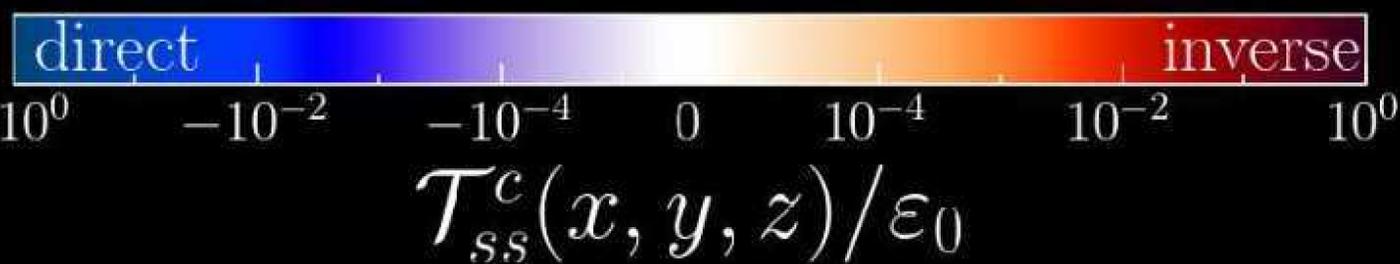
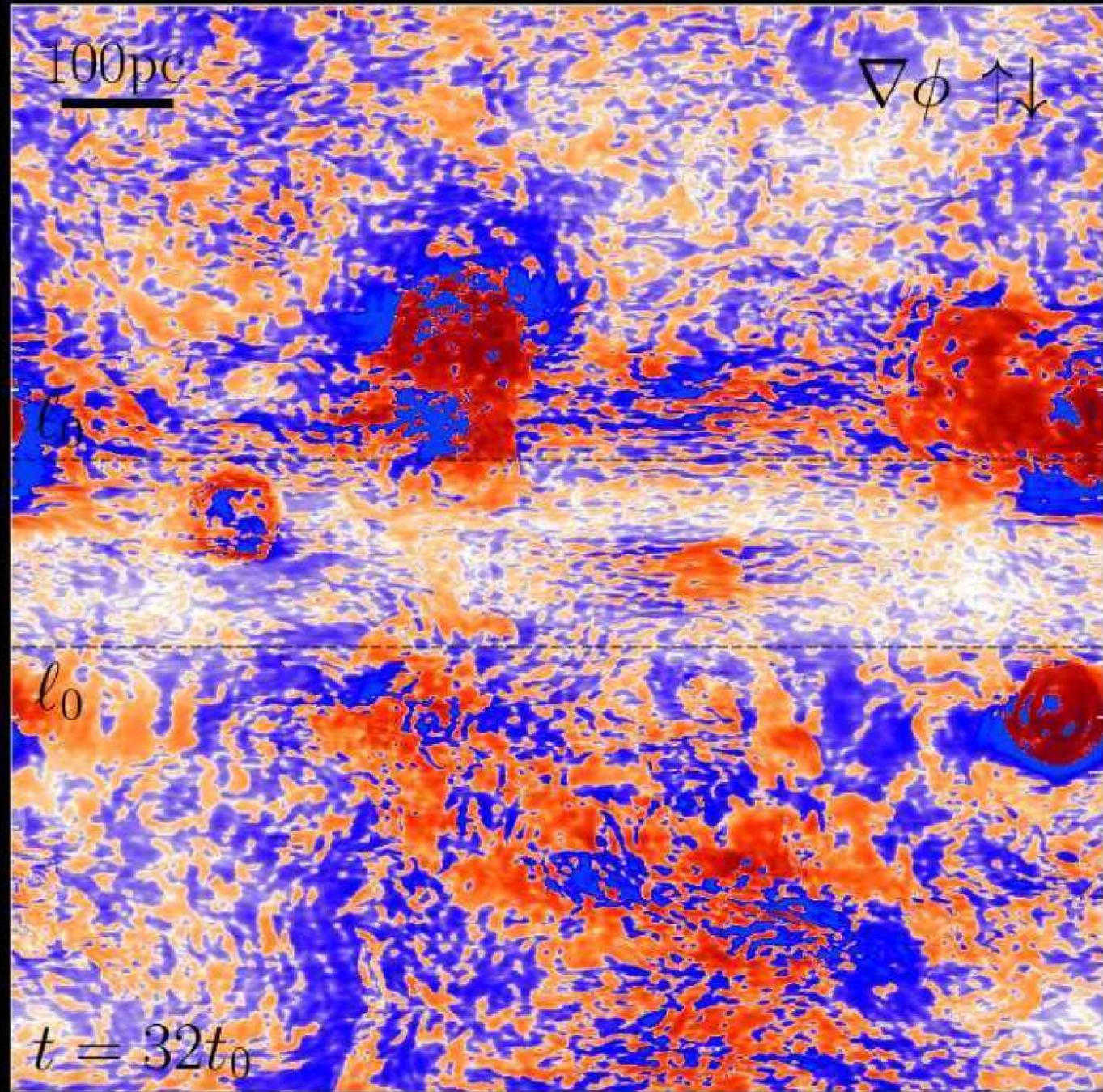
# The incompressible turbulence ( $u_s \rightarrow u_s$ cascade)



# The incompressible turbulence ( $u_s \xrightarrow{u_c} u_s$ cascade)



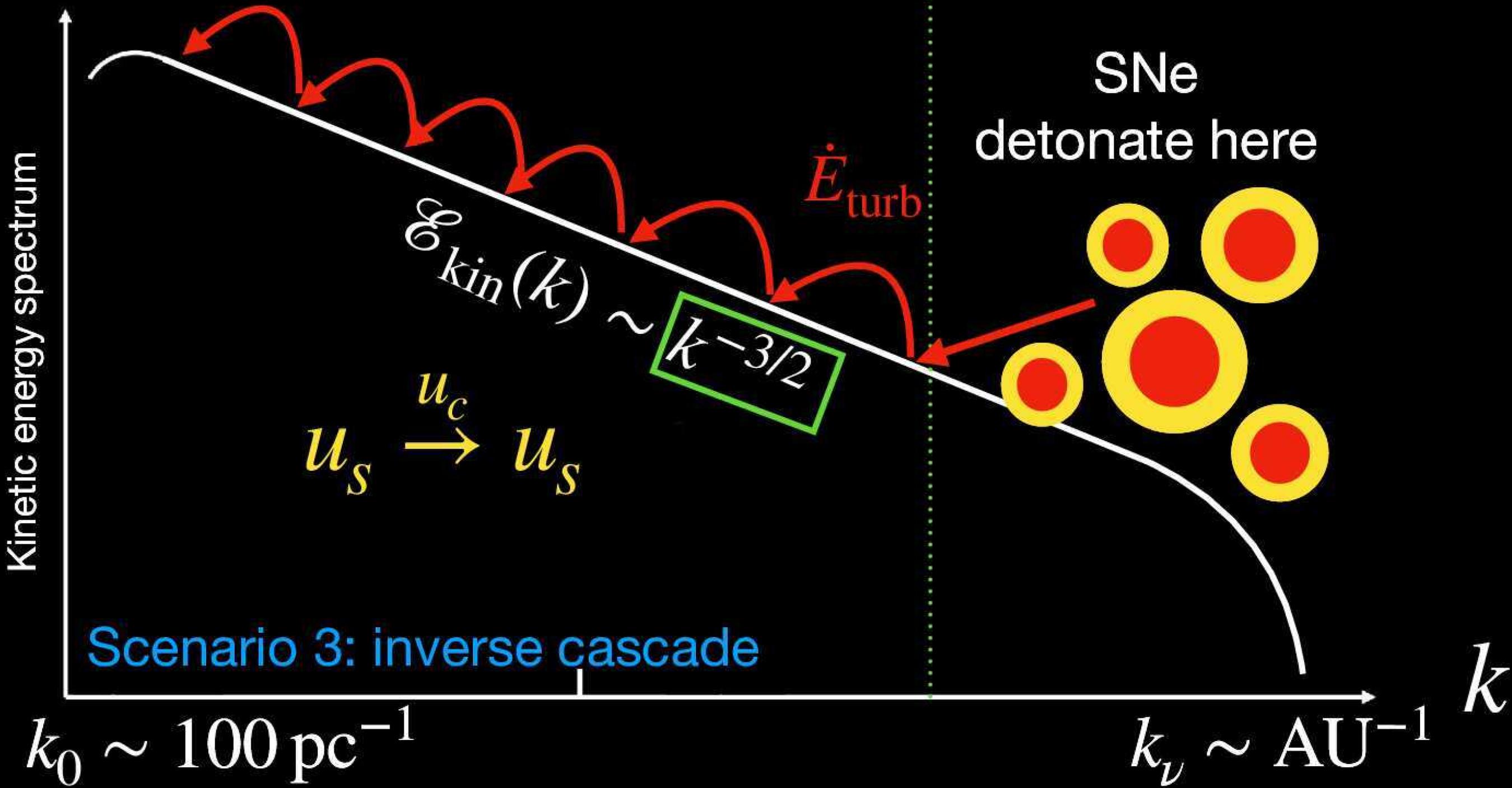
# The incompressible turbulence ( $u_s \xrightarrow{u_c} u_s$ cascade)



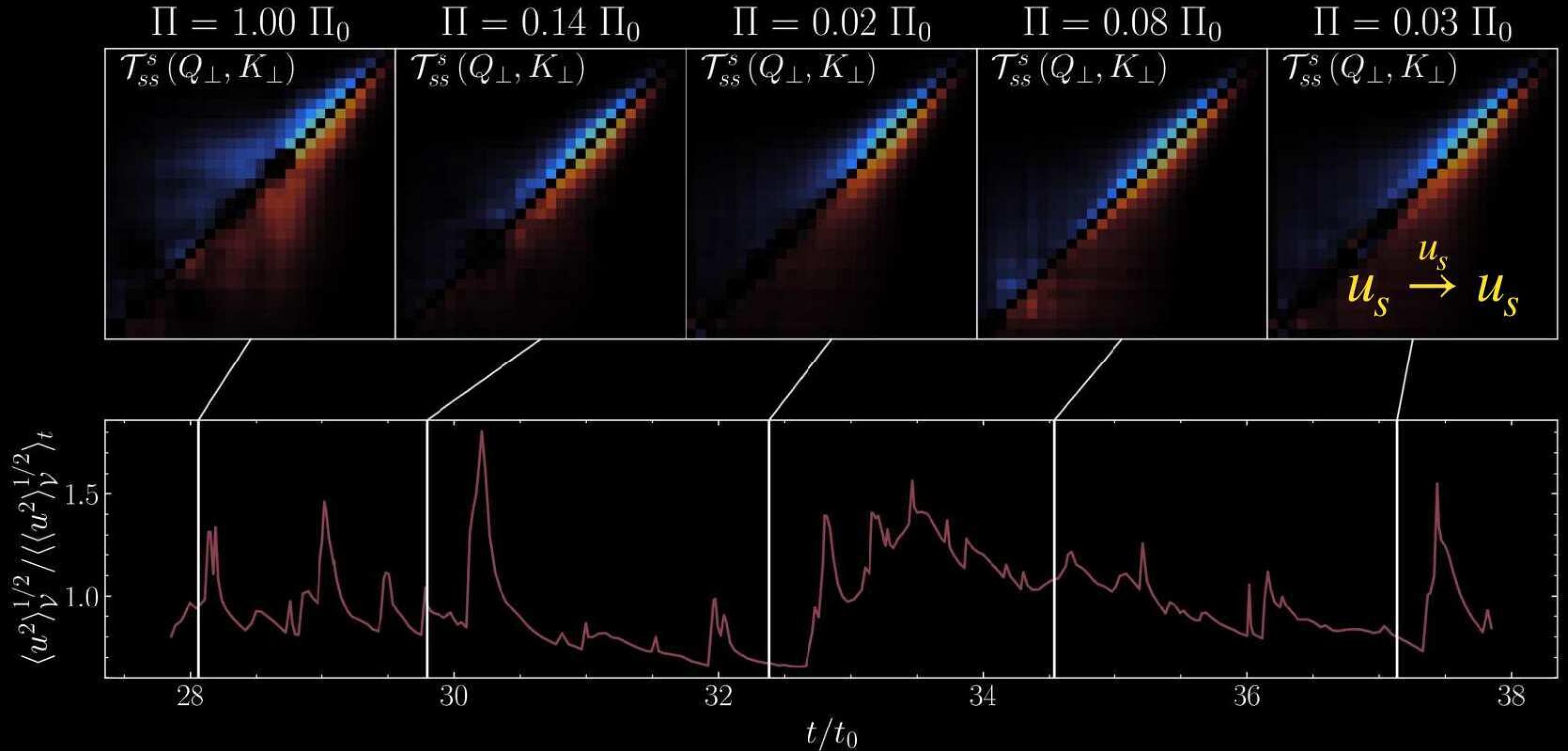
# The naive ( $\mathcal{M} \approx 2$ ) warm ionized medium spectrum (the volume-filling spectrum)

$$R_{\text{cool}}^{-1} \approx 14 \text{ pc}^{-1}$$

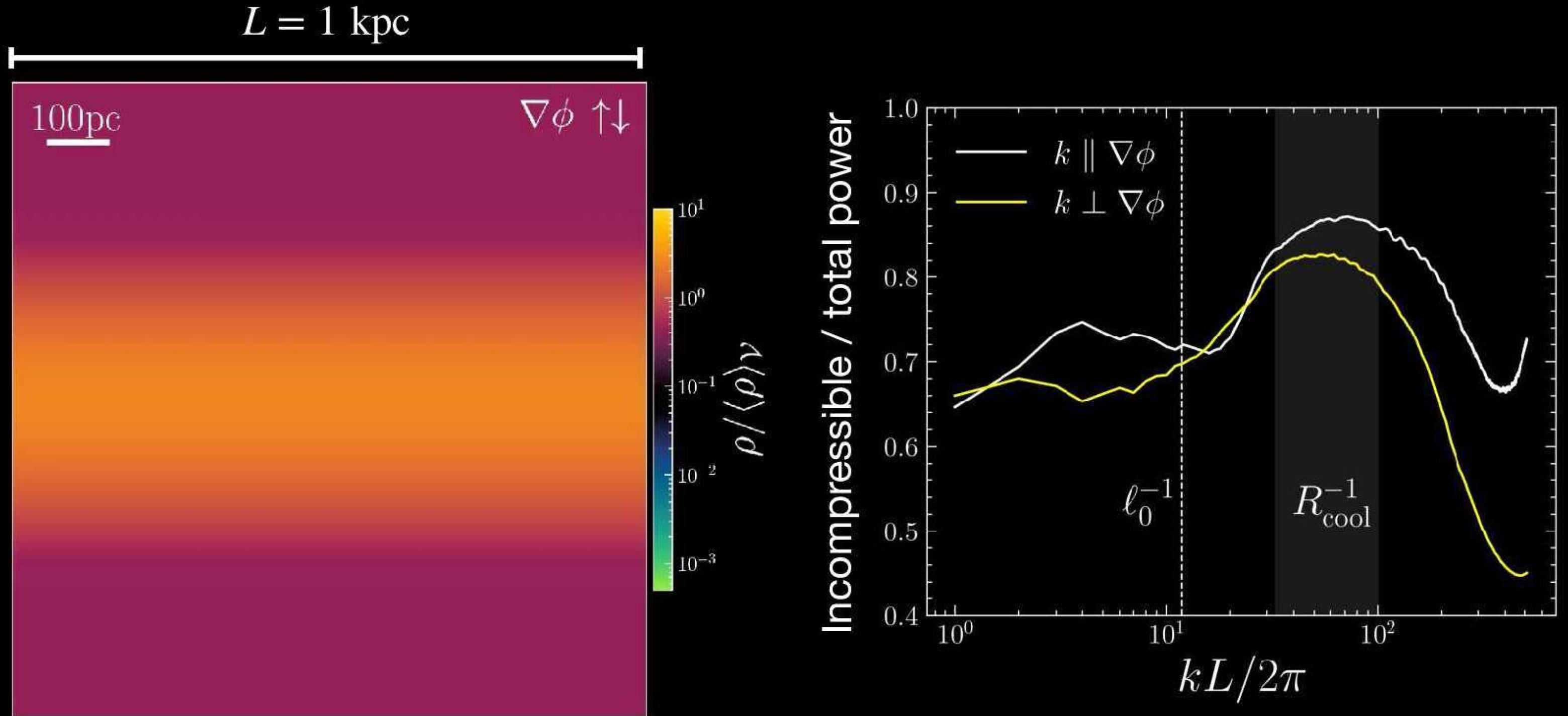
Cioffi + 1998  
Blondin + 1998



# The entire incompressible cascade comes and goes



# Where does the incompressible turbulence come from?



# Where does the incompressible turbulence come from?

No vorticity!

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u} + P \mathbb{I}) = - \rho \nabla \phi + \dot{m}_{\text{SNe}} \mathbf{p}_{\text{SNe}}(Z, n_{\text{H}})$$

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take the curl, Fourier transform and square

$$\frac{1}{2} \frac{\partial \omega(\mathbf{k}) \cdot \omega^\dagger(\mathbf{k})}{\partial t}$$

# Where does the incompressible turbulence come from?

No vorticity!

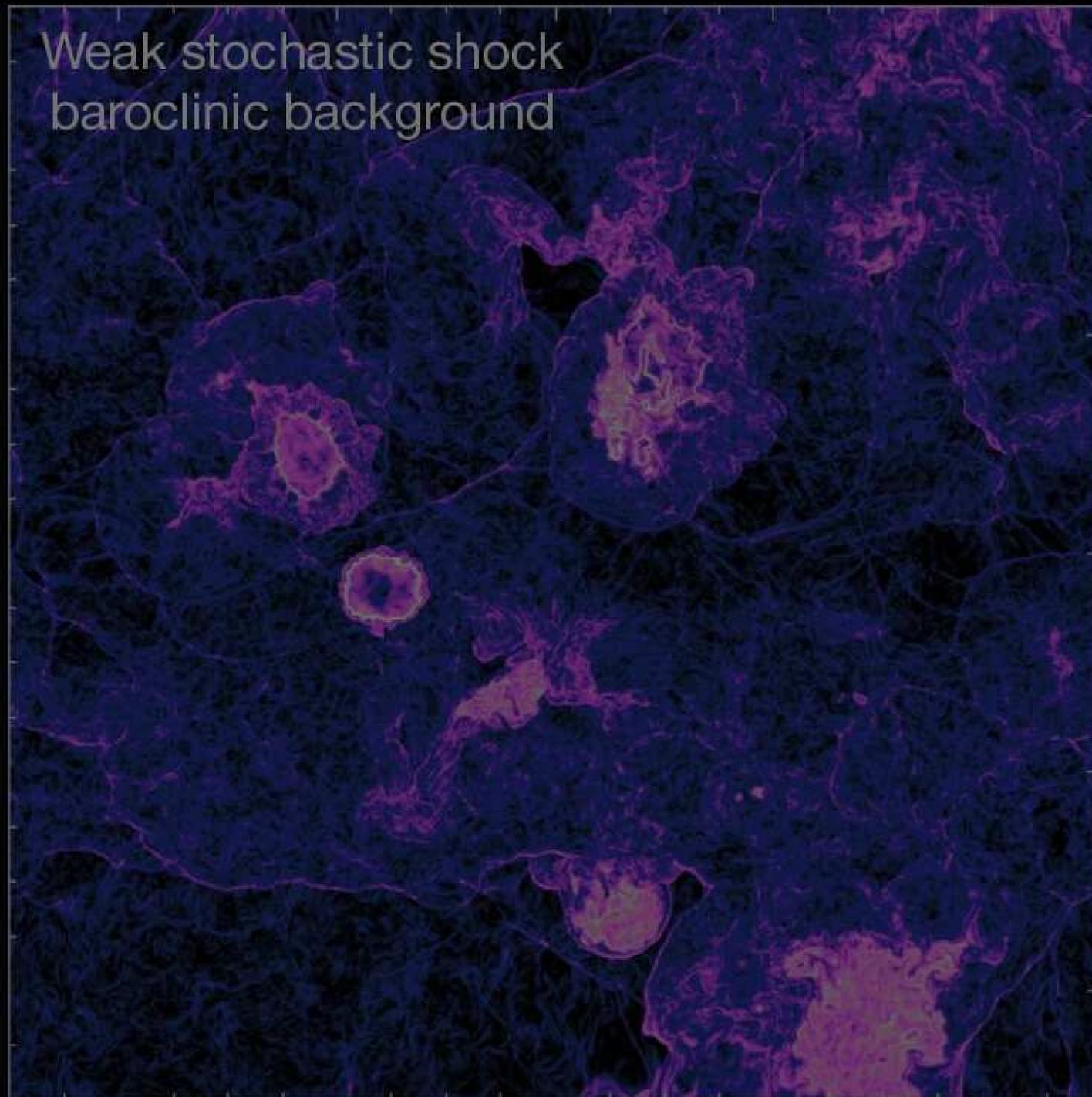
$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u} + P \mathbb{I}) = - \boxed{\rho \nabla \phi + \dot{m}_{\text{SNe}} \mathbf{P}_{\text{SNe}}(Z, n_{\text{H}})}$$



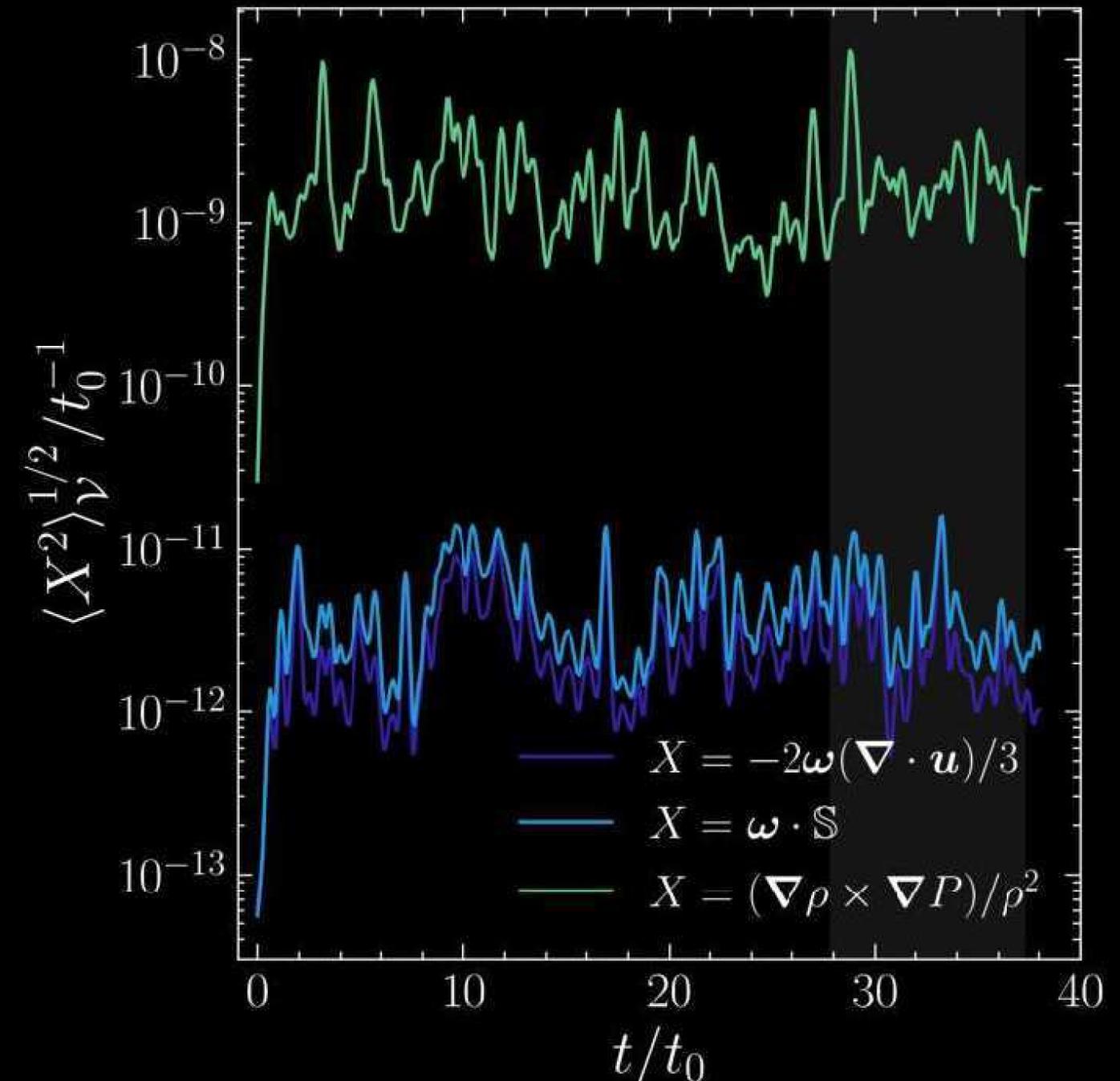
$$\frac{1}{2} \frac{\partial \omega(\mathbf{k}) \cdot \omega^\dagger(\mathbf{k})}{\partial t} = \boxed{\text{Cascade}} \left[ - \frac{d\Pi_\omega(\mathbf{k})}{d\mathbf{k}} \right] + \boxed{\text{Source}} \left[ \text{Re} \left\{ \omega(\mathbf{k}) \cdot (\nabla \rho \times \nabla P / \rho^2)^\dagger(\mathbf{k}) \right\} \right] - \boxed{\text{Sink}} \left[ \mathcal{D}(\mathbf{k}) \right]$$



# Where does the incompressible turbulence come from?

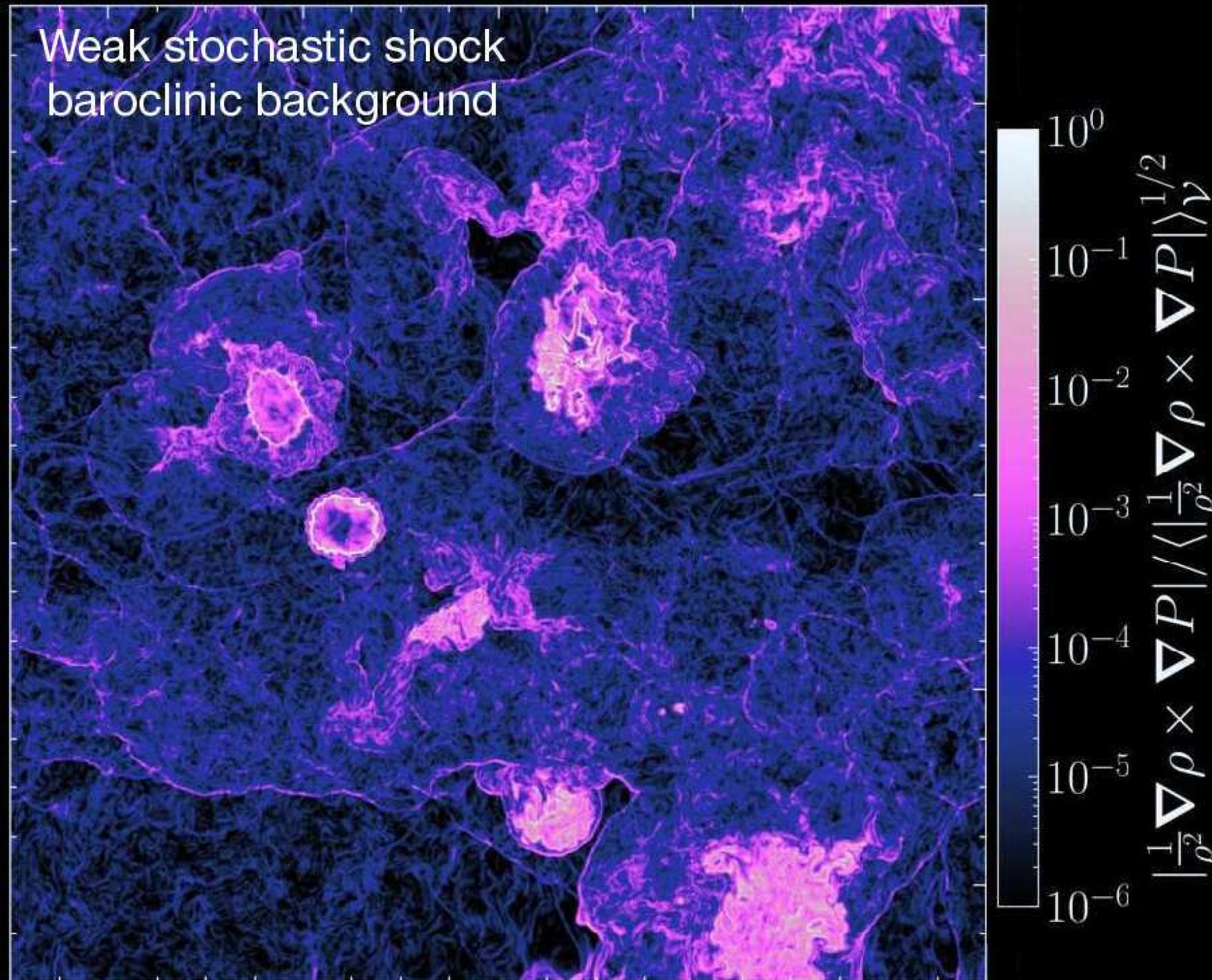


## Vorticity sources

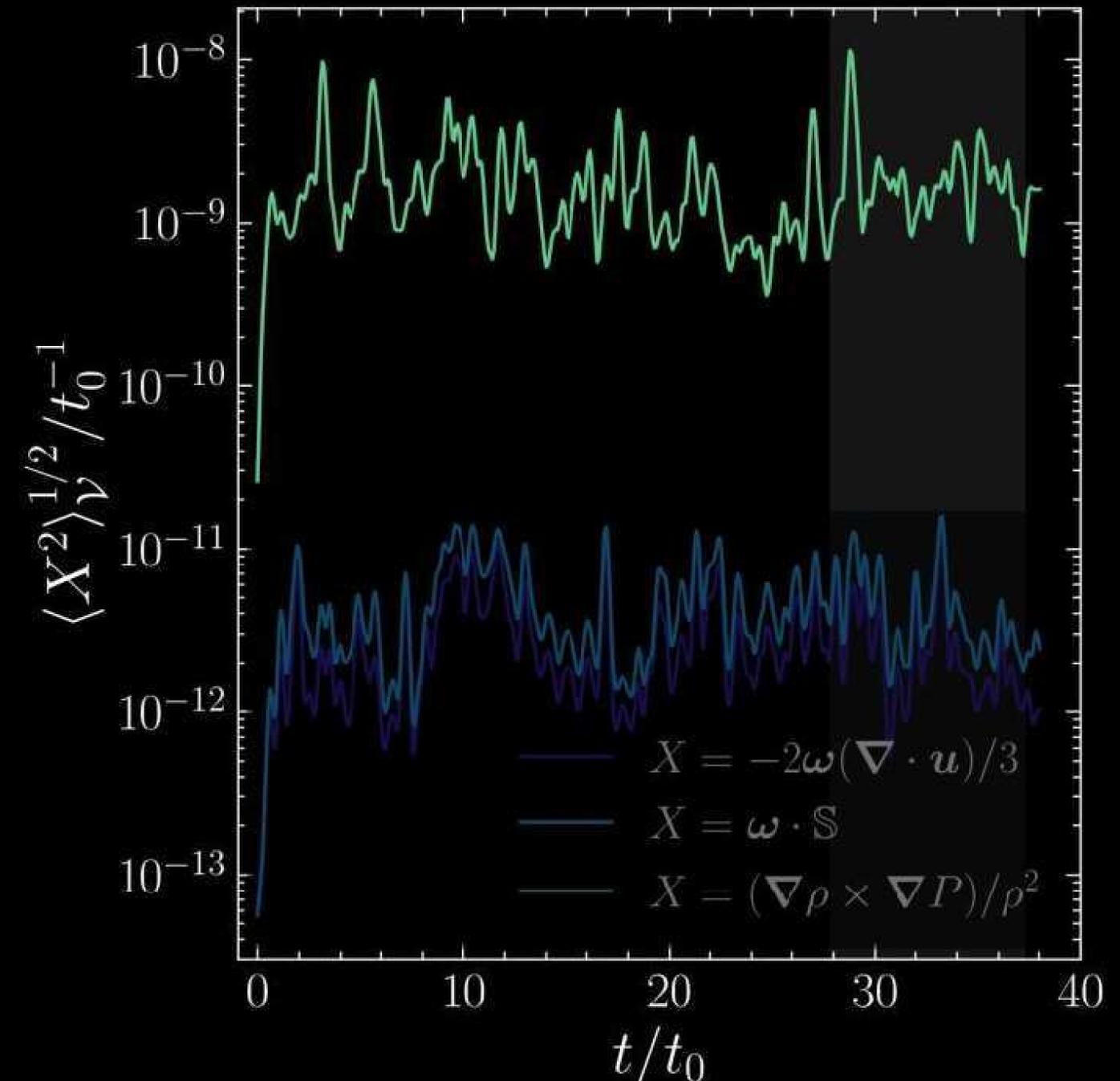


Completed dominated by baroclinic  
battery at all times.

# Where does the incompressible turbulence come from?

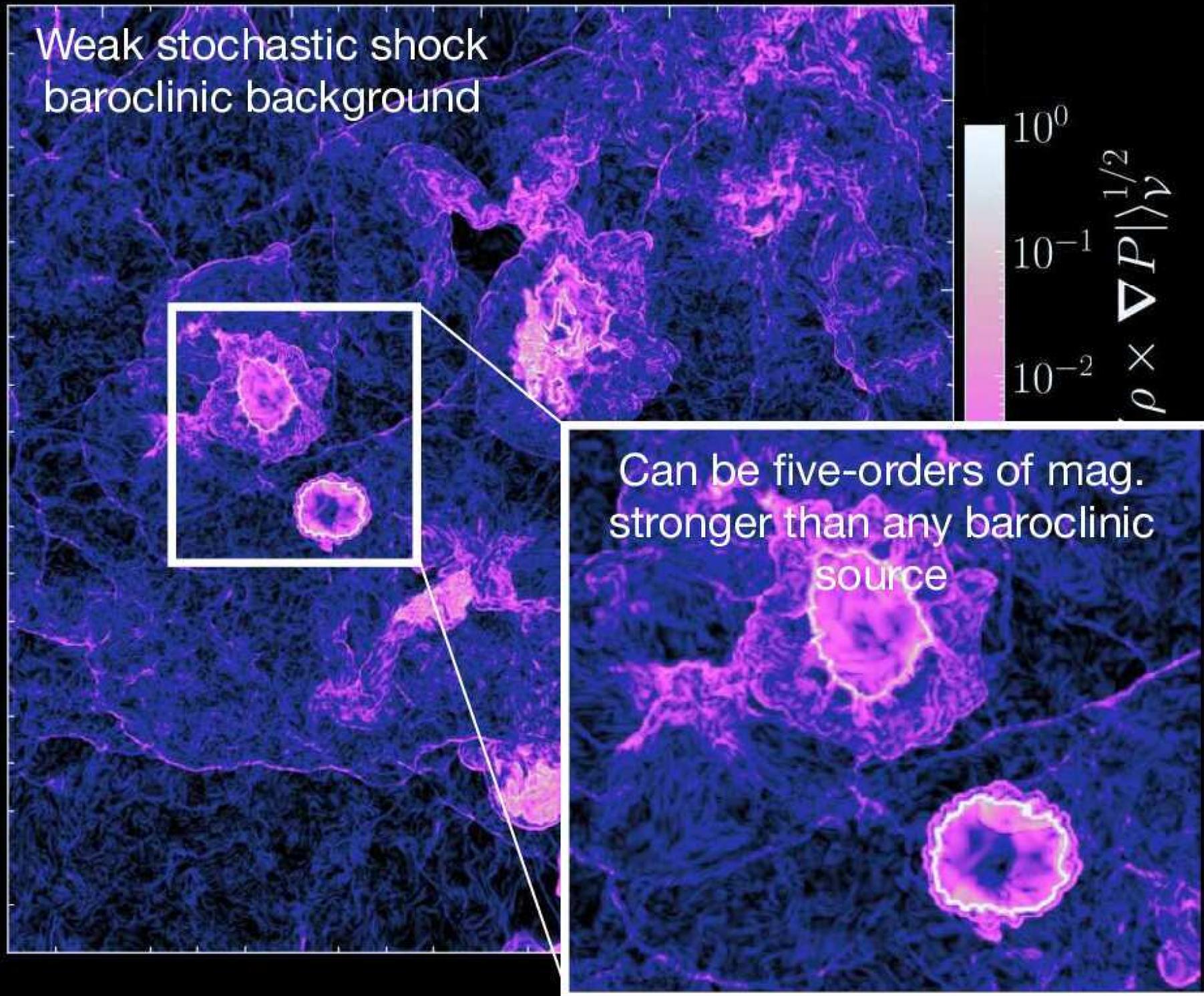


## Vorticity sources

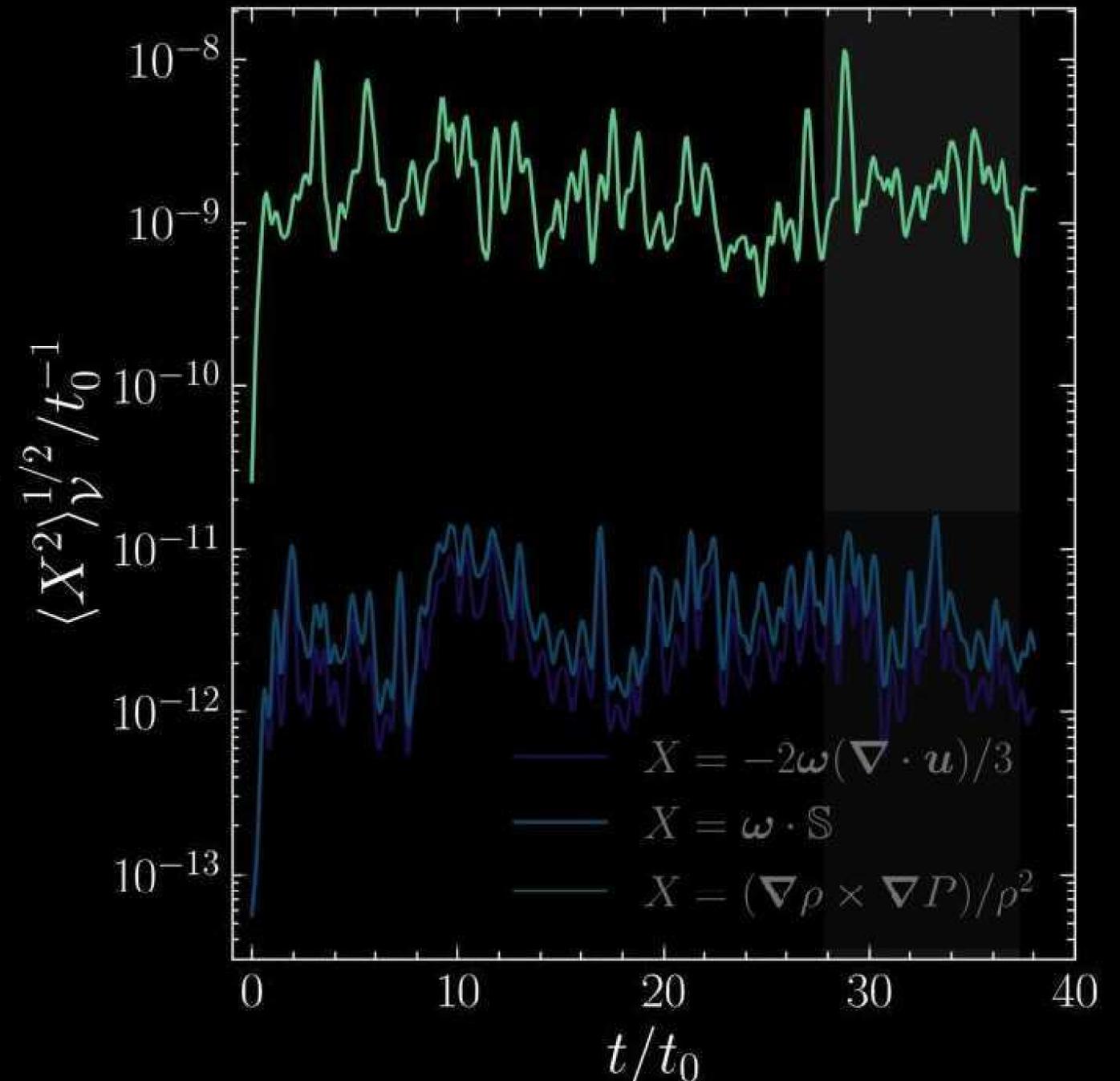


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# Where does the incompressible turbulence come from?



## Vorticity sources



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# Where does the incompressible turbulence come from?

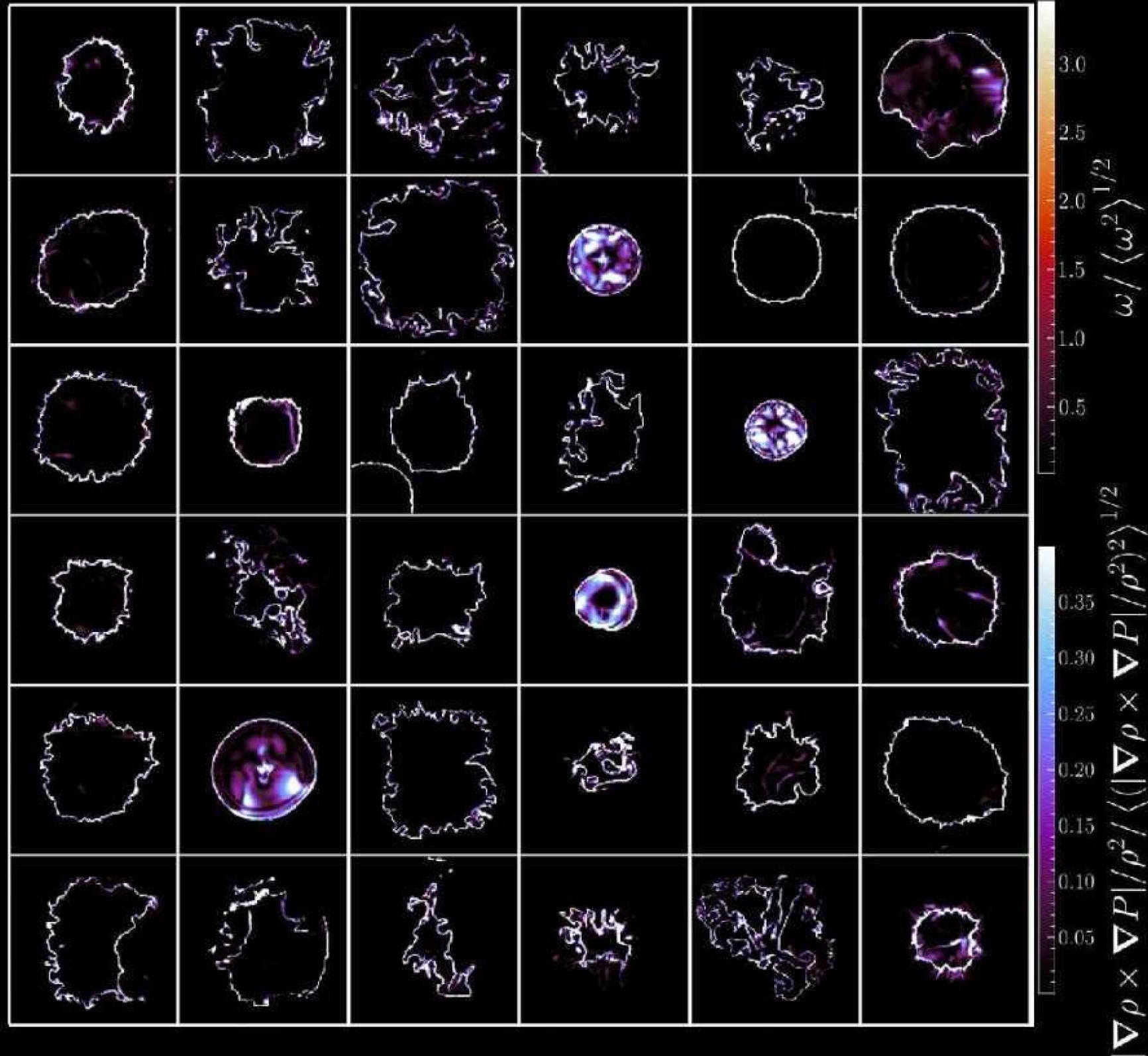
$$\frac{1}{2} \frac{\partial \omega(\mathbf{k}) \cdot \omega^\dagger(\mathbf{k})}{\partial t} = \underbrace{-\frac{d\Pi_\omega(\mathbf{k})}{d\mathbf{k}}}_{\text{Cascade}} + \underbrace{\text{Re} \left\{ \omega(\mathbf{k}) \cdot (\nabla \rho \times \nabla P / \rho^2)^\dagger(\mathbf{k}) \right\}}_{\text{Source}} - \underbrace{\mathcal{D}(\mathbf{k})}_{\text{Sink}}$$

Assuming isotropic, stationary, high-Re

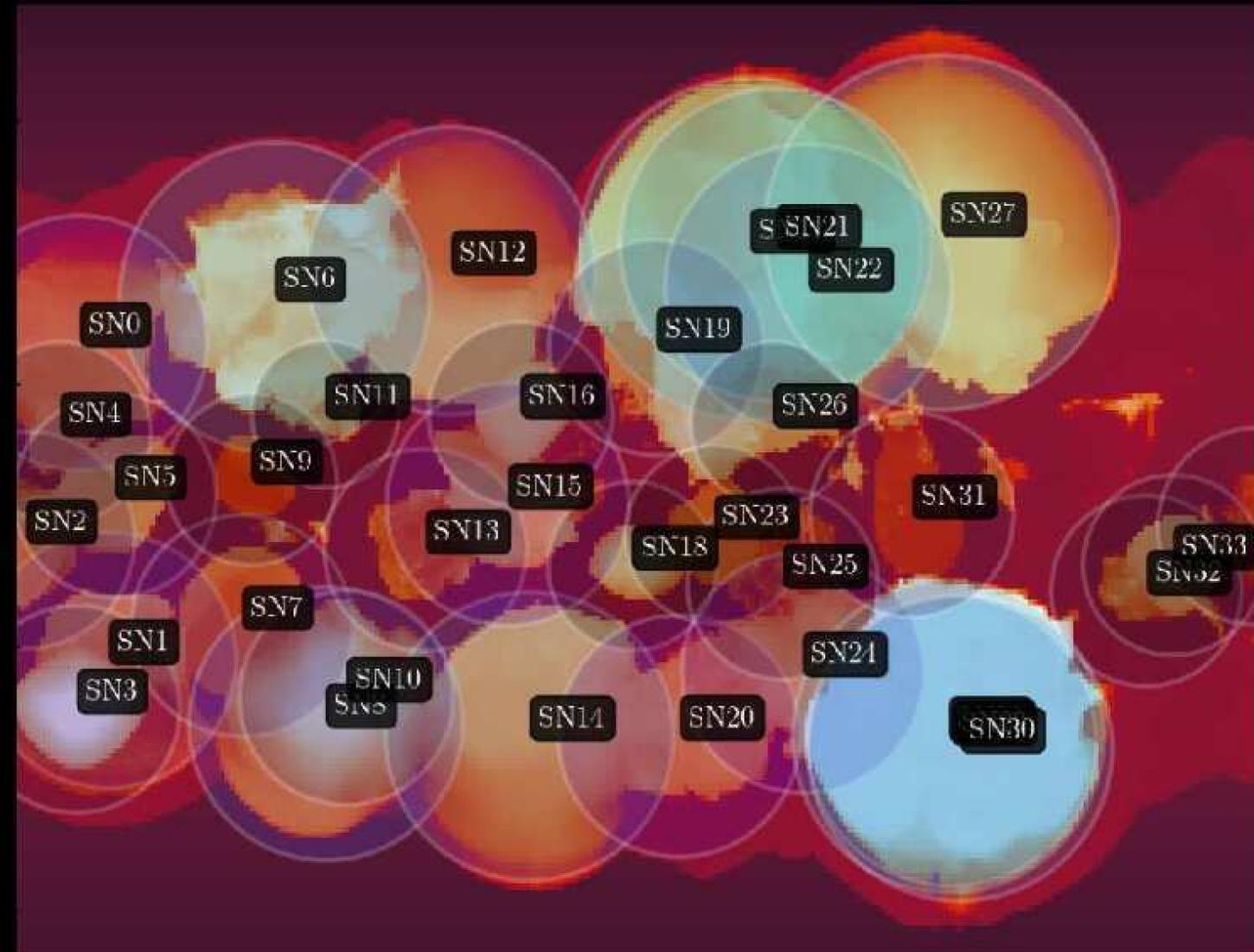
$$\underbrace{\frac{d\Pi_\omega(\mathbf{k})}{d\mathbf{k}}}_{\text{Cascade}} \approx \underbrace{\text{Re} \left\{ \omega(\mathbf{k}) \cdot (\nabla \rho \times \nabla P / \rho^2)^\dagger(\mathbf{k}) \right\}}_{\text{Source}}$$
$$\mathcal{P}_{\omega\text{B}}(k)$$

# Where does the incompressible turbulence come from?

Beattie (2026; submitted ApJL) Supernovae drive large-scale incompressible turbulence from small-scale instabilities



Cluster all SNR in 3D to extract local statistics



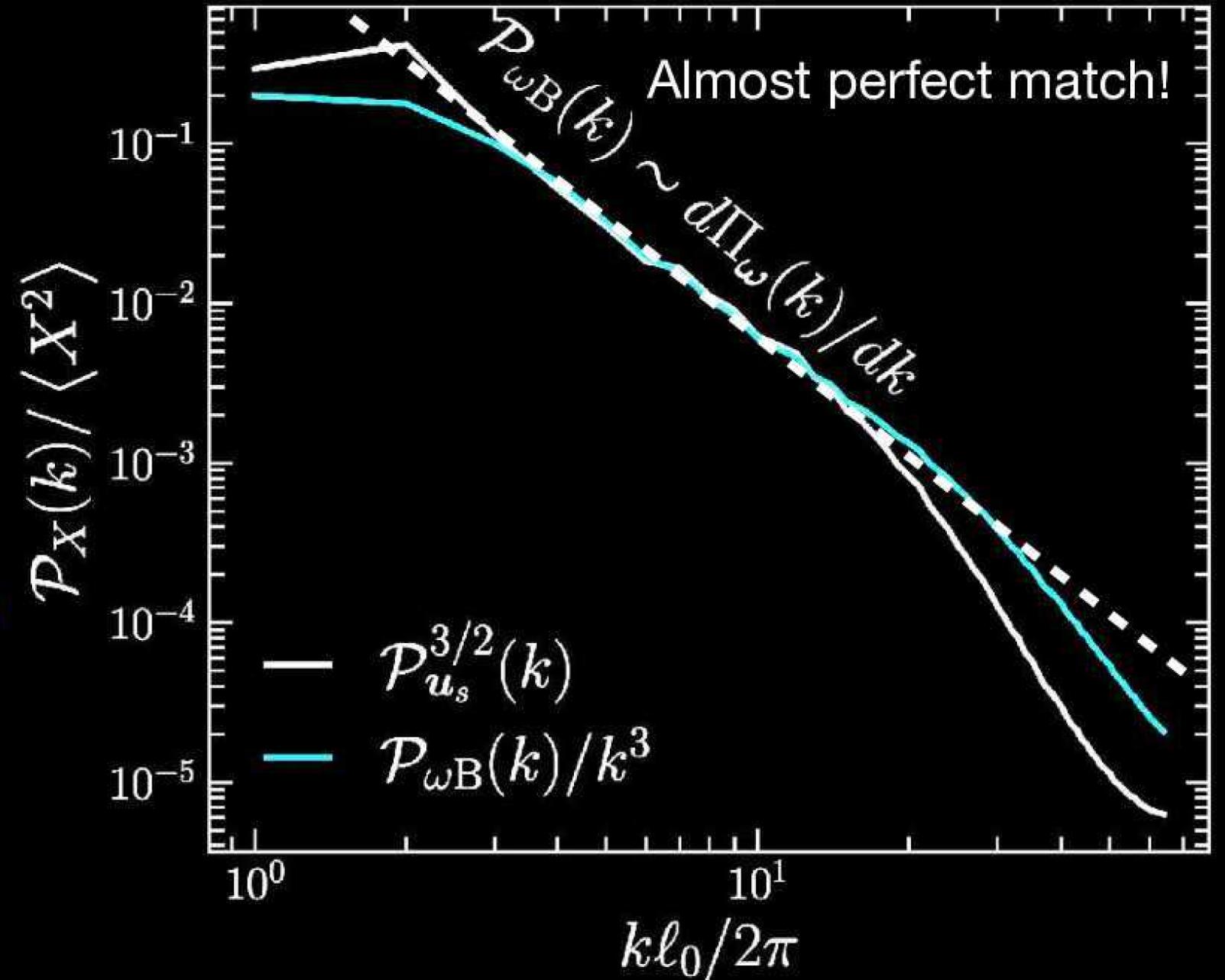
# Where does the incompressible turbulence come from?

Assuming isotropic, stationary, high-Re

$$\mathcal{P}_{\omega\text{B}}(k) \approx \frac{d\Pi_{\omega}(k)}{dk}$$

relation between incompressible mode spectrum and baroclinic source spectrum

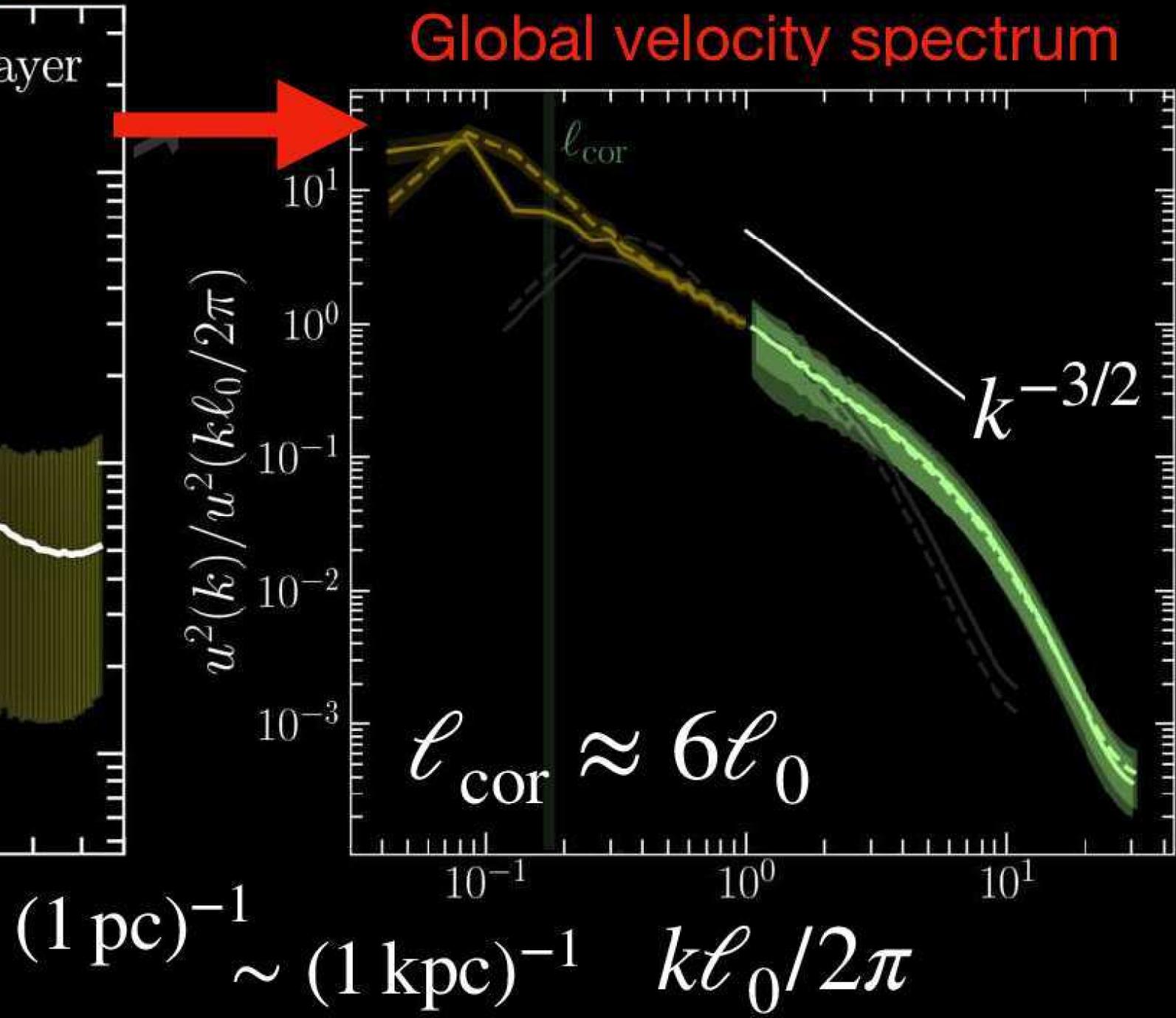
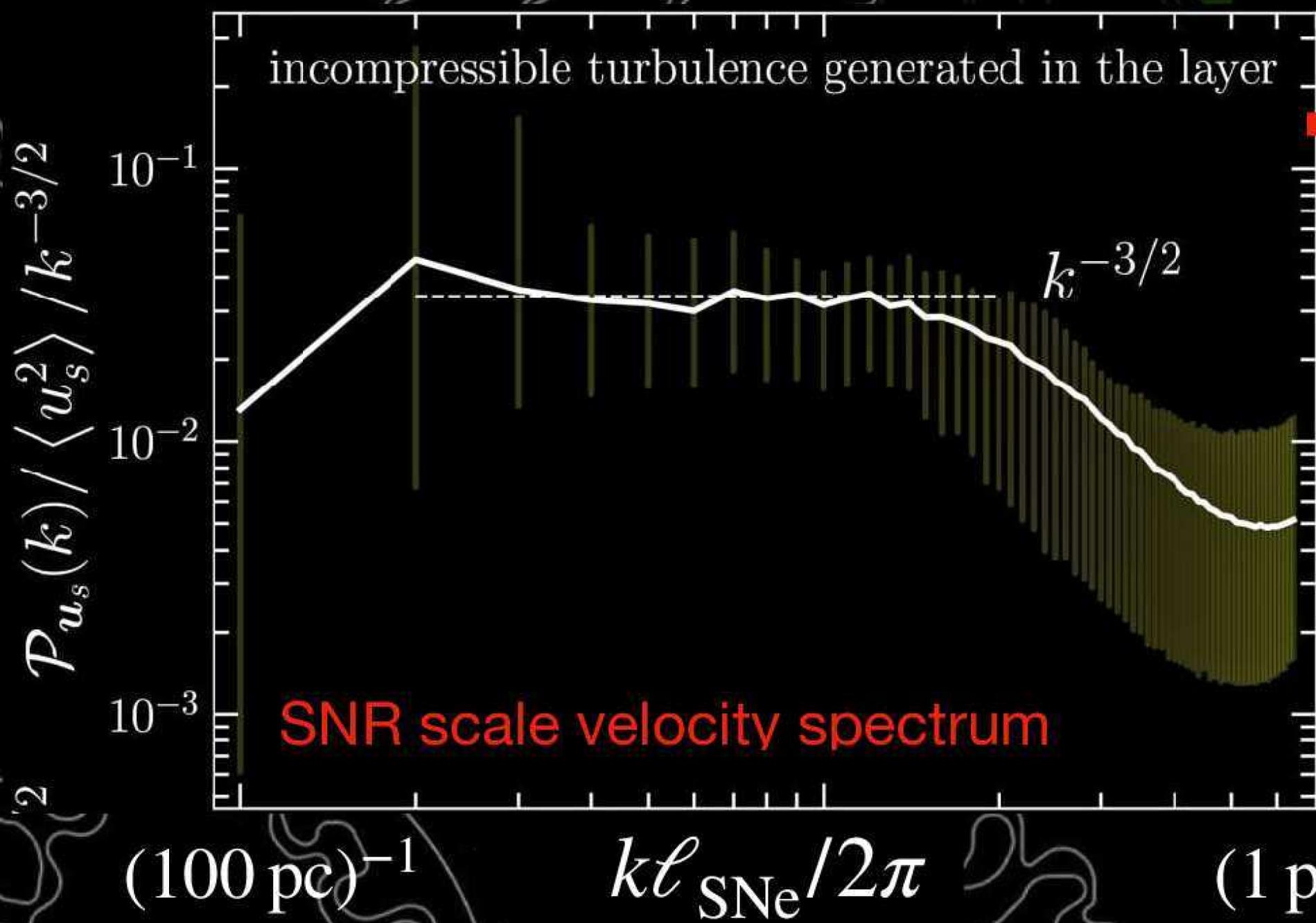
$$\mathcal{P}_{\omega\text{B}}(k) \sim k^3 \mathcal{P}_{u_s}^{3/2}(k)$$



$$k \ell_{\text{SNe}} / 2\pi$$

# Incompressible modes generated on SNR scale imprinted on all scales

(scales larger than gaseous scale height!!!)

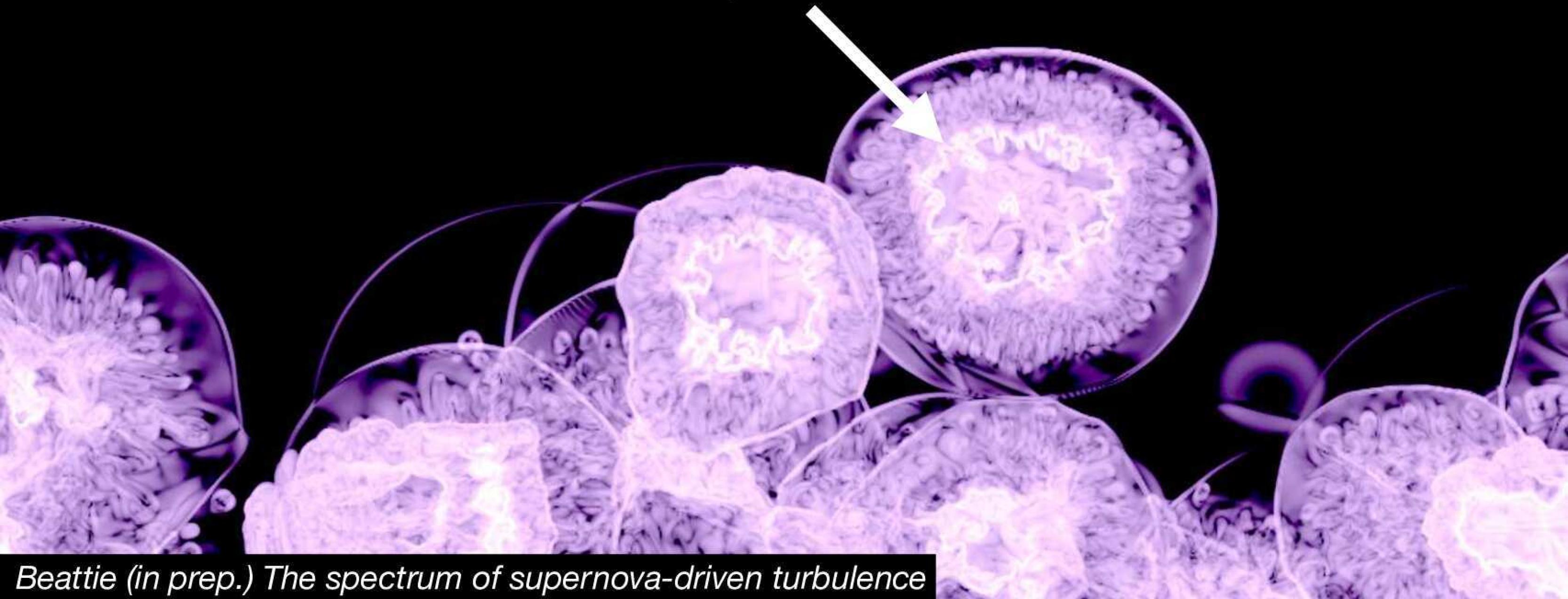


# Can we predict the spectrum?

Baroclinicity is born from thin shell instabilities (RT & overstability)

$$\gamma \sim \sqrt{k}$$

Linear RTI  
Vishniac (1983)



*Beattie (in prep.) The spectrum of supernova-driven turbulence*

# Can we predict the spectrum?

Baroclinicity is born from thin shell instabilities (RT & overstability)

$$\gamma \sim \sqrt{k} \quad \text{Linear RTI} \quad \text{Vishniac (1983)}$$
$$t_{\text{nl}} \sim \gamma^{-1} \sim k^{-1/2}$$


Motivated by reconnection-mediated turbulence models (e.g., Boldyrev+2017; Dong+2022)

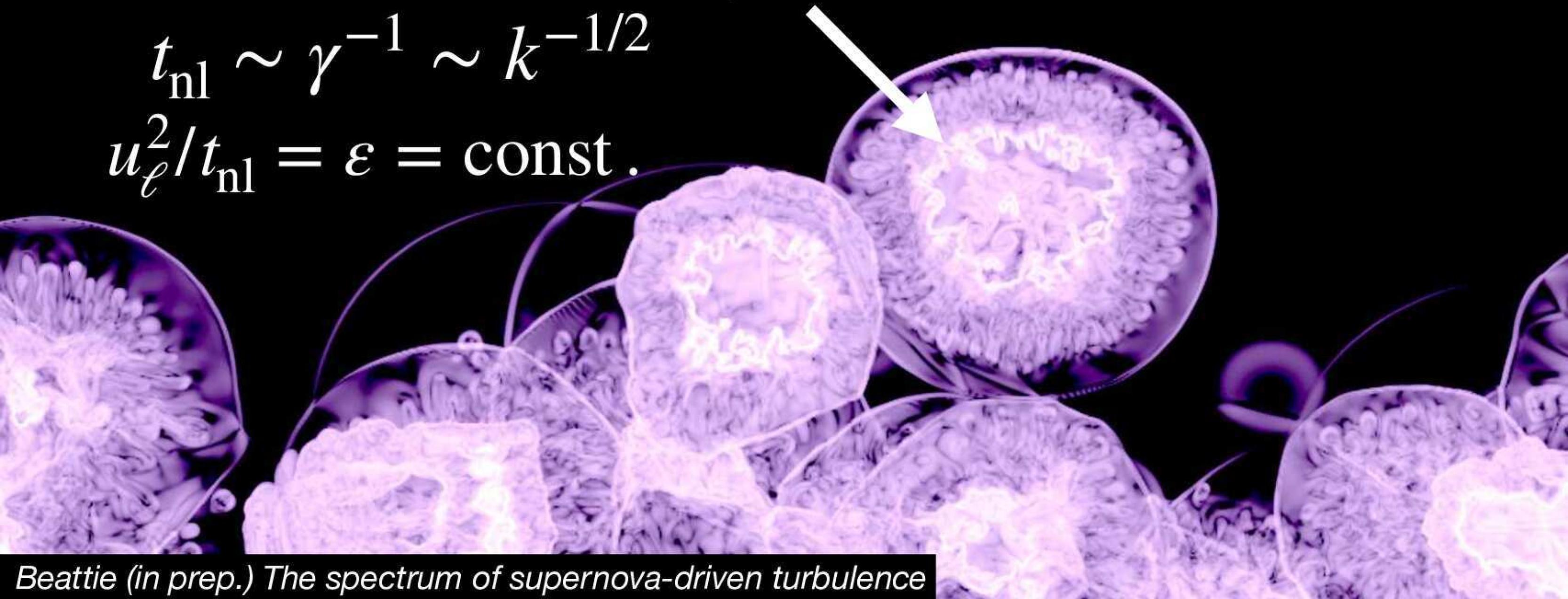
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$$\gamma \sim \sqrt{k} \quad \text{Linear RTI} \\ \text{Vishniac (1983)}$$

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$$u_{\ell}^2 / t_{n1} = \varepsilon = \text{const.}$$



# Can we predict the spectrum?

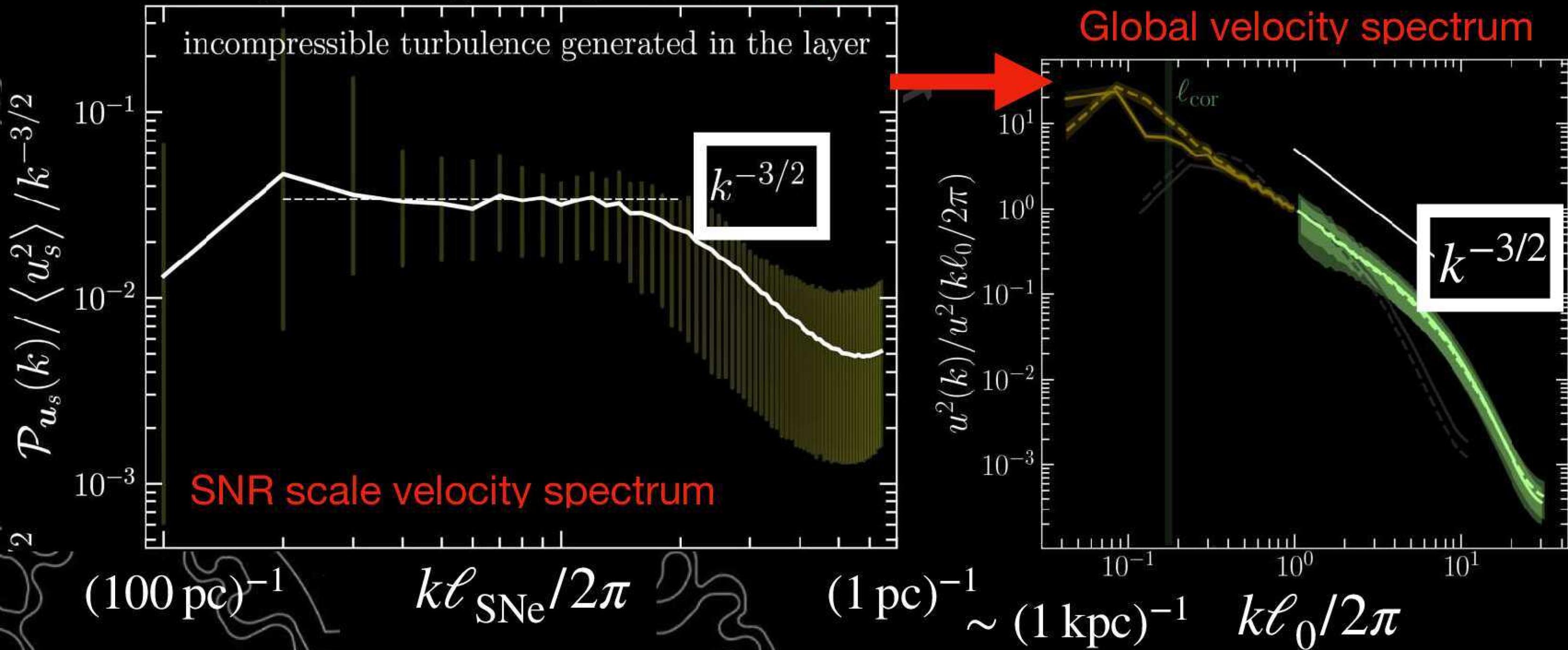
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$$u_\ell \sim \ell^{1/4} \iff u^2(k) \sim k^{-3/2} dk$$

# Shell instabilities drive and imprint themselves on the spectrum!



# What's the driving scale of SN-driven turbulence?

From baroclinic feeding

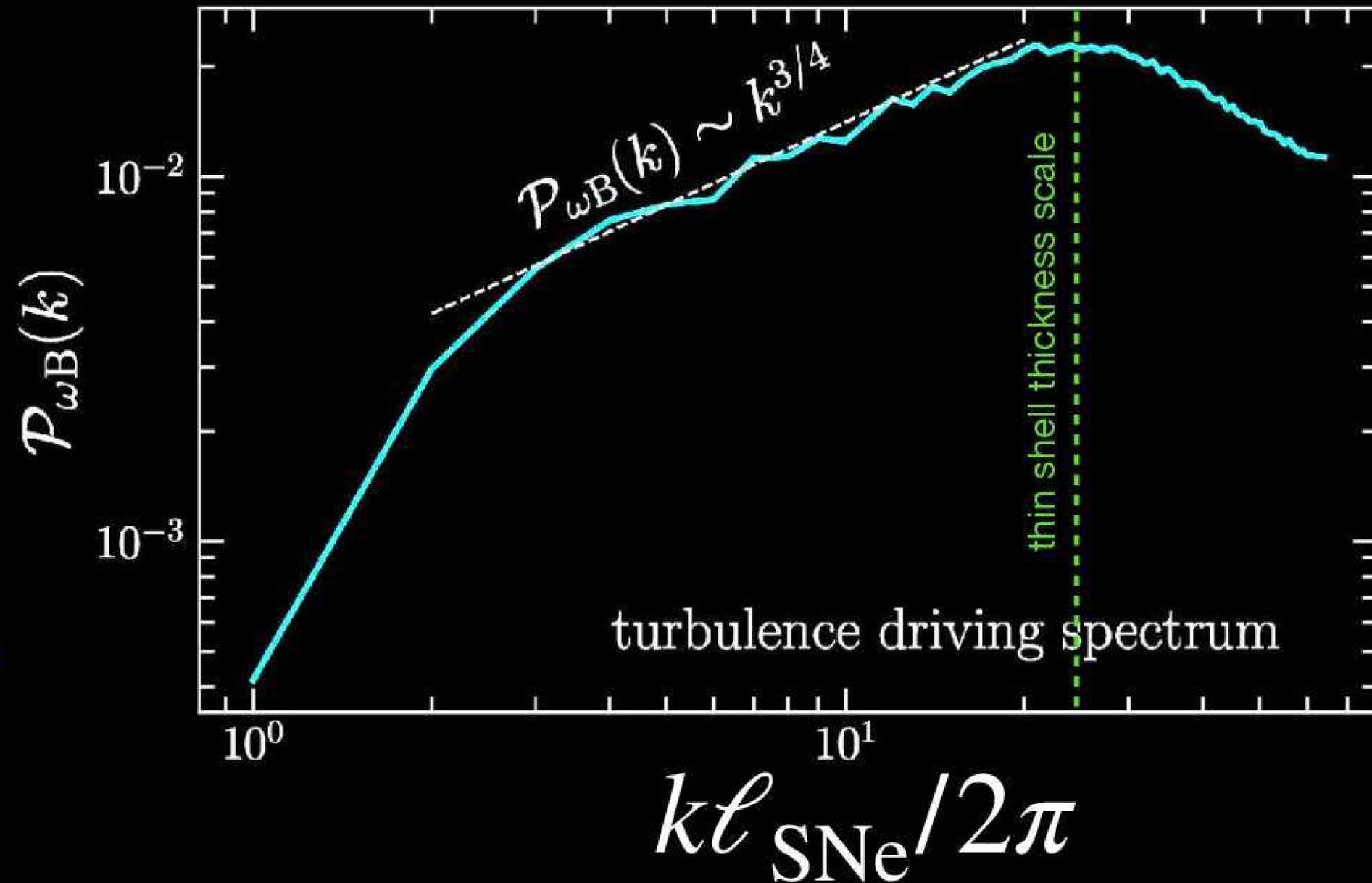
$$\mathcal{P}_{\omega_B}(k) \sim k^3 \mathcal{P}_{u_s}^{3/2}(k)$$

From RTI

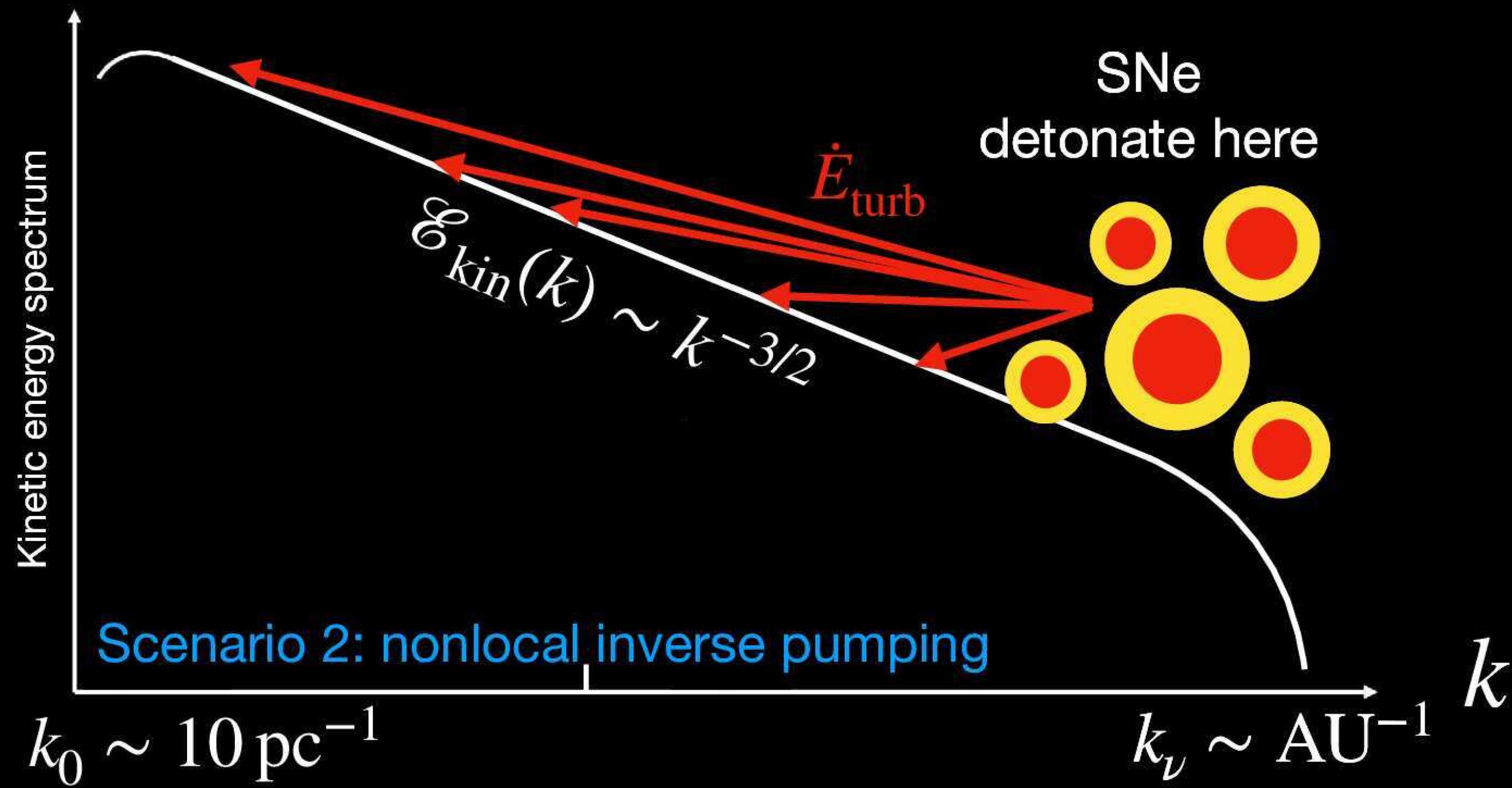
$$\mathcal{P}_{u_s}(k) \sim k^{-3/2}$$

Spectrum that drives the turbulence is therefore:

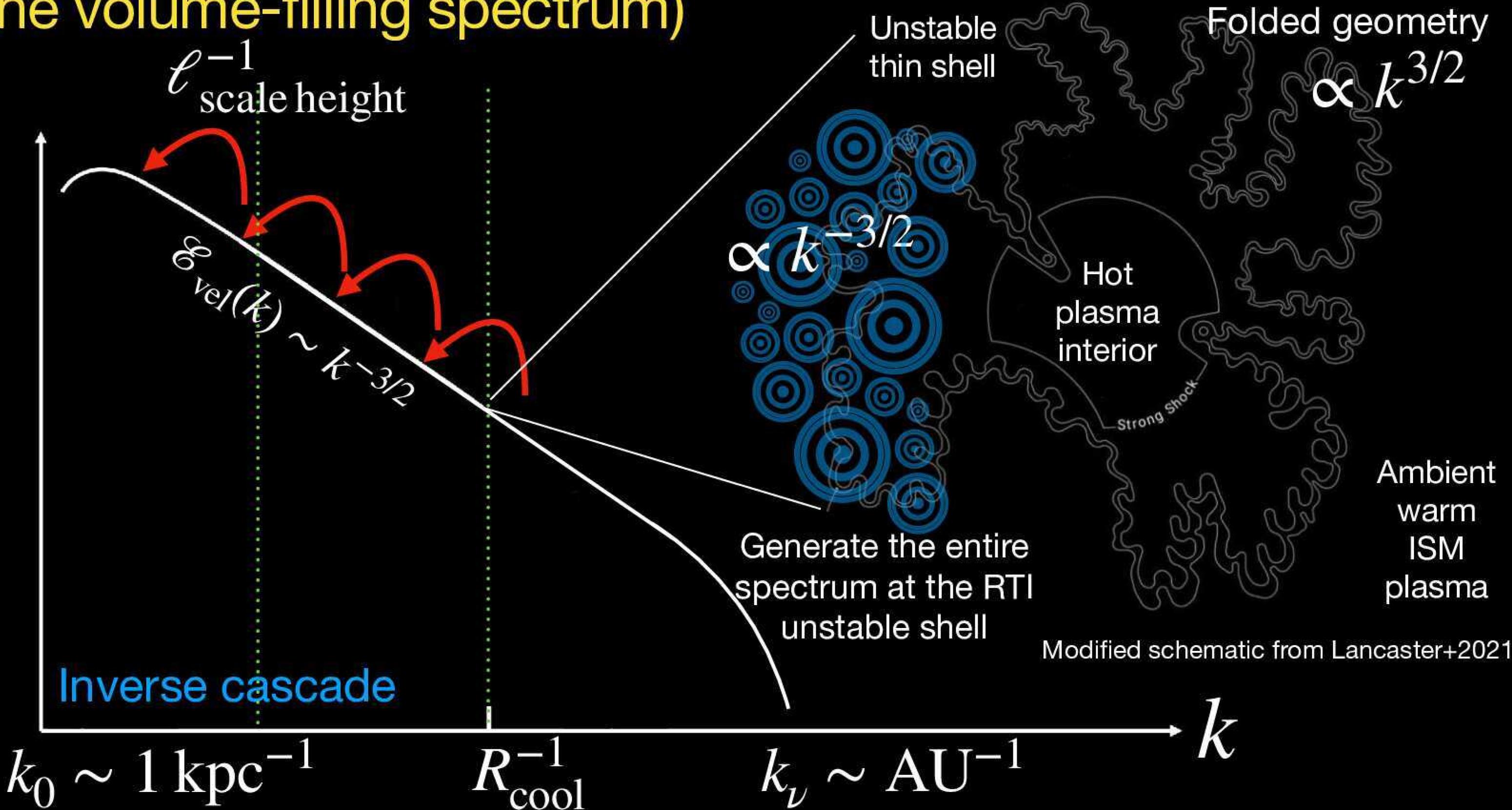
$$\mathcal{P}_{\omega_B}(k) \sim k^{3/4}$$

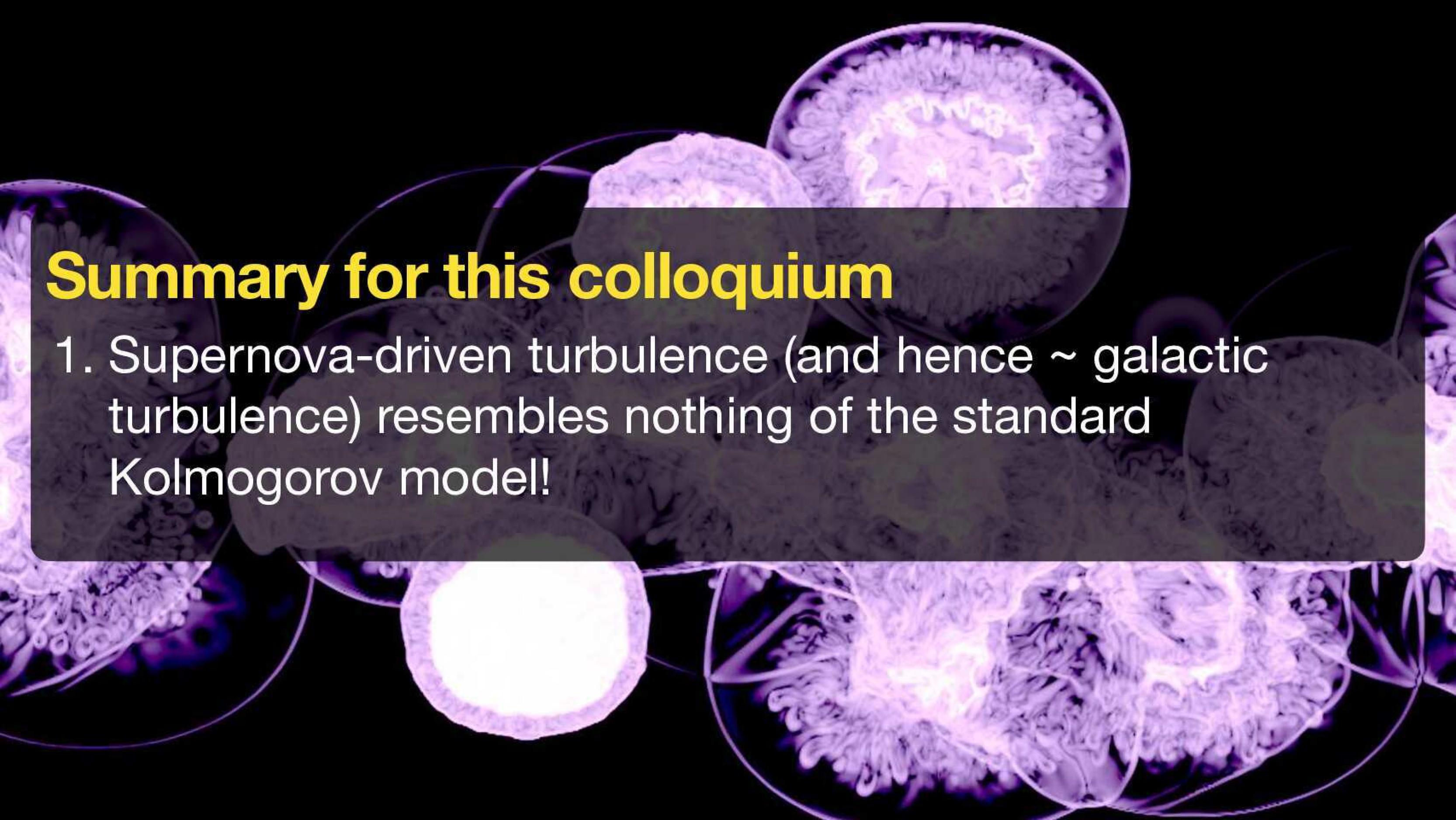


The naive ( $\mathcal{M} \approx 2$ ) warm ionized medium spectrum  
(the volume-filling spectrum)



# The SN-driven warm ionized medium spectrum (the volume-filling spectrum)





## Summary for this colloquium

1. Supernova-driven turbulence (and hence  $\sim$  galactic turbulence) resembles nothing of the standard Kolmogorov model!

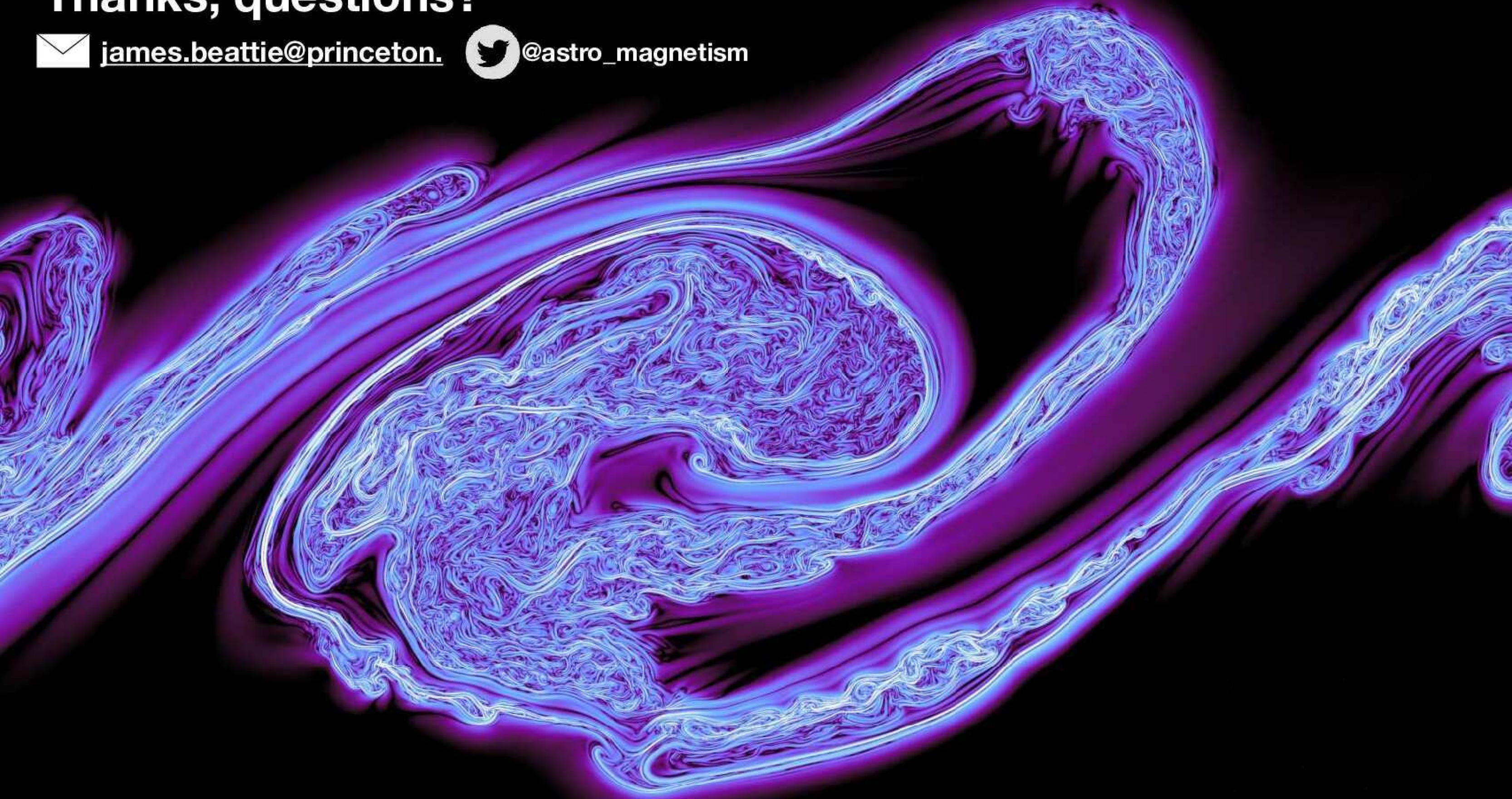
# Thanks, questions?



[james.beattie@princeton.edu](mailto:james.beattie@princeton.edu)



[@astro\\_magnetism](https://twitter.com/astro_magnetism)



# Compressible modes and solenoidal modes live very different lives!

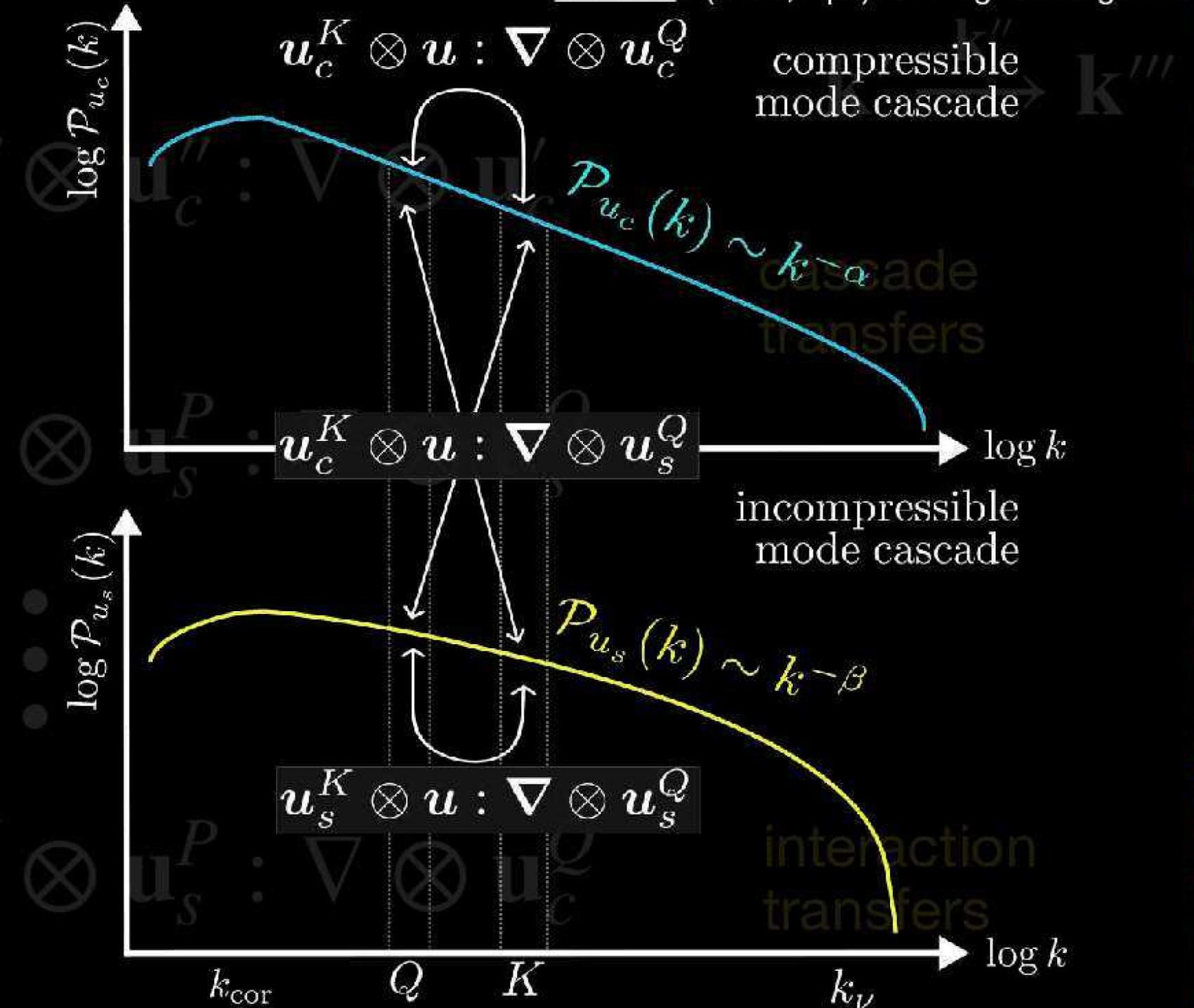
We should treat them differently!

Beattie + (2025, ApJ) So long Kolmogorov.

$$\mathcal{T}_{cc}^c(Q, K) = - \int dV \mathbf{u}'_c$$

$$\mathcal{T}_{ss}^s(Q, K) = - \int dV \mathbf{u}_s^K$$

$$\mathcal{T}_{cs}^s(Q, K) = - \int dV \mathbf{u}_s^K$$



Compressible modes and solenoidal modes live very different lives!  
 We should treat them differently!

$$Q \xrightarrow{P} K$$

$$\mathcal{T}_{cc}^c(Q, K) = - \int dV \mathbf{u}_c^K \otimes \mathbf{u}_c^P : \nabla \otimes \mathbf{u}_c^Q$$

cascade transfers

$$\mathcal{T}_{ss}^s(Q, K) = - \int dV \mathbf{u}_s^K \otimes \mathbf{u}_s^P : \nabla \otimes \mathbf{u}_s^Q$$

$$\begin{aligned} \mathbf{u}_s \cdot \mathbf{u}_c &= 0 \\ \nabla \cdot \mathbf{u}_s &= 0 \\ |\nabla \times \mathbf{u}_c| &= 0 \\ \mathbf{u} &= \mathbf{u}_c + \mathbf{u}_s \end{aligned}$$

$$\mathcal{T}_{cs}^s(Q, K) = - \int dV \mathbf{u}_s^K \otimes \mathbf{u}_s^P : \nabla \otimes \mathbf{u}_c^Q$$

interaction transfers

But to understand the turbulence we must understand the flux from mode to mode!

kinetic energy density

$$\frac{1}{2} \overbrace{\partial_t \rho \mathbf{u}^Q \cdot \mathbf{u}^K} + \underbrace{\mathbf{u}^K \cdot \nabla \cdot \mathbb{F}_{\rho \mathbf{u}}(\mathbf{u}^P, \mathbf{u}^Q)} = 0 \quad Q \xrightarrow{P} K$$

energy flux rate density  
from transport between  $\mathbf{u}^Q$  and  $\mathbf{u}^K$

$$\mathbf{u}^K \cdot \nabla \cdot \mathbb{F}(\mathbf{u}^P, \mathbf{u}^Q)_{\rho \mathbf{u}} = \mathbf{u}^K \otimes \mathbf{u}^P : \nabla \otimes \mathbf{u}^Q + \dots$$

$$\varepsilon \sim u^3 / \ell$$

But to understand the turbulence we must understand the flux from mode to mode!

$$\mathbf{u}^K \cdot \nabla \cdot \mathbb{F}(\mathbf{u}^P, \mathbf{u}^Q)_{\rho \mathbf{u}} = \mathbf{u}^K \otimes \mathbf{u}^P : \nabla \otimes \mathbf{u}^Q + \dots$$

$$\mathcal{T}_{uu}(Q, K | P) = - \int dV \mathbf{u}^K \otimes \mathbf{u}^P : \nabla \otimes \mathbf{u}^Q$$
$$\varepsilon \sim u^3 / \ell$$