The Supersonic Turbulent Dynamo

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High Energy Density Laboratory Astrophysics
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Bold goals for this (10min!) talk

1. Comparison of the **fast growth stages** in supersonic and subsonic turbulent dynamos.

2. Time permitting: new results on the **dynamo saturation** with $10{,}080^3$ supersonic FLASH simulations at $\mathrm{Re} \sim \mathrm{Rm} \sim 10^6$

This talk will be about magnetic dynamos in the context of visco/resistive Newtonian collisional MHD fluids.


$L \sim \mathcal{O}(\mathrm{kpc})$
M82 (Cigar Galaxy)
M51 (Whirlpool Galaxy)
small scale, disordered magnetic fields
$\mathcal{E}_{\mathrm{kin}} \sim \mathcal{E}_{\mathrm{mag}}$
Beck (2015)
$\mathbf{B} = \langle \mathbf{B} \rangle + \mathbf{b}$
large scale dynamo theory
small scale dynamo theory
large scale, ordered magnetic fields
Lopez-Rodriguez + (2021)
Credit: NASA, the SOFIA science team, A. Borlaff; NASA, ESA, S. Beckwith (STScI) and the Hubble Heritage Team (STScI/AURA)

$L \sim \mathcal{O}(\mathrm{pc})$

Soler, 2019, A&A

Irreducible — add magnetic flux to the cascade on all scales

straight, strong fields ($\kappa \propto B^{-1/2}$; Schekochihin+2004) penetrating through Orion A

Orion A in infrared; ESA/Herschel/Planck; J. D. Soler, MPIA

The cold ISM: A supersonic laboratory for interesting nonlinear physics

- ~ 20% MW ISM gas is molecular hydrogen, organised into MCs (low-volume filling ~ 1-2%).
- cold, $T \sim 10\,K$, c_s is low, approximately isothermal
- $\sigma_v / c_s = \mathcal{M} \sim 10$, supersonic (compressible), $\mathrm{Re} \sim 10^9$
- $L \sim 10\,\mathrm{pc} \implies T = L/\sigma_v \sim \mathcal{O}(\mathrm{Myr})$
- $n \sim 10^3 - 10^{7+}$, huge density contrasts.
- weakly bounded (not virialised) by their own self-gravity $\alpha_{\mathrm{vir}} = 2|E_{\mathrm{kin}}|/|E_{\mathrm{grav}}| > 2$.
- threaded by dynamically important B fields, Ohmic $\mathrm{Rm} \sim 10^{16}$.

Krumholz & McKee (2005); Federrath & Klessen (2012); Tritsis+(2016); Xu & Lazarian (2016); Soler & Hennebelle (2017); Squire & Hopkins (2017); Mocz & Burkhart (2018), Burkhart & Mocz (2018); Heyer+(2012); Tritsis+(2018); Hu+(2019); Heyer+(2020); Krumholz+(2020); Beattie & Federrath (2020); Beattie+(2020); Körtgen & Soler (2020); Skalidis & Tassis (2020); Federrath+(2021); Bukhart (2021); Barreto-Mota+(2021); Hopkins+(2022); Beattie+(2022); Fielding+(2022); Sampson+ (2022); Kriel+(2022); Galishnikova+(2022); Beattie+(2023)

W3/W4/W5 MCs and star forming region complex.

Simplest possible supersonic dynamo simulations

- Modified version of finite volume code *FLASH*, second-order in space approximate Riemann (PPM) solver with framework outlined in Bouchut+(2010), tested in *FLASH* in Waagen+(2011).

- Compressible non-helical, visco/resistive MHD turbulence driven with finite correlation time t_0 (OU process; Federrath2022) on $L/2$ in triply periodic box.

- No net magnetic flux $\langle b \rangle = 0$. Pure turbulent magnetic field.

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$d_t(\rho \mathbf{u}) + \nabla \cdot \mathbb{F} = \frac{1}{\mathrm{Re}} \nabla \cdot \sigma_{\mathrm{viscous}} + \rho \mathbf{f}$$

$$\partial_t \mathbf{b} = \nabla \times (\mathbf{u} \times \mathbf{b}) + \frac{1}{\mathrm{Rm}} \nabla^2 \mathbf{b}$$

$$\nabla \cdot \mathbf{b} = 0 \quad p = c_s \rho$$

~200 DNS simulations

Grids up to $10{,}080^3$

$1 \leq \mathrm{Pm} \leq 300$

$10 \leq \mathrm{Re} \leq 10^6$

$500 \leq \mathrm{Rm} \leq 10^6$

$0.1 \leq \mathscr{M} \leq 10$

Simplest possible supersonic dynamo simulations

Broad parameter study

~200 DNS simulations

Grids up to $1{,}152^3$

$1 \leq \mathrm{Pm} \leq 300$

$10 \leq \mathrm{Re} \leq 10^4$

$500 \leq \mathrm{Rm} \leq 10^4$

$0.1 \leq \mathscr{M} \leq 10$

One hero simulation

$\mathscr{M} \sim 4$

$10{,}080^3$

$\mathrm{Re} \sim 10^6$

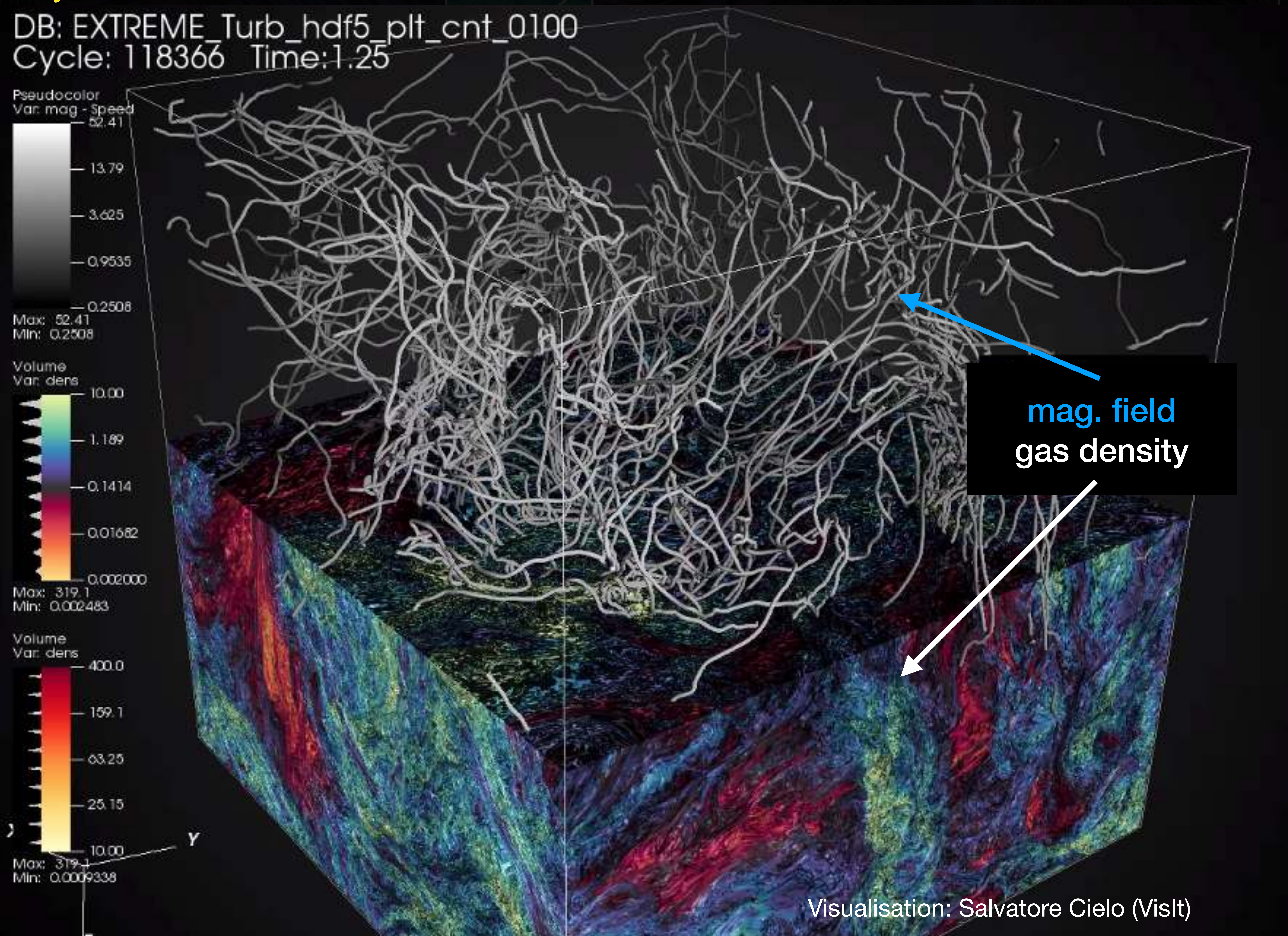
$\mathrm{Rm} \sim 10^6$

150,000 compute cores

10^8 compute hours

2PB data products

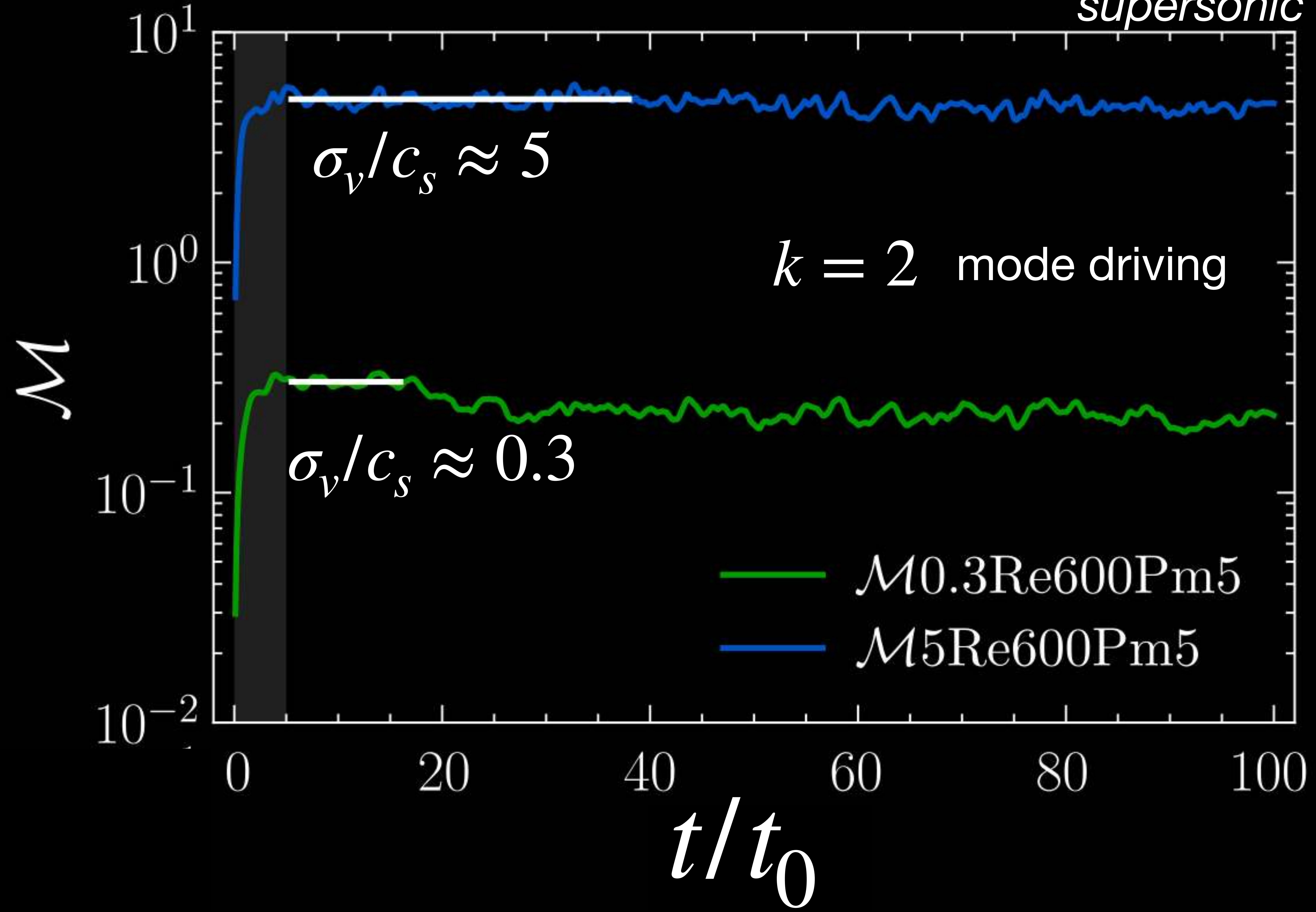
Beattie et al. (2024, in subm.). *Supersonic, magnetised turbulence at extreme Reynold's numbers*



Visualisation: Salvatore Cielo (VisIt)

Integral statistics

Kriel, Beattie+ (2024). *Fundamental scales II: the kinematic stage of the supersonic dynamo*

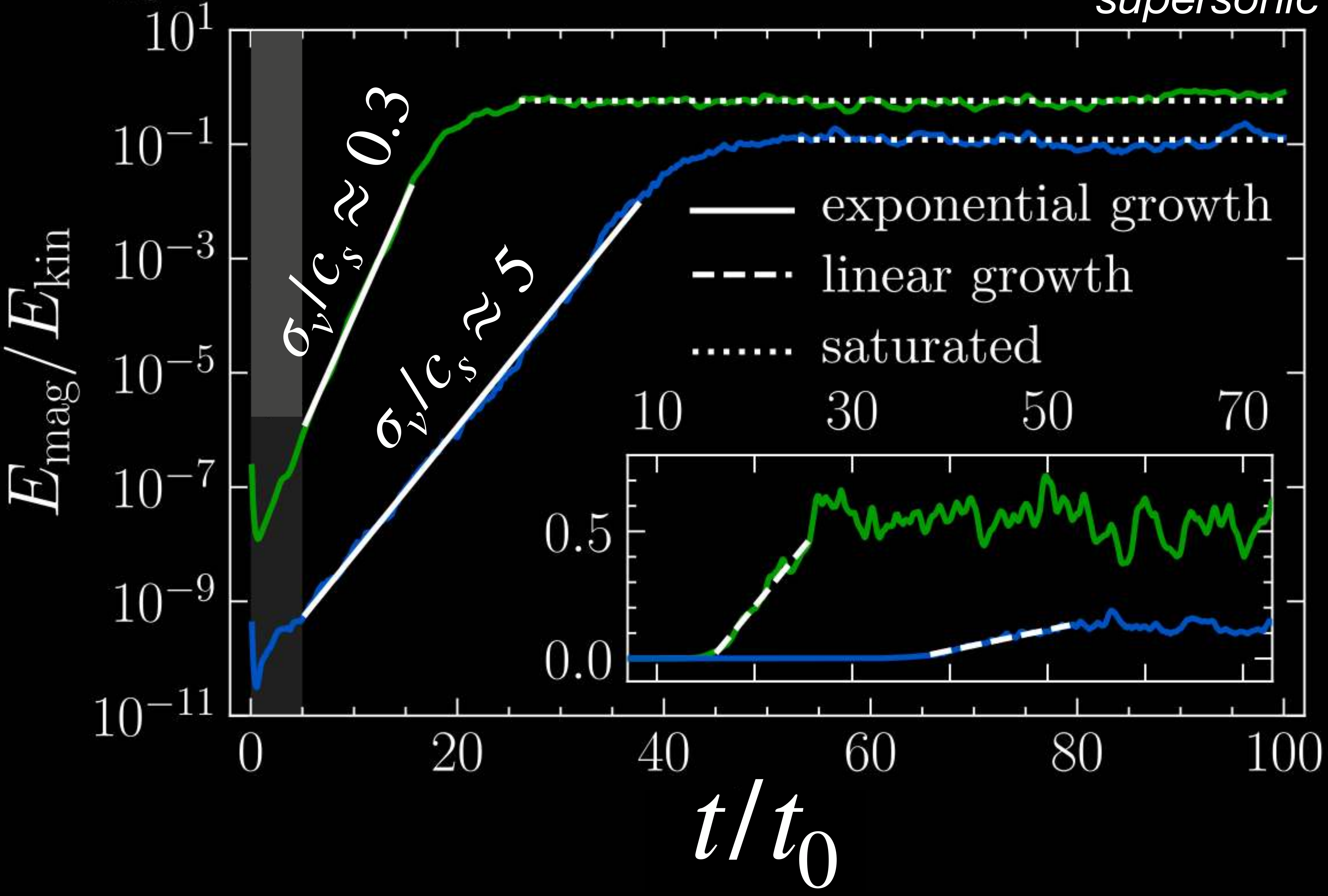


$$\mathrm{Pm} = \frac{\nu}{\eta}$$

$$\mathscr{M} = \frac{\sigma_v}{c_s}$$

Integral statistics

Kriel, Beattie+ (2024). *Fundamental scales II: the kinematic stage of the supersonic dynamo*



For fixed material properties, saturation and growth rate always lower and less efficient than in subsonic dynamos!

$$E_{\rm mag} = \langle b^2 \rangle / (2\mu_0)$$

$$E_{\rm kin} = \langle \rho v^2 \rangle / 2$$

Integral statistics

Kriel, Beattie+ (2024). *Fundamental scales II: the kinematic stage of the supersonic dynamo*

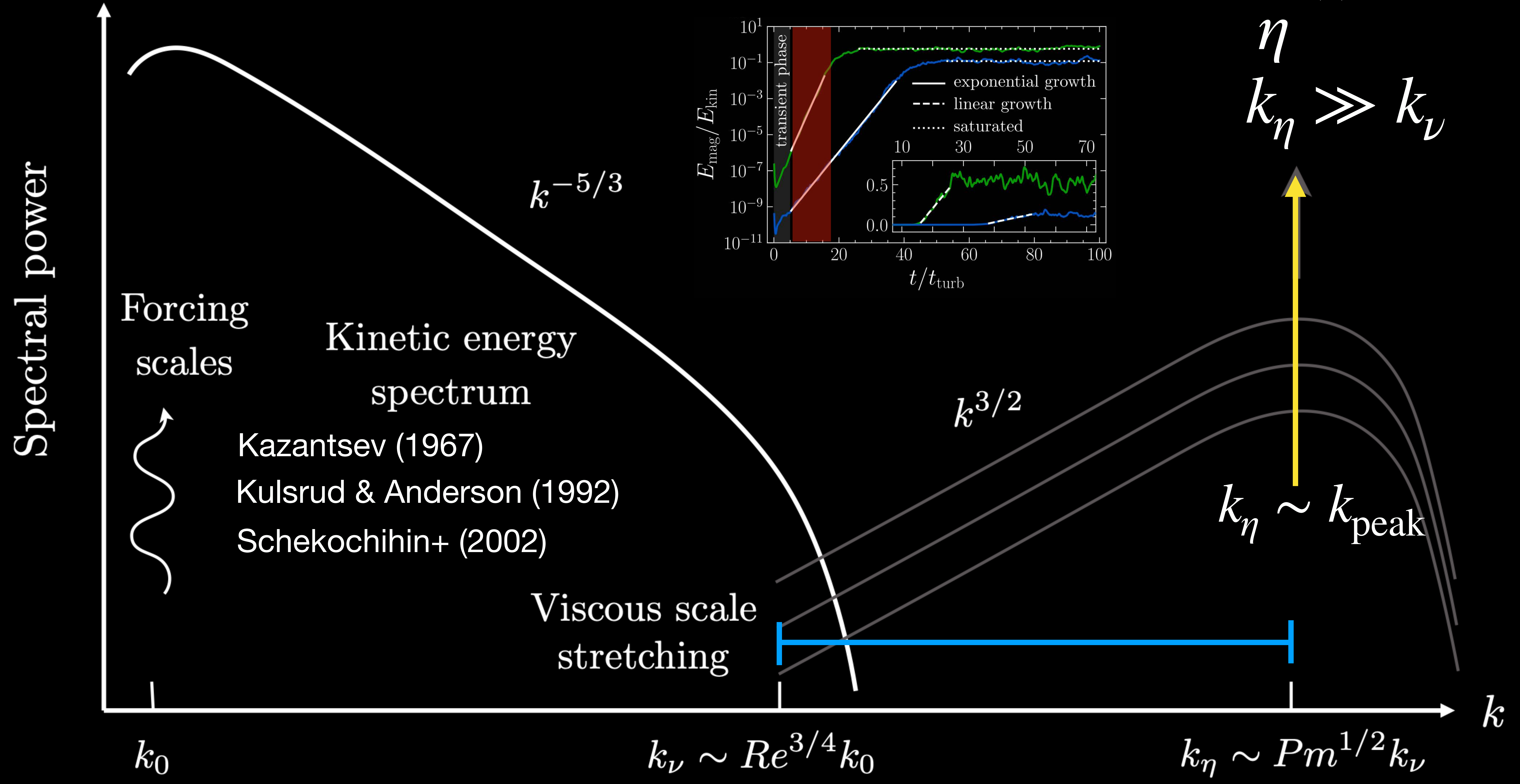
Turbulent dynamo

kinematic regime: dissipation scales

Modified from Rincon (2019)

$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

$$k_\eta \gg k_\nu$$



Turbulent dynamo

kinematic regime: dissipation scales

Modified from Rincon (2019)

$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

$$k_\eta \gg k_\nu$$

stretching at the viscous scale

$$\frac{u_\nu}{\ell_\nu} \sim \frac{\eta}{\ell_\eta^2}$$

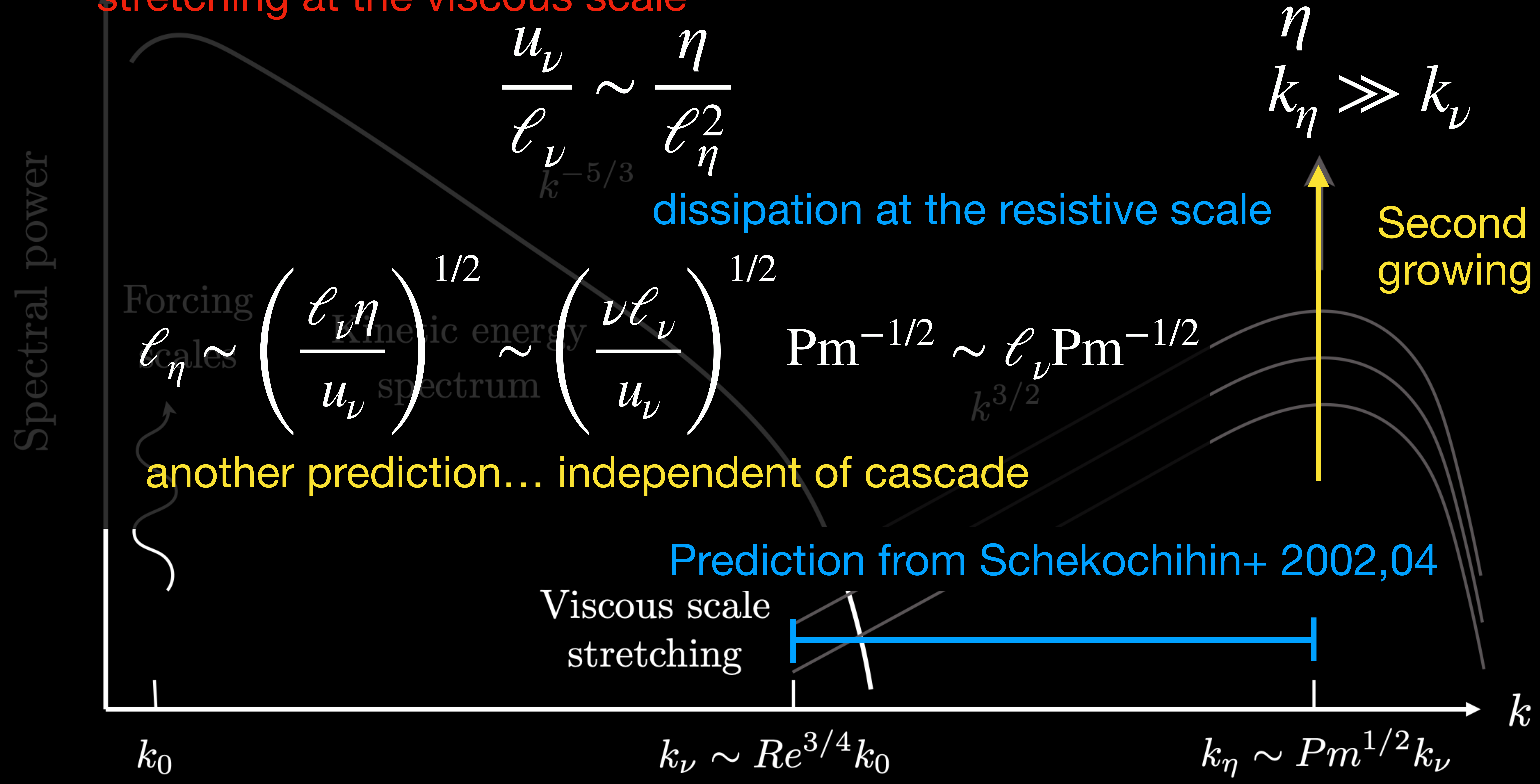
dissipation at the resistive scale

Second fastest growing stage

$$\ell_\eta \sim \left(\frac{\ell_\nu \eta}{u_\nu}\right)^{1/2} \sim \left(\frac{\nu \ell_\nu}{u_\nu}\right)^{1/2} \mathrm{Pm}^{-1/2} \sim \ell_\nu \mathrm{Pm}^{-1/2}$$

another prediction… independent of cascade

Prediction from Schekochihin+ 2002,04



Turbulent dynamo
kinematic regime: the resistive scale

Neco Kriel
Grad. Student (ANU)

stretching at the viscous scale

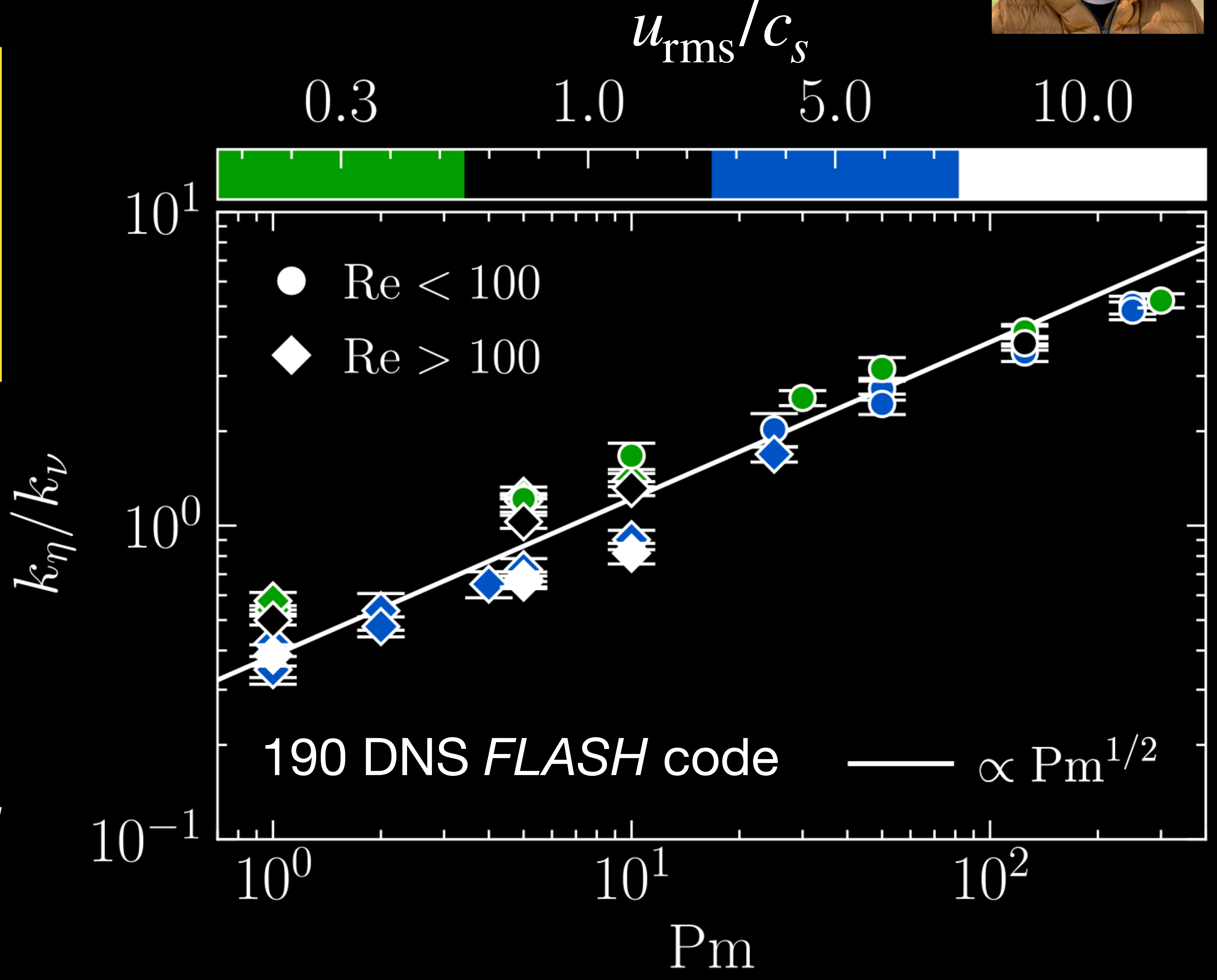
$$\frac{u_\nu}{\ell_\nu} \sim \frac{\eta}{\ell_\eta^2}$$

dissipation at the resistive scale

1. universal of super-sub-sonic gas motions.

2. implies the viscous scale eddies the engine for kinematic dynamo in both regimes

Kriel, Beattie+ (2024). *Fundamental scales II: the kinematic stage of the supersonic dynamo*



Turbulent dynamo

kinematic regime: the peak scale

Modified from Rincon (2019)

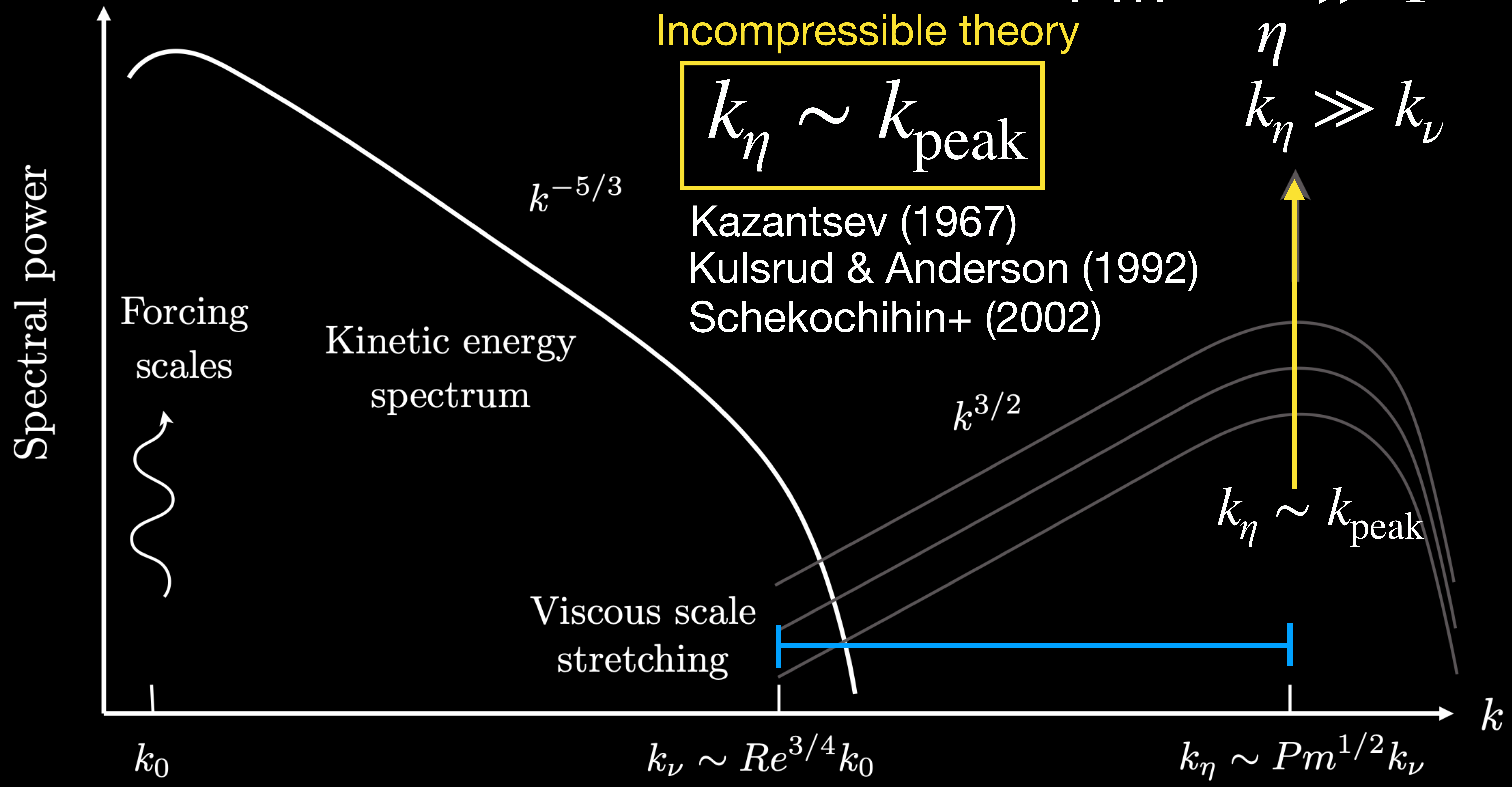
$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

$$k_\eta \gg k_\nu$$

Incompressible theory

$$\boxed{k_\eta \sim k_{\mathrm{peak}}}$$

Kazantsev (1967)
Kulsrud & Anderson (1992)
Schekochihin+ (2002)

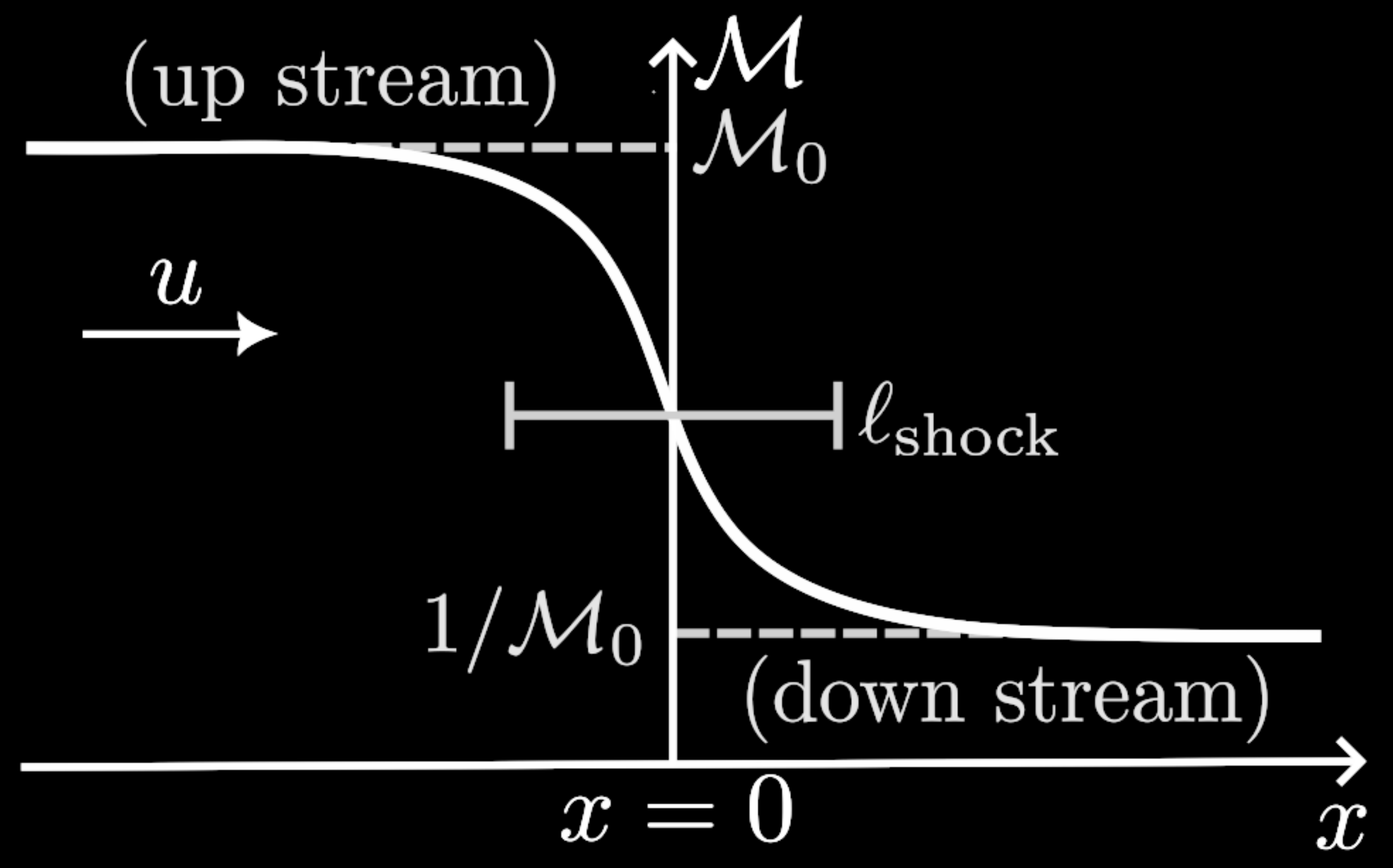


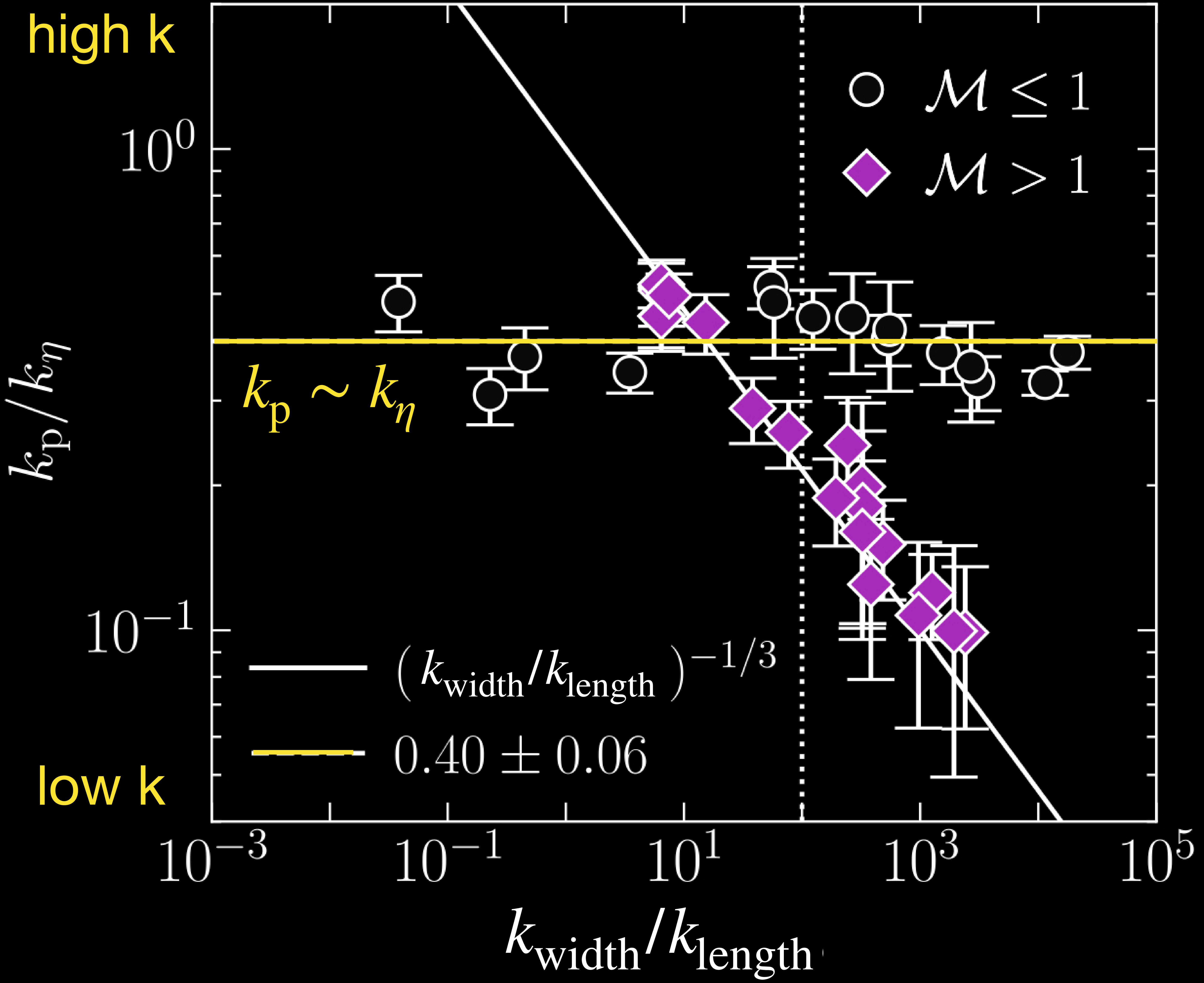
Turbulent dynamo
kinematic regime: the peak scale

Neco Kriel
Grad. Student (ANU)

Characteristic shock geometry

$$k_{\text{length}} \sim k_0$$
$$k_{\text{width}} \sim \mathscr{M}^2/\text{Re}(\mathscr{M} - 1)^2$$





Kriel, Beattie+ (2024). *Fundamental scales II: the kinematic stage of the supersonic dynamo*

Turbulent dynamo

kinematic regime: the peak scale

Neco Kriel
Grad. Student (ANU)

Characteristic shock geometry

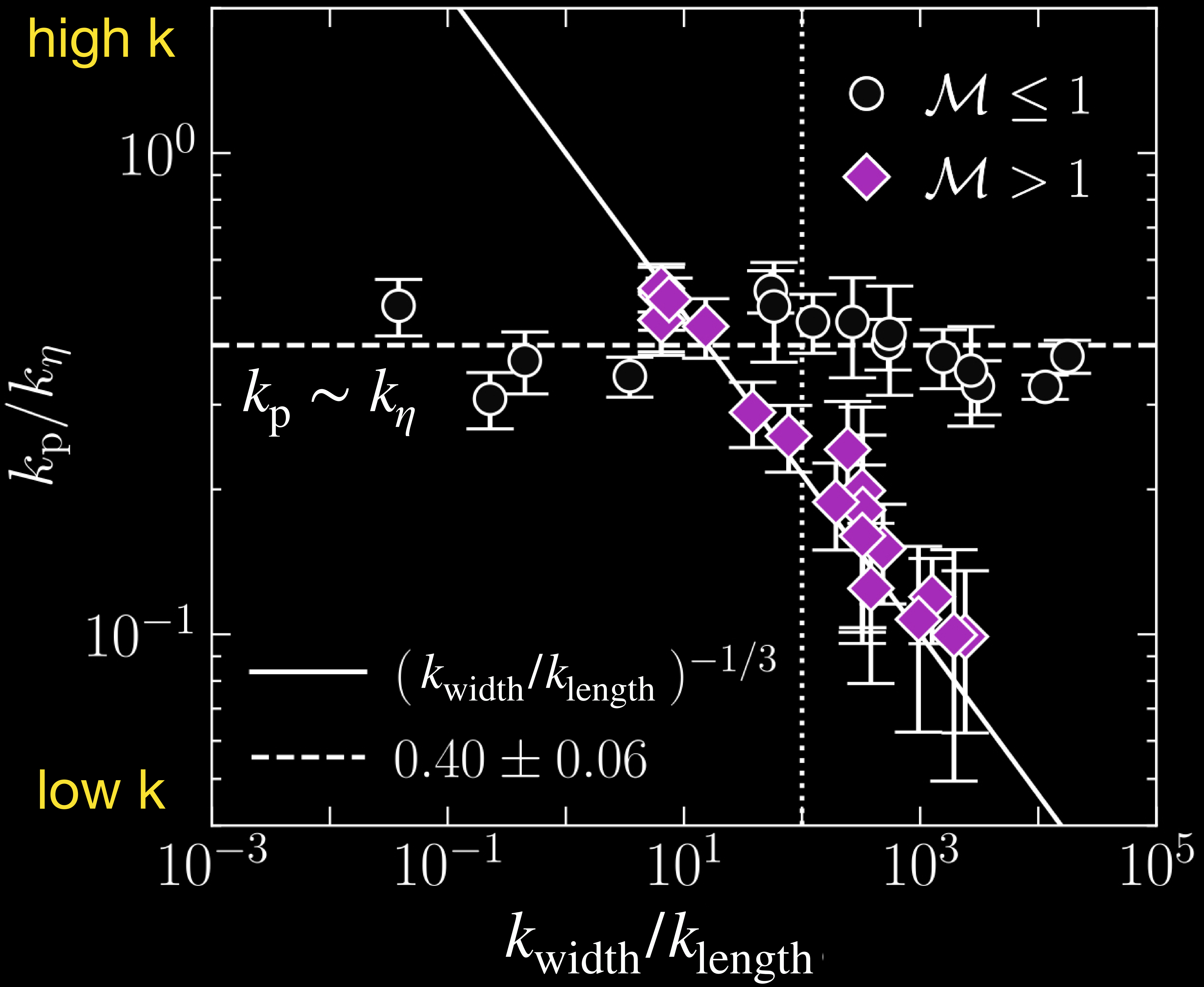
$$k_{\mathrm{length}} \sim k_0$$
$$k_{\mathrm{width}} \sim \mathcal{M}^2/\mathrm{Re}(\mathcal{M} - 1)^2$$

Peak magnetic energy scale moves to lower k modes, away from resistive scales.

Supersonic dynamo builds larger scale b fields compared to subsonic.

high k

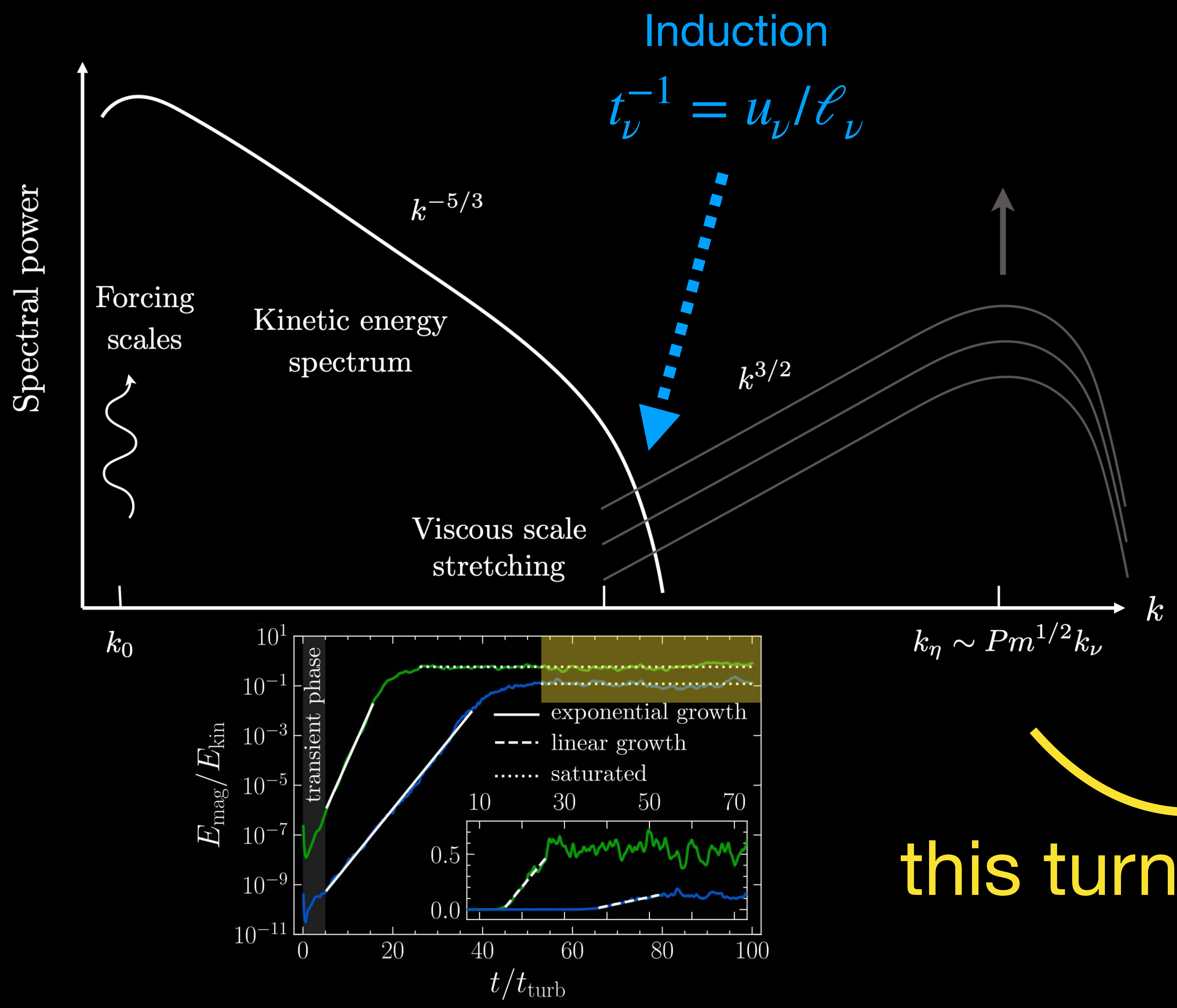
low k



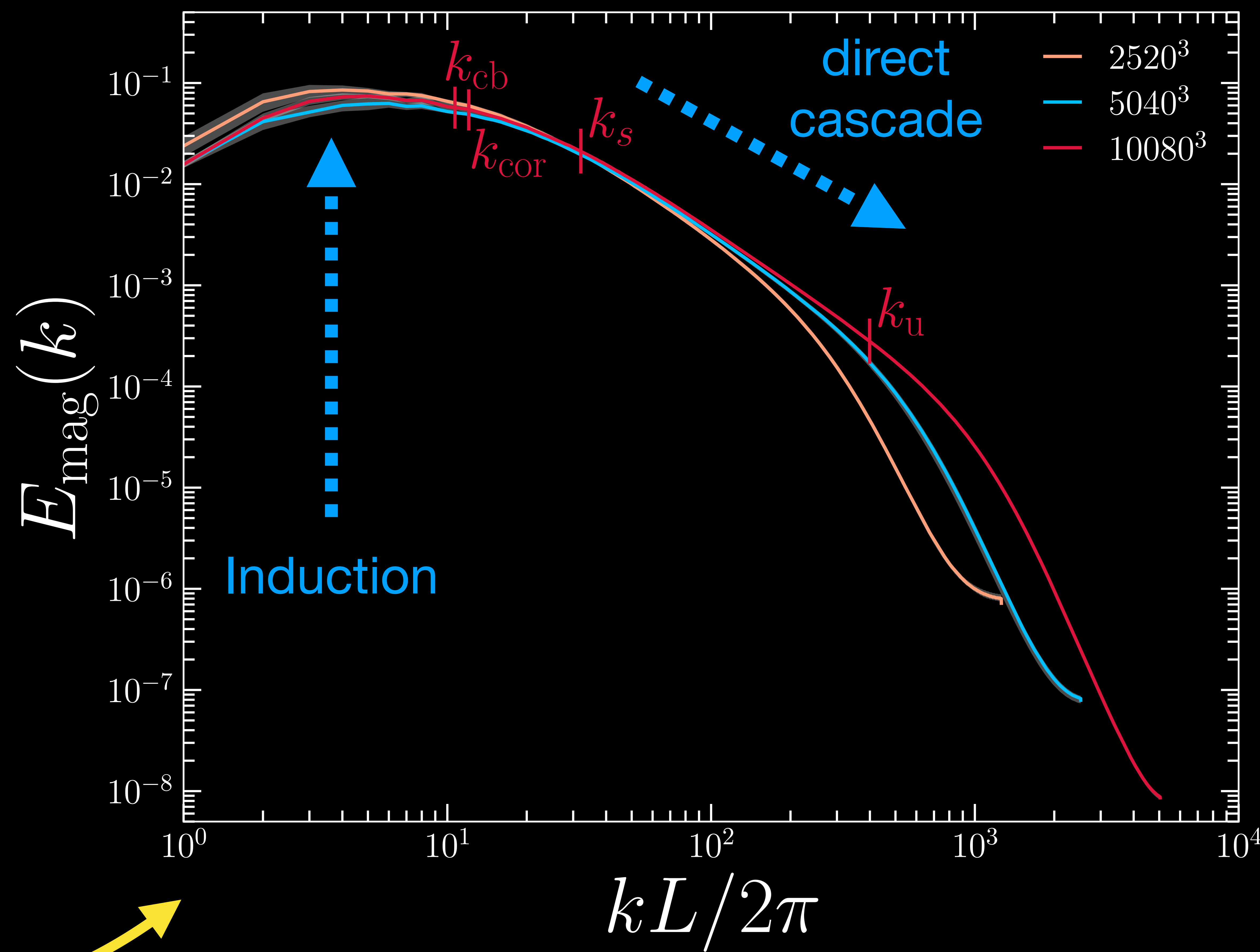
Kriel, Beattie+ (2024). *Fundamental scales II: the kinematic stage of the supersonic dynamo*

Saturated supersonic turbulent dynamo

Kinematic (linear) dynamo



Saturated (nonlinear) dynamo

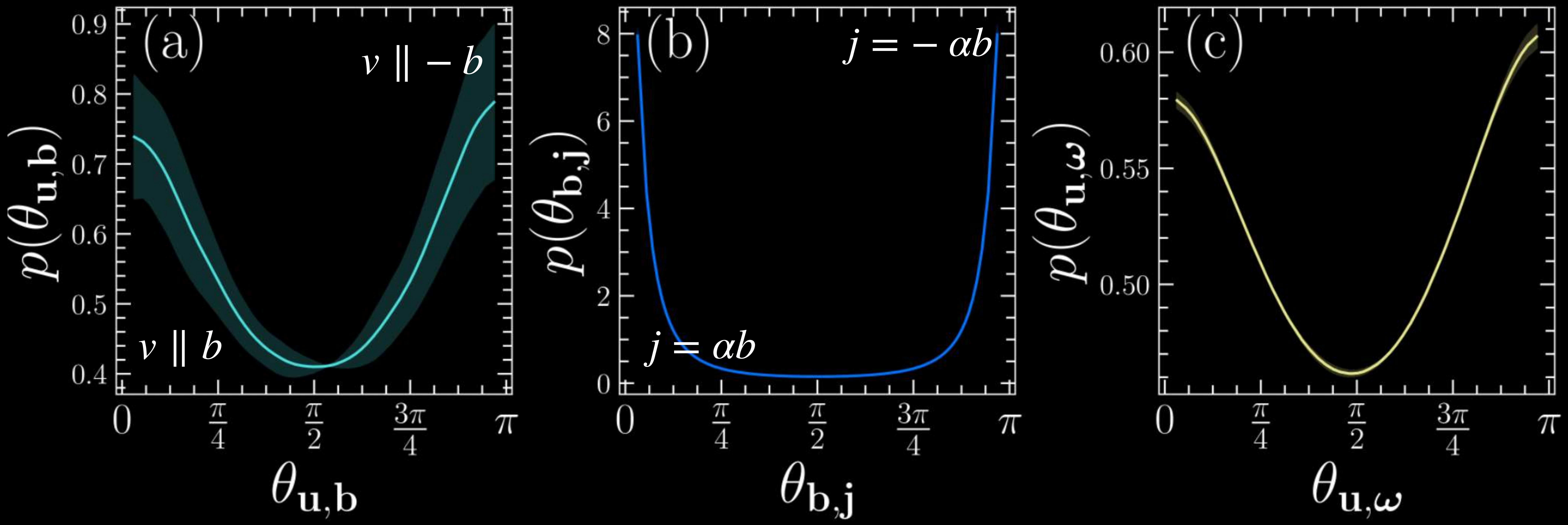


this turns into this

Supersonic turbulent dynamo at $\mathrm{Re} \sim 10^6$

$\mathscr{M} \sim 4$
$\mathrm{Re} \sim 10^6$
$\mathrm{Pm} \sim 1$

Saturated regime: global alignment



$\nabla \times (\mathbf{u} \times \mathbf{b})$ — induction

$\mathbf{j} \times \mathbf{b}$ — Lorentz force

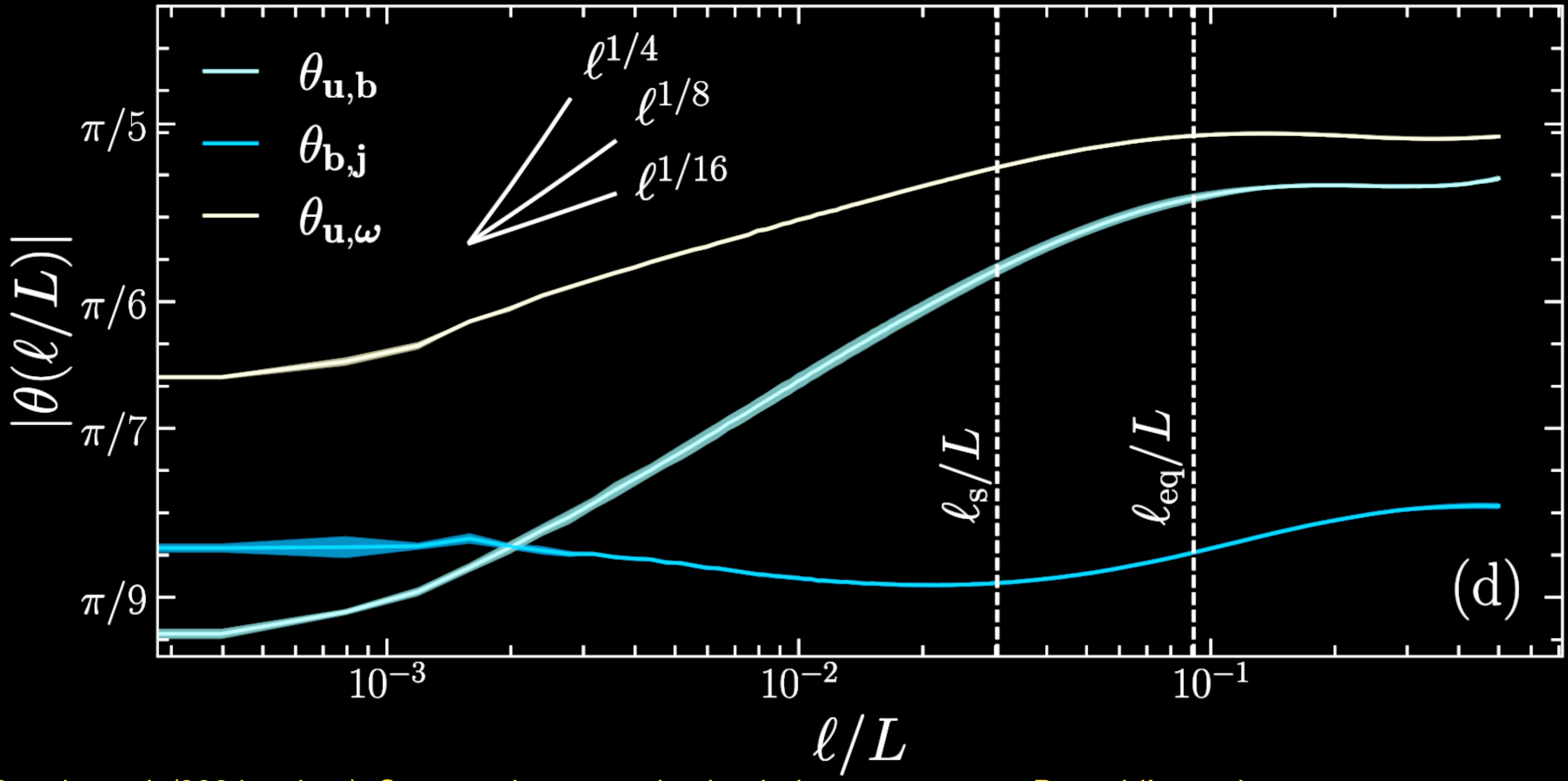
$\nabla \cdot (\mathbf{u} \otimes \mathbf{u}) \sim \omega \times \mathbf{u}$ — quadratic nonlinearity

Beattie et al. (2024, subm.). *Supersonic, magnetised turbulence at extreme Reynold's numbers*

Supersonic turbulent dynamo at $\mathrm{Re} \sim 10^6$

Saturated regime: scale-dependent alignment

$\mathcal{M} \sim 4$
$\mathrm{Re} \sim 10^6$
$\mathrm{Pm} \sim 1$



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Supersonic turbulent dynamo at $\mathrm{Re} \sim 10^6$

Saturated regime: scale-dependent alignment

$\mathscr{M} \sim 4$
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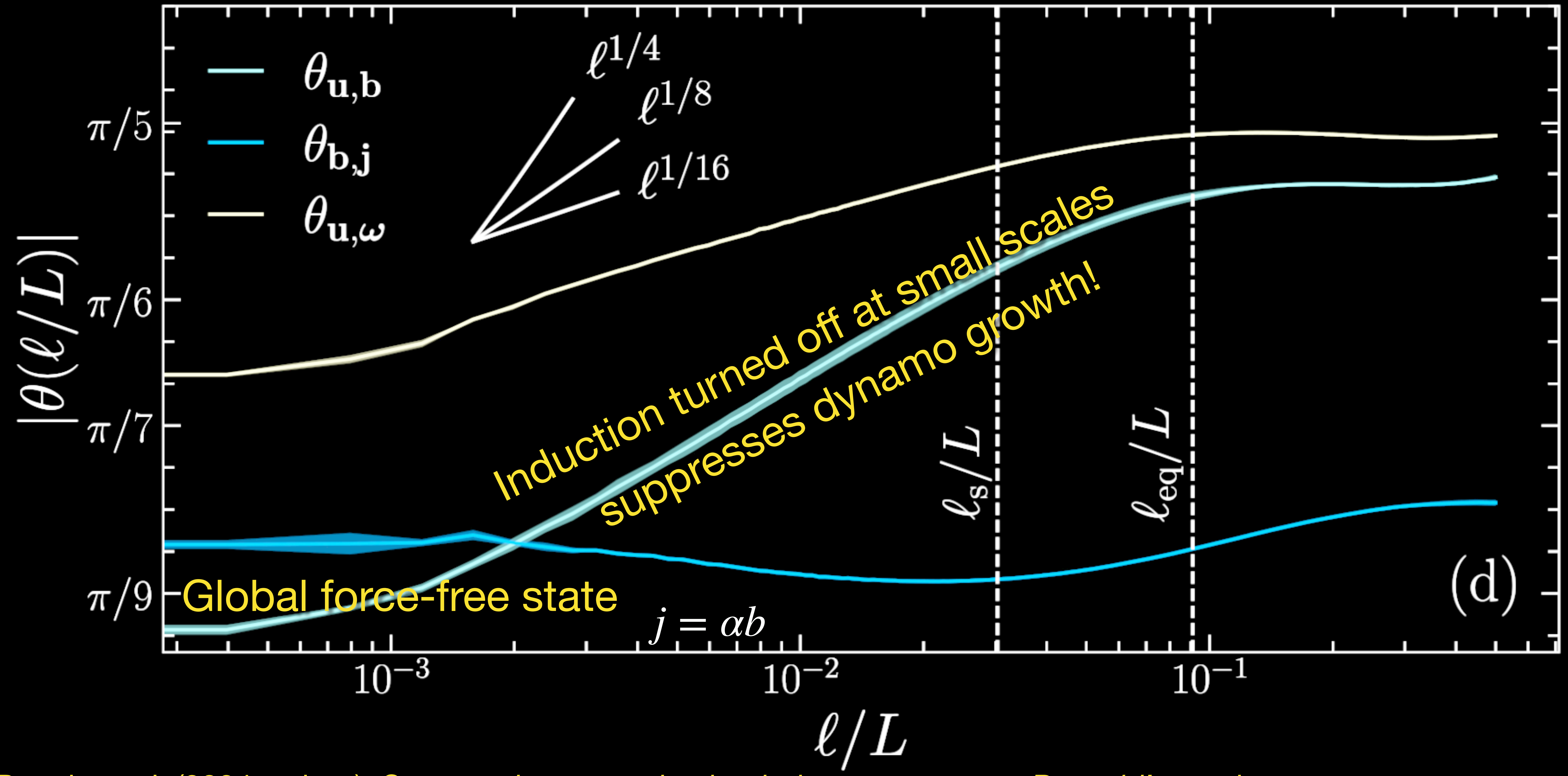
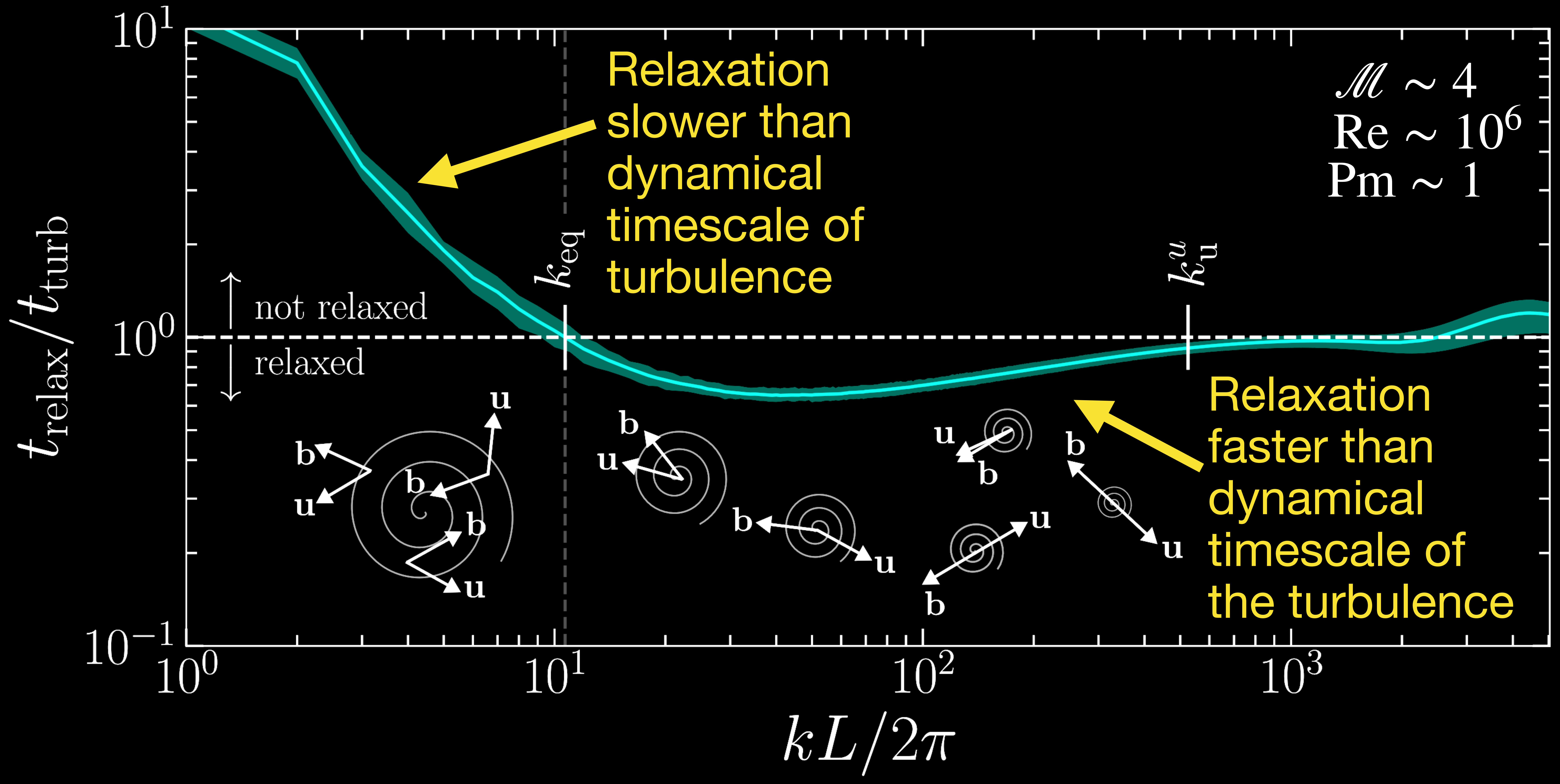


Beattie et al. (2024, subm.). *Supersonic, magnetised turbulence at extreme Reynold's numbers*

Plasma relaxation deep in the cascade



Beattie et al. (2024, subm). *Supersonic, magnetised turbulence at extreme Reynold's numbers*


$v \parallel b$
$v \parallel -b$
Saturated dynamo creates patches of locally relaxed plasma
$\mathbf{v} \cdot \mathbf{b}$

Summary

Thanks, questions?

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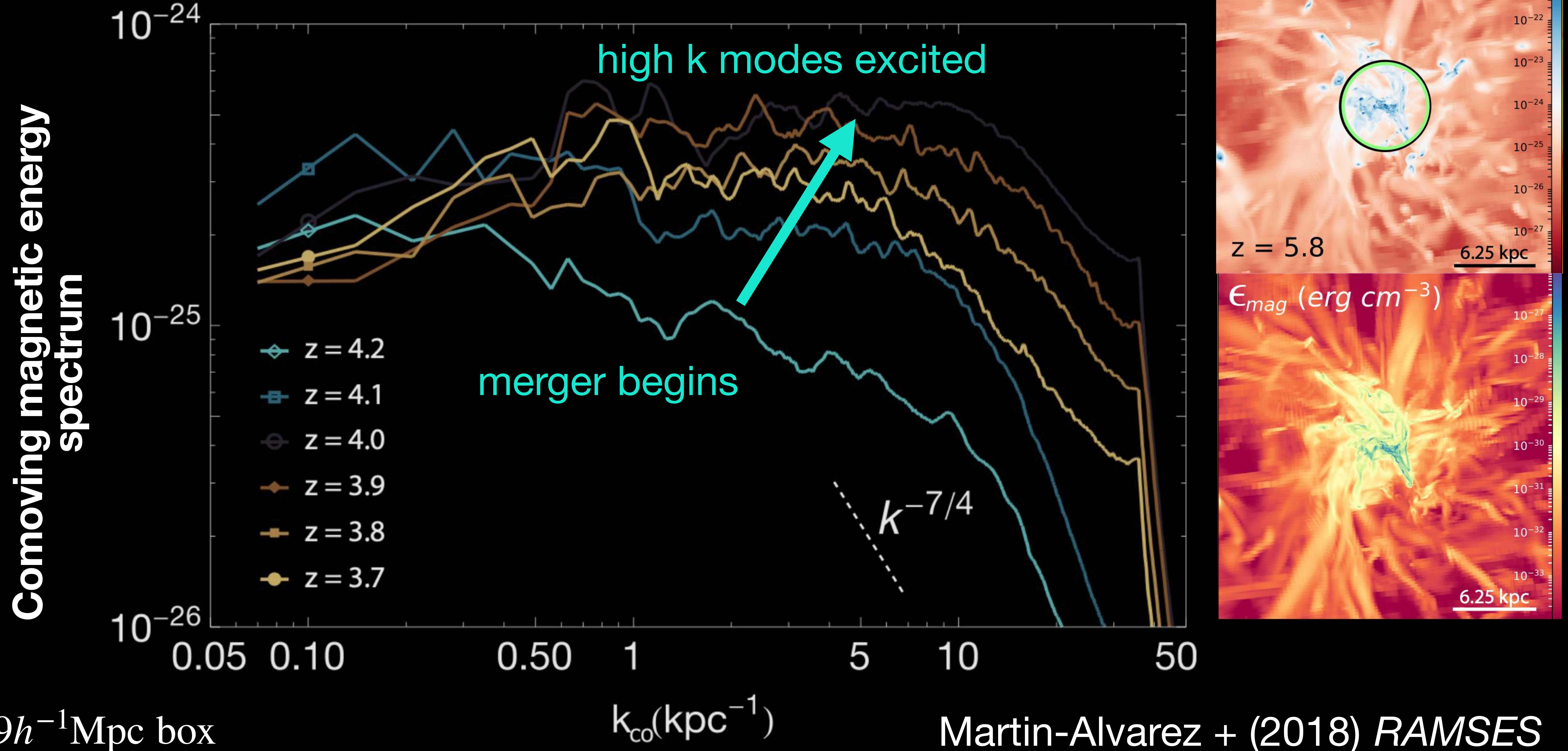
1. The **viscous scale is the engine for the small-scale dynamo**, supersonic or subsonic (universality of $\mathrm{Pm}^{1/2}$ relation).
2. In kinematic supersonic dynamo **magnetic energy spectrum deviates from Kazantsev theory** ($k_{\mathrm{peak}} \neq k_\eta$) and peak energy becomes sensitive to aspect ratio of shocks.
3. Small scale dynamos **saturate through an alignment process** **due to local plasma relaxation.**

Extra slides

Examples of small scale dynamos

Galaxy mergers in cosmological sims

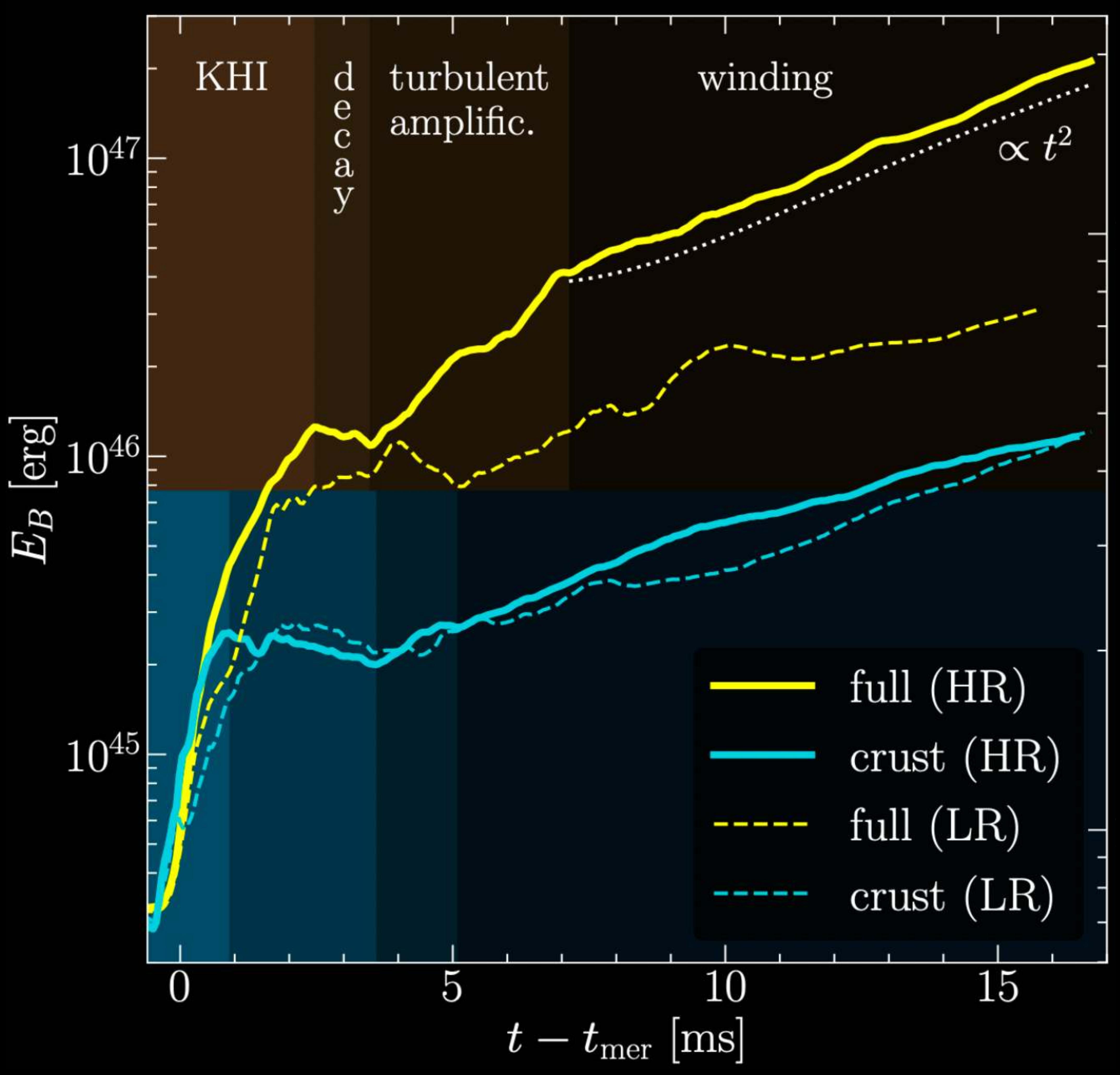


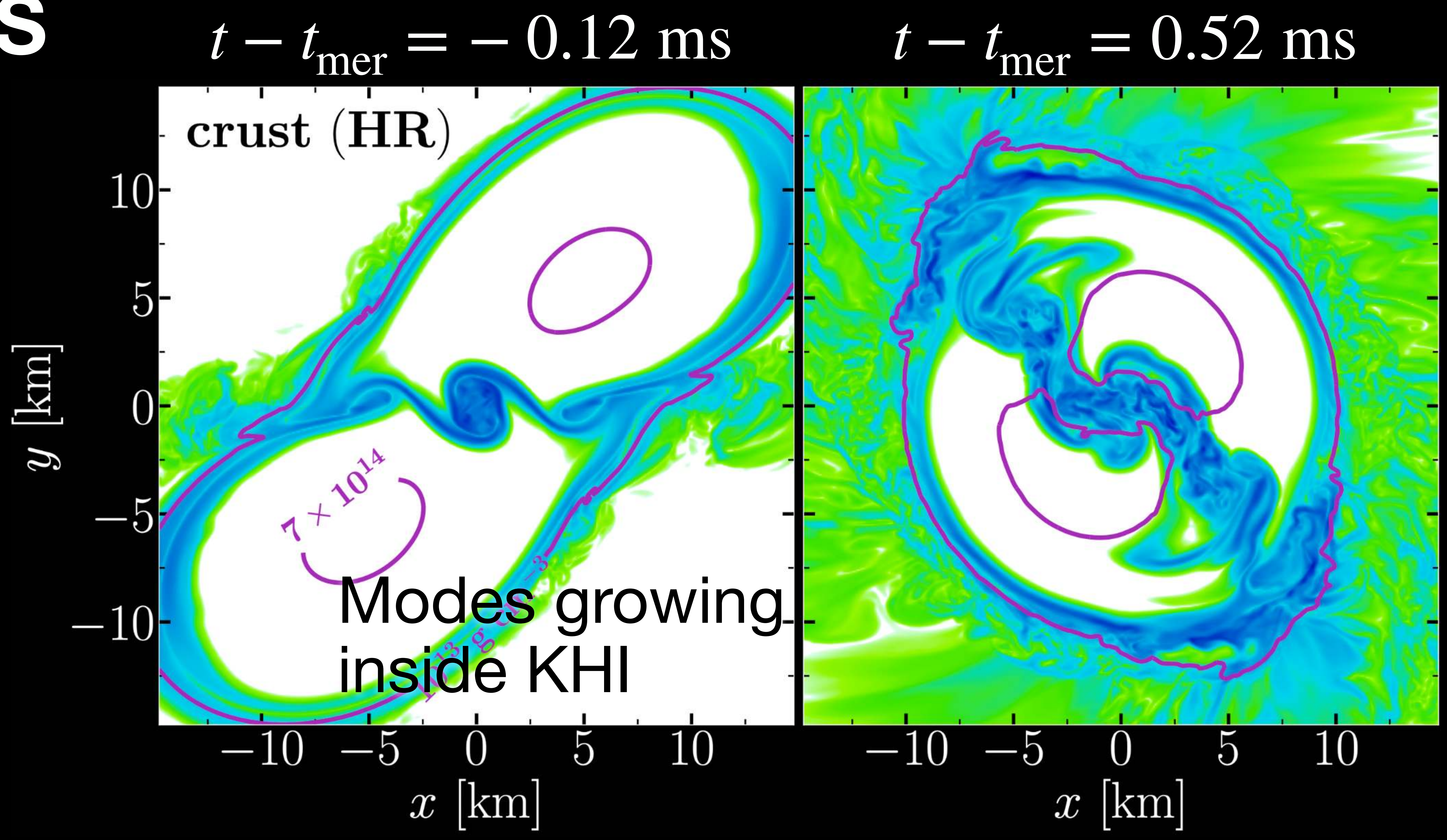
$9h^{-1}$Mpc box

Martin-Alvarez + (2018) *RAMSES*

Examples of small scale dynamos

KHI instabilities in merging NS





exponential growth → pause

exponential growth → LSD

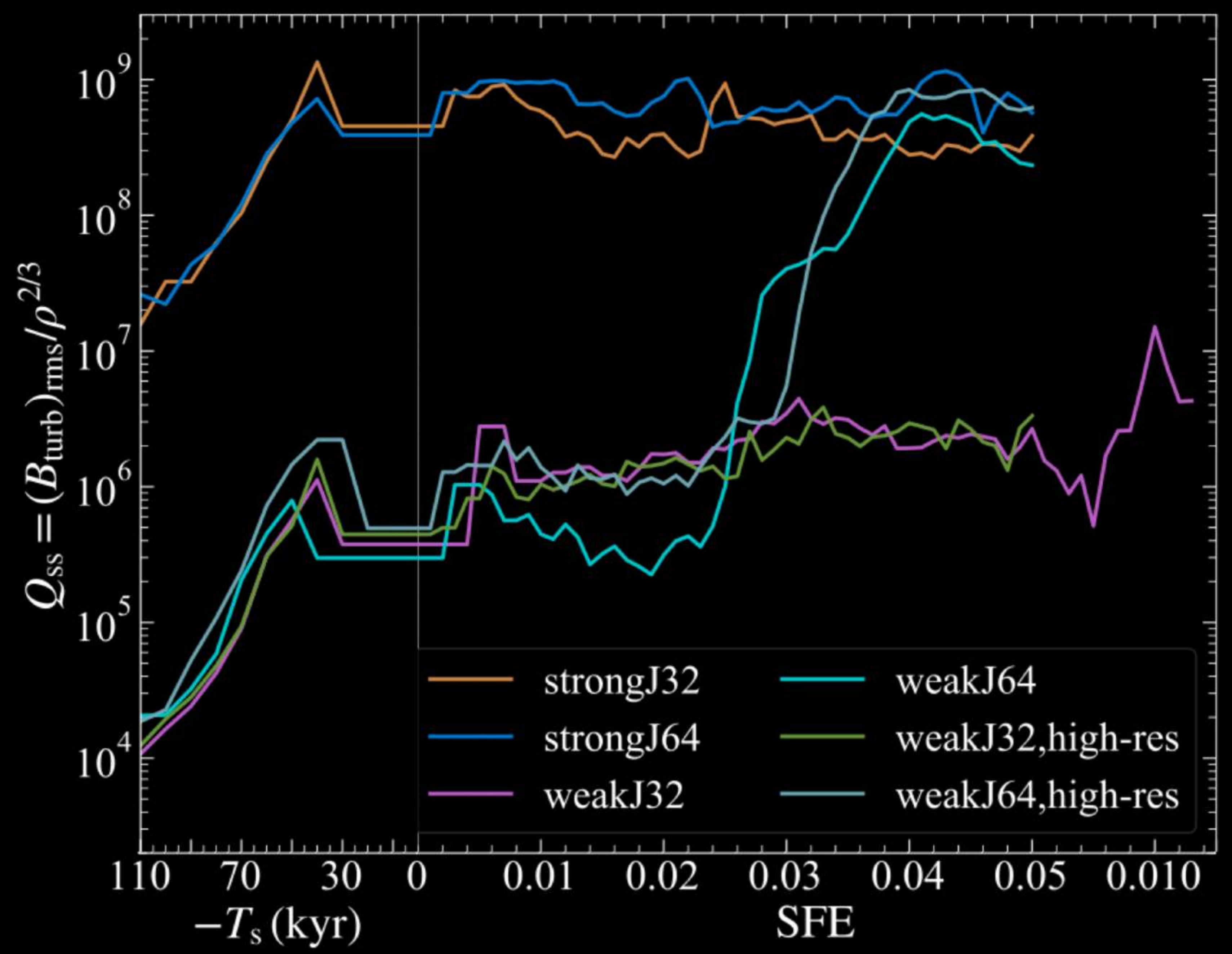
Chabanov+ (2023)

ILES
FIL

Examples of small scale dynamos

There are many, across all scales (all MHD)

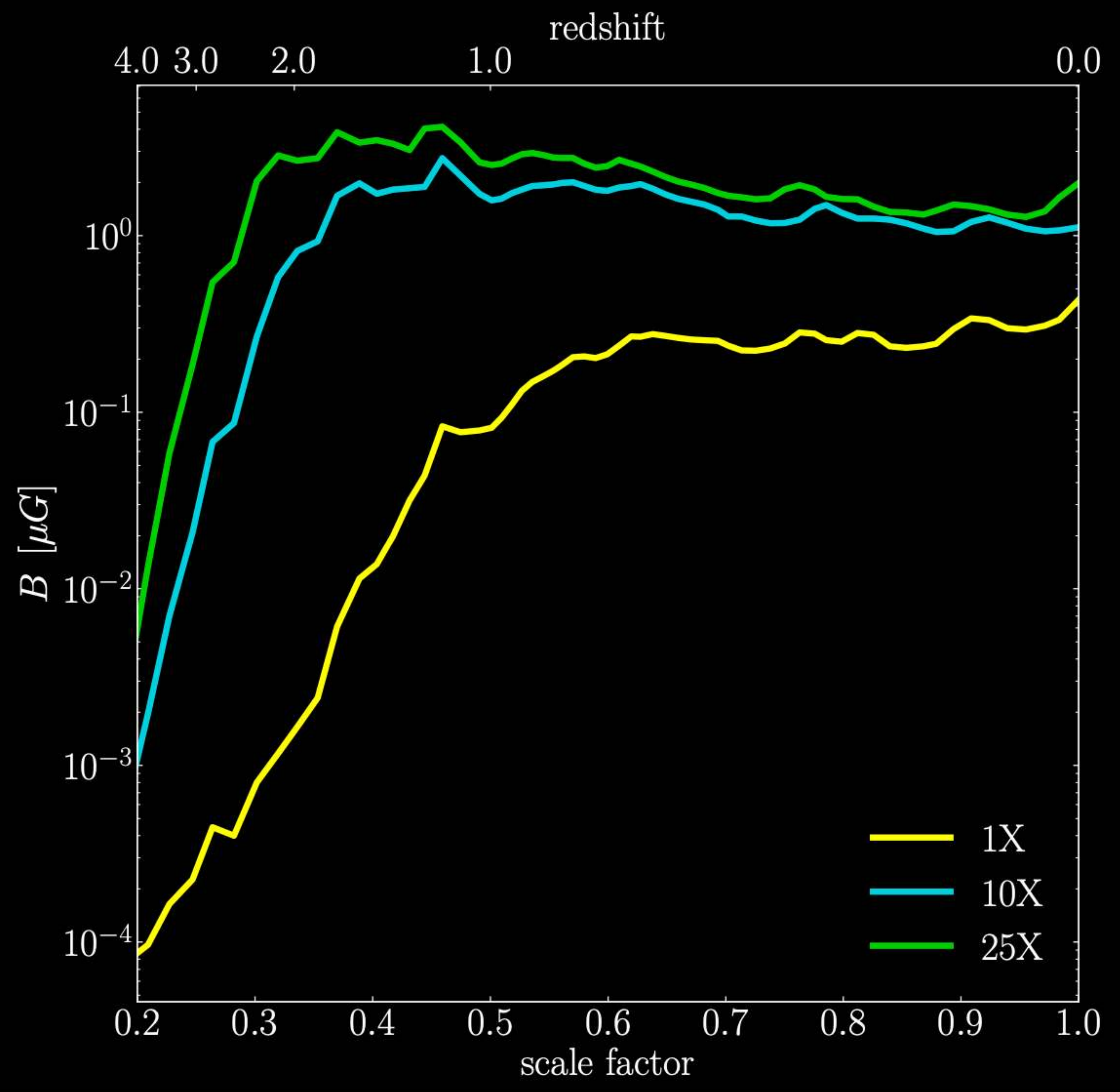
Sharda+2021



Molecular clouds in first generation stars

FLASH

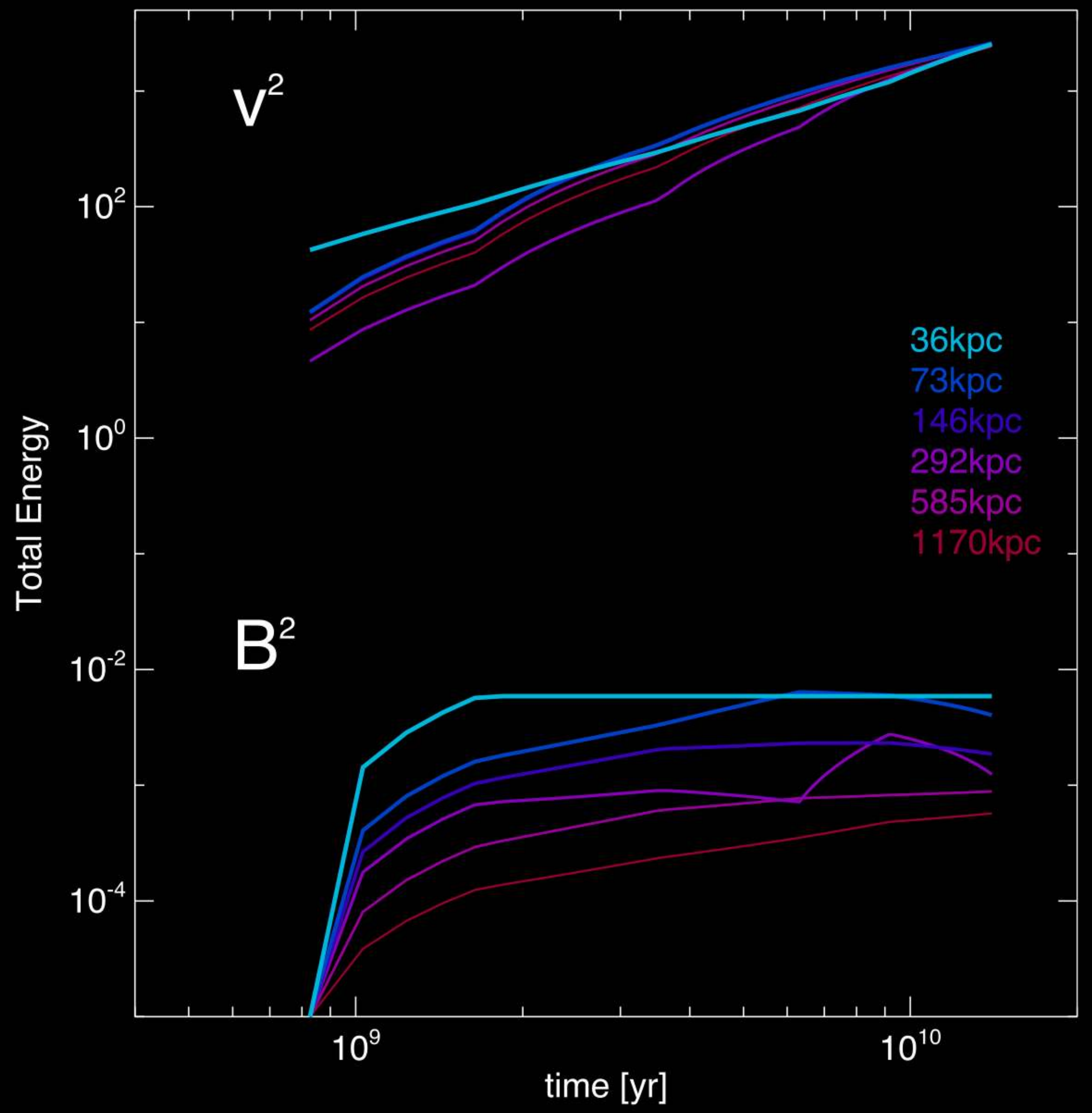
Steinwandel+2021



Intracluster medium

RAMSES

Vazza+2014

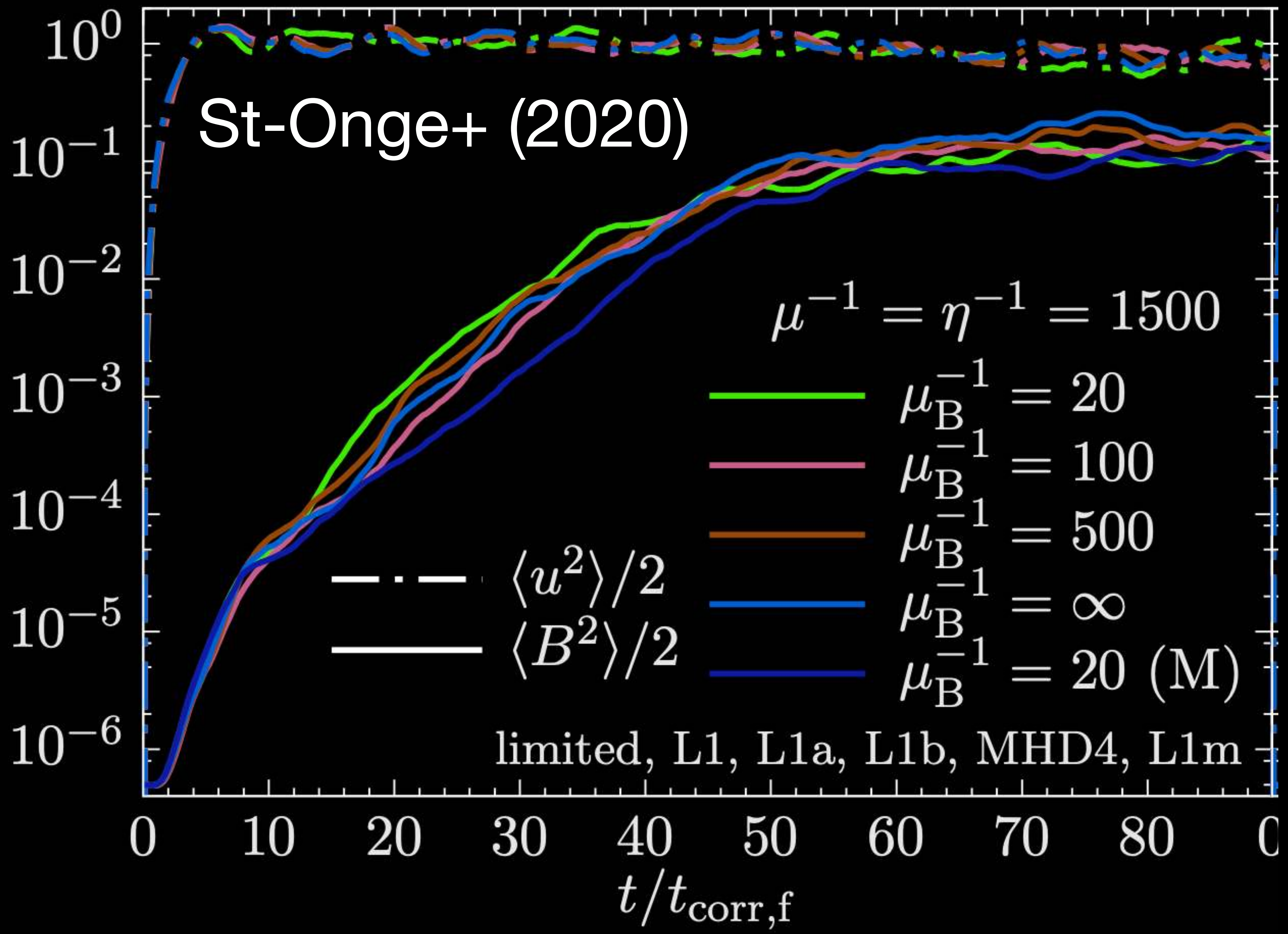


Cosmic filaments

ENZO

Examples of small scale dynamos
and plasma regimes

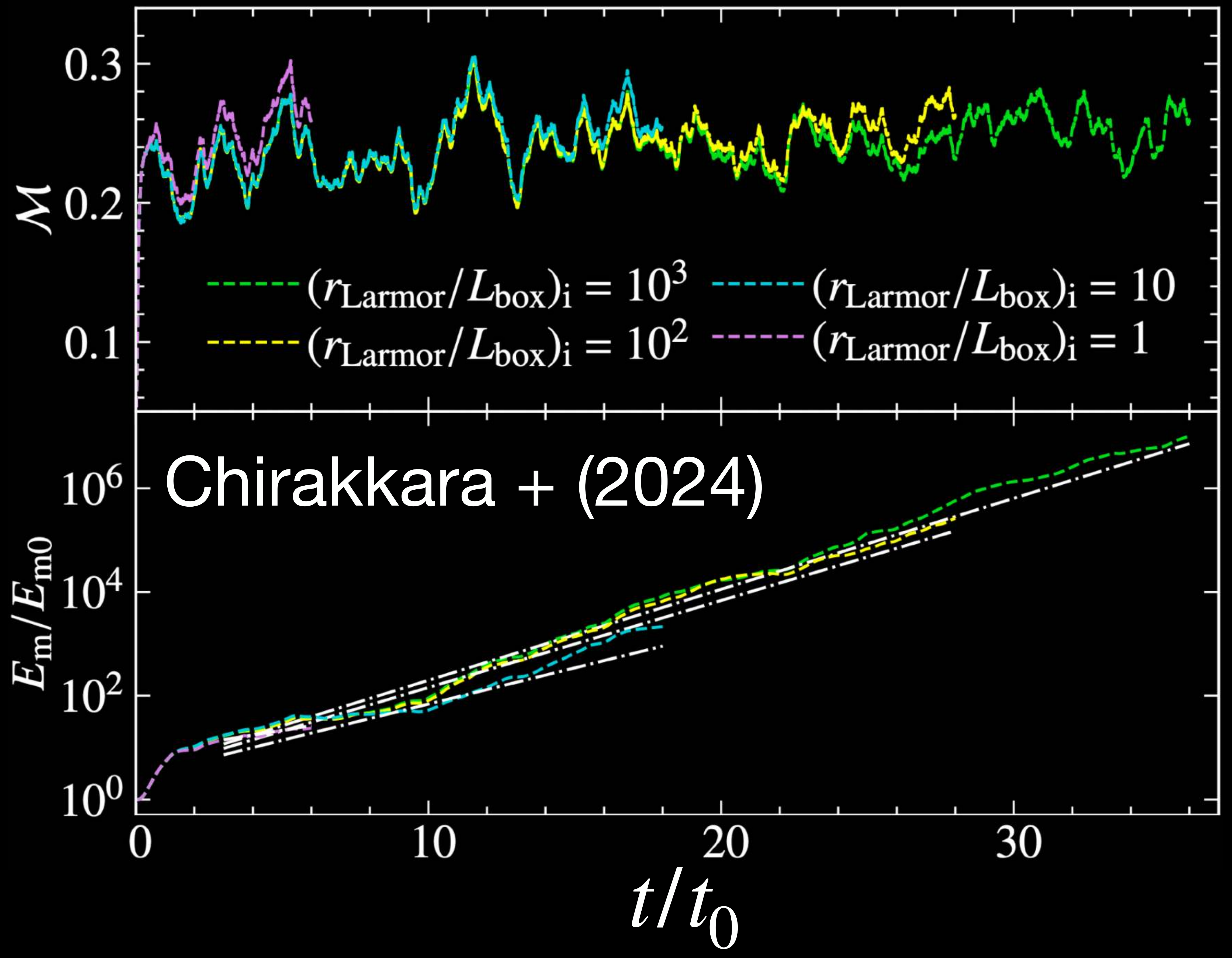
Weakly collisional Braginksii MHD



(added anisotropic viscous Braginskii stress term into MHD)

$\nabla \cdot (\hat{\mathbf{b}} \otimes \hat{\mathbf{b}} (\hat{\mathbf{b}} \otimes \hat{\mathbf{b}} : \nabla \mathbf{v}))$ *Snoopy*

Collisionless plasma



(Hybrid-Kinetic PIC: electron fluid + ion PIC)

FLASH

Stochastically forced Compressible MHD equations (ILES)

$$\partial_t \rho + \nabla \cdot \left(\rho \mathbf{u}\right) = 0$$

$$\partial_t \left(\rho \mathbf{u}\right) + \nabla \cdot \left(\rho \mathbf{u} \otimes \mathbf{u} + p \mathbb{I} - \frac{1}{4\pi} \mathbf{b} \otimes \mathbf{b}\right) = \boxed{\rho \mathbf{f}} + \nabla \cdot \mathbb{D}_\nu (\rho \mathbf{u})$$

$$\partial_t \mathbf{b} + \nabla \cdot (\mathbf{u} \otimes \mathbf{b} - \mathbf{b} \otimes \mathbf{u}) = \nabla \cdot \mathbb{D}_\eta (\mathbf{b})$$

$$\nabla \cdot \mathbf{b} = 0$$

$$p = c_s^2 \rho + \frac{1}{8\pi} \mathbf{b} \cdot \mathbf{b}$$

the turbulence source function

Stochastically forced Compressible MHD equations (ILES)

$$d\hat{\mathbf{f}}(\mathbf{k},t) = f_0(\mathbf{k})\mathbb{P}(\mathbf{k}) \cdot d\mathbf{W}(t) - \hat{\mathbf{f}}(\mathbf{k},t)\frac{dt}{t_0}$$

$d\mathbf{W}(t)$ Weiner process that draws delta correlated from $\sim \mathcal{N}(0,1)$

K space projection tensor

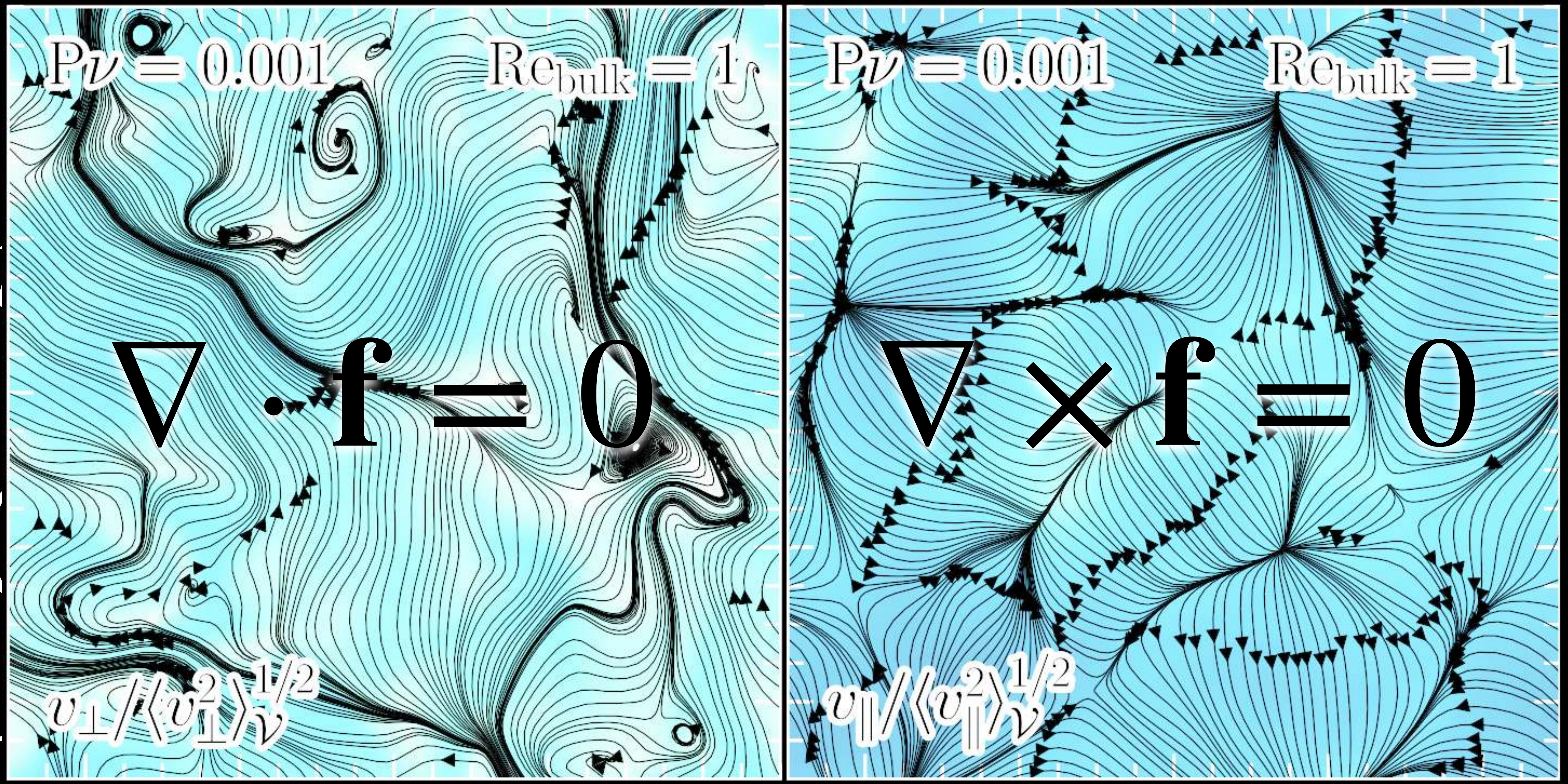
$$\mathbb{P} = \zeta\mathbb{P}^{\perp} + (1-2\zeta)\mathbb{P}^{\parallel} = \zeta\mathbb{I} + (1-2\zeta)\frac{\mathbf{k}\otimes\mathbf{k}}{|\mathbf{k}|^2}$$

$$\zeta = 1 \implies \nabla\cdot\mathbf{f} = 0$$

$$\zeta = 0 \implies \nabla\times\mathbf{f} = 0$$

t_0 e-folding time of the forcing / correlation time / outer-scale turbulent turnover time

Stochastically forced Compressible MHD equations (ILES)



$$d\hat{\mathbf{f}}(\mathbf{k}, t \quad \ldots \quad \mathbf{k}, t)\frac{dt}{t_0}$$

$$d\mathbf{W}(\quad \ldots$$ ed from $\sim \mathcal{N}(0,1)$

K space projectior

$$\mathbb{P} = \zeta \mathbb{P}^{\perp} + (1 - 2\zeta)\mathbb{P}^{\parallel} = \zeta \mathbb{I} + (1 - 2\zeta)\frac{\mathbf{k} \otimes \mathbf{k}}{|\mathbf{k}|^2}$$

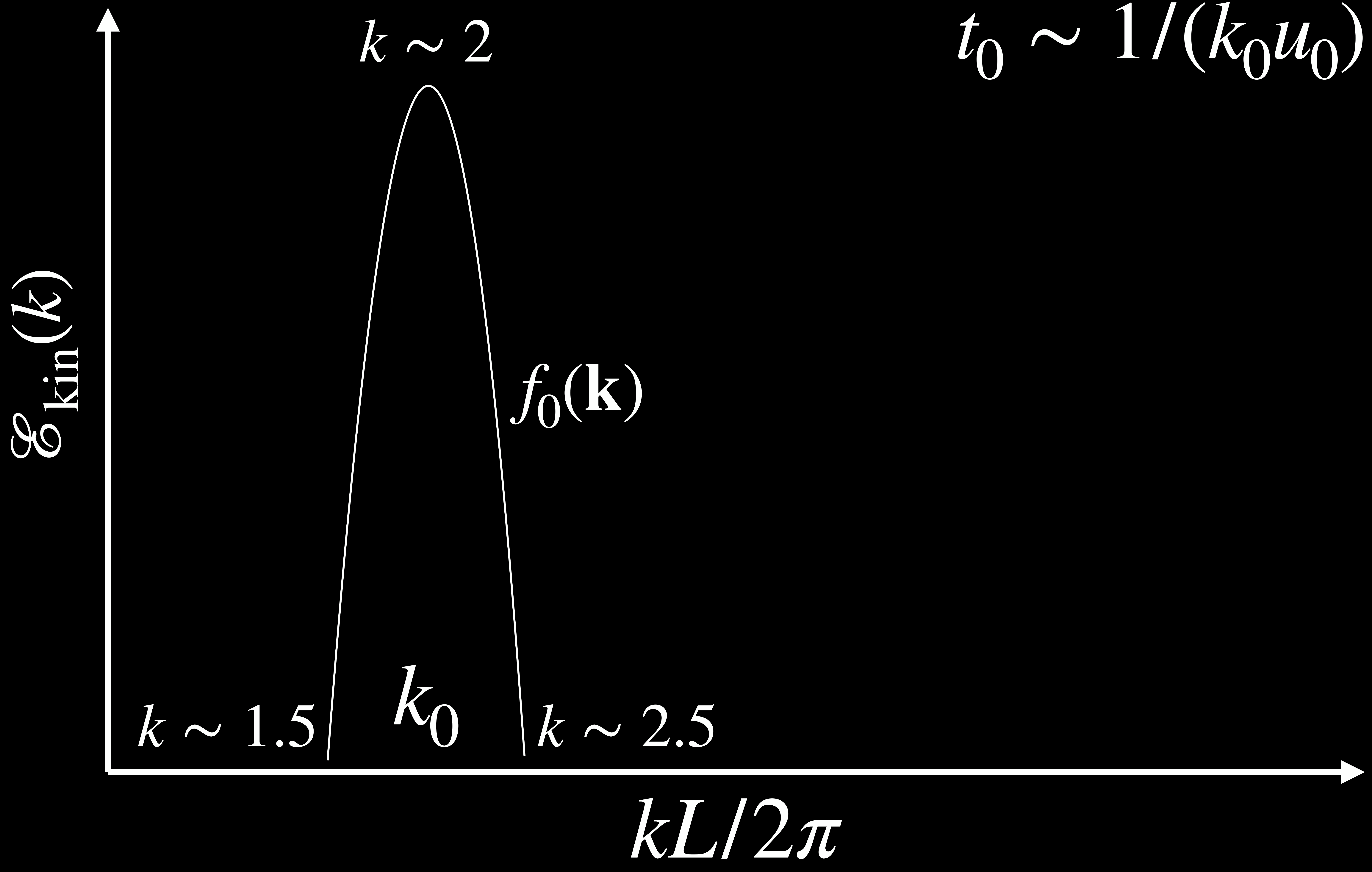
$$\zeta = 1 \implies \nabla \cdot \mathbf{f} = 0$$

$$\zeta = 0 \implies \nabla \times \mathbf{f} = 0$$

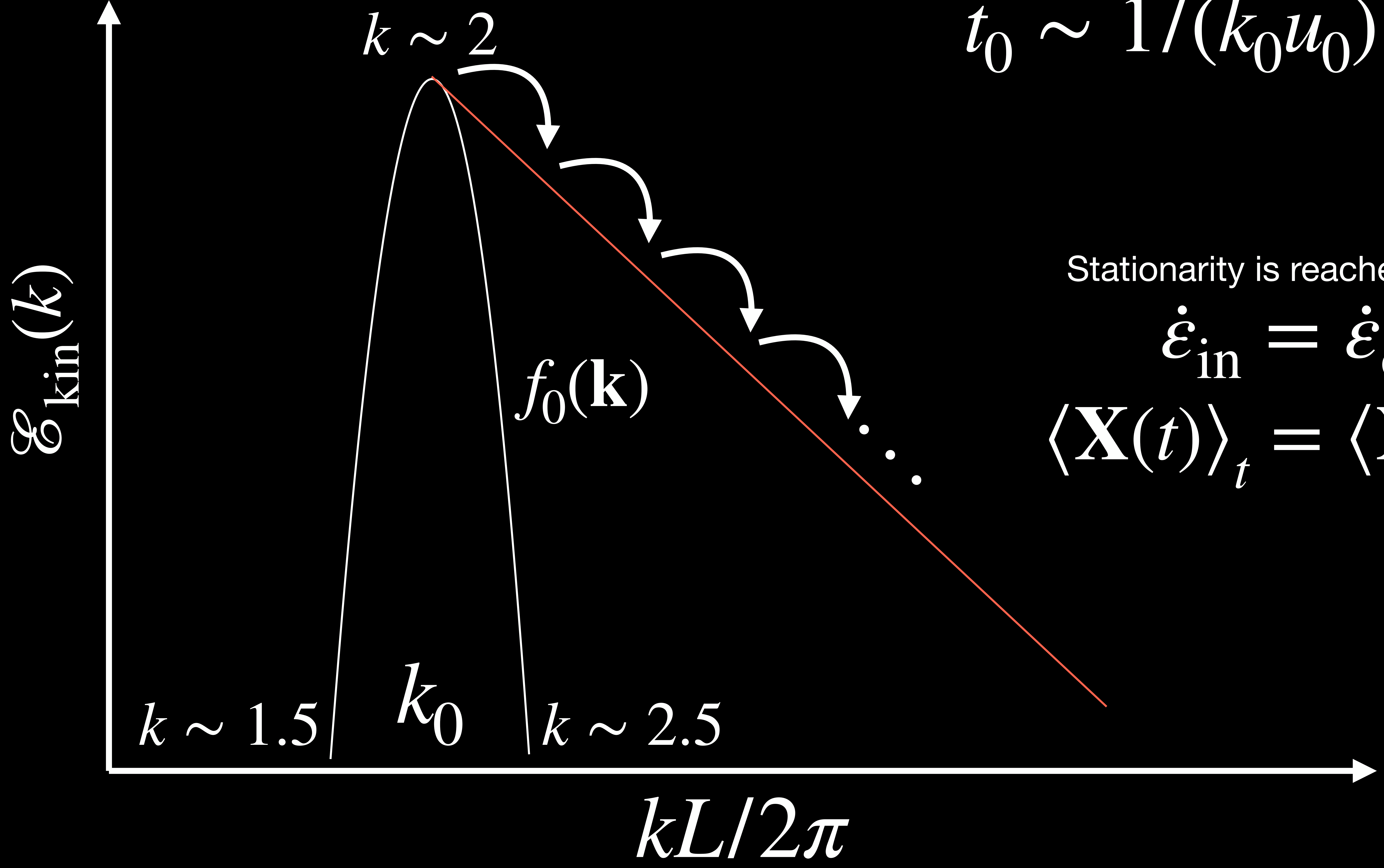
t_0 e-folding time of the forcing / correlation time / outer-scale turbulent turnover time

Stochastically forced Compressible MHD equations (ILES)

$t_0 \sim 1/(k_0 u_0)$



Stochastically forced Compressible MHD equations (ILES)



$$t_0 \sim 1/(k_0 u_0)$$

Stationarity is reached when

$$\dot{\varepsilon}_{\mathrm{in}} = \dot{\varepsilon}_{\mathrm{out}}$$

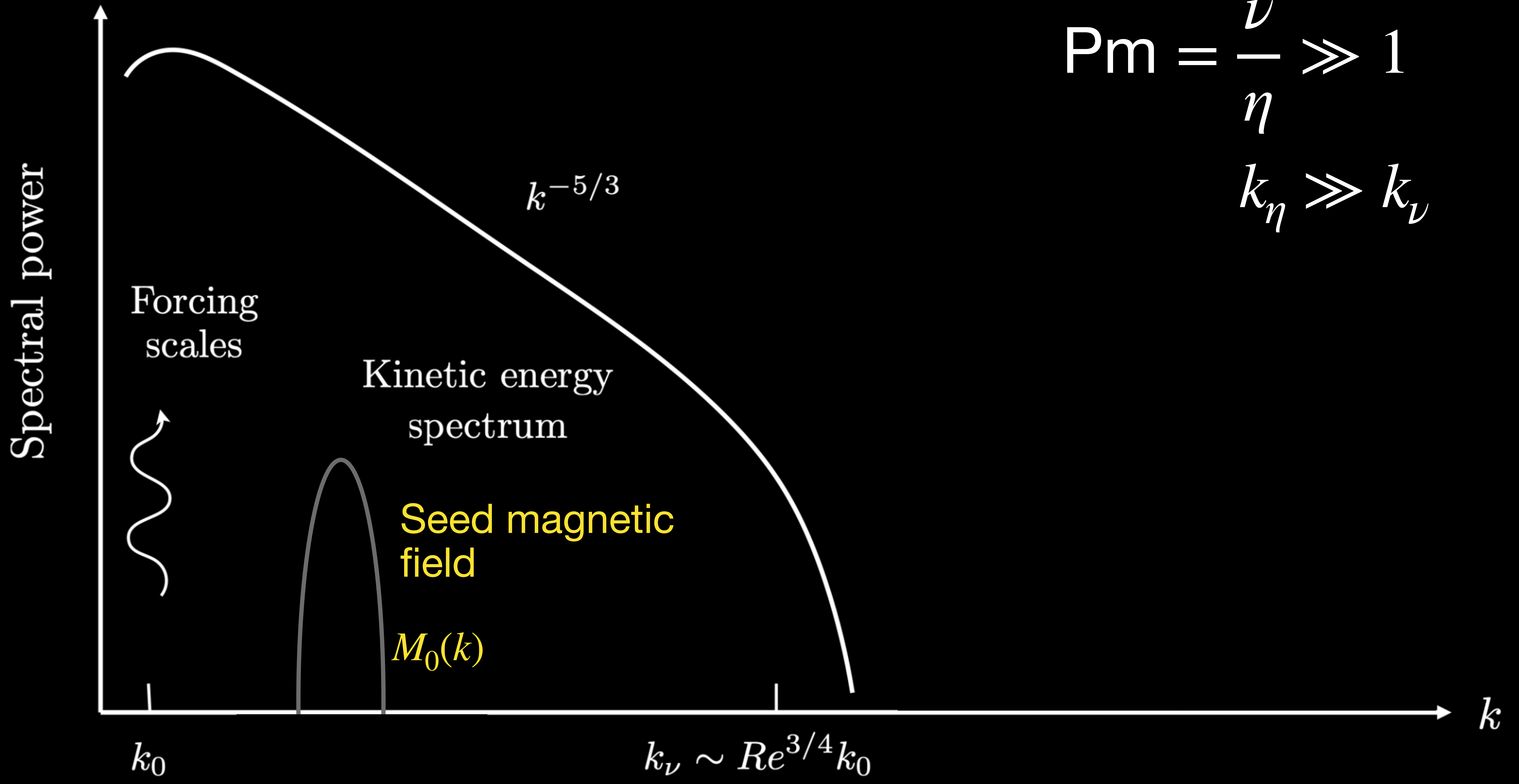
$$\langle \mathbf{X}(t) \rangle_t = \langle \mathbf{X}(t+\tau) \rangle_t$$

Turbulent dynamo

Modified from Rincon (2019)

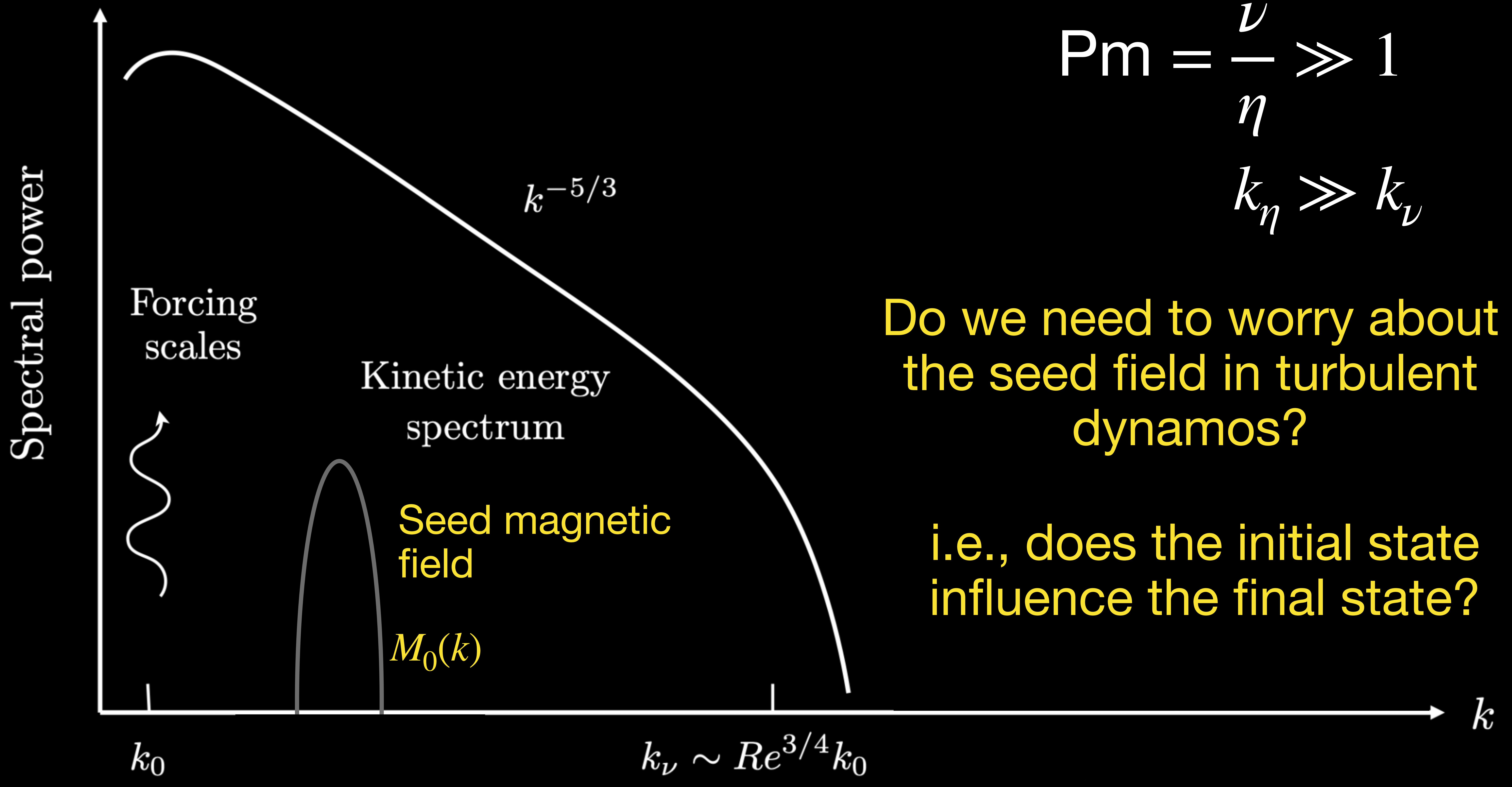
$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

$$k_\eta \gg k_\nu$$



Turbulent dynamo

Modified from Rincon (2019)



$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

$$k_\eta \gg k_\nu$$

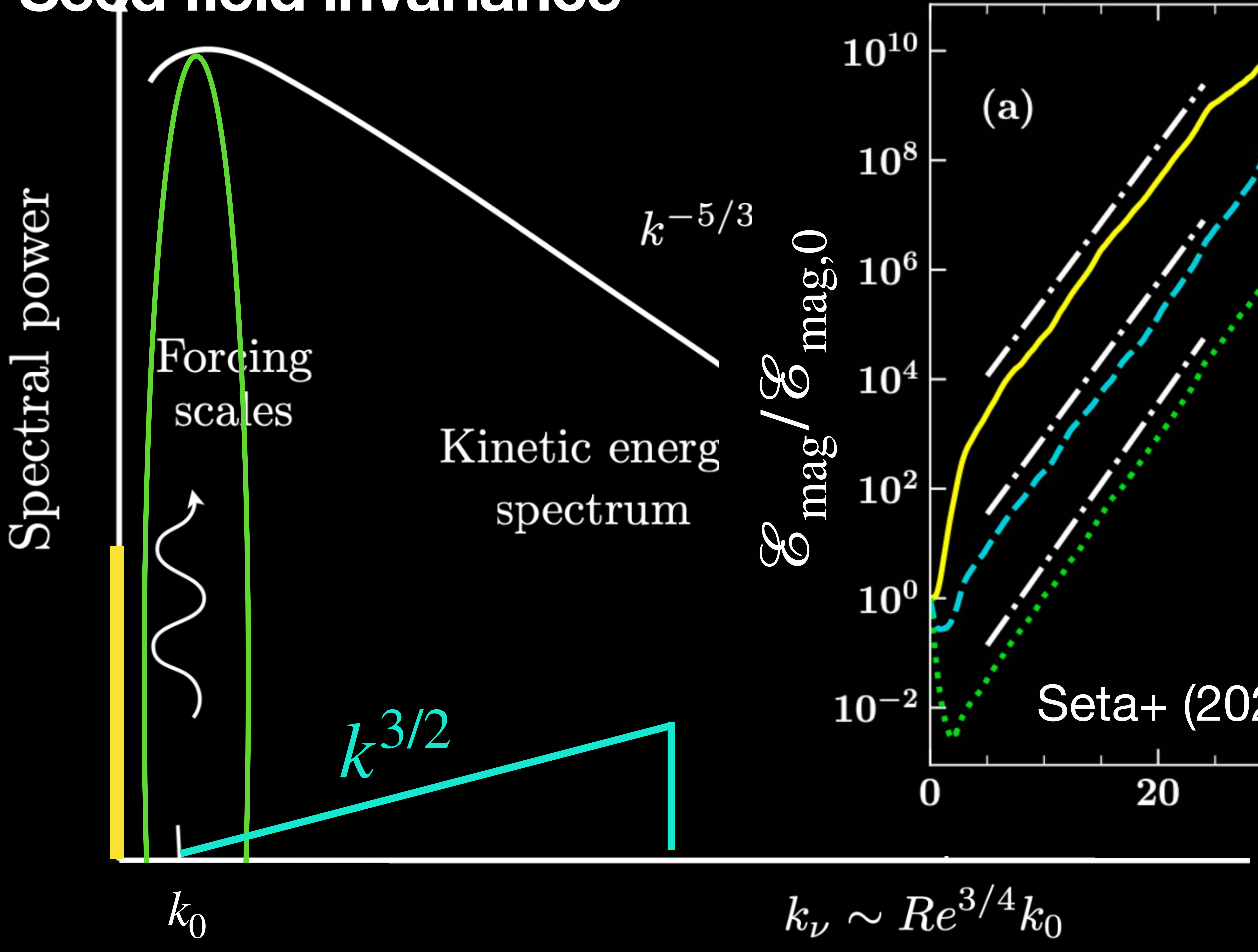
Do we need to worry about the seed field in turbulent dynamos?

i.e., does the initial state influence the final state?

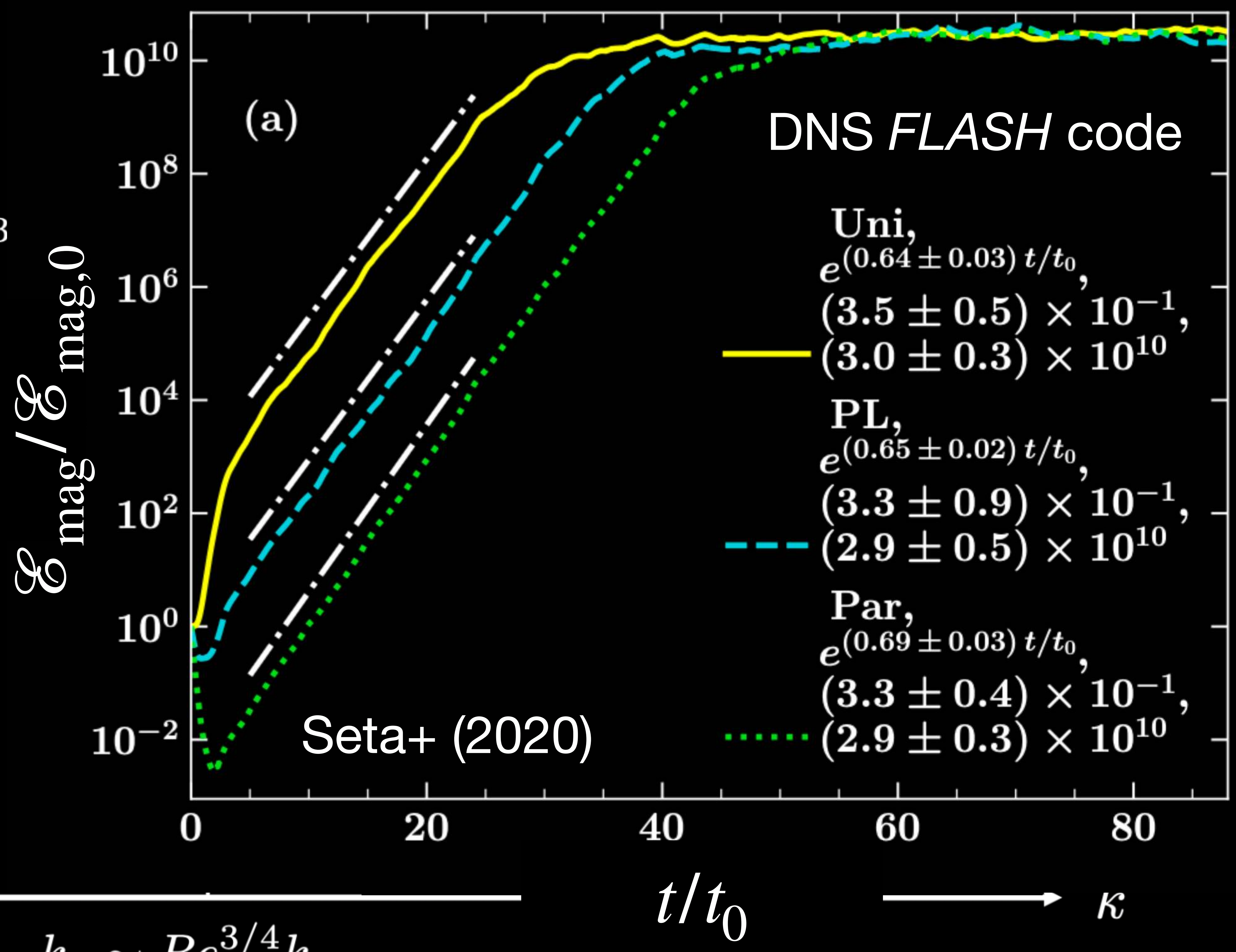
Turbulent dynamo

Seed field invariance

Modified from Rincon (2019)

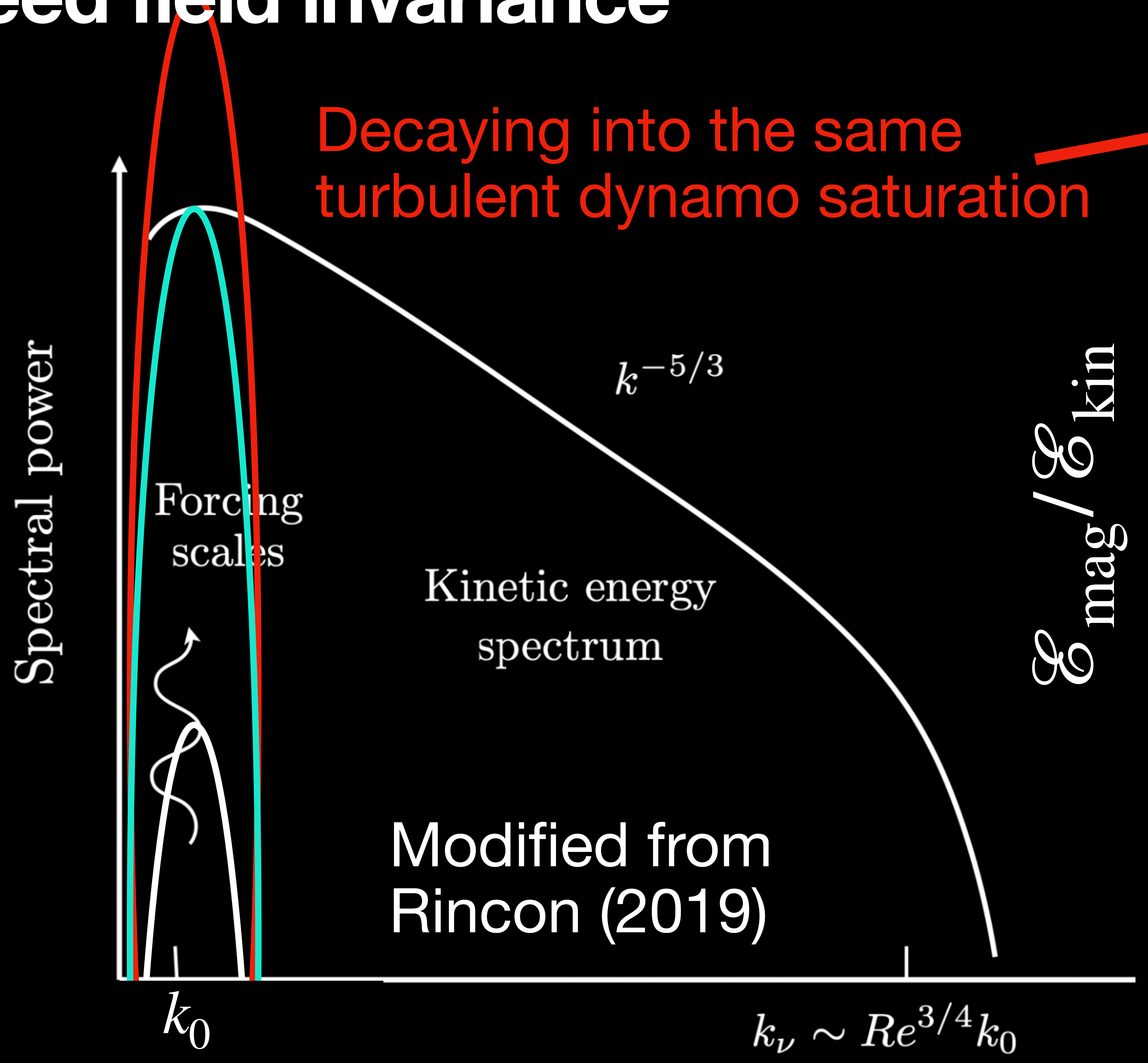


Invariant to structure

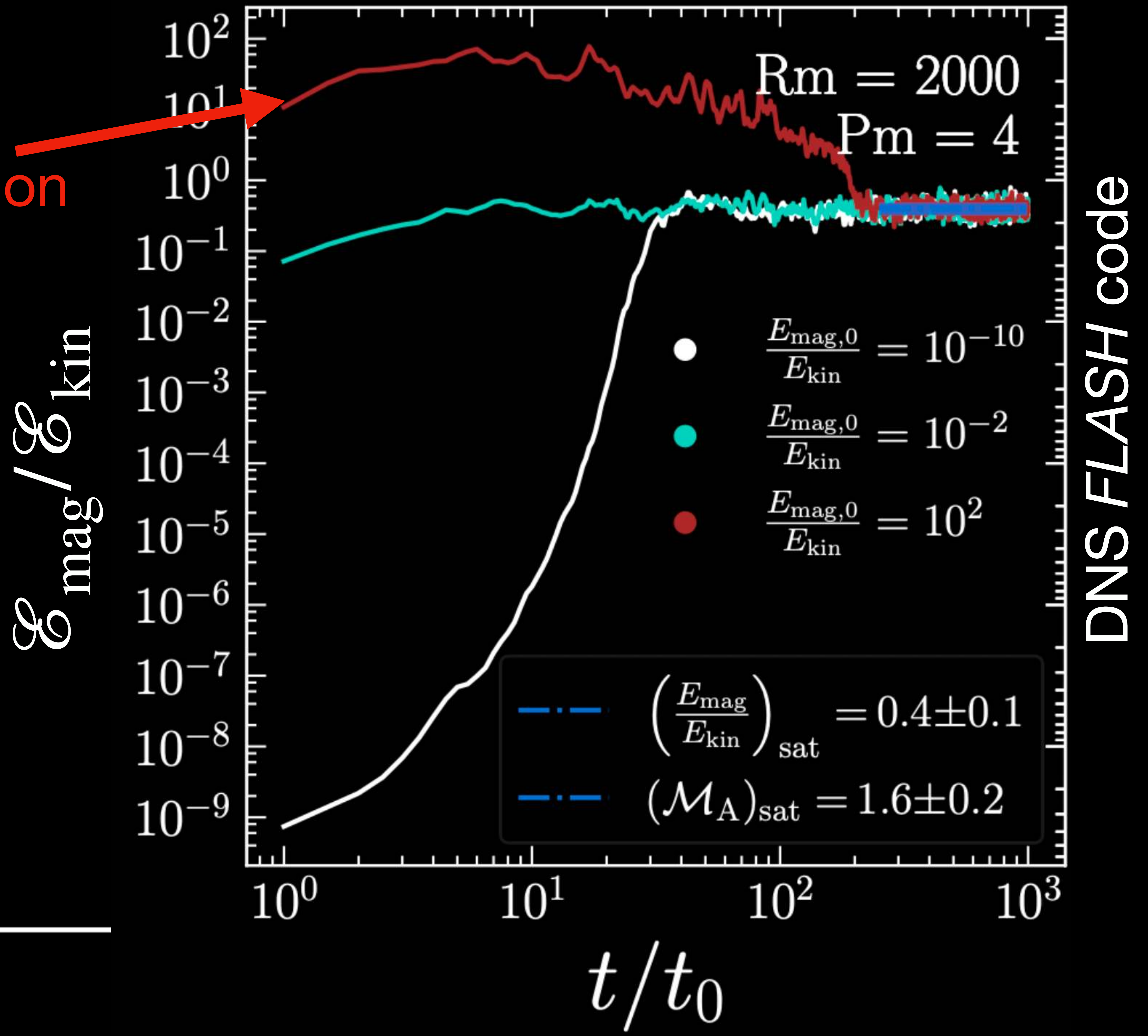


Turbulent dynamo

Seed field invariance



Invariant to amplitude



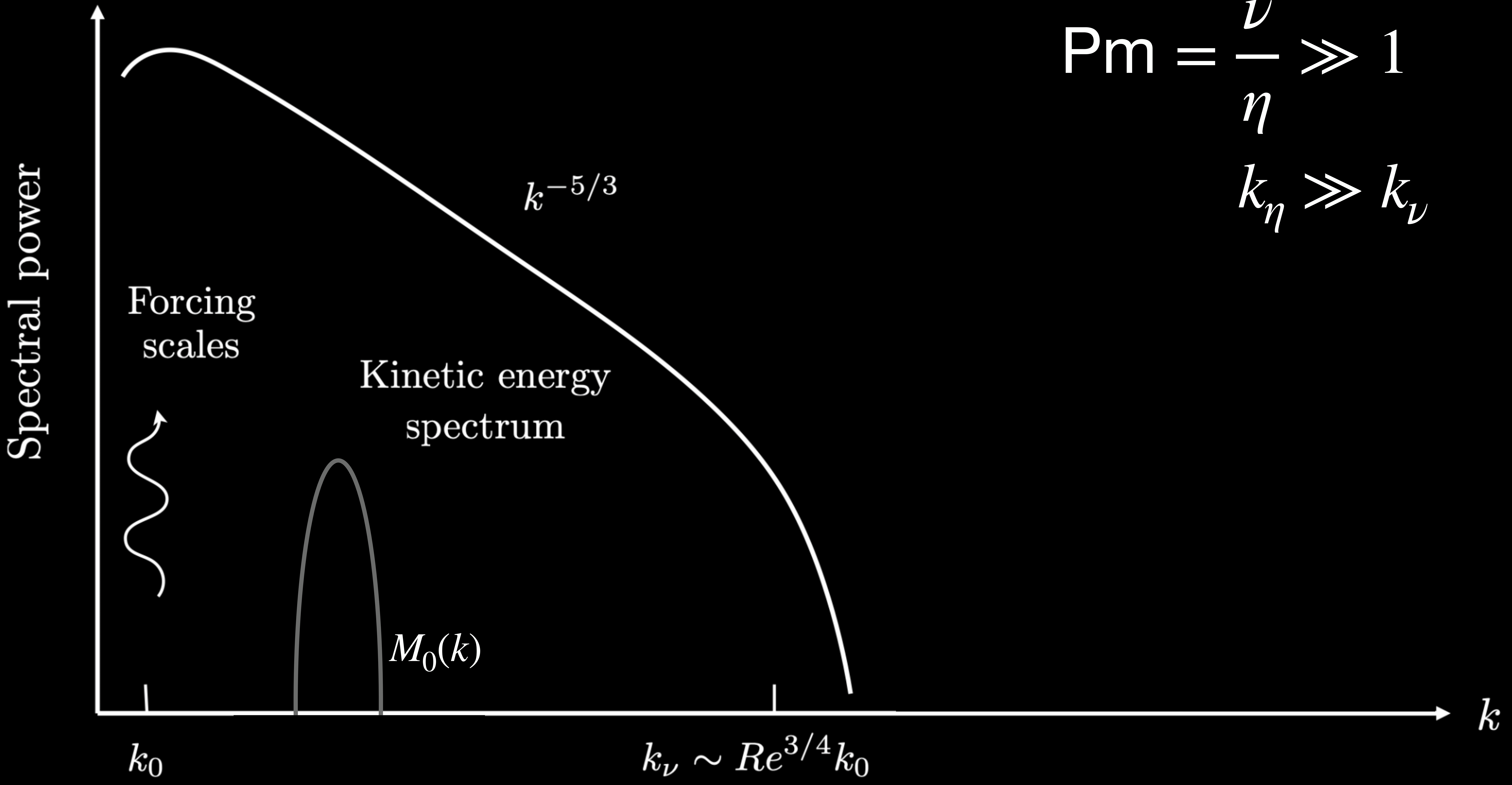
Beattie+ (2023). *Growth or Decay I: Universality of the turbulent dynamo saturation*

Turbulent dynamo

Modified from Rincon (2019)

$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

$$k_\eta \gg k_\nu$$



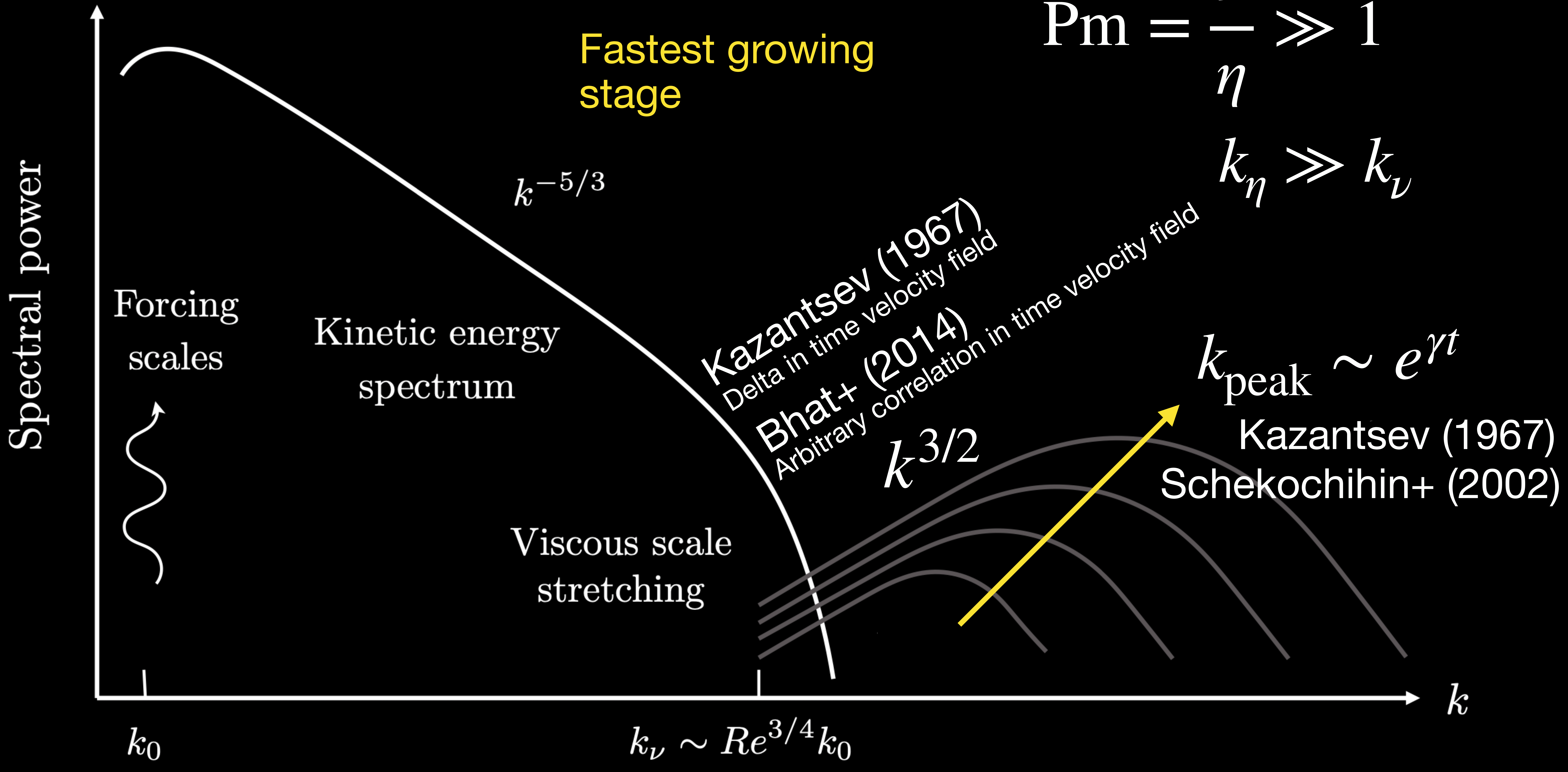
Dynamo in k space

First growth stage: diffusion-free regime

Modified from Rincon (2019)

$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

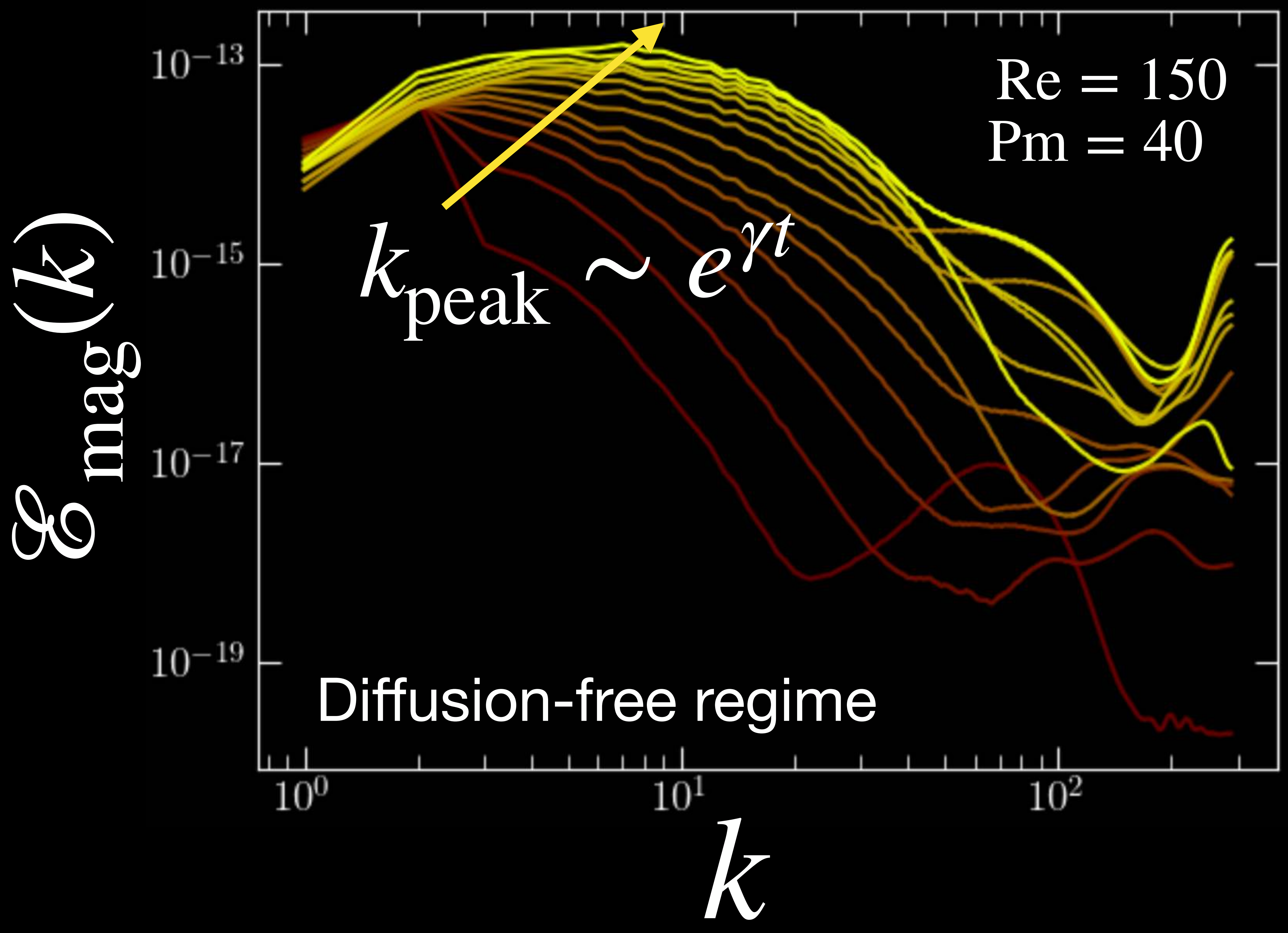
$$k_\eta \gg k_\nu$$



Turbulent dynamo

Diffusion-free regime: growing + populating

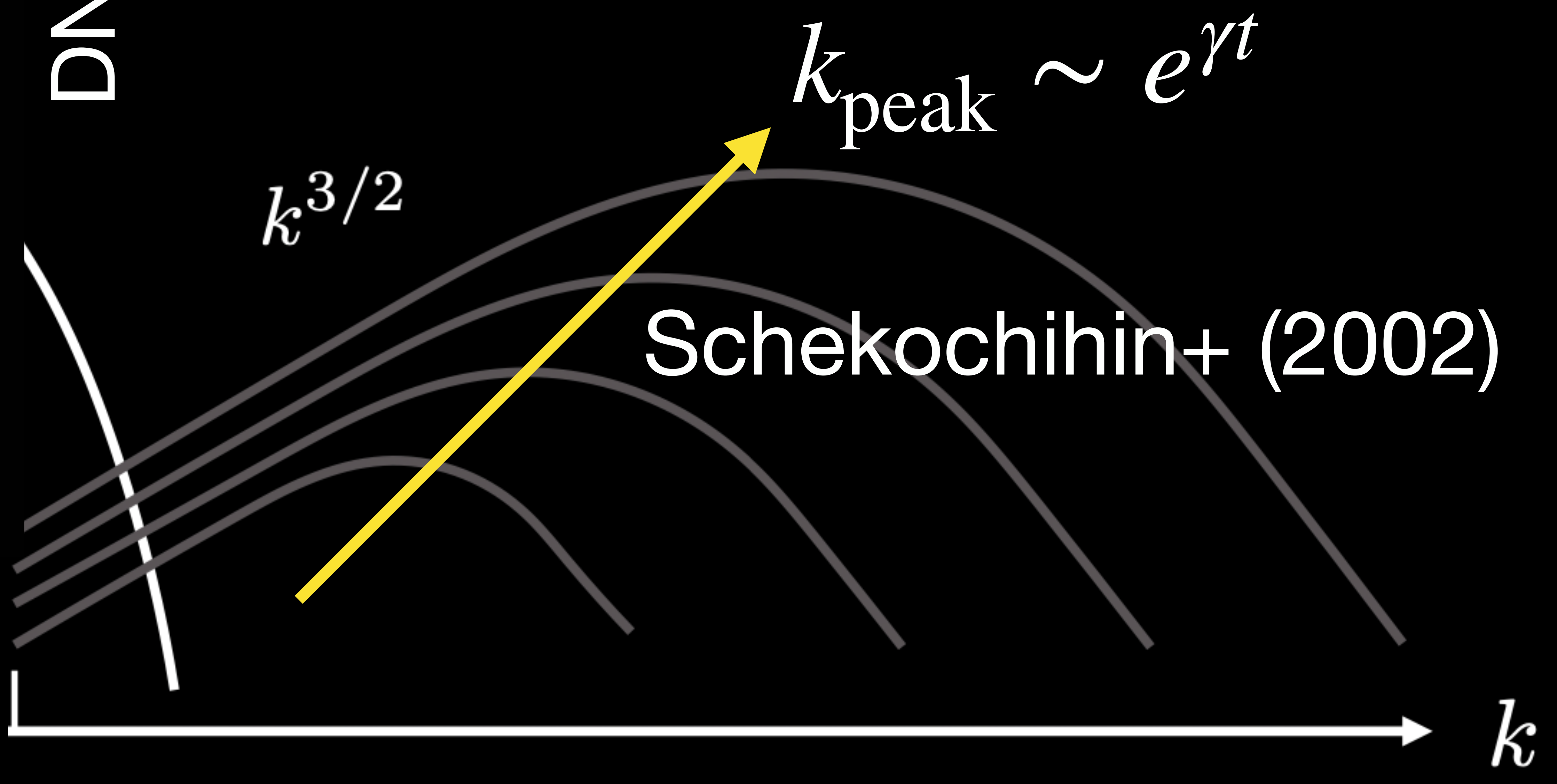
Modified from Rincon (2019)



DNS *FLASH* code

$$d_t(\mathscr{E}_{\rm mag}) \propto (\hat{\mathbf{b}} \otimes \hat{\mathbf{b}} : \nabla \mathbf{v})\mathscr{E}_{\rm mag}$$

$$\gamma t_0 \sim (\hat{\mathbf{b}} \otimes \hat{\mathbf{b}} : \nabla \mathbf{v})t_0 \sim t_0/t_\nu \sim {\rm Re}^{1/2}$$

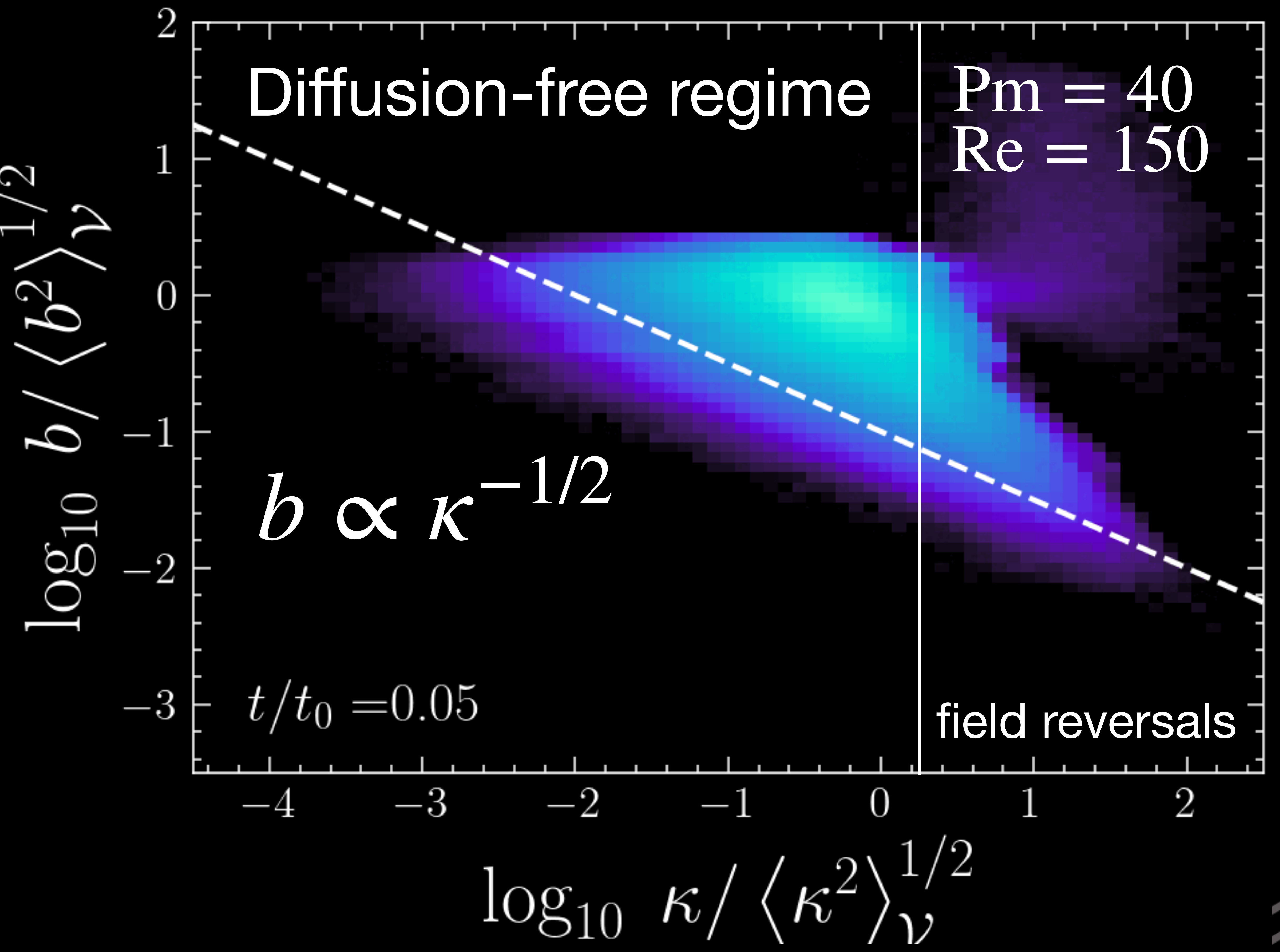


Varma, Beattie, Kriel, Ripperda (*in prep.*)

Turbulent dynamo

Modified from Rincon (2019)

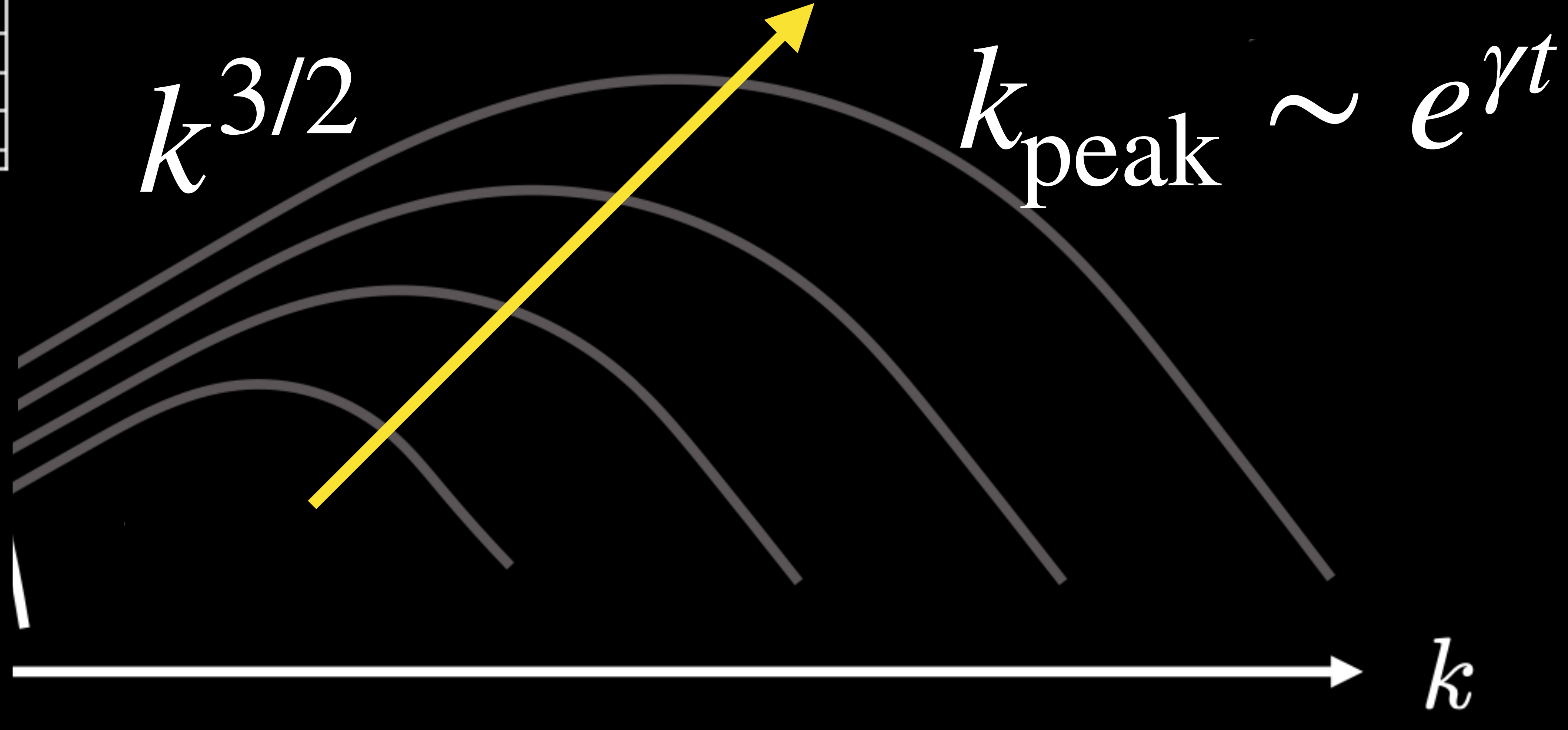
Diffusion-free regime: onset of folding



$$d_t(b\kappa^\alpha) = \left(\frac{1}{2} - \alpha\right)\hat{\mathbf{b}} \otimes \hat{\mathbf{b}} : \nabla \mathbf{v}$$
$$+\alpha \hat{\mathbf{n}} \otimes \hat{\mathbf{n}} : \nabla \mathbf{v}$$

$\alpha = \dfrac{1}{2}$ special cases where stretching makes relation stationary

Schekochihin+ (2004)



$$\kappa = |\hat{b} \cdot \nabla \hat{b}|$$

Varma, Beattie, Kriel, Ripperda (*in prep.*)

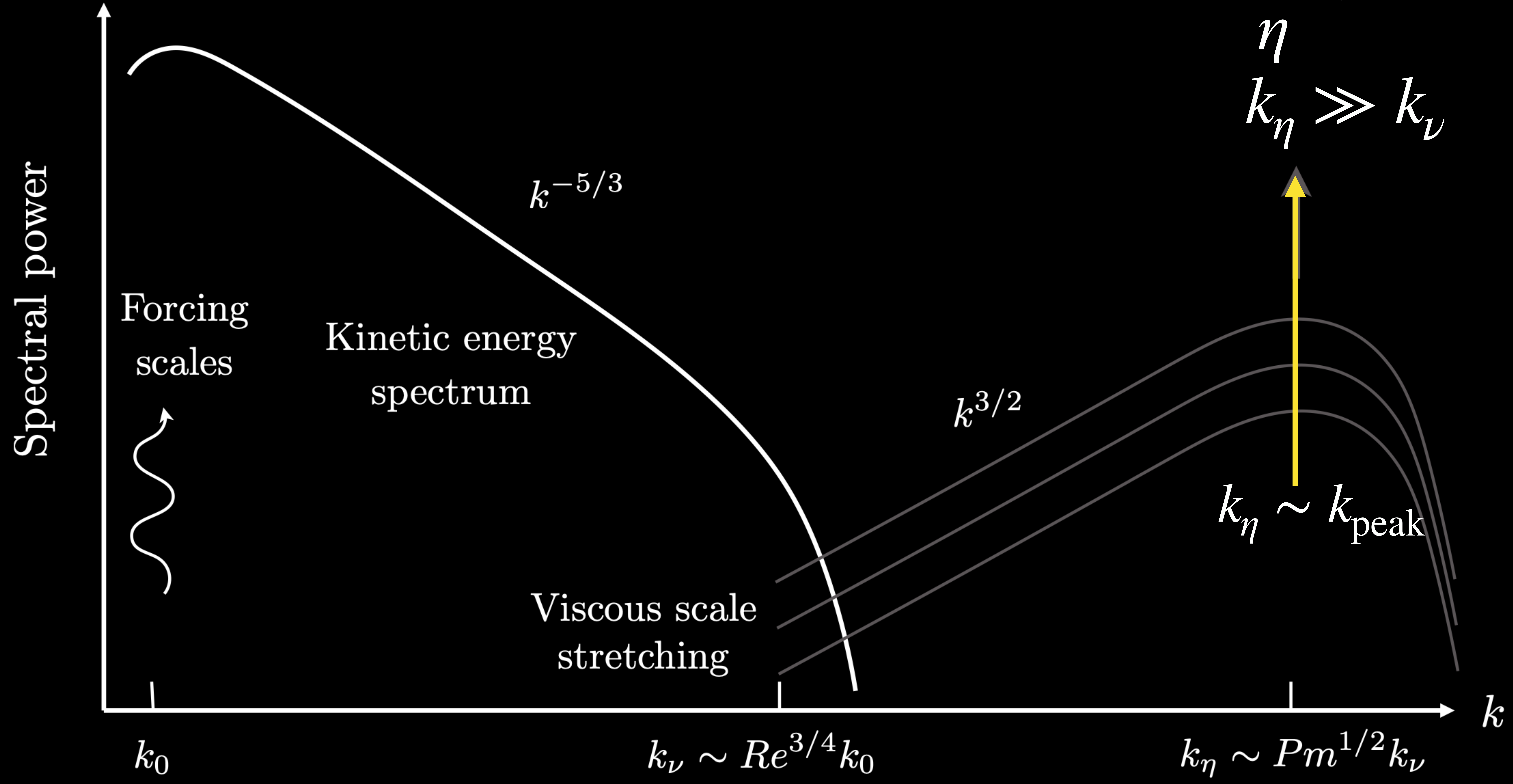
Turbulent dynamo

Second growth stage: kinematic regime

Modified from Rincon (2019)

$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

$$k_\eta \gg k_\nu$$



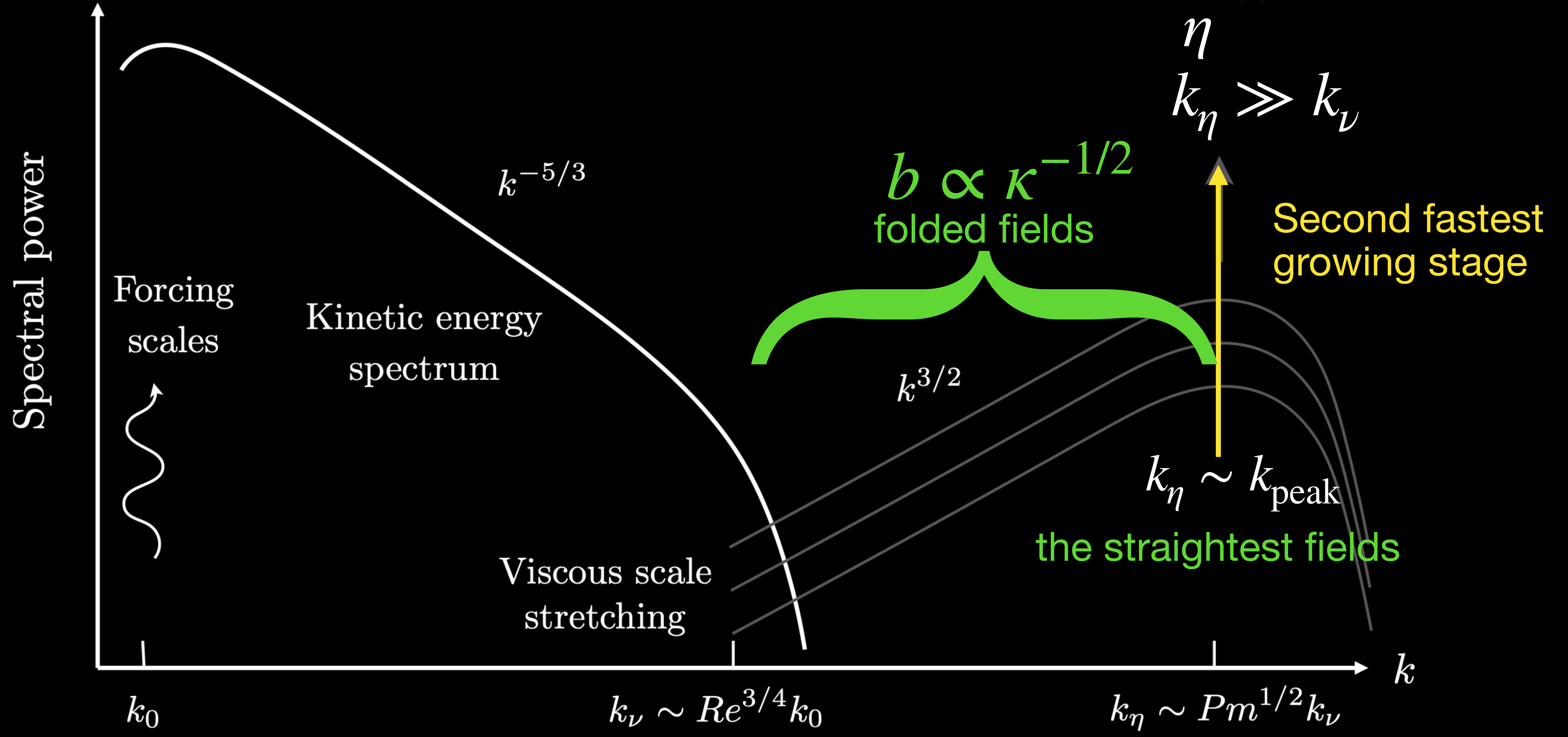
Turbulent dynamo

Second growth stage: kinematic regime

Modified from Rincon (2019)

$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

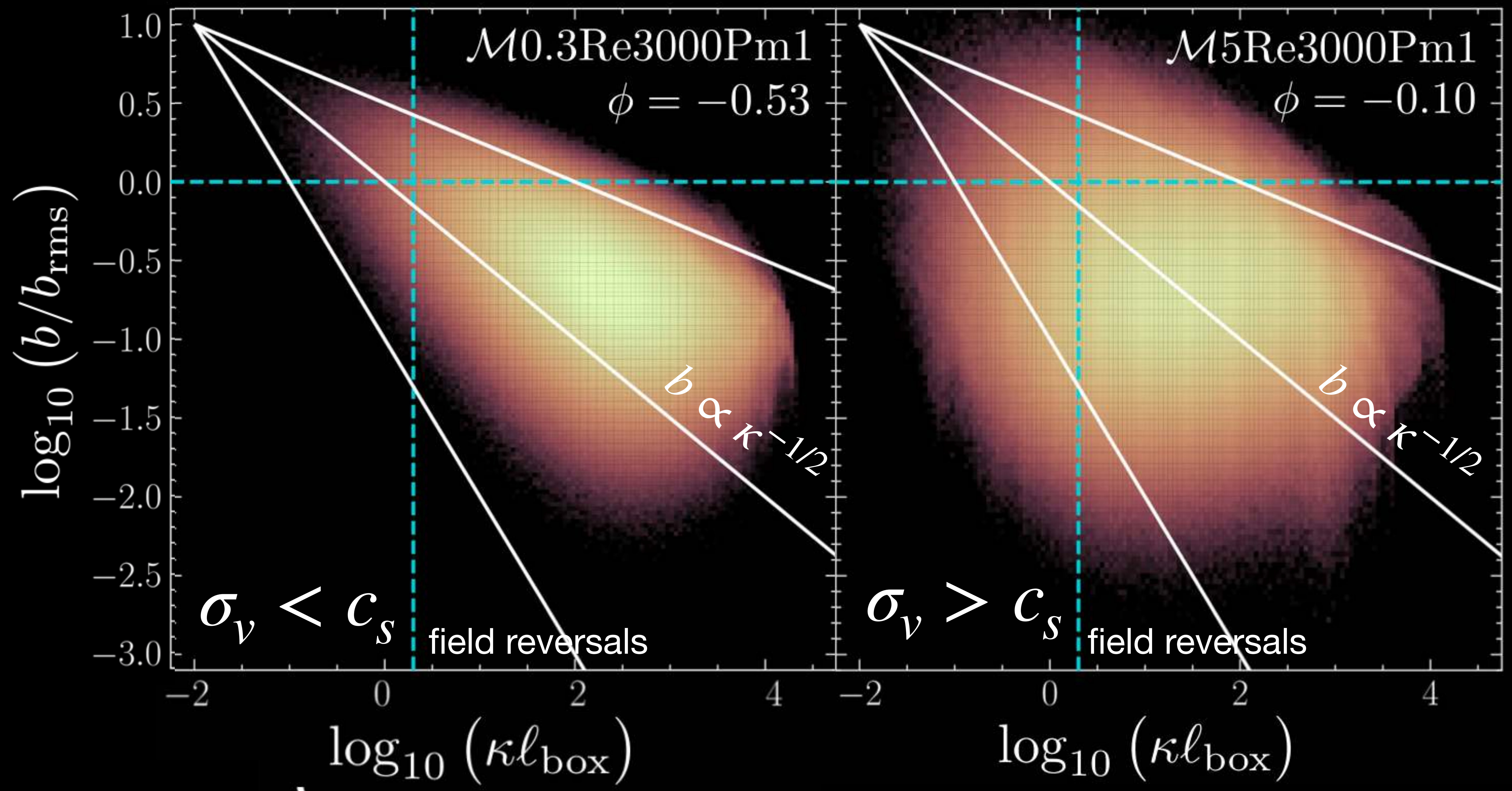
$$k_\eta \gg k_\nu$$



Turbulent dynamo

kinematic regime: folded fields

Neco Kriel
Grad. Student (ANU)

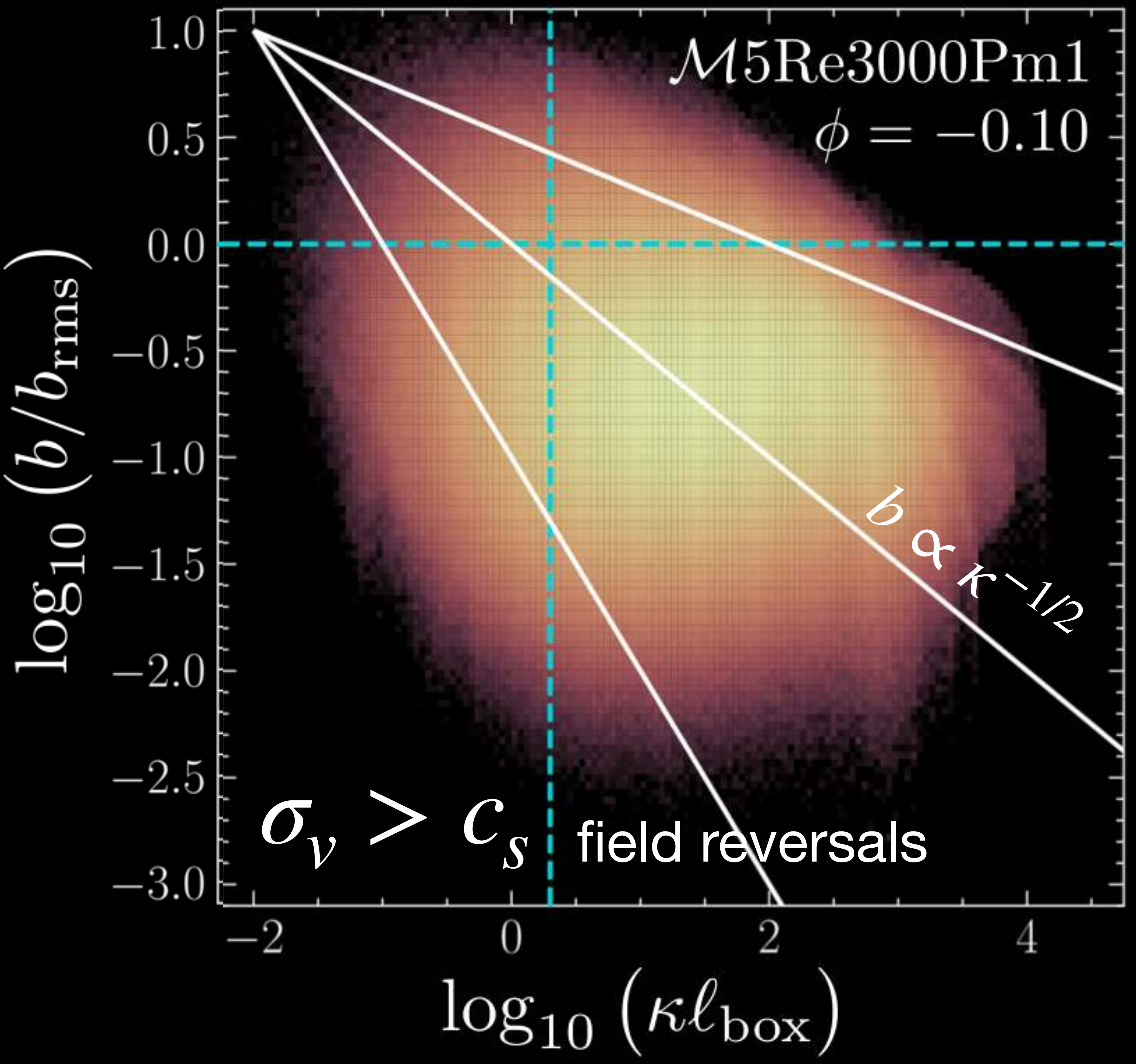


Kriel, Beattie+ (2024). *Fundamental scales II: the kinematic stage of the supersonic dynamo*

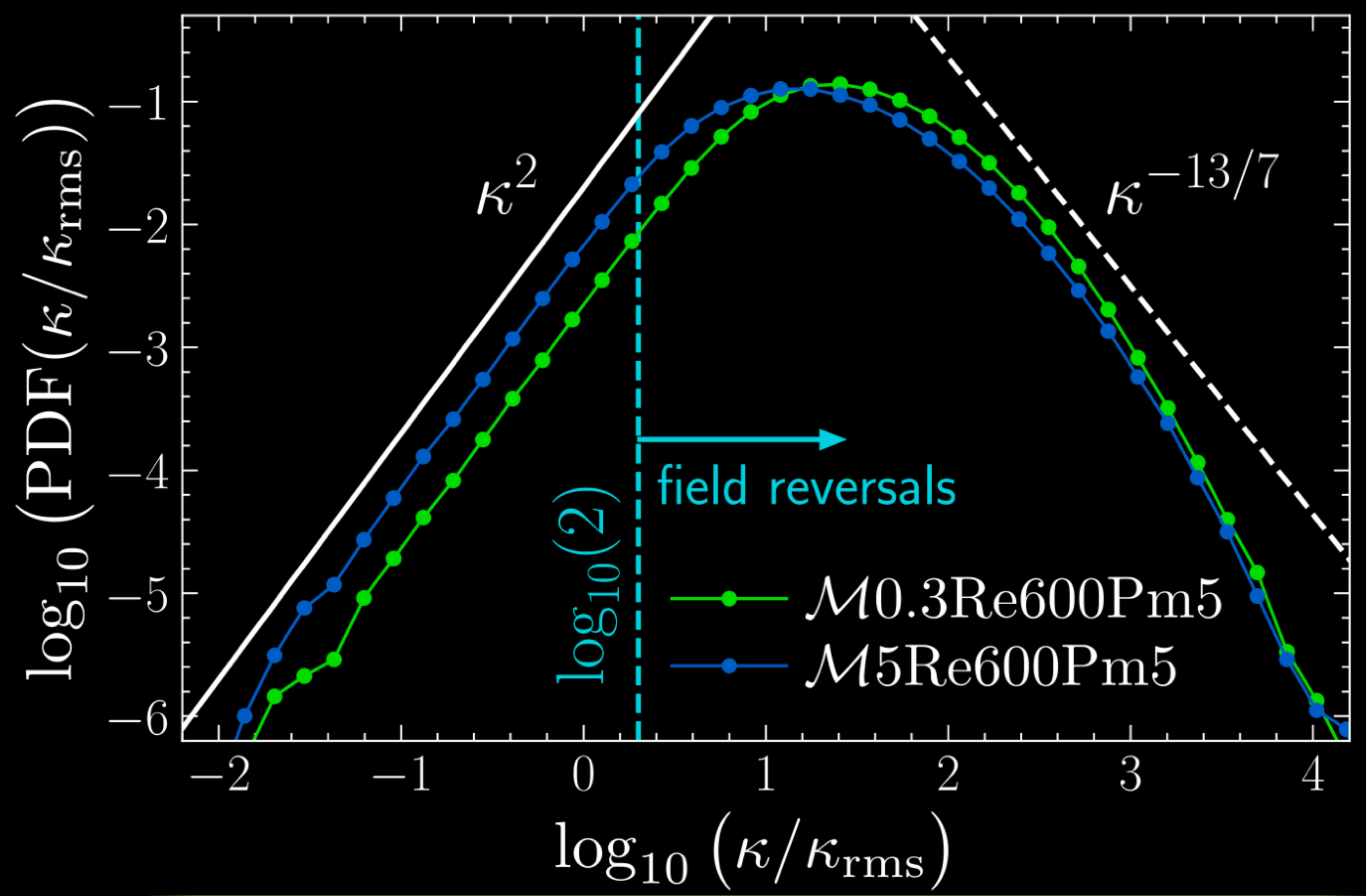
Turbulent dynamo

kinematic regime: folded fields

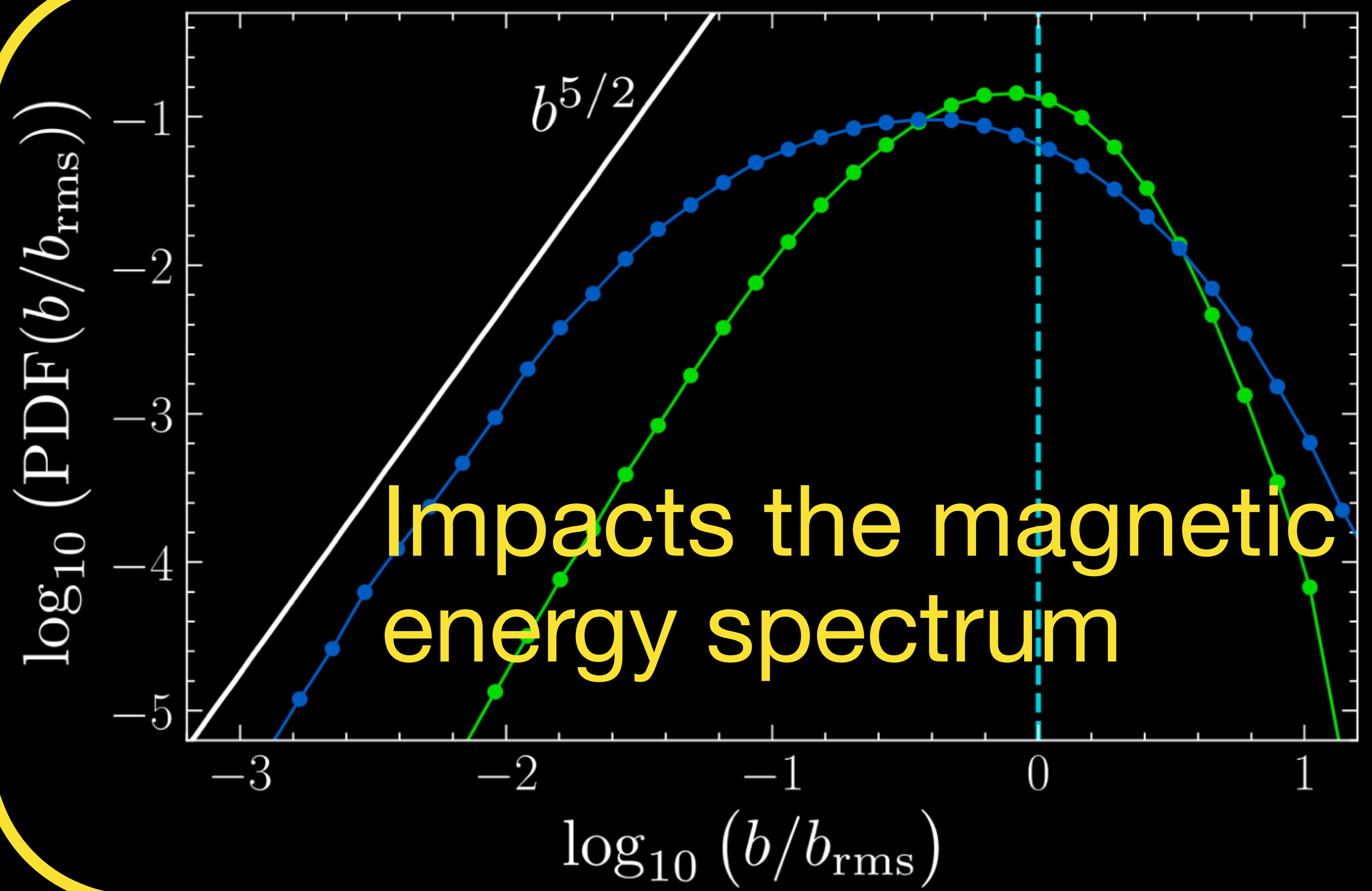
Neco Kriel
Grad. Student (ANU)



Folded field retained



Amplitudes change (flux compressions)



Impacts the magnetic energy spectrum

Kriel, Beattie+ (2024). *Fundamental scales II: the kinematic stage of the supersonic dynamo*

Turbulent dynamo

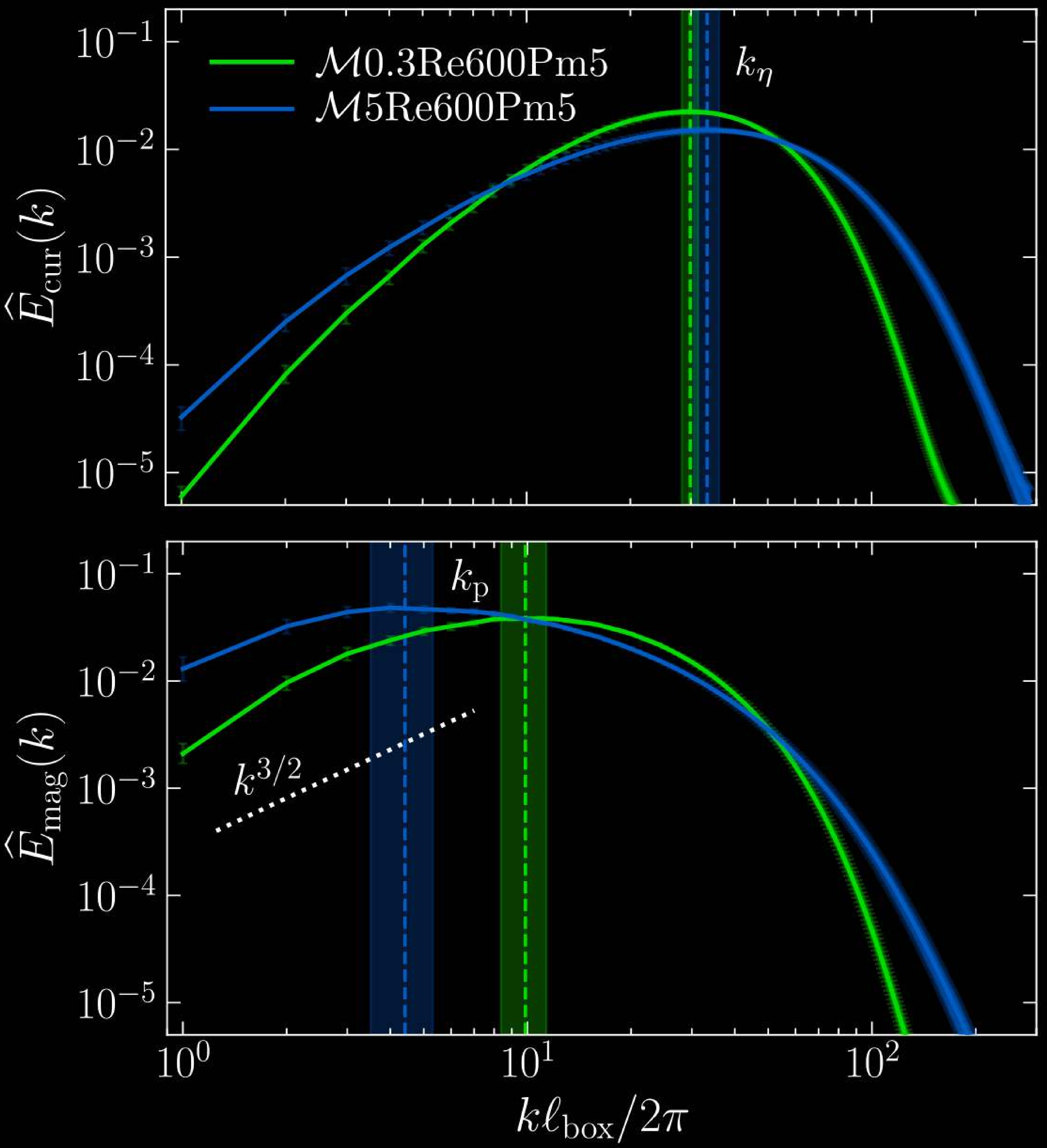
kinematic regime: the peak scale

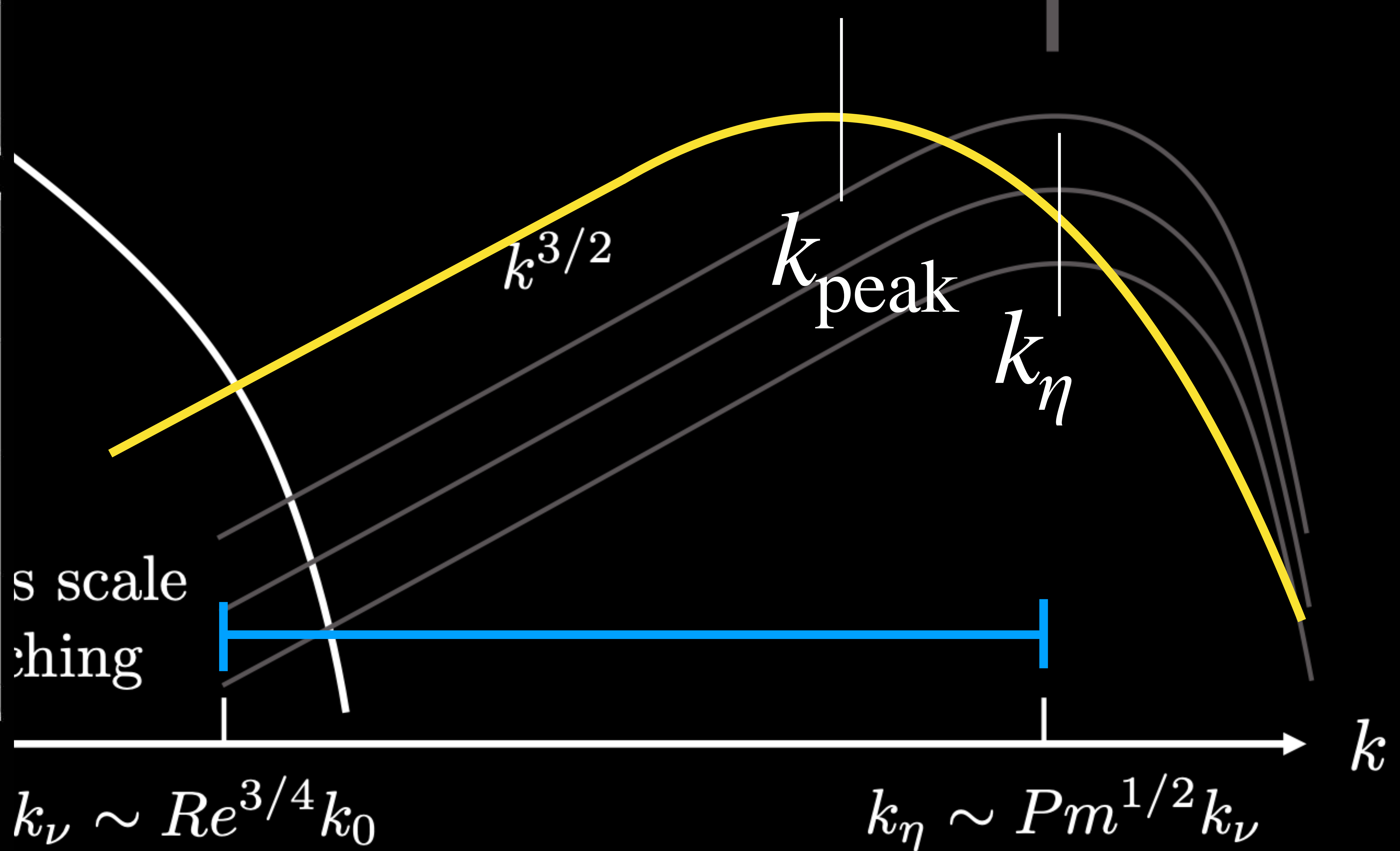
Modified from Rincon (2019)

$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

$$k_\eta \gg k_\nu$$

$$\boxed{k_{\mathrm{peak}}/k_\eta \sim (k_{\mathrm{width}}/k_{\mathrm{length}})^{-1/3}}$$





Turbulent dynamo

kinematic regime: viscous scale

Neco Kriel
Grad. Student (ANU)

Derived from $k^{-5/3}$ velocity spectrum

$$k_\nu \sim \mathrm{Re}^{3/4}$$

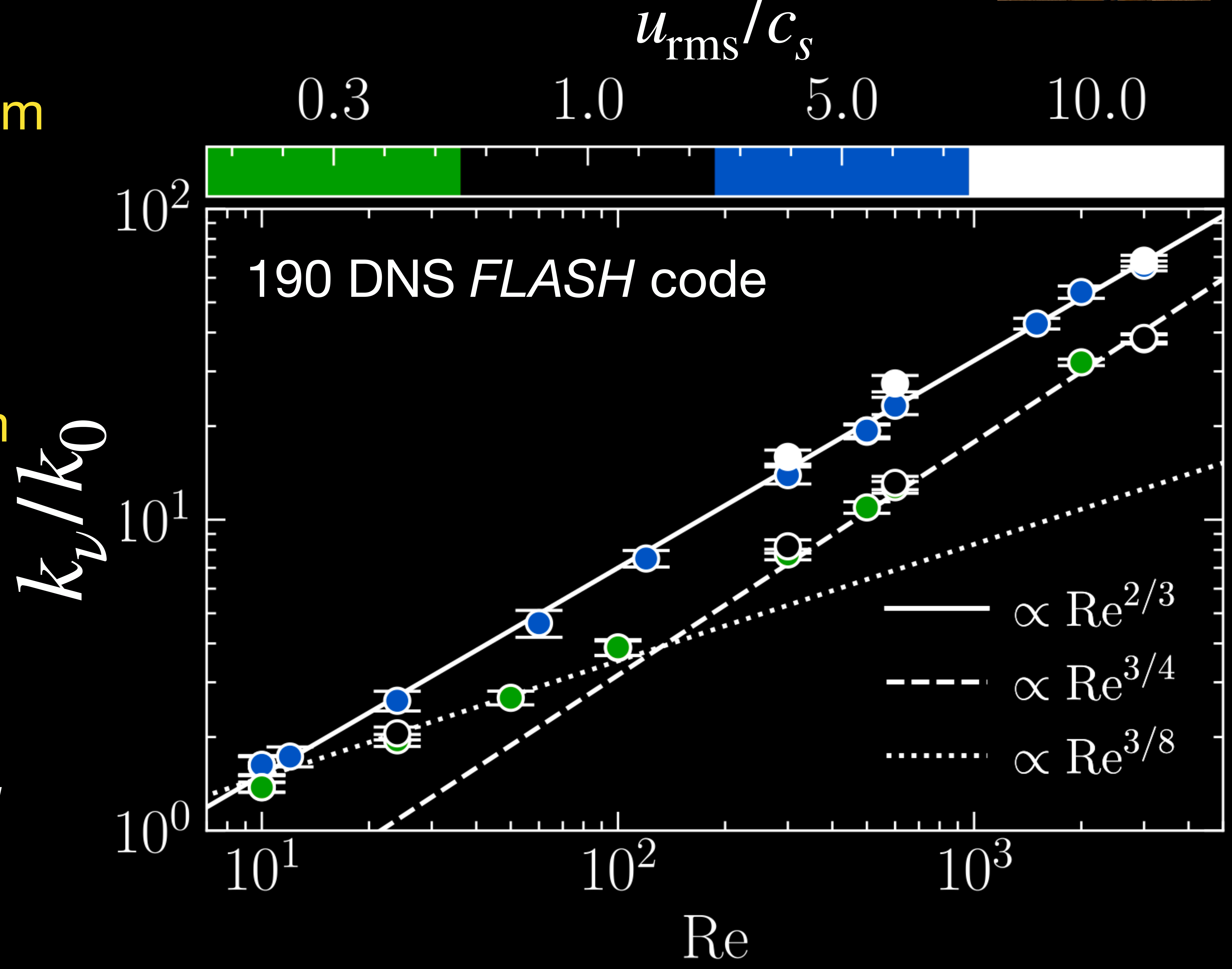
Kolmogorov41

Derived from k^{-2} velocity spectrum

$$k_\nu \sim \mathrm{Re}^{2/3}$$

Schober+(2015)

Kriel, Beattie+ (2024). *Fundamental scales II: the kinematic stage of the supersonic dynamo*



Turbulent dynamo

kinematic regime: viscous scales

Neco Kriel
Grad. Student (ANU)

Kriel, Beattie+ (2024). *Fundamental scales II: the kinematic stage of the supersonic dynamo*