The chronology of high-k magnetic modes: from dynamo to stationary turbulence

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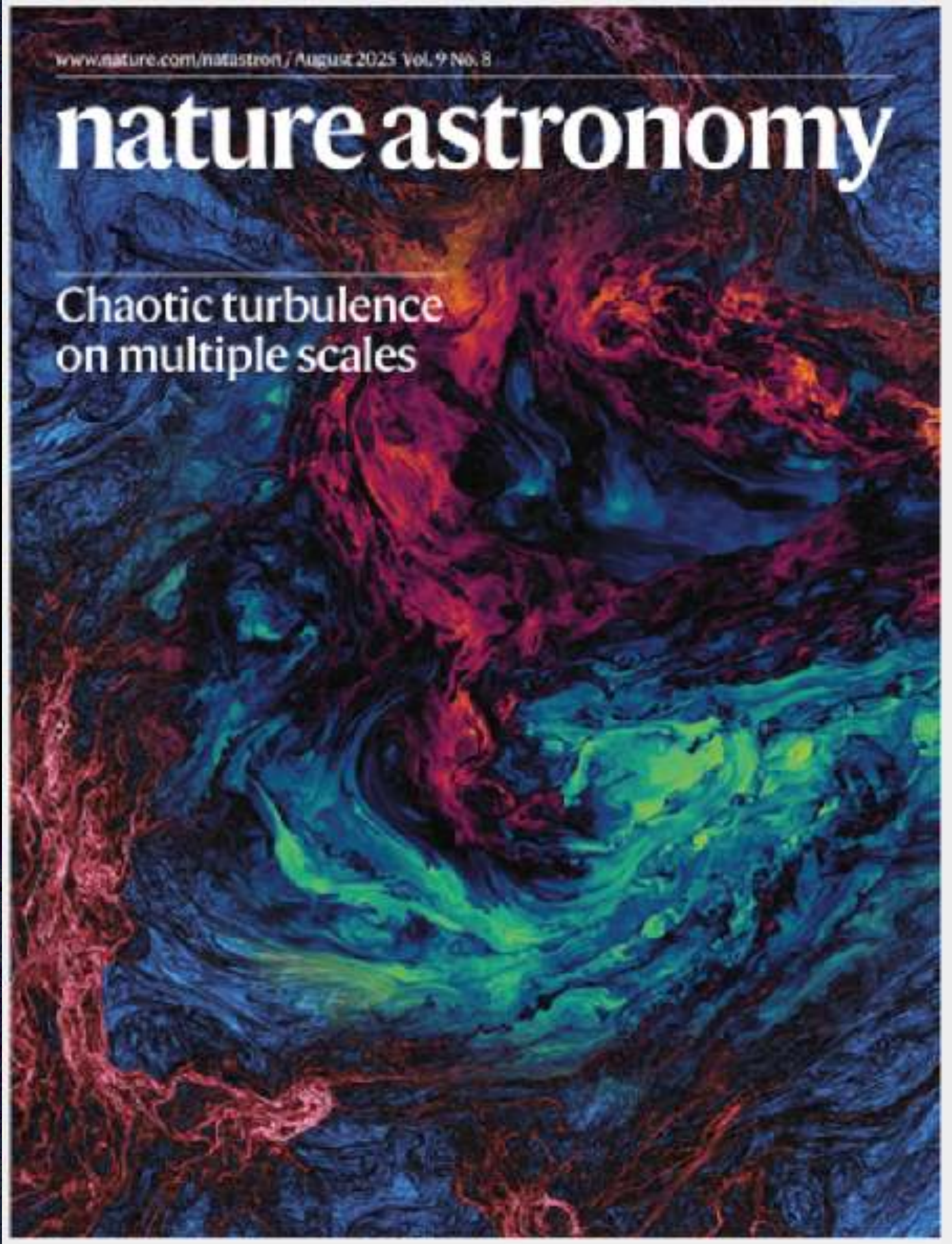
KITP: Frontiers of Relativistic Plasmas Physics in Astrophysics and Laboratory Experiments | 20th August 2025

Collaborators for this work: Neco Kriel (ANU), Amitava Bhattacharjee (PU), Christoph Federrath (ANU), Ralf Klessen (UH), Salvatore Cielo (LRZ)

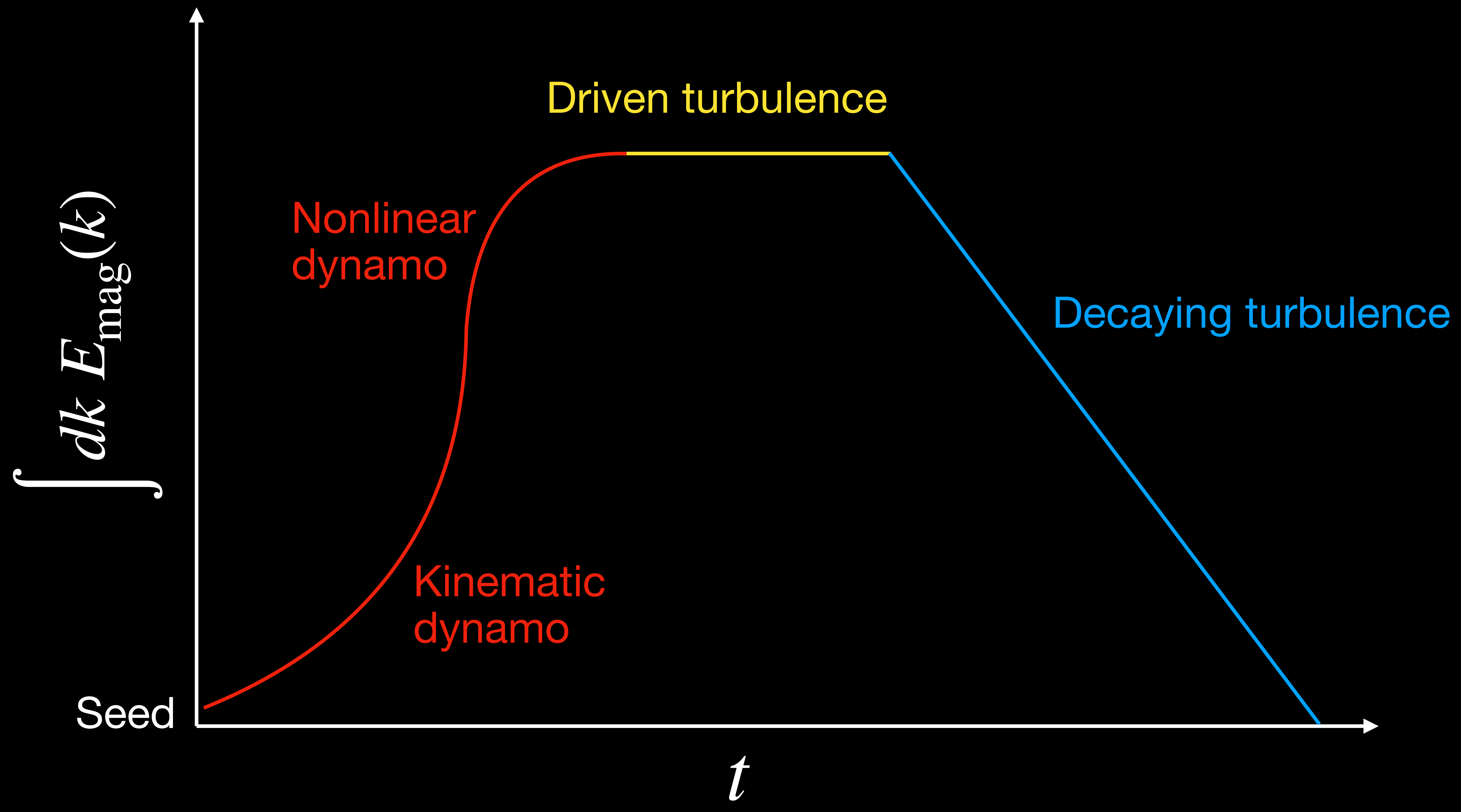
Goals for this early morning talk

1. Wake everyone up.
2. Grow a small-scale (kinematic + nonlinear) field in the KITP theatre.
3. Venture into the dynamo saturation with $10{,}080^3$ domain that can resolve inertial ranges, presumably in the MHD asymptotic state, focussing on the role of alignment.

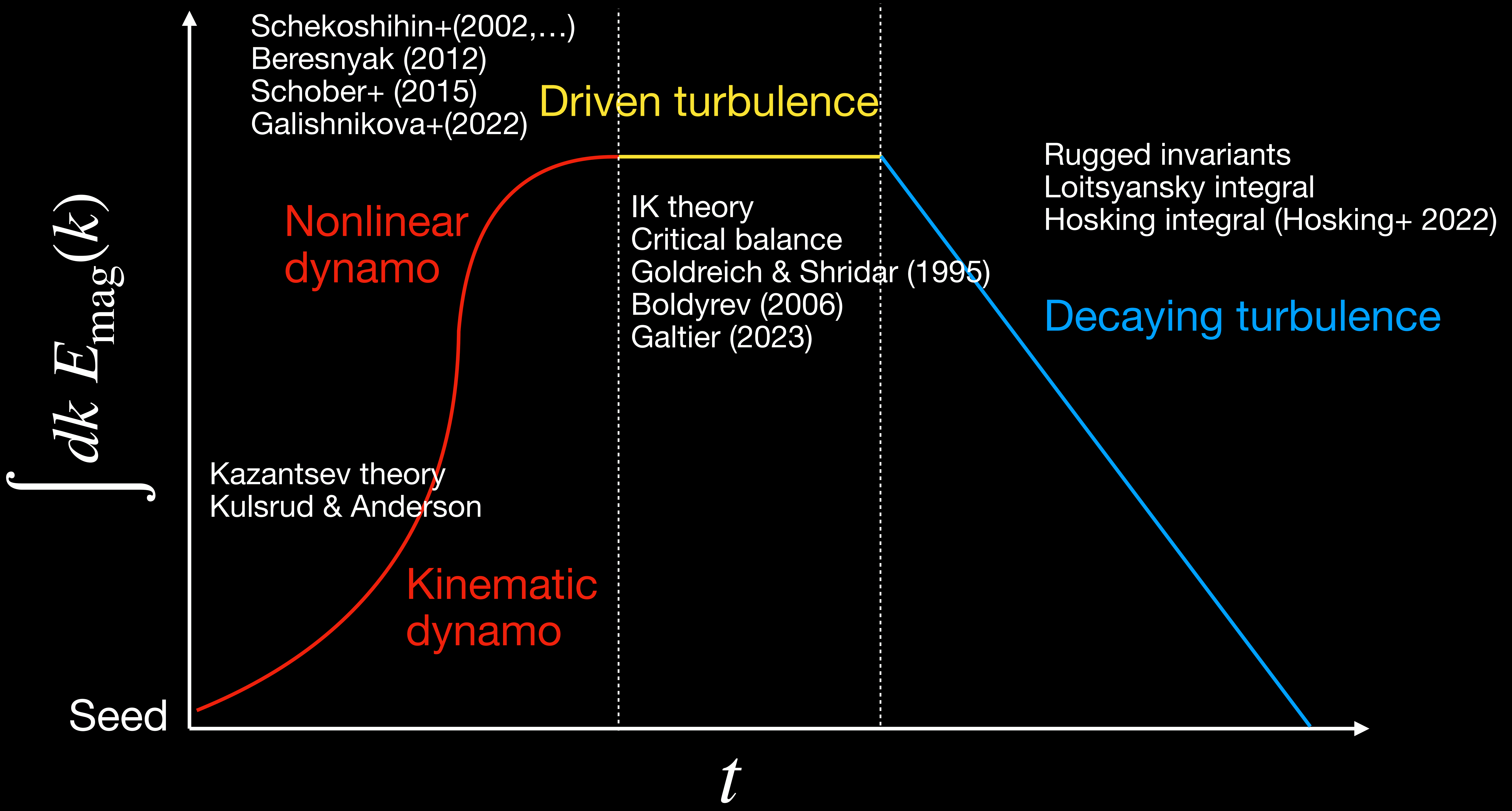
Came out 2 days ago!



The lifecycle of a high-k mode spans across a patchwork quilt of theories



The lifecycle of a high-k mode spans across a patchwork quilt of theories



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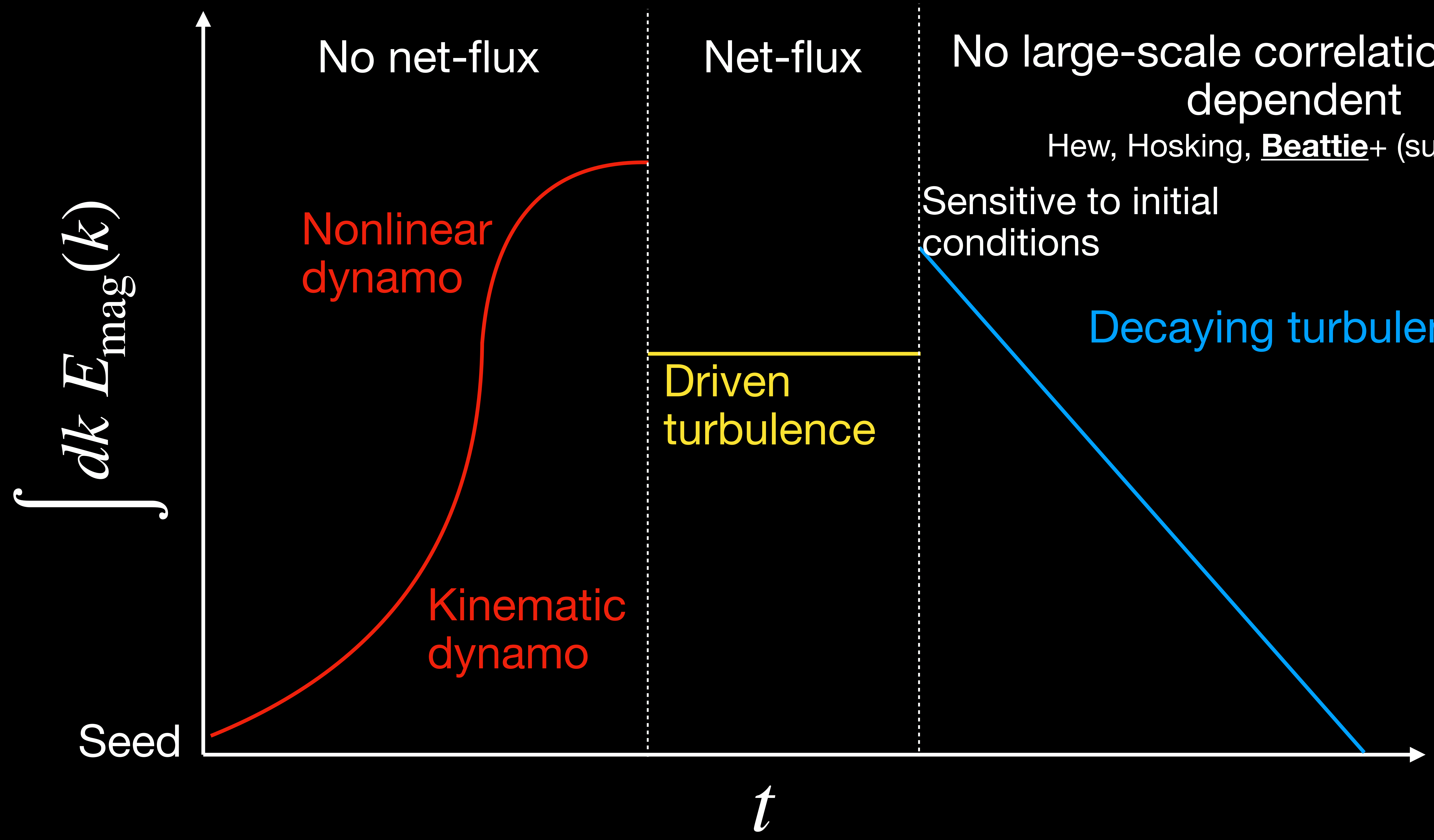
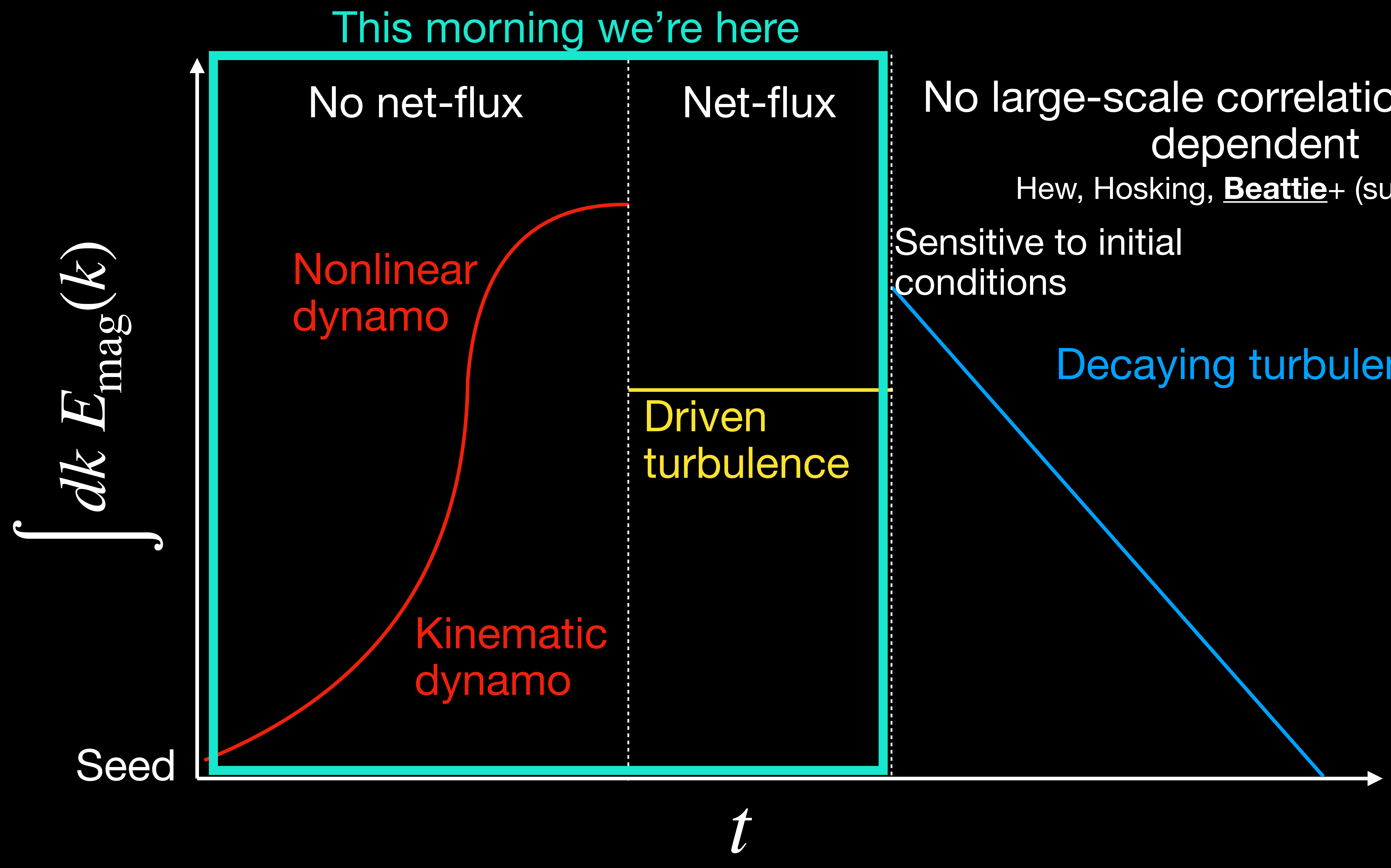
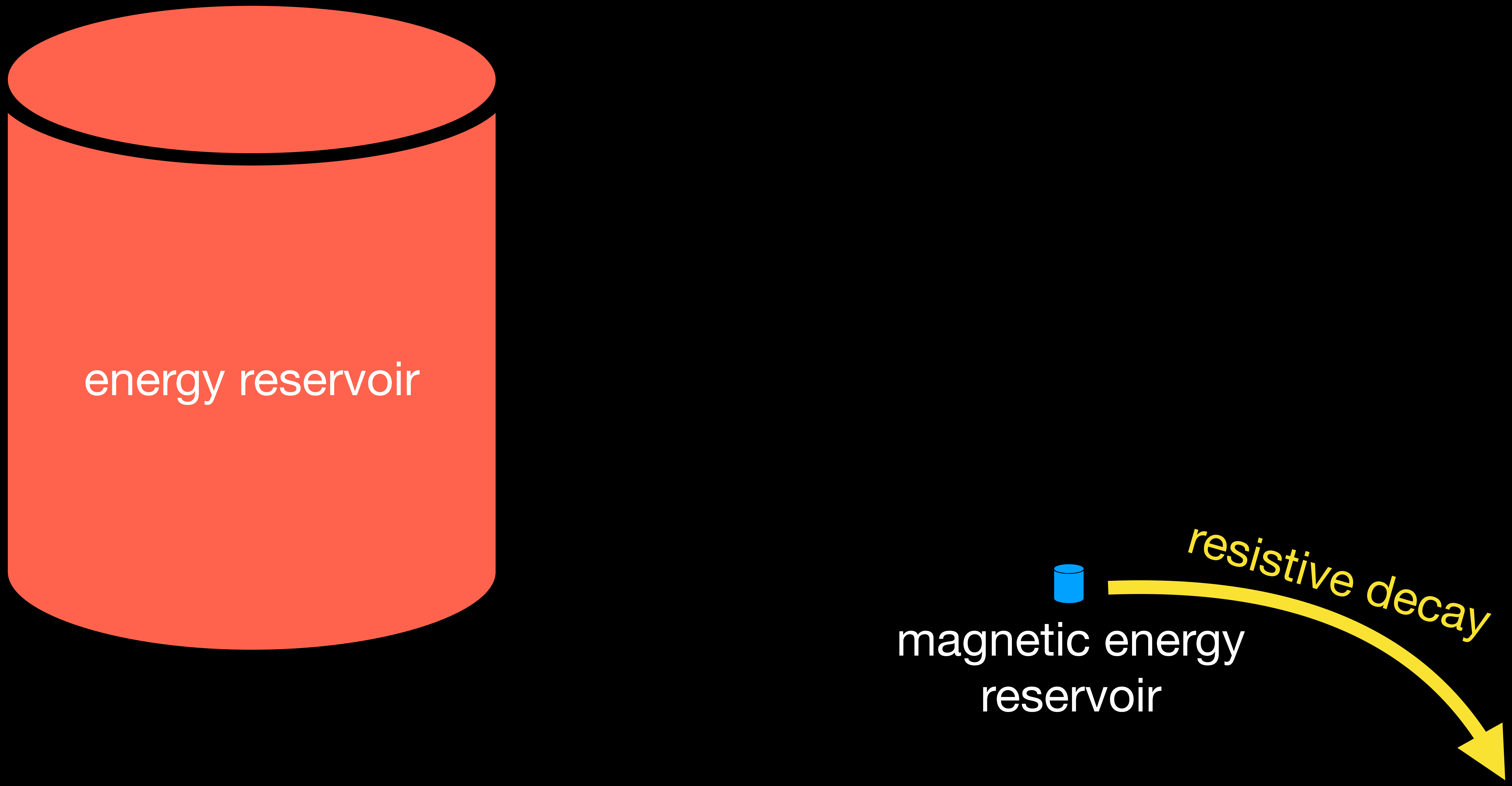


The lifecycle of a high-k mode spans across a patchwork quilt of theories



What is a magnetic dynamo?

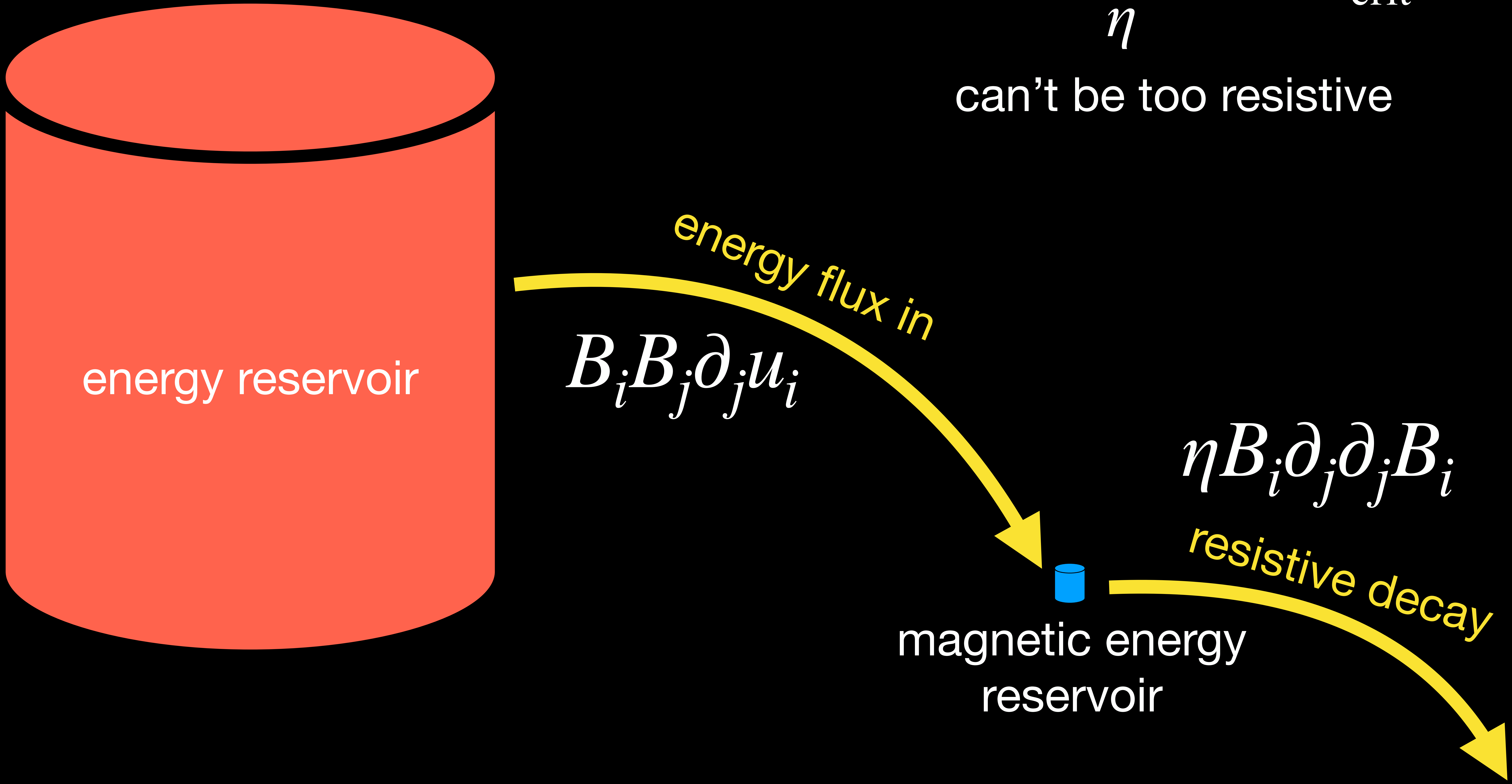
Starting with a seed magnetic field



What is a magnetic dynamo?
Growth

$$\mathrm{Rm} \sim \frac{U_0 L}{\eta} > \mathrm{Rm}_{\mathrm{crit}}$$

can't be too resistive



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Growth

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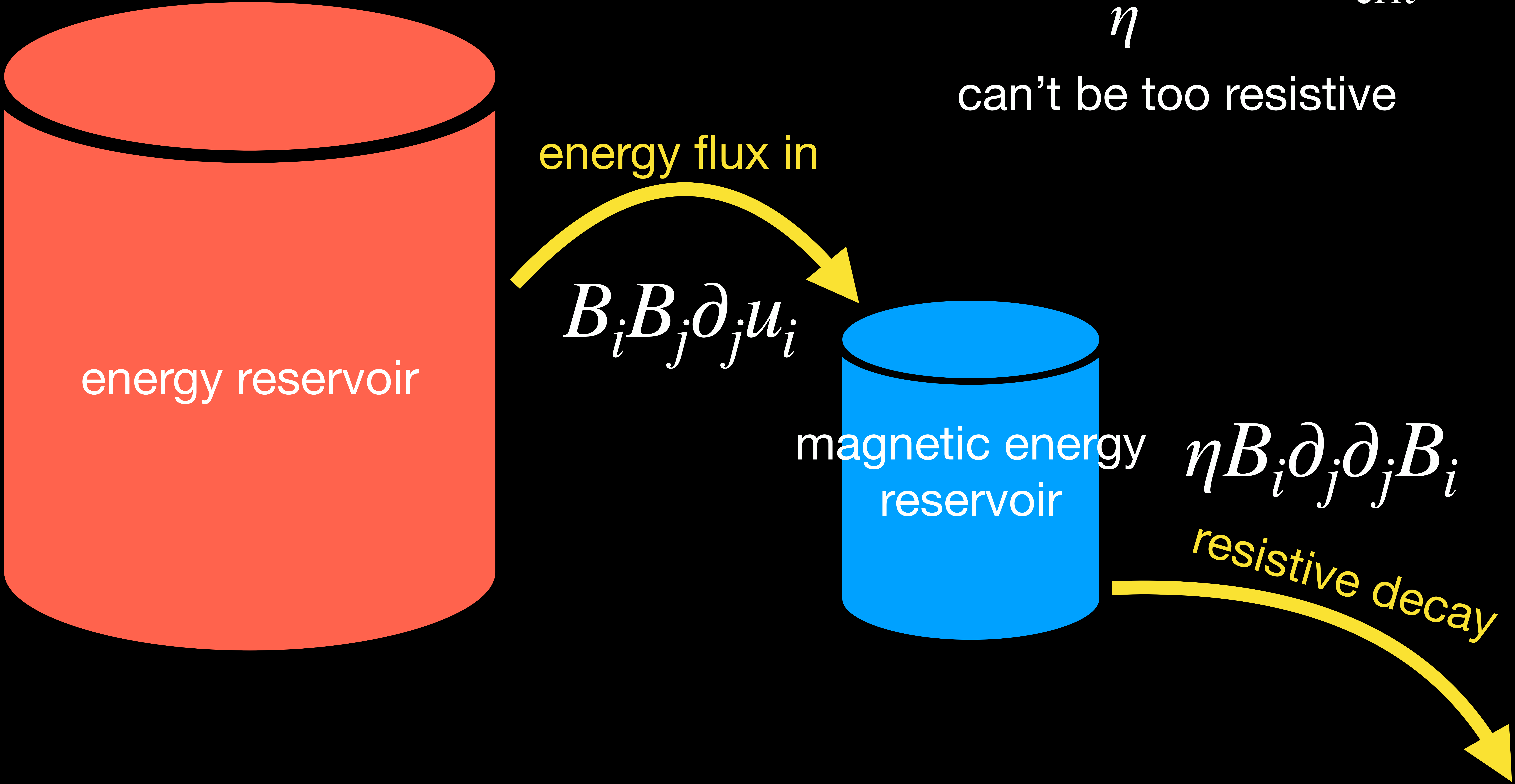
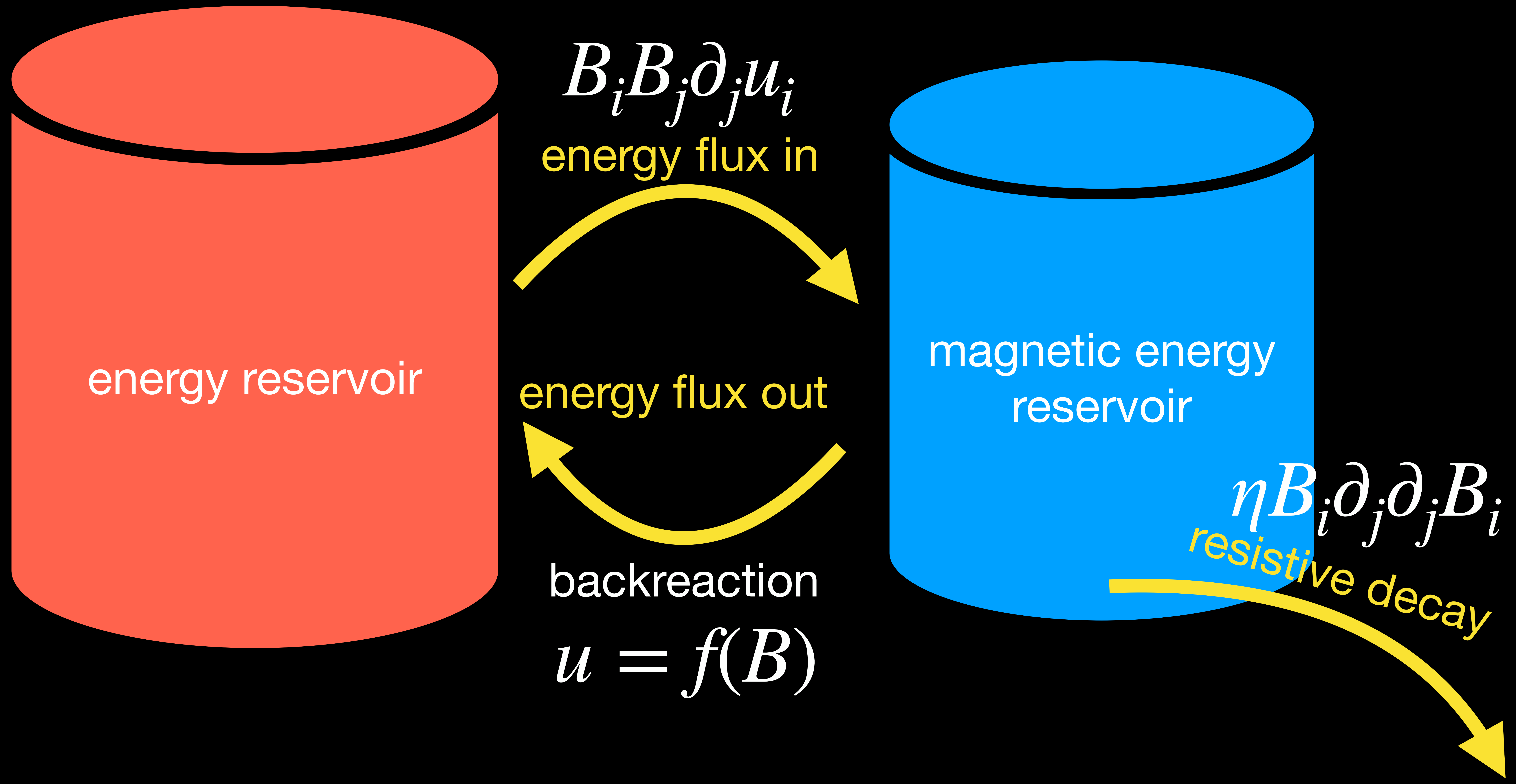


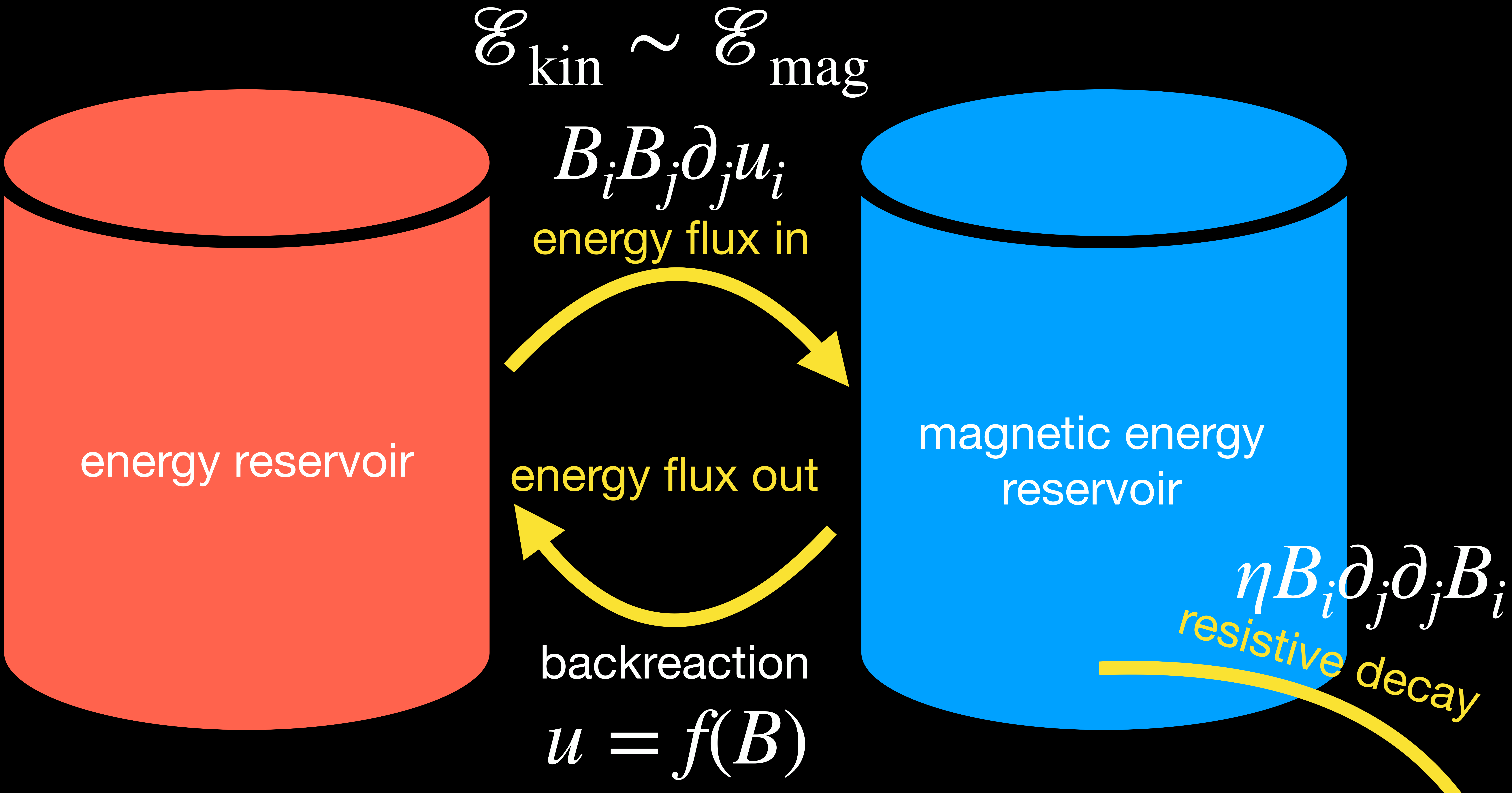
What is a magnetic dynamo?

Nonlinearities and backreaction



What is a magnetic dynamo?

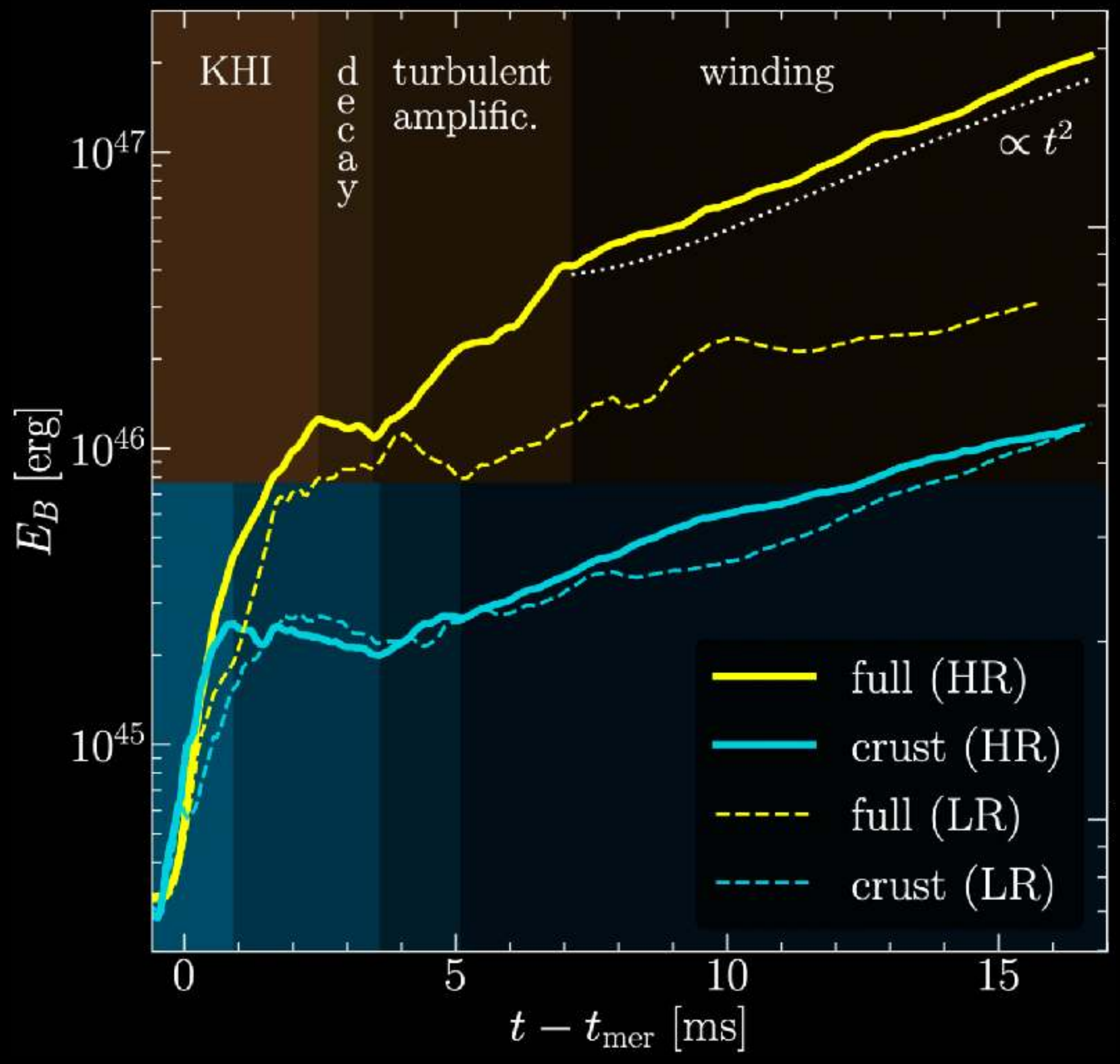
Saturation

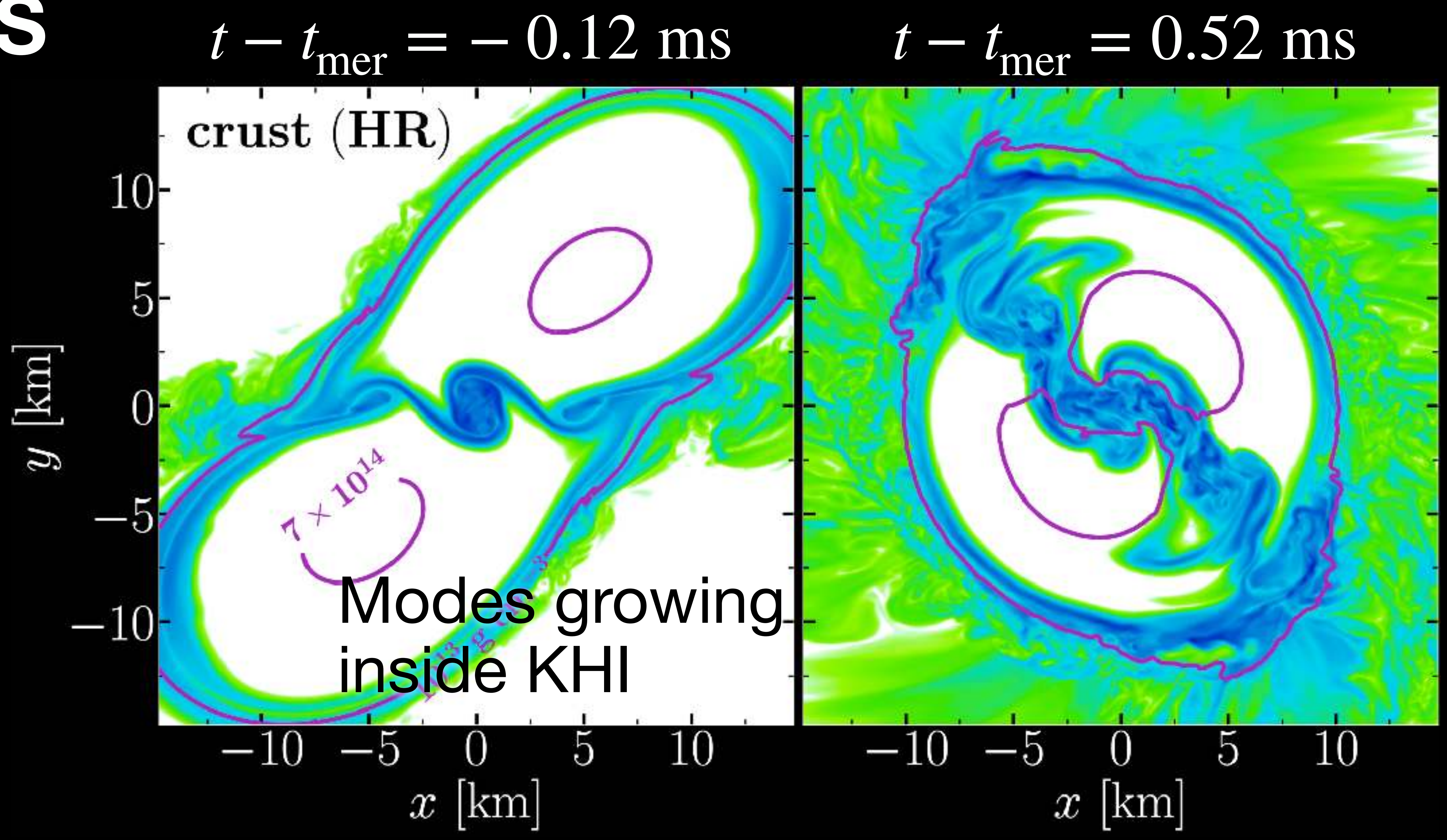


energy flux in = energy flux out + resistive decay

Examples of small scale dynamos

KHI instabilities in merging NS





exponential growth → pause

exponential growth → LSD

Chabanov+ (2023) *ILES* *FIL*

Examples of small scale dynamos

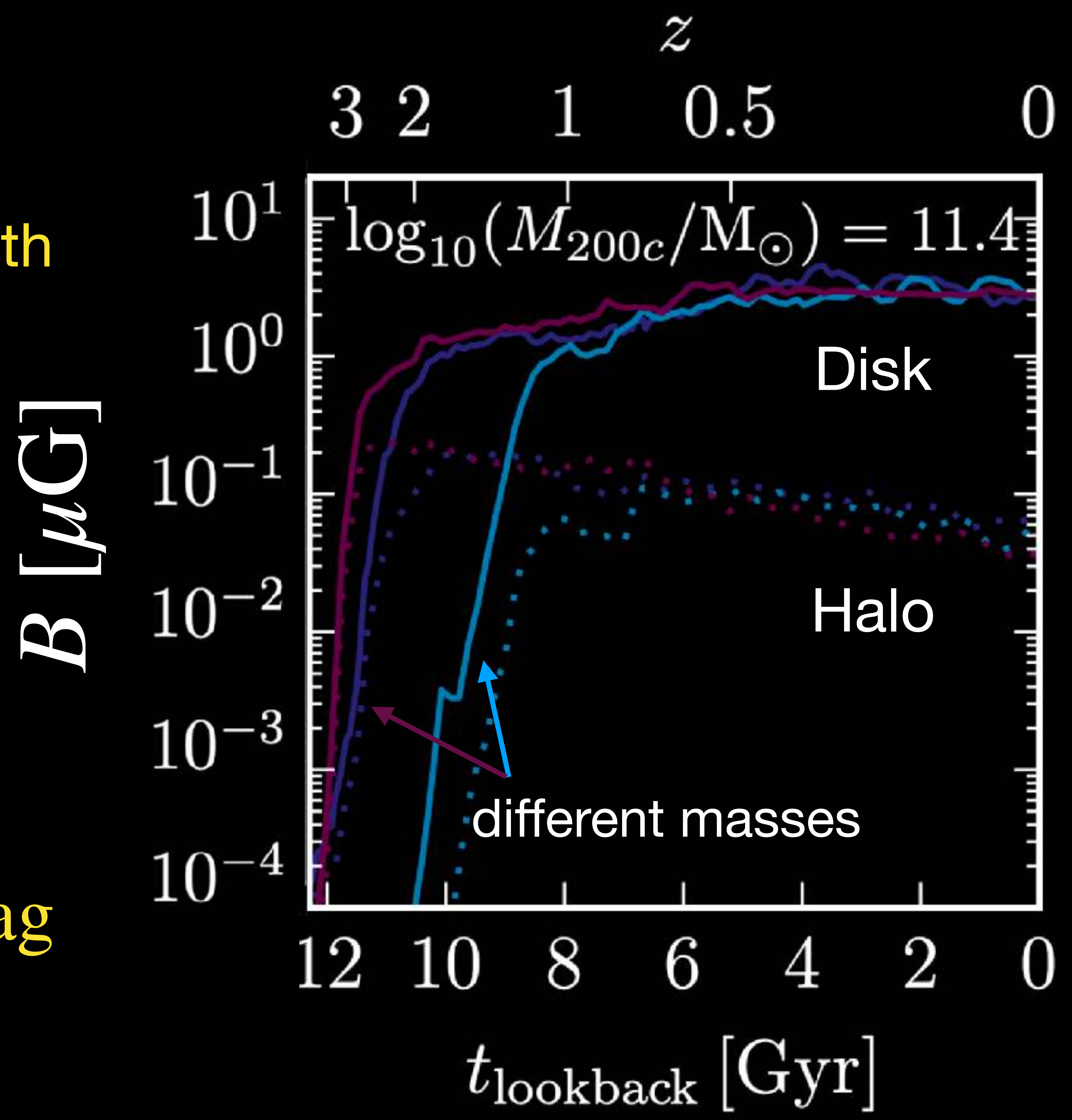
Milky Way-type galaxies in cosmological sims

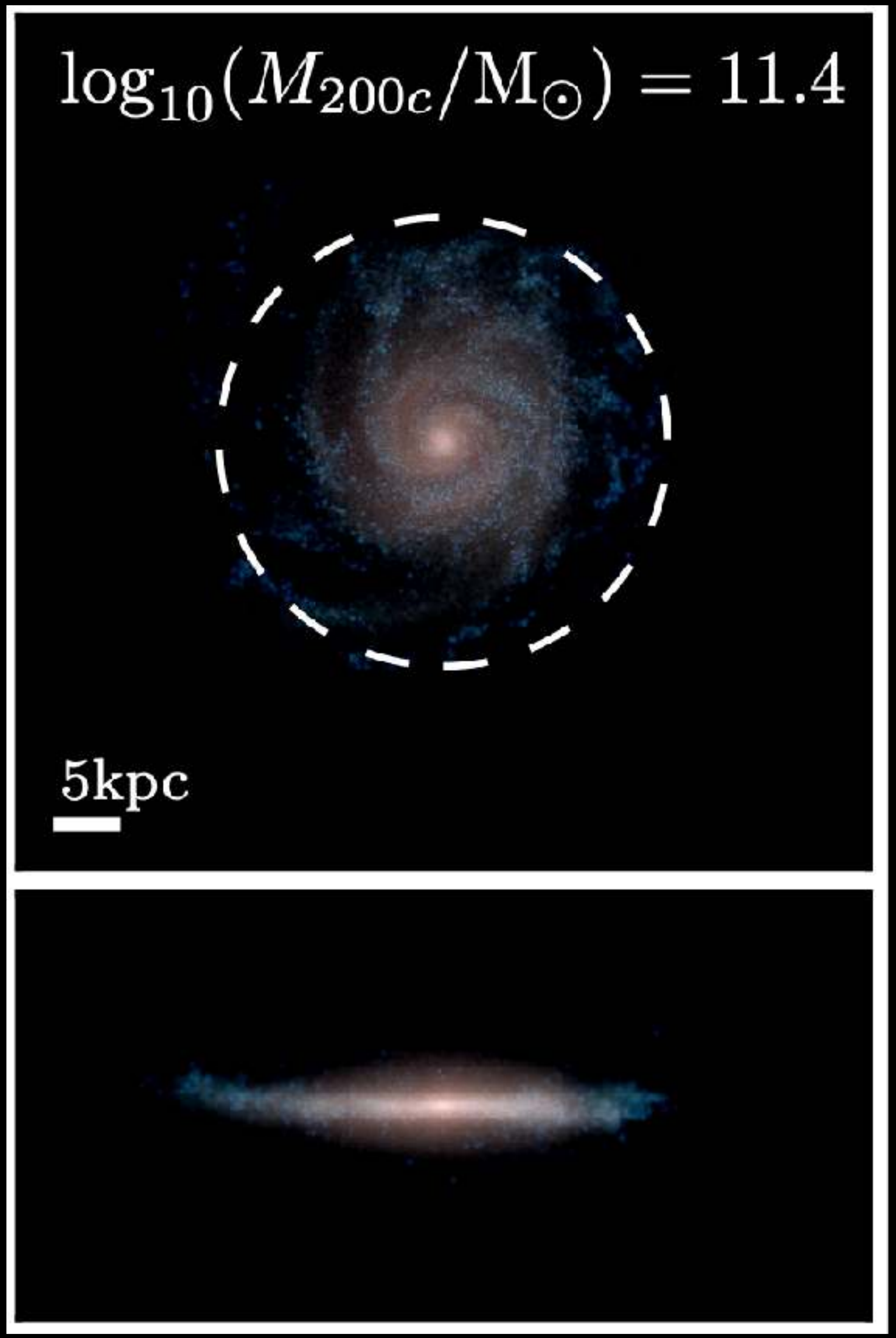
exponential growth

linear growth

saturation

$\mathscr{E}_{\mathrm{kin}} \sim \mathscr{E}_{\mathrm{mag}}$



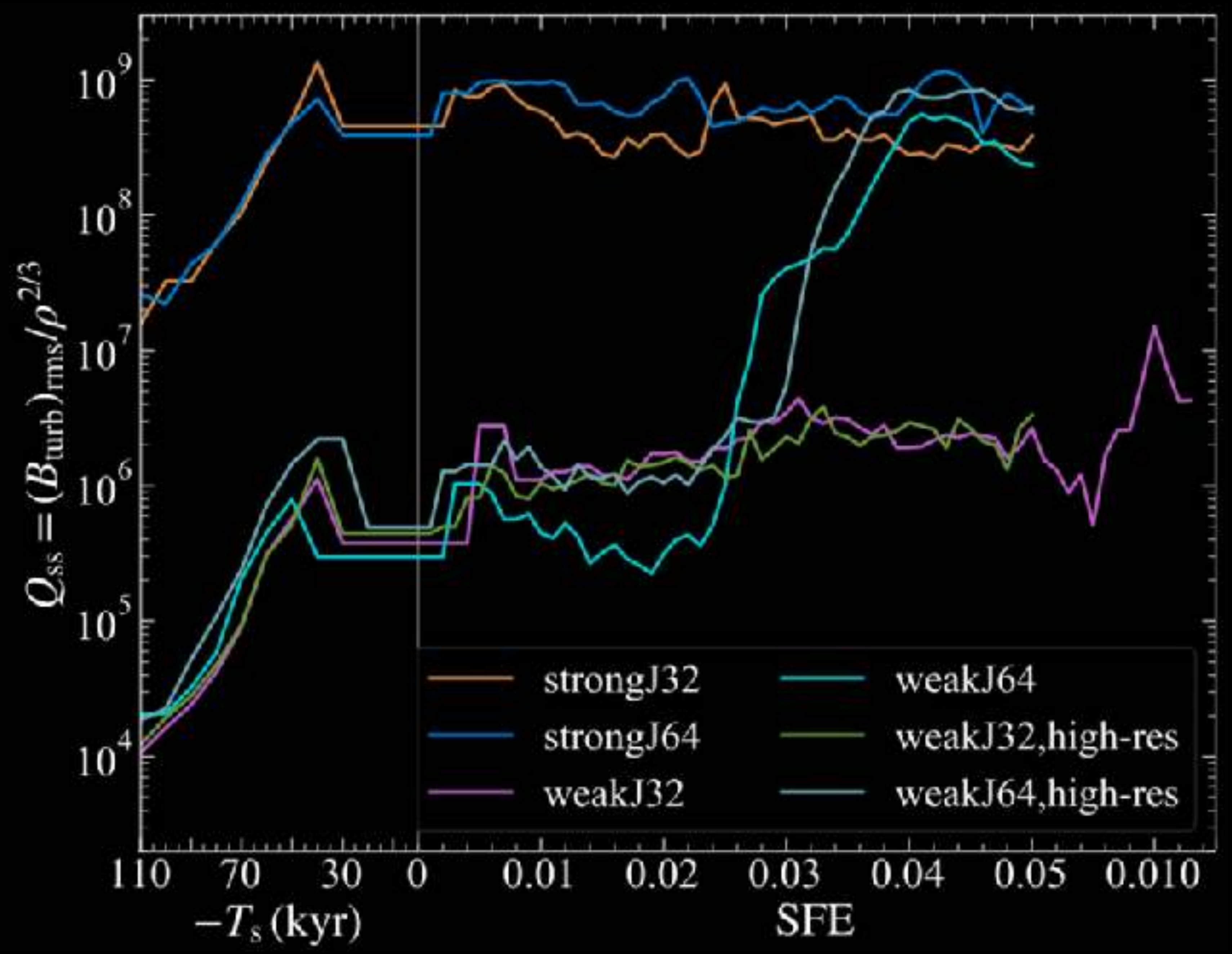


Pakmor+ (2024) *AREPO*

Examples of small scale dynamos

There are many, across all scales (all MHD)

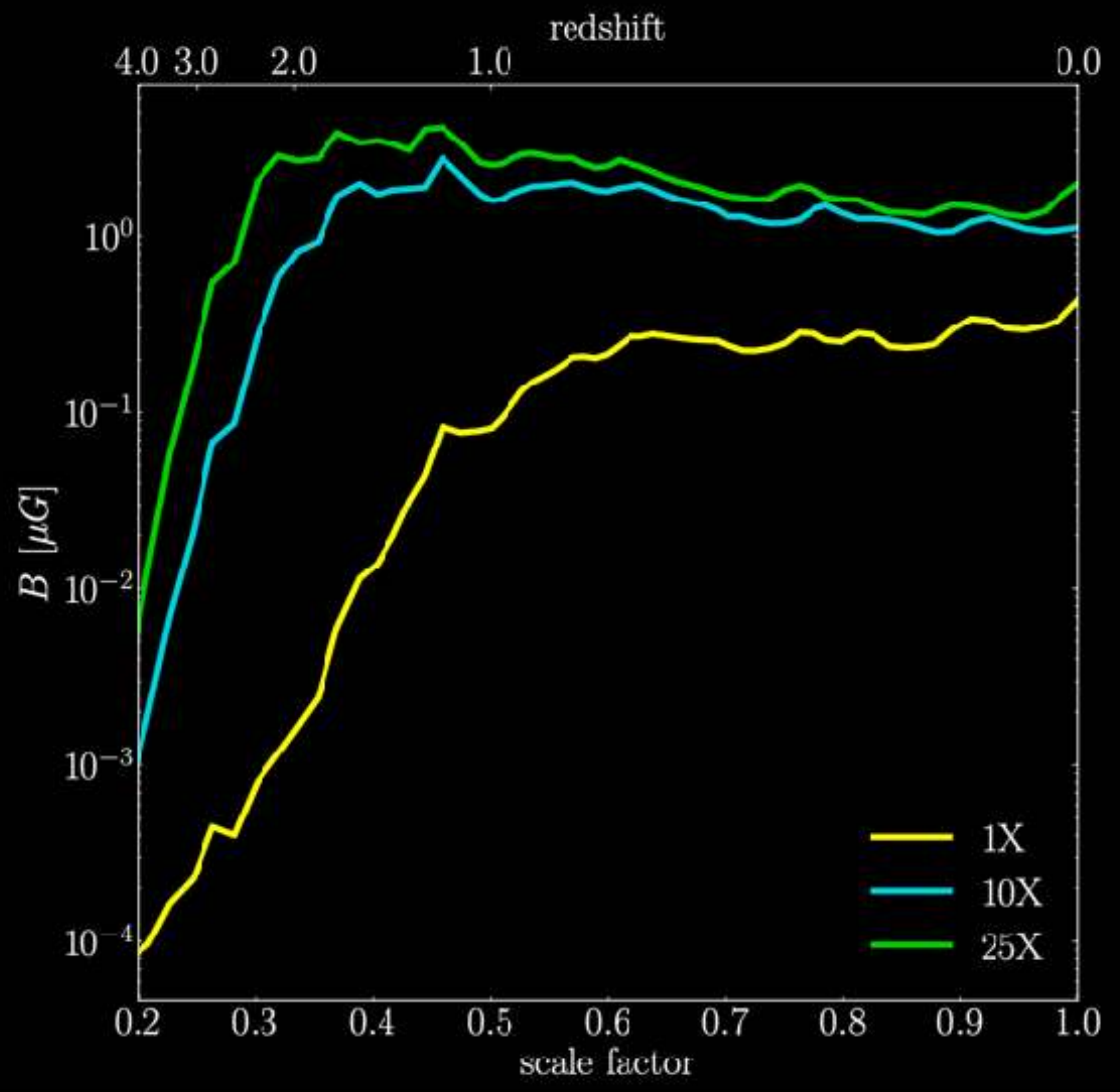
Sharda+2021



Molecular clouds in first generation stars

FLASH

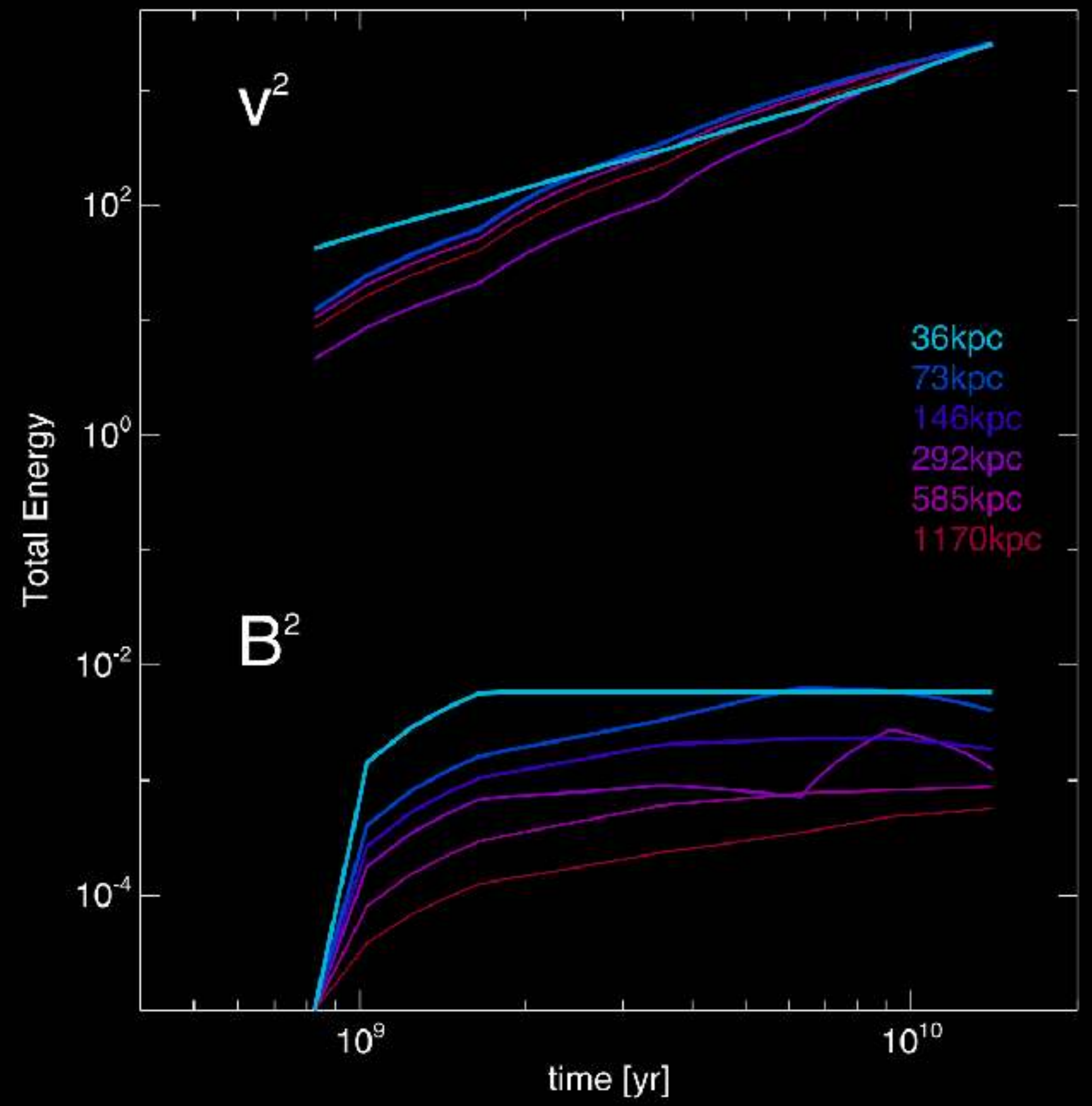
Steinwandel+2021



Intracluster medium

RAMSES

Vazza+2014



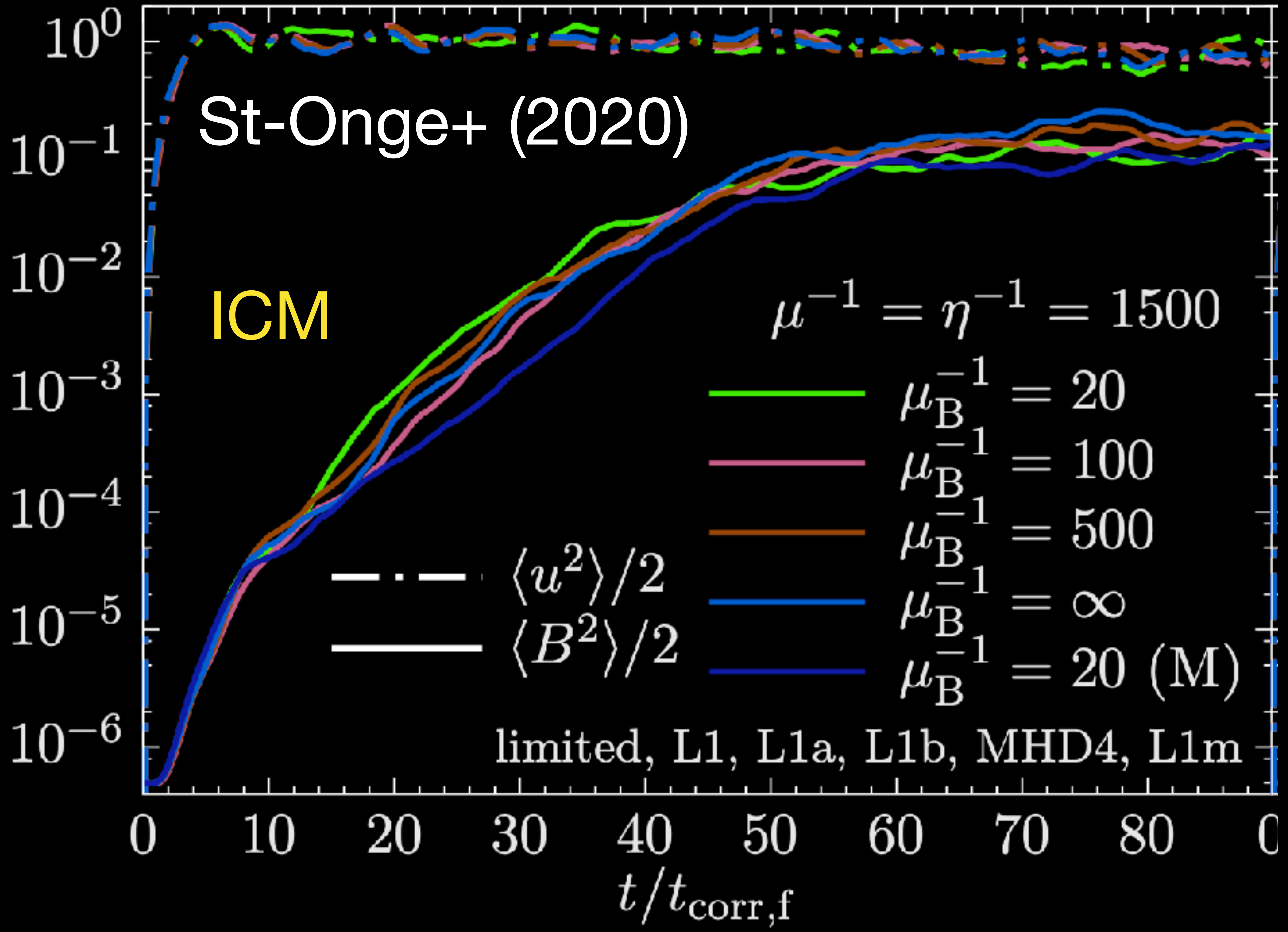
Cosmic filaments

ENZO

Examples of small scale dynamos

and plasma regimes

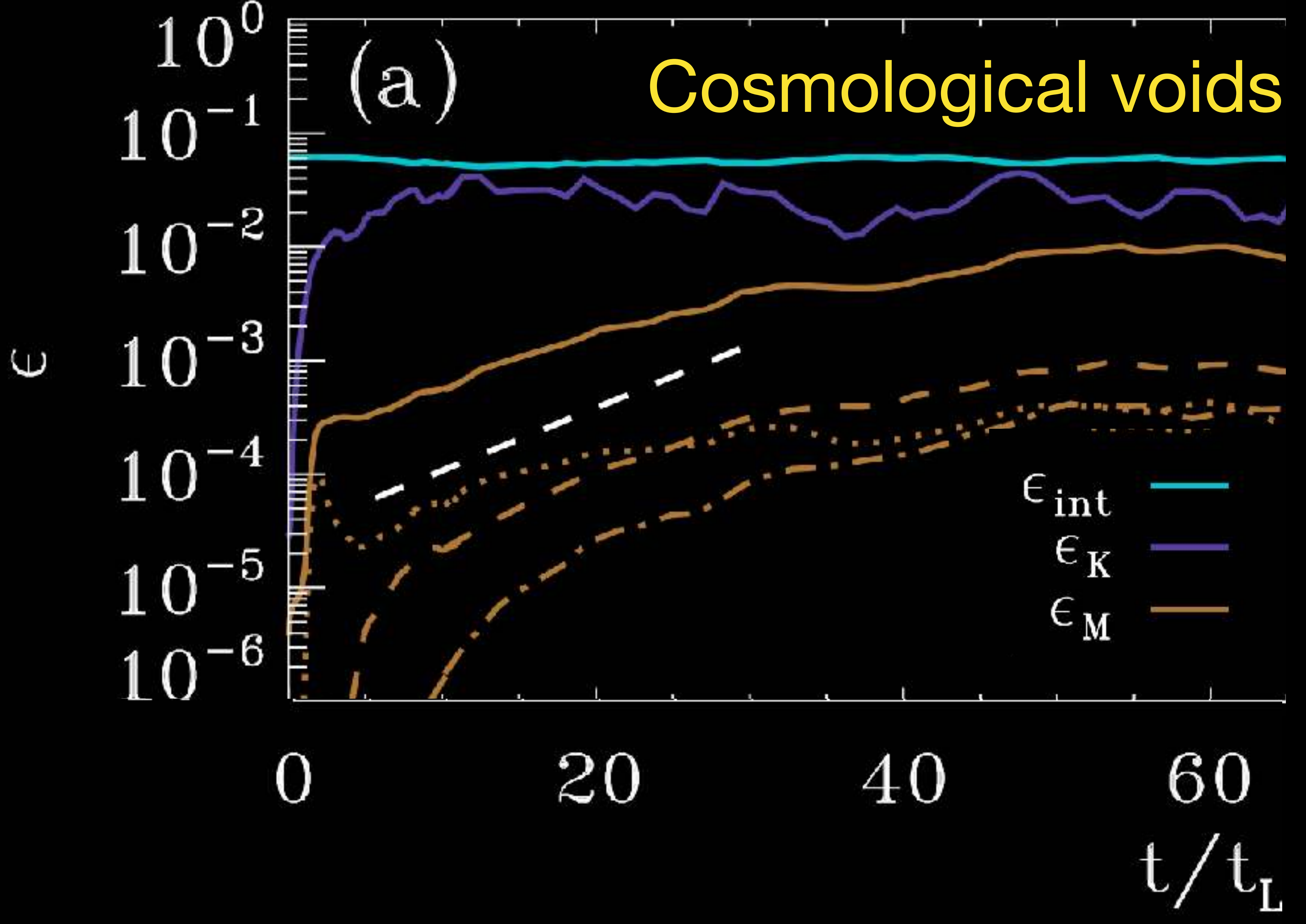
Weakly collisional Braginksii MHD



(added anisotropic viscous Braginskii stress term into MHD)

$\nabla \cdot (\hat{\mathbf{b}} \otimes \hat{\mathbf{b}}(\hat{\mathbf{b}} \otimes \hat{\mathbf{b}} : \nabla \mathbf{v}))$ *Snoopy*

Collisionless plasma magnetogenesis coupled to dynamo



Sironi+2023

(PIC: pair plasma)

TRISTAN—MP

Small scale dynamos

Ubiquitous across multiple phenomena / regimes

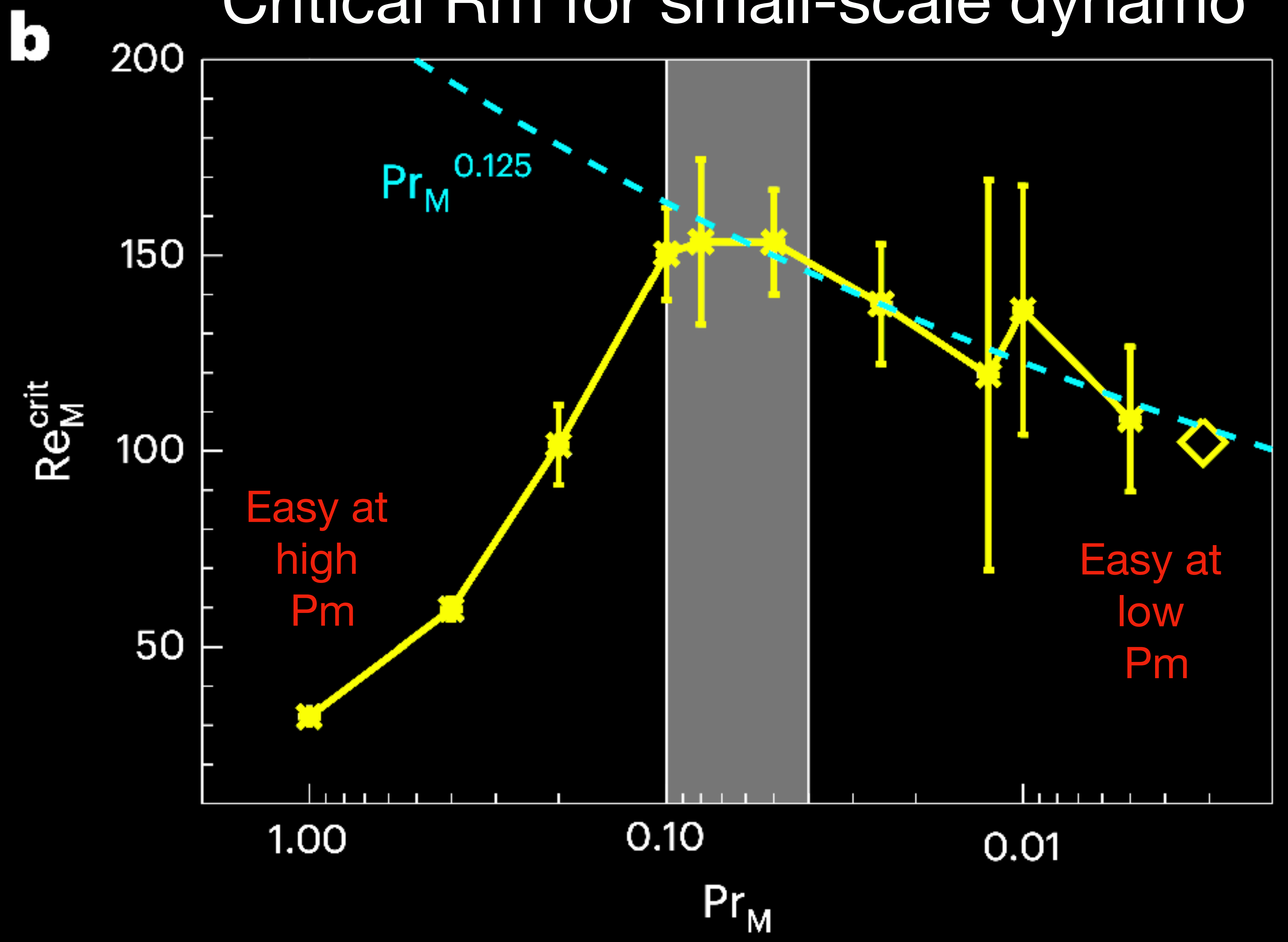
Extremely simple ingredients

1. $\nabla \otimes \mathbf{u}$

(dynamo is $B_j B_i \partial_i u_j$)

2. $\mathrm{Rm} = \dfrac{U_0 L}{\eta} \gtrsim 100$

Critical Rm for small-scale dynamo



Warnecke+2023. *Numerical evidence for a small-scale dynamo approaching solar magnetic Prandtl numbers.*

Simulations in this talk: 100s of no net-flux boxes

- 400 or so highly-modified version of finite volume code *FLASH*, second-order in space approximate Riemann (PPM) solver with framework outlined in Bouchut+ (2010), tested in *FLASH* in Waagen+ (2011).

- Compressible non-helical, visco/resistive MHD turbulence driven with finite correlation time OU process on $L/2$.

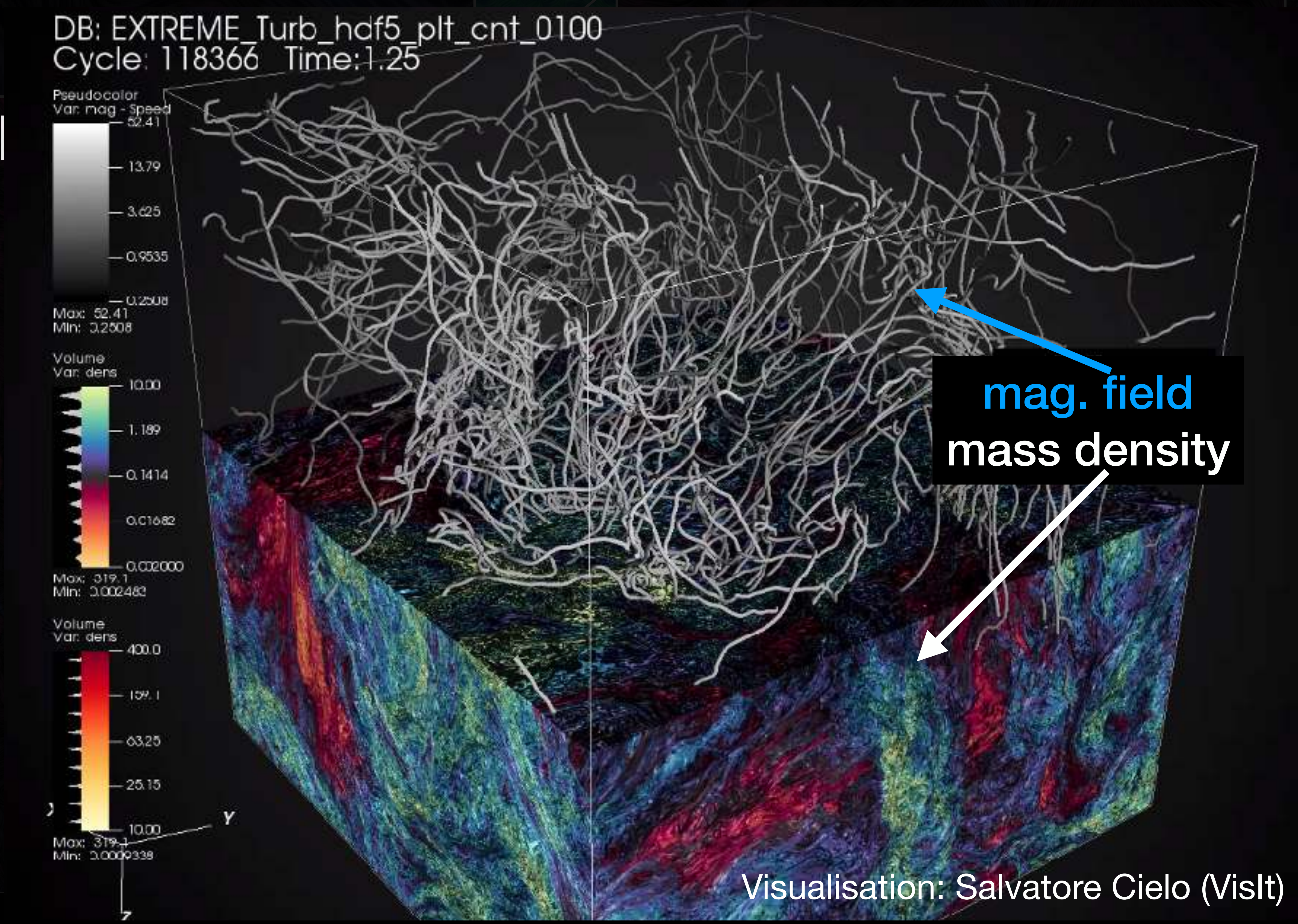
- No net magnetic flux. Pure turbulent magnetic fiel

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$d_t(\rho \mathbf{u}) + \nabla \cdot \mathbb{F} = \frac{1}{\mathrm{Re}} \nabla \cdot \sigma_{\text{viscous}} + \rho \mathbf{f}$$

$$\partial_t \mathbf{b} = \nabla \times (\mathbf{u} \times \mathbf{b}) + \frac{1}{\mathrm{Rm}} \nabla^2 \mathbf{b}$$

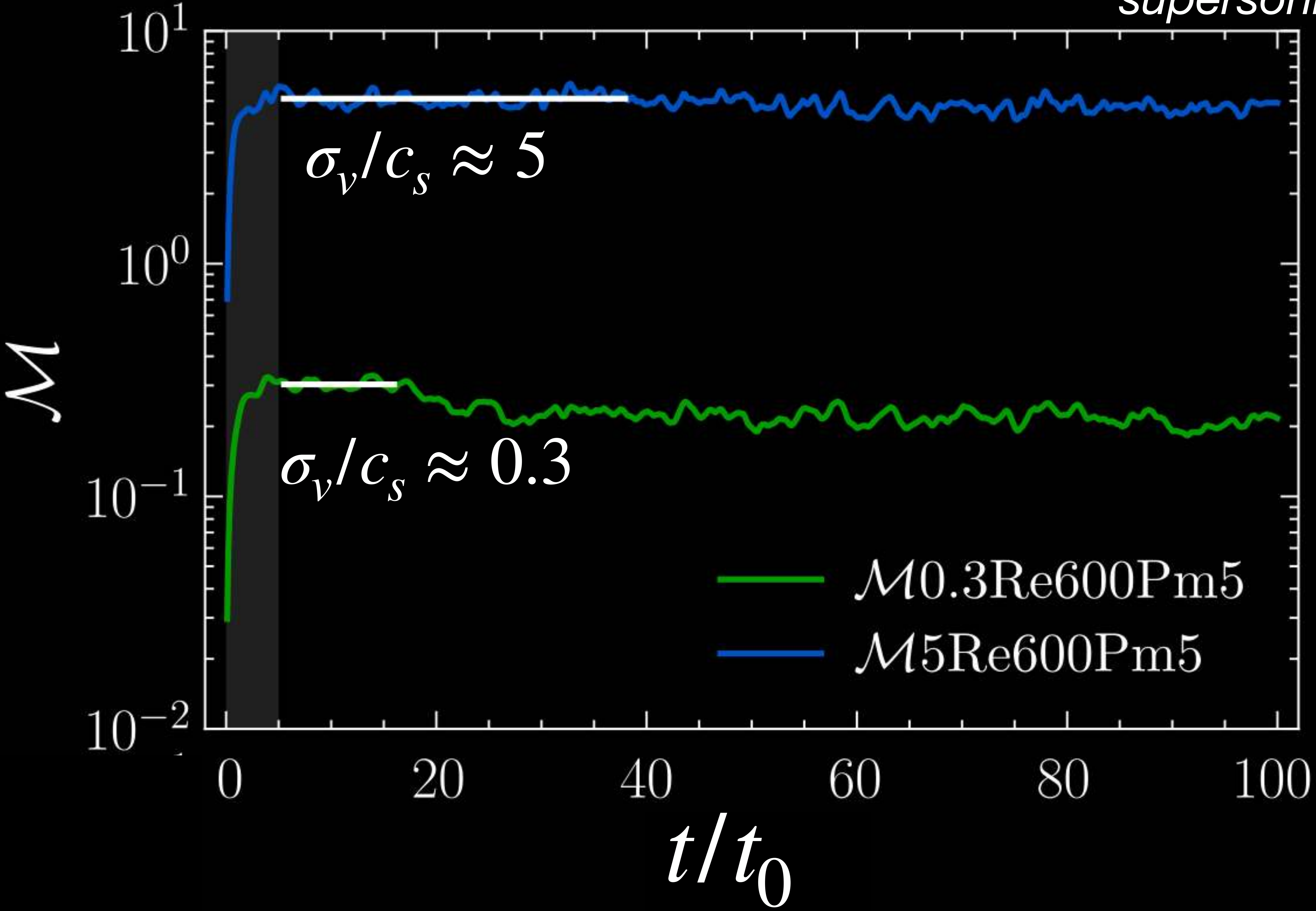
$$\nabla \cdot \mathbf{b} = 0$$



Visualisation: Salvatore Cielo (VisIt)

Simulations in this talk

Kriel, **Beattie**+ (2025). *Fundamental scales II: the kinematic stage of the supersonic dynamo*



$$\mathrm{Pm} = \frac{\nu}{\eta} \propto \frac{T^4}{n_e}$$

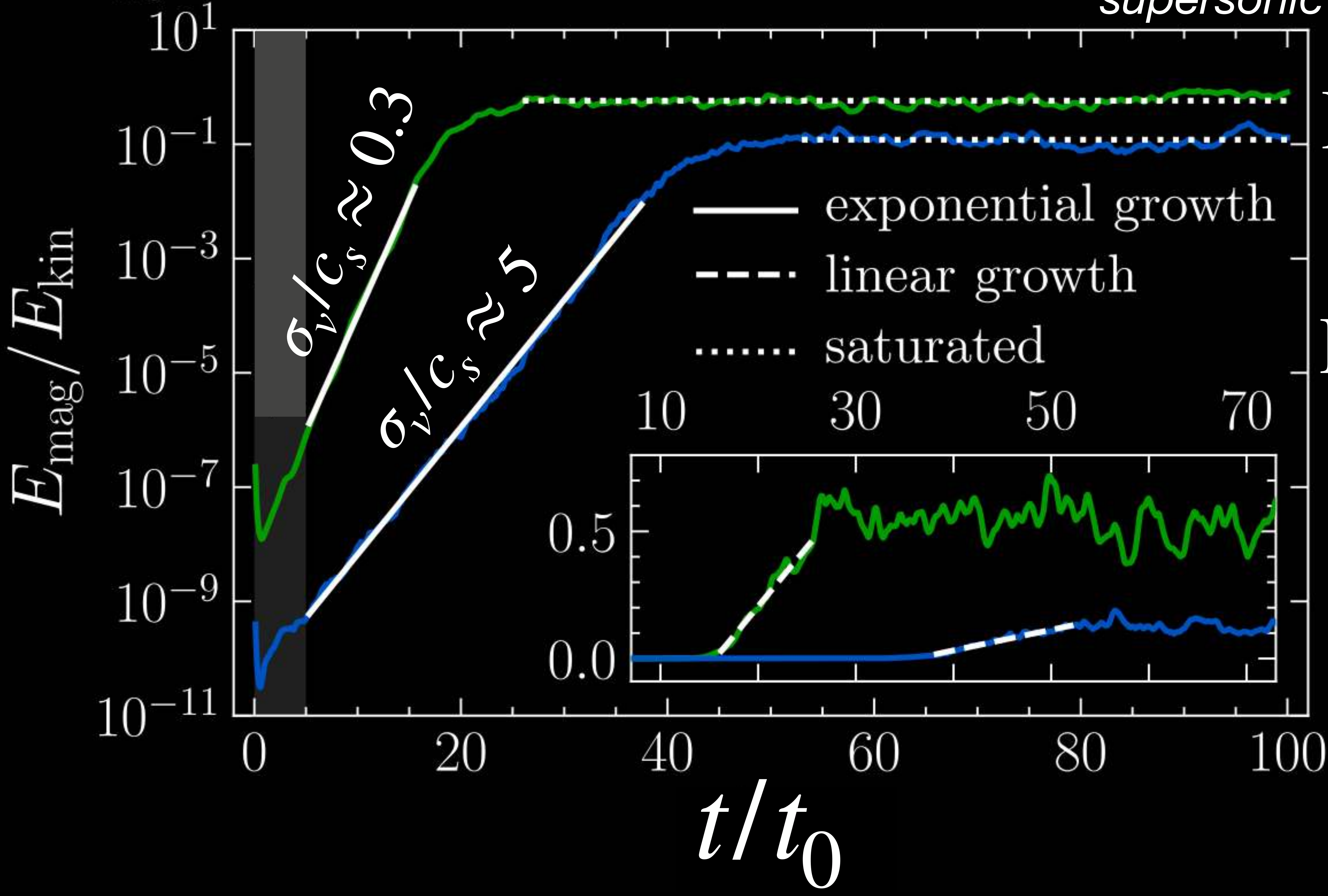
$$\mathrm{Rm} = \frac{U_0 L}{\eta}$$

$$\mathrm{Re} = \frac{U_0 L}{\nu}$$

$$\mathscr{M} = \frac{\langle u^2 \rangle^{1/2}}{\langle c_s \rangle}$$

Simulations in this talk

Kriel, **Beattie**+ (2025). *Fundamental scales II: the kinematic stage of the supersonic dynamo*



$$\mathrm{Pm} = \frac{\nu}{\eta} \propto \frac{\mathrm{T}^4}{\mathrm{n_e}}$$

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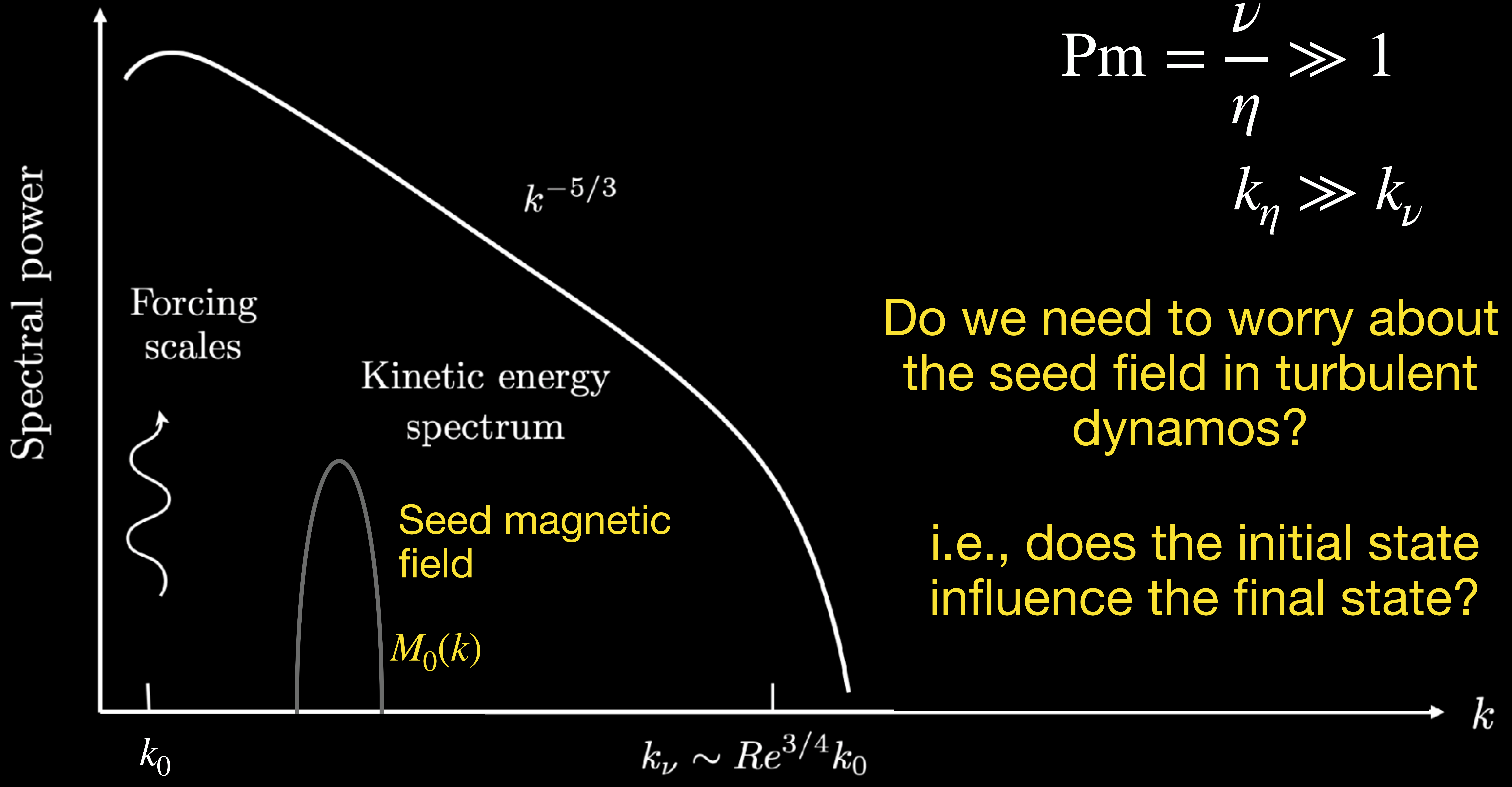
$$\mathscr{M} = \frac{\langle u^2 \rangle^{1/2}}{\langle c_s \rangle}$$

Small-scale dynamo

Modified from Rincon (2019)



$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

$$k_\eta \gg k_\nu$$

Do we need to worry about the seed field in turbulent dynamos?

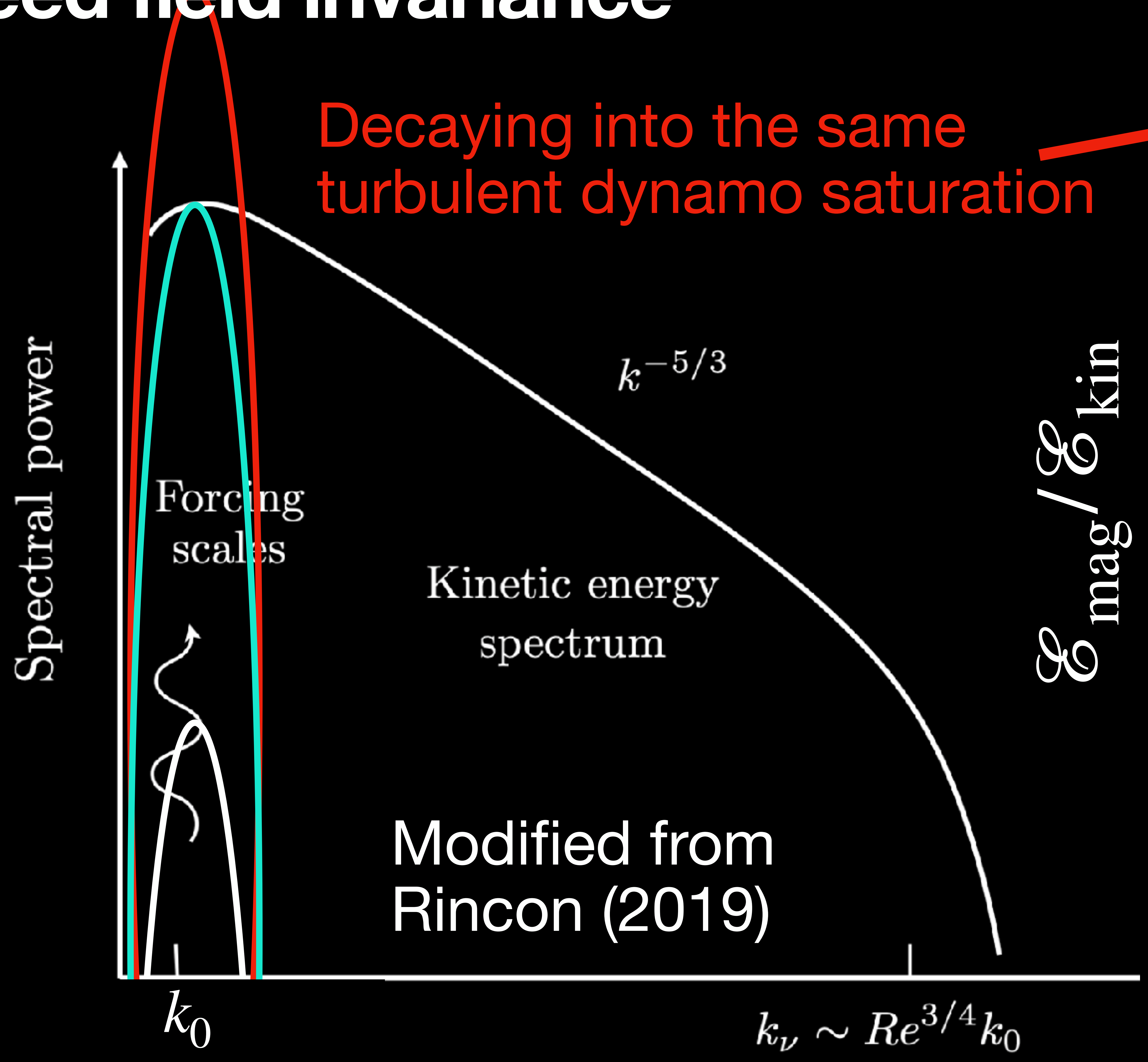
i.e., does the initial state influence the final state?

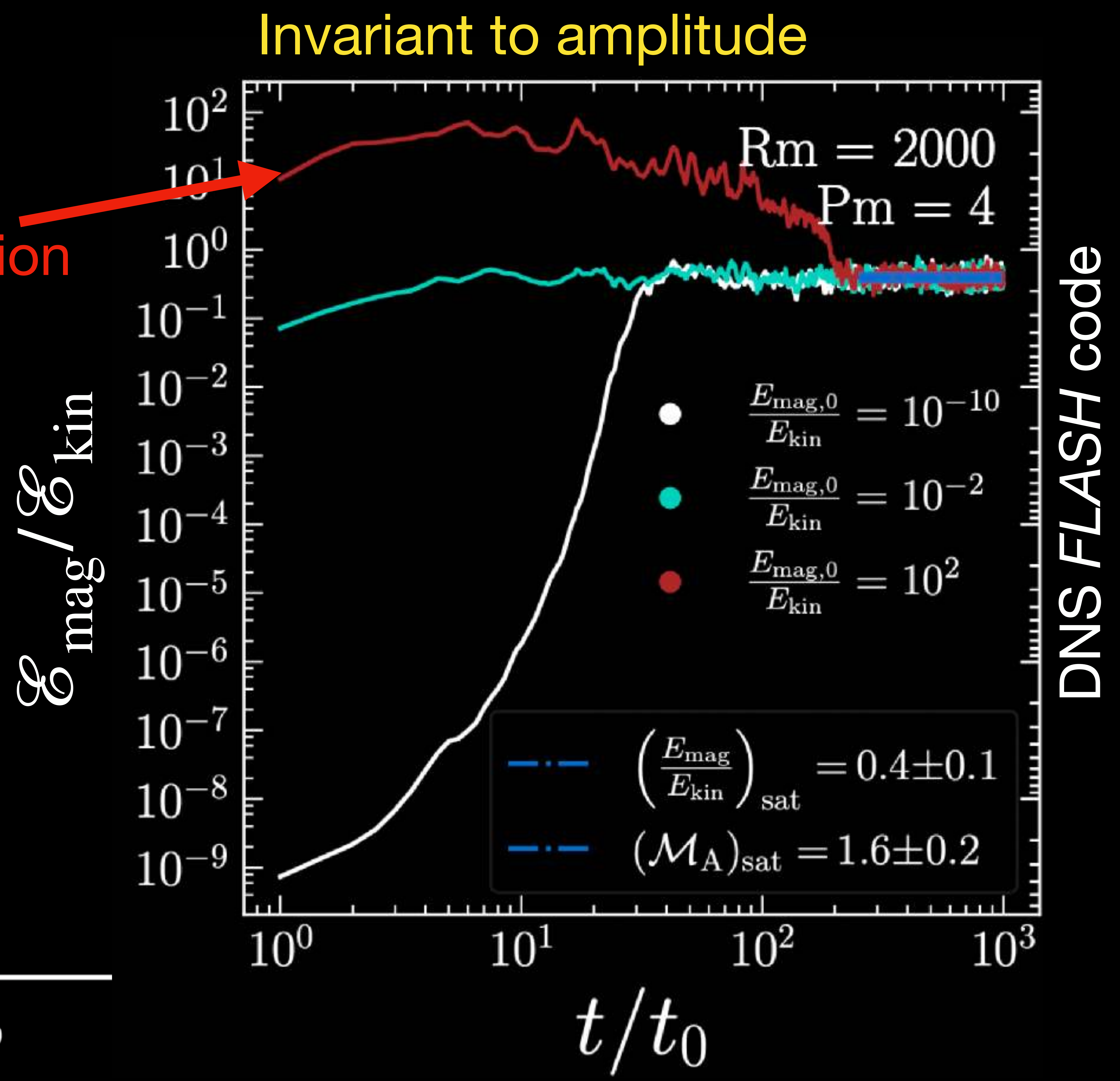
Small-scale dynamo

Seed field invariance

Modified from Rincon (2019)

Small-scale dynamo
Seed field invariance





Beattie+ (2023). *Growth or Decay I: Universality of the turbulent dynamo saturation*

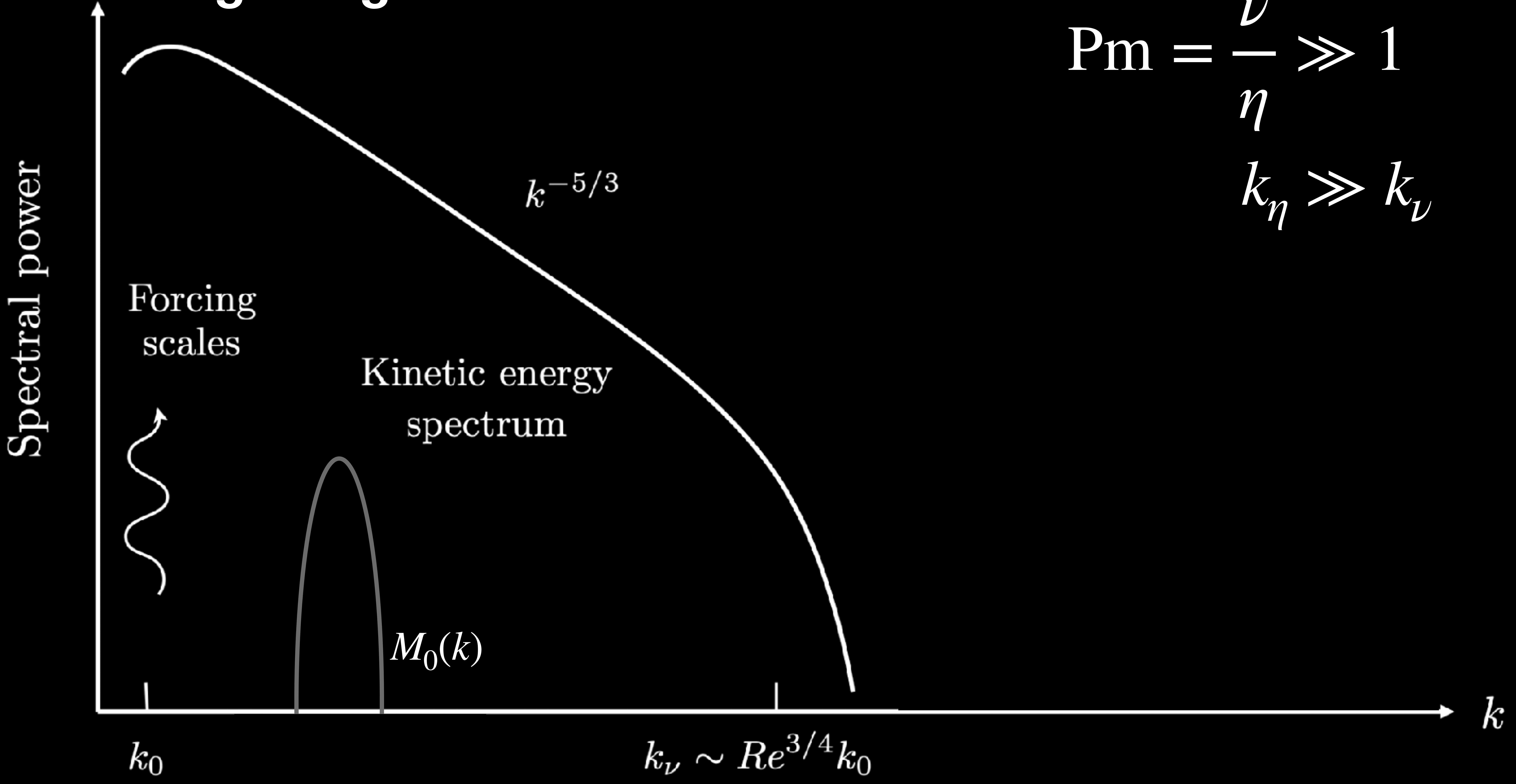
Small-scale dynamo

In the beginning…

Modified from Rincon (2019)

$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

$$k_\eta \gg k_\nu$$

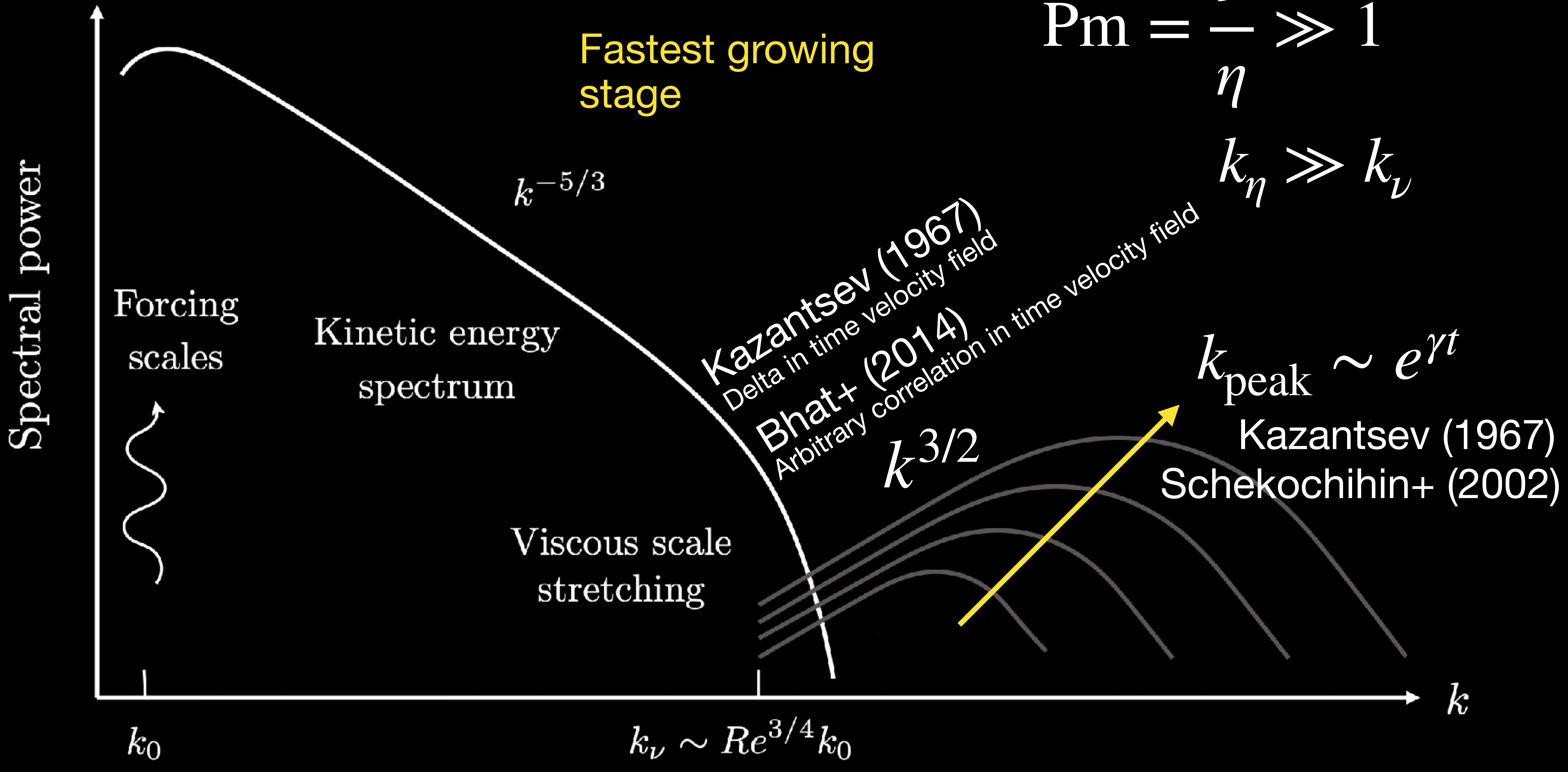


Small-scale dynamo
First growth stage: kinematic regime

Modified from Rincon (2019)

$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

$$k_\eta \gg k_\nu$$



Small-scale dynamo
First growth stage: kinematic regime

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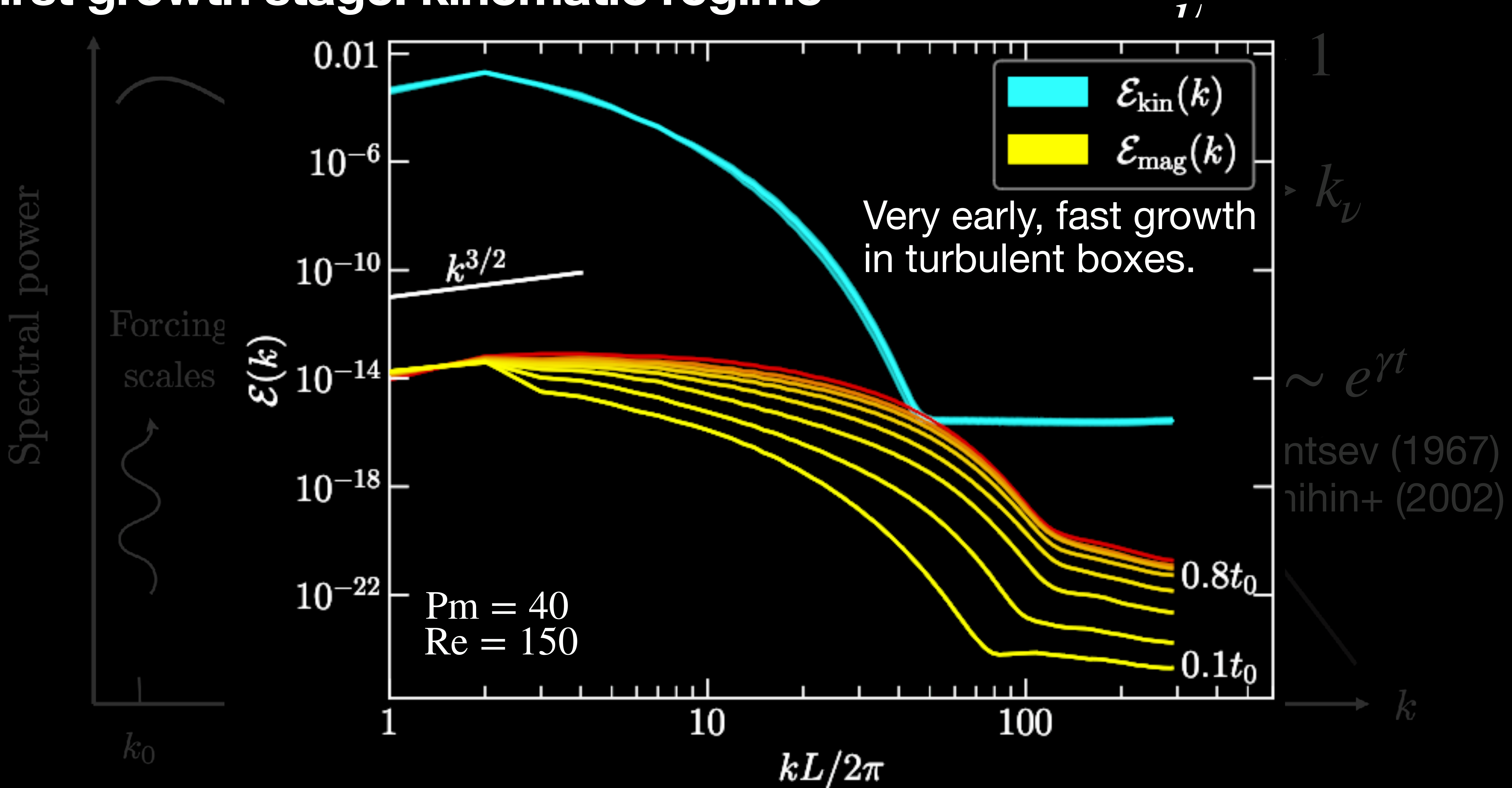
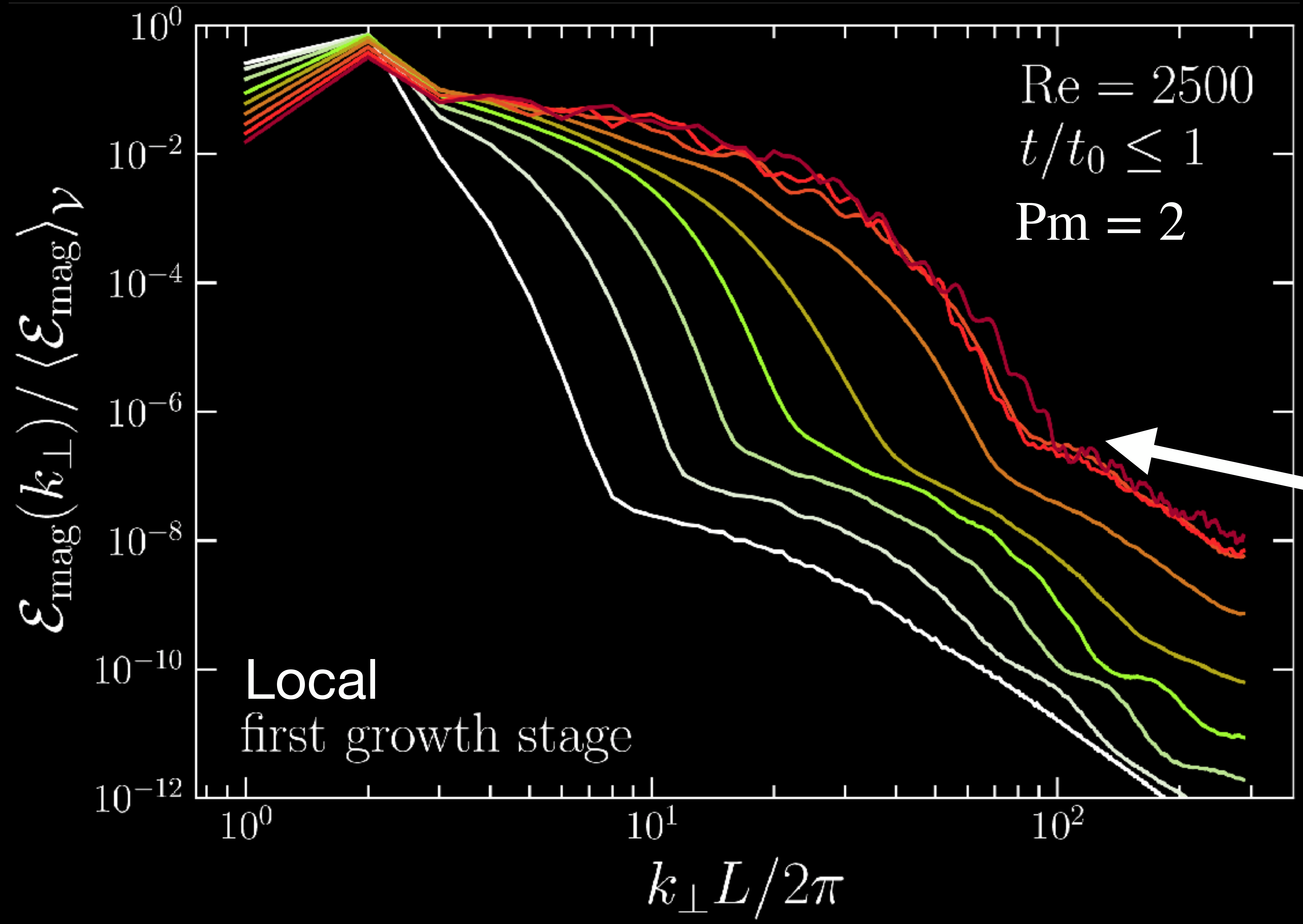
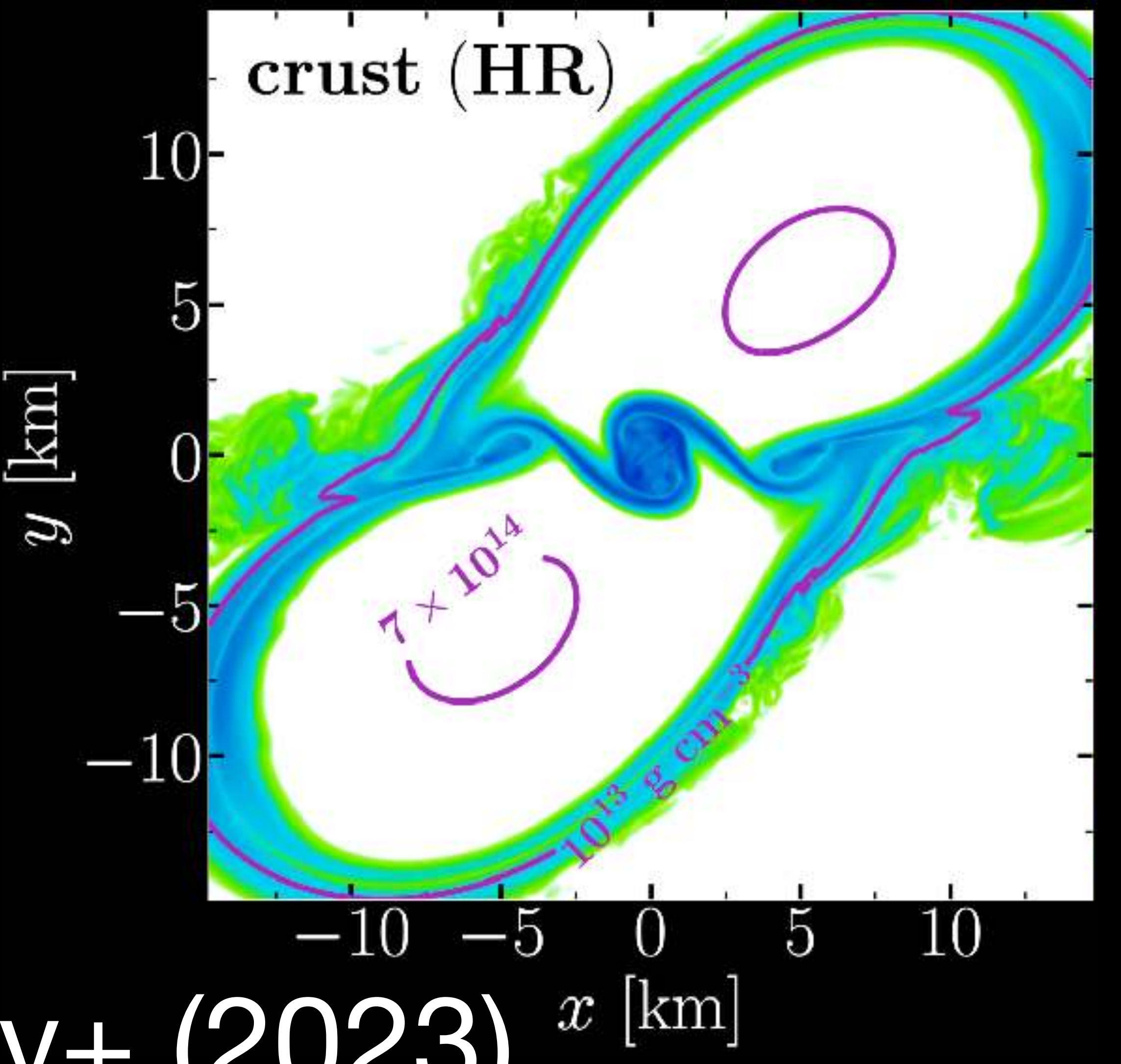
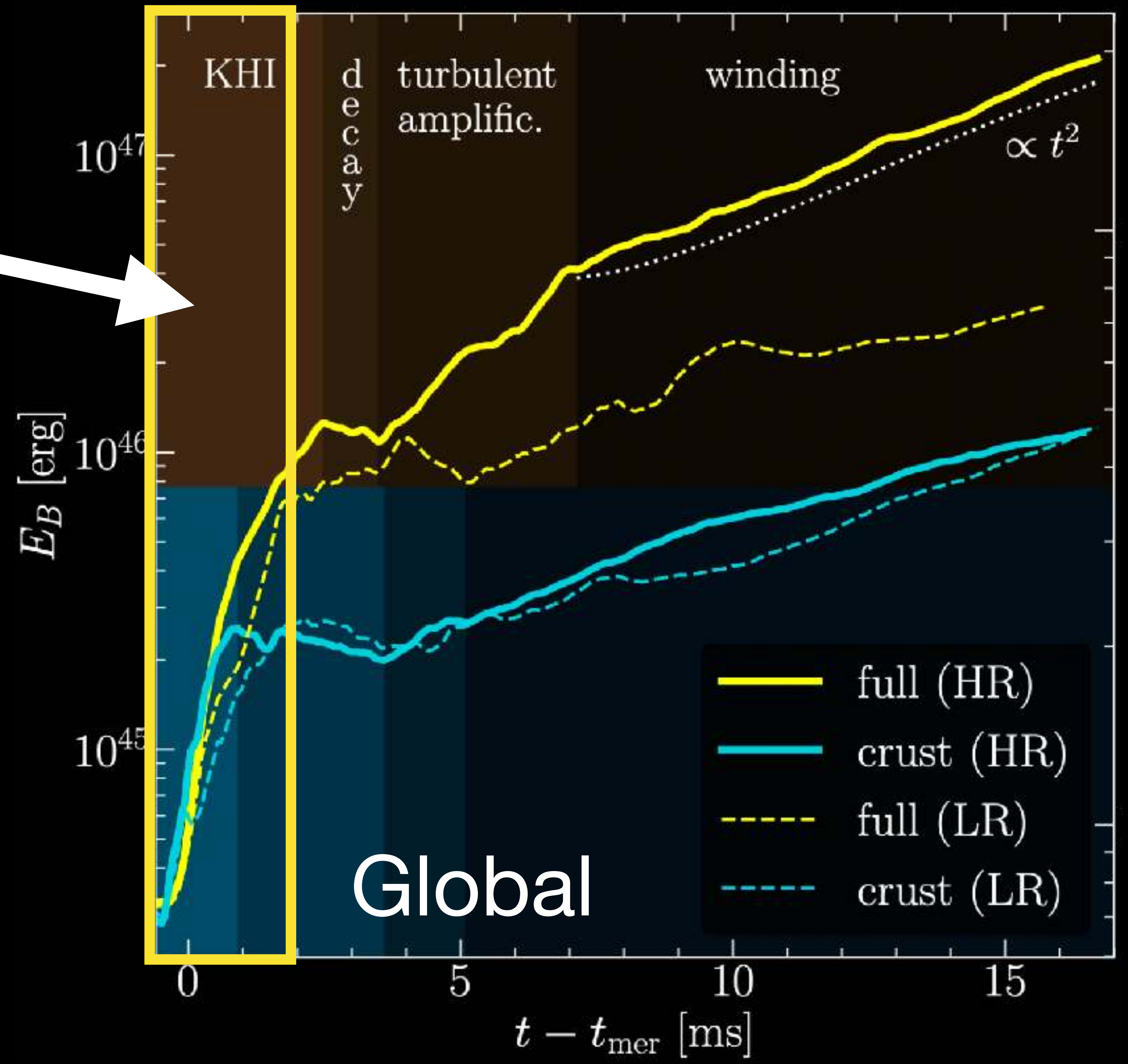


Small-scale dynamo
Local KHI dynamo simulations





Chabanov+ (2023)



Beattie+ *(in prep.) The chronology of the KHI dynamo. Implications for merging compact objects*

Small-scale dynamo
The engine of the kinematic dynamo

Growth rate dominated by the smallest possible scales of the flow gradients

$$\gamma = \langle \hat{\mathbf{b}} \otimes \hat{\mathbf{b}} : \nabla \otimes \mathbf{u} \rangle \sim u_\nu/\ell_\nu \sim 1/t_\nu,$$

$$u_\ell/\ell \sim \varepsilon^{1/3}\ell^{-2/3}, \quad t_\ell \sim \ell^{2/3}.$$

put in units of outer scale turnover time $t_0 = \ell_0/u_0$

$$t_0\gamma \sim t_0/t_\nu,$$

and to summarise,

K41 spectrum (incompresible)

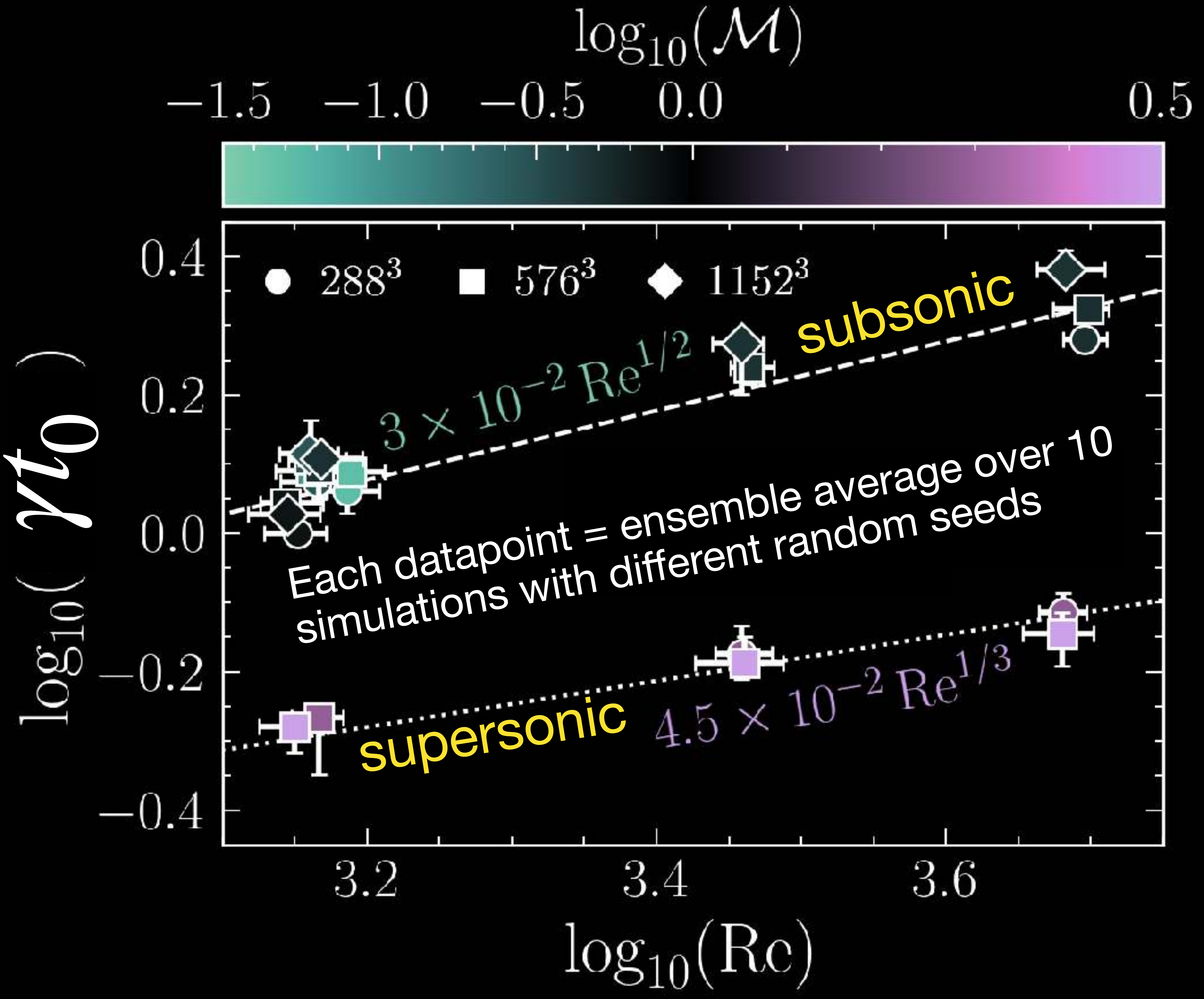
$$t_0\gamma \sim (\ell_0/\ell_\nu)^{2/3} \sim \left(\mathrm{Re}^{3/4}\right)^{2/3} \sim \mathrm{Re}^{1/2},$$

Burgers spectrum (supersonic)

$$t_0\gamma \sim (\ell_0/\ell_\nu)^{1/2} \sim \left(\mathrm{Re}^{2/3}\right)^{1/2} \sim \mathrm{Re}^{1/3}.$$

Small-scale dynamo

Confronting γ with data



The engine of the kinematic dynamo is k_ν

K41 spectrum (incompressible)

$$t_0\gamma \sim \mathrm{Re}^{1/2},$$

Burgers spectrum (supersonic)

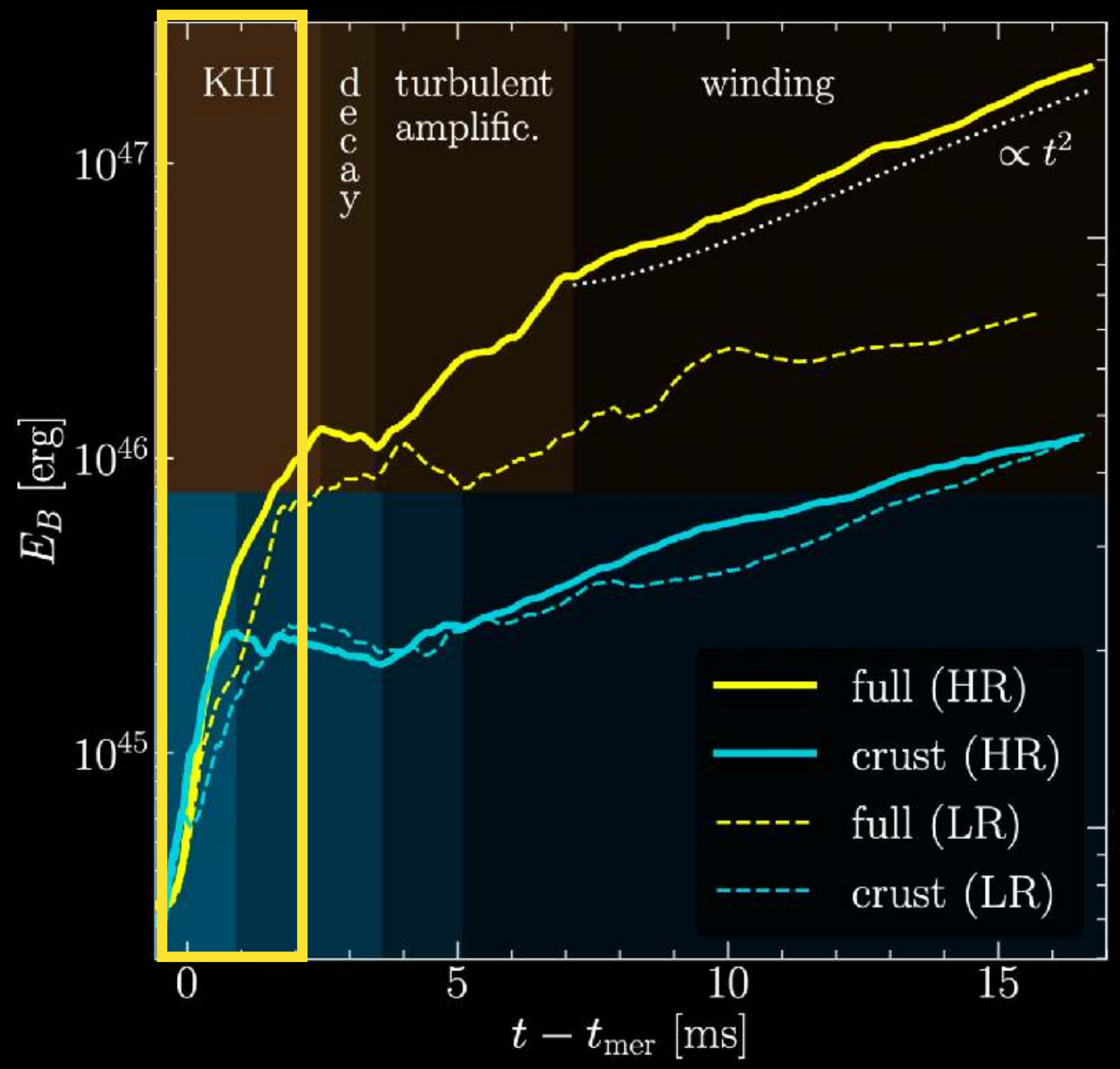
$$t_0\gamma \sim \mathrm{Re}^{1/3}.$$

Kriel, **Beattie**+ (in prep.). *Growth rate of magnetic energy doing the nonlinear small-scale dynamo*

Small-scale dynamo
Local KHI dynamo simulations

Most+(2024)

Chabanov+ (2023)



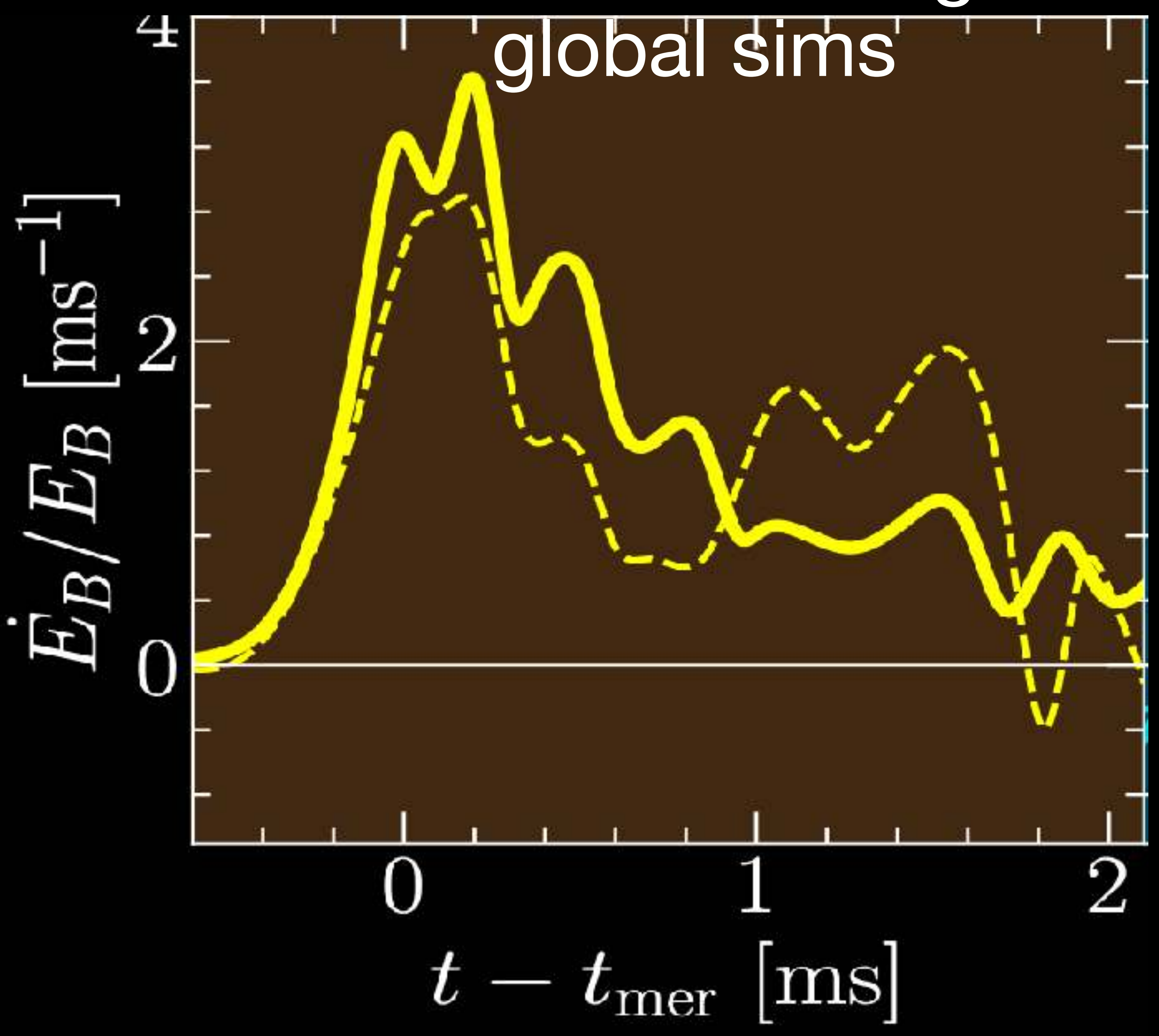
Weak interactions create a bulk viscosity

$$\mathrm{Re} \sim 10^{6+}$$

$$\Updownarrow$$

$$\gamma \sim 10^3\,\mathrm{ms}^{-1}$$

An extreme challenge for global sims



(bulk viscosity can act as an effective shear viscosity for the turbulent dynamo)

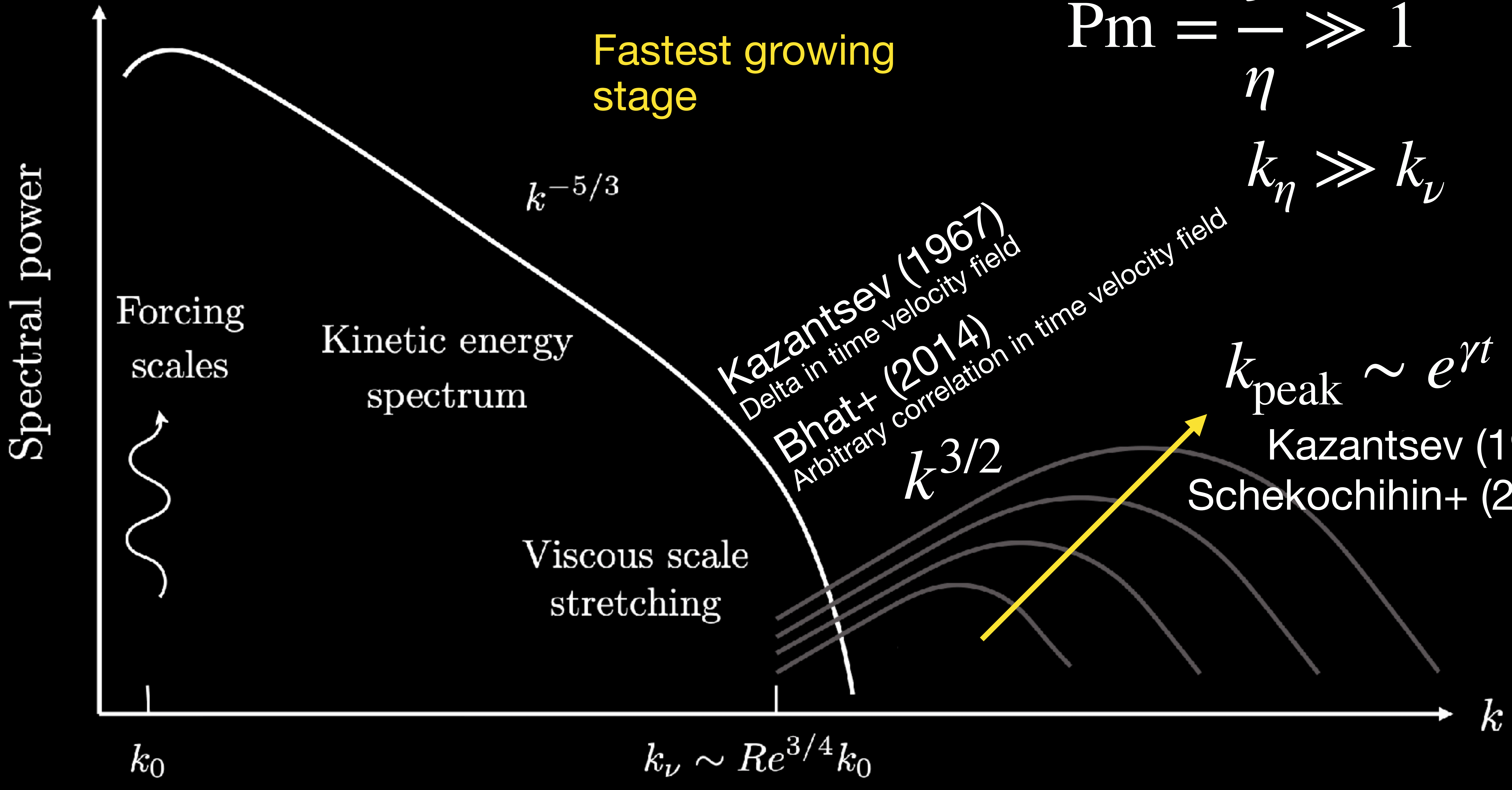
***Beattie**+ (2025, MNRAS) Taking control of compressible modes: bulk viscosity and the turbulent dynamo*

Small-scale dynamo
First growth stage: kinematic regime

Modified from Rincon (2019)

$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

$$k_\eta \gg k_\nu$$



Small-scale dynamo
Kinematic regime: the sub-viscous range

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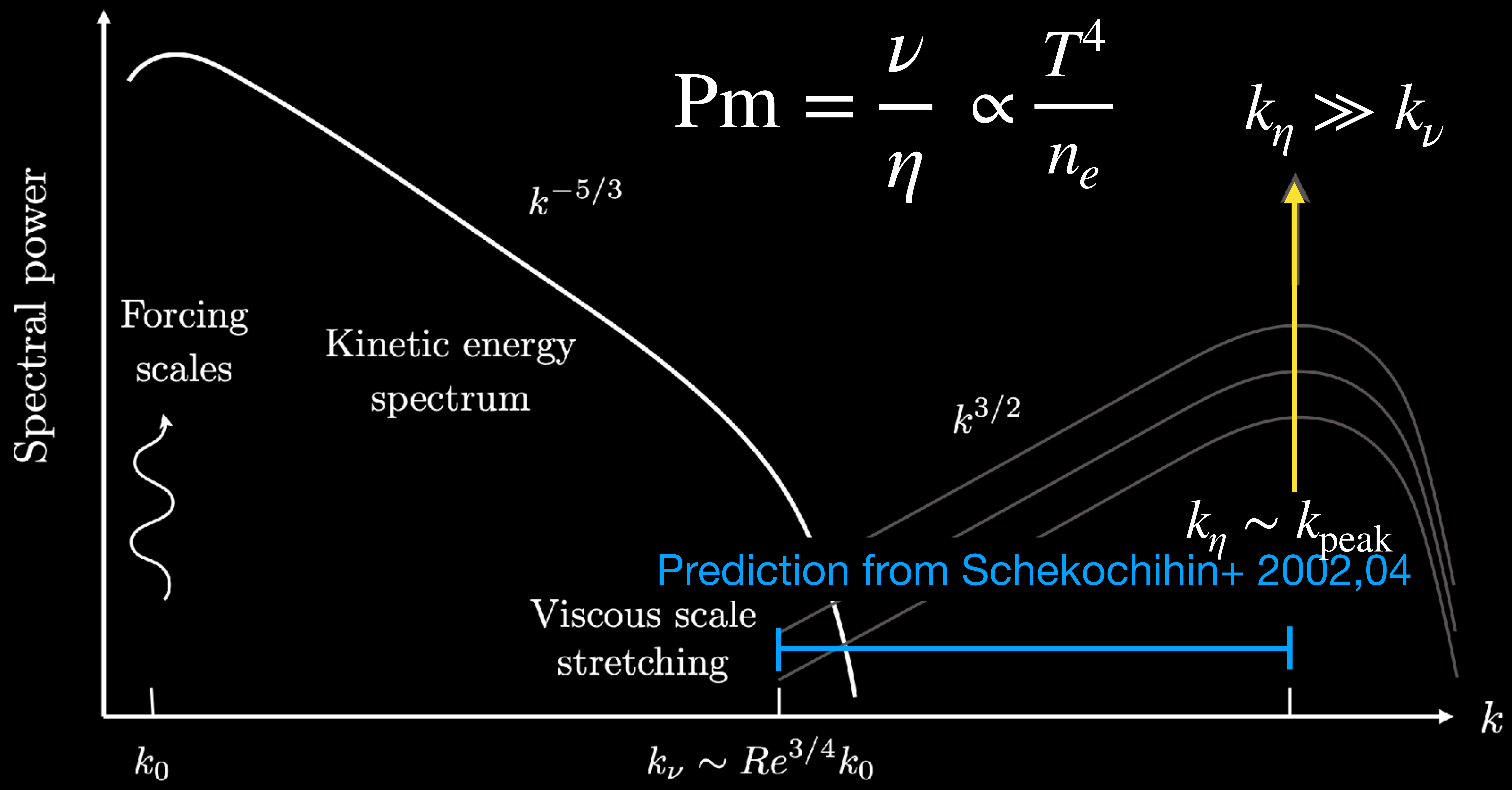
$k_\eta \gg k_\nu$



Small-scale dynamo

Kinematic regime: the sub-viscous range

Modified from Rincon (2019)

$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

$$k_\eta \gg k_\nu$$

stretching at the viscous scale

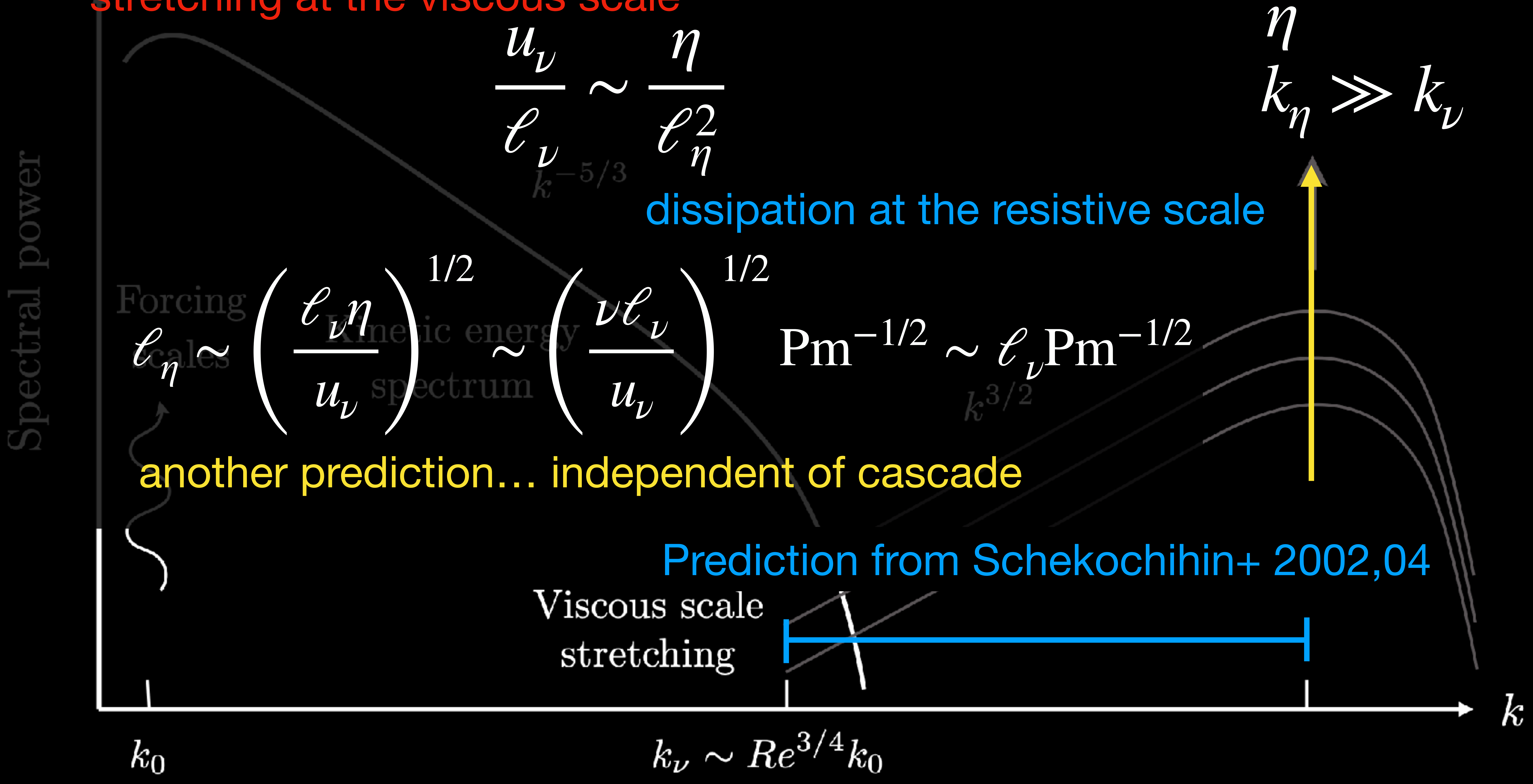
$$\frac{u_\nu}{\ell_\nu} \sim \frac{\eta}{\ell_\eta^2}$$

dissipation at the resistive scale

$$\ell_\eta \sim \left(\frac{\ell_\nu \eta}{u_\nu}\right)^{1/2} \sim \left(\frac{\nu \ell_\nu}{u_\nu}\right)^{1/2} \mathrm{Pm}^{-1/2} \sim \ell_\nu \mathrm{Pm}^{-1/2}$$

another prediction… independent of cascade

Prediction from Schekochihin+ 2002,04



Small-scale dynamo
Kinematic regime: the sub-viscous range

Derived from $k^{-5/3}$ velocity spectrum

$$k_\nu \sim \mathrm{Re}^{3/4}$$

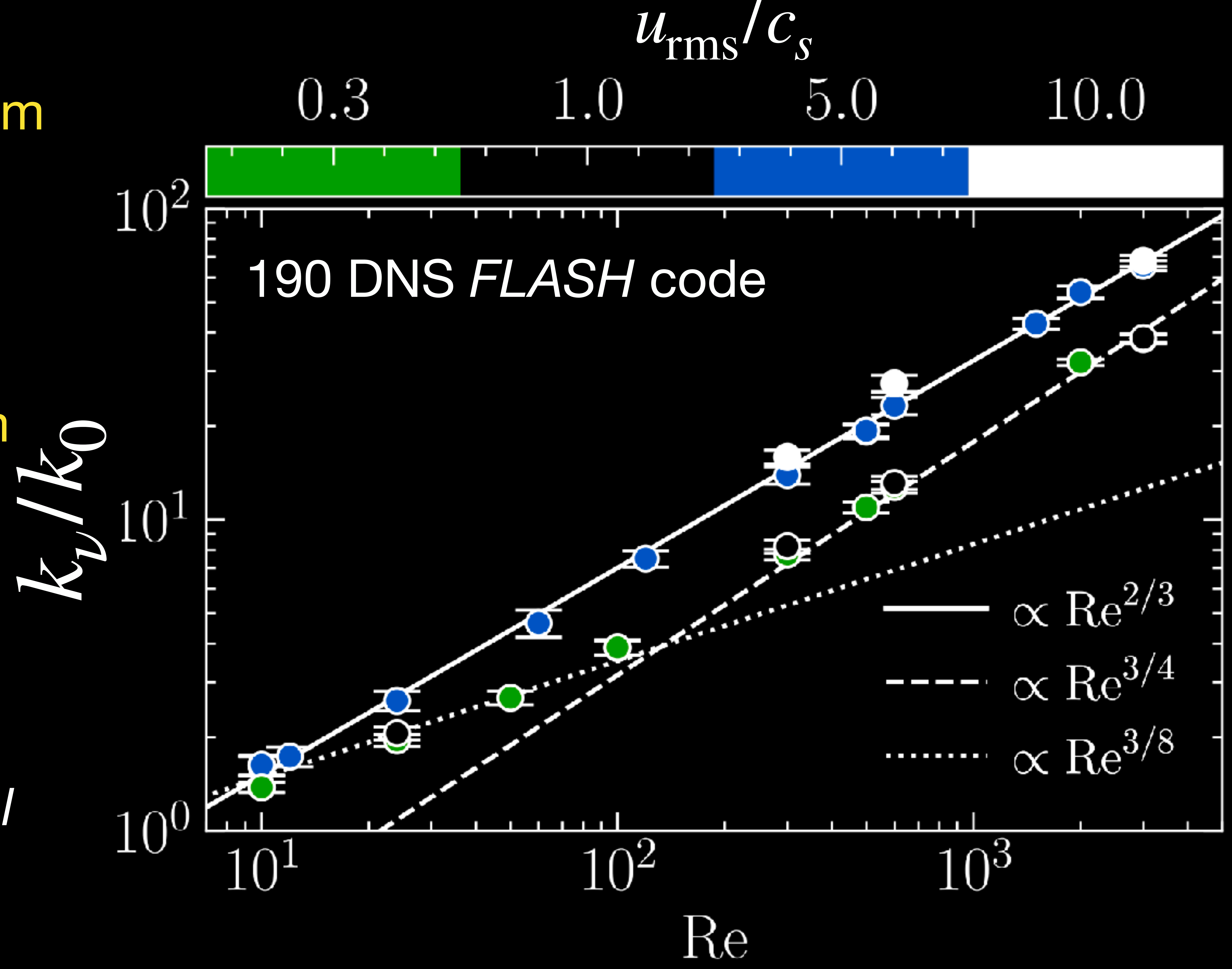
Kolmogorov41

Derived from k^{-2} velocity spectrum

$$k_\nu \sim \mathrm{Re}^{2/3}$$

Schober+(2015)

Kriel, **Beattie**+ (2025). *Fundamental scales II: the kinematic stage of the supersonic dynamo*



Small-scale dynamo
Kinematic regime: the sub-viscous range

Kriel, **Beattie**+ (2025). *Fundamental scales II: the kinematic stage of the supersonic dynamo*

Small-scale dynamo

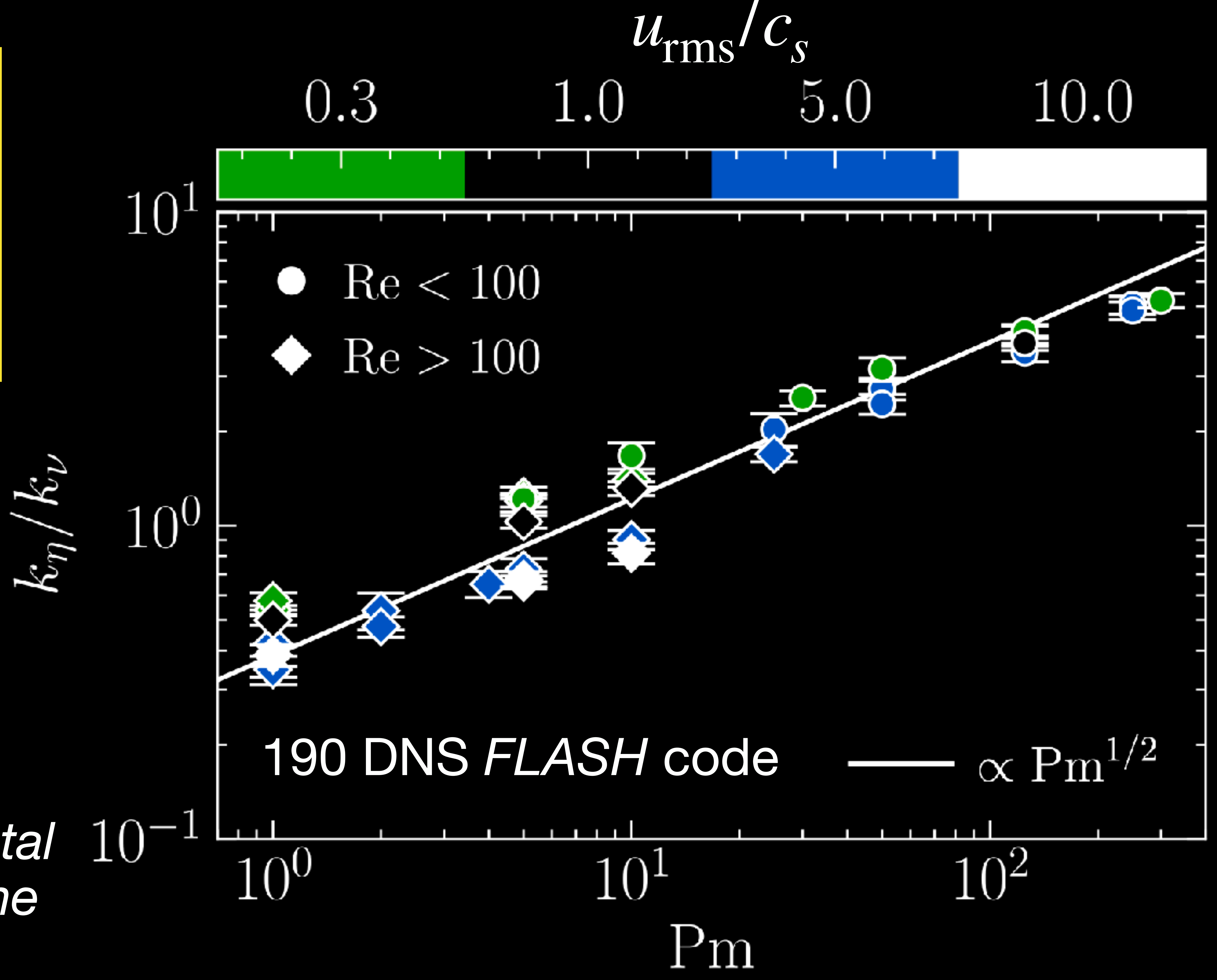
Most of the energy is in the sub-viscous modes!

stretching at the viscous scale

$$\frac{u_\nu}{\ell_\nu} \sim \frac{\eta}{\ell_\eta^2}$$

dissipation at the resistive scale

$$\frac{k_\eta}{k_\nu} \sim \mathrm{Pm}^{1/2}$$

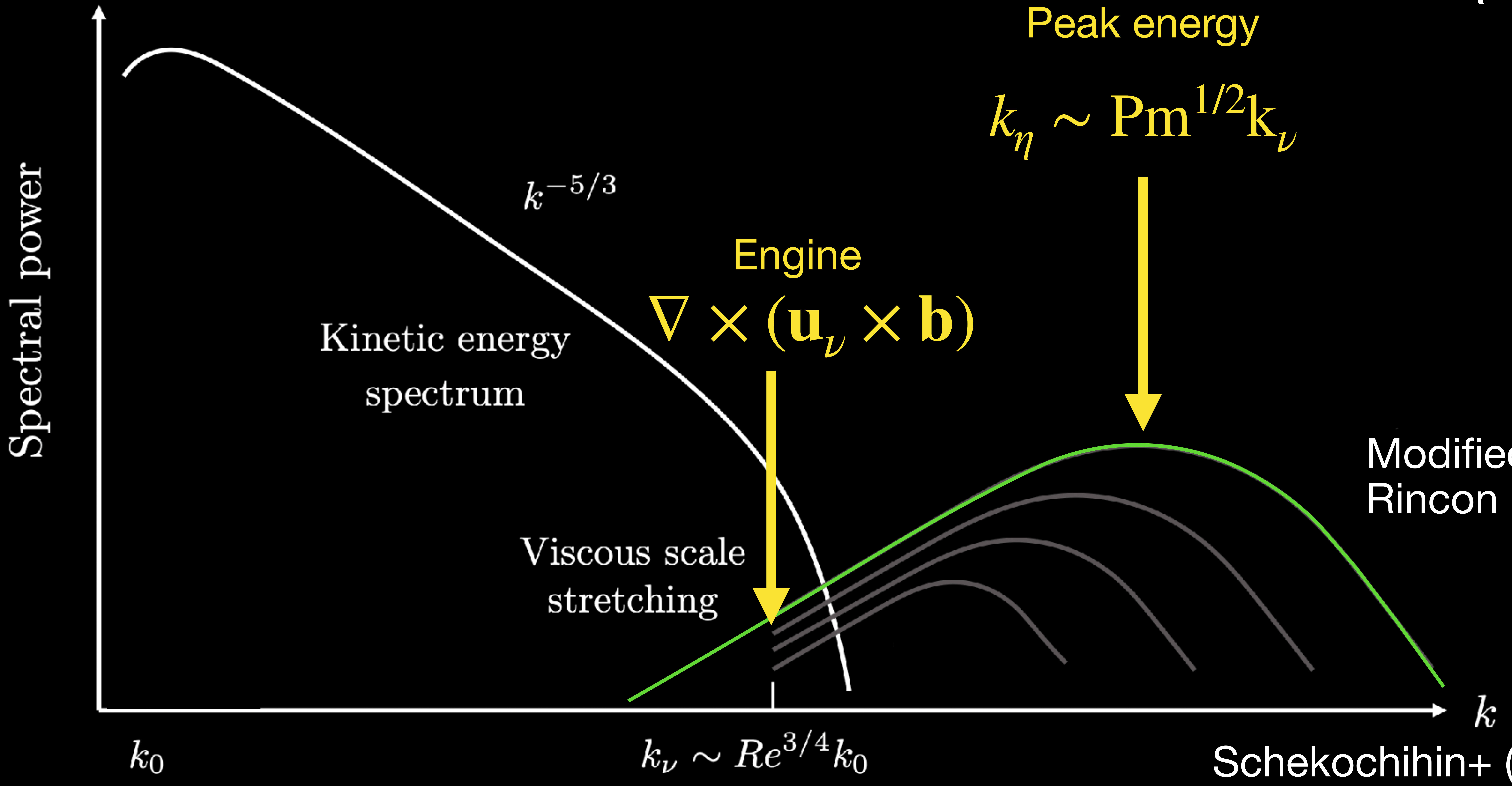


Kriel, **Beattie**+ (2025). *Fundamental scales II: the kinematic stage of the supersonic dynamo*

Small-scale dynamo

Nonlinear dynamo and backreaction

$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

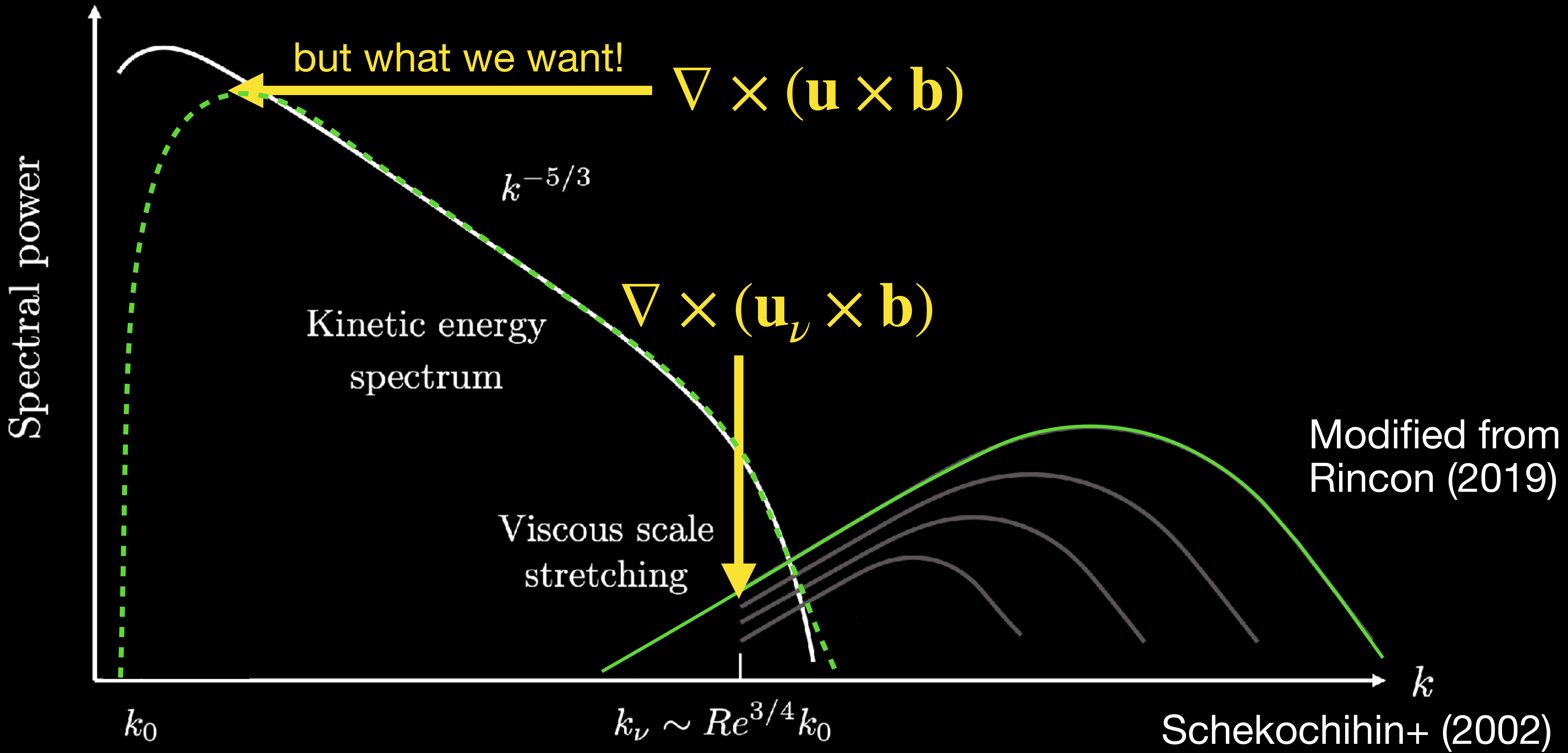


Modified from Rincon (2019)

Schekochihin+ (2002)

Small-scale dynamo

Nonlinear dynamo and backreaction



Schekochihin+ (2002)

Small-scale dynamo

Nonlinear dynamo and backreaction

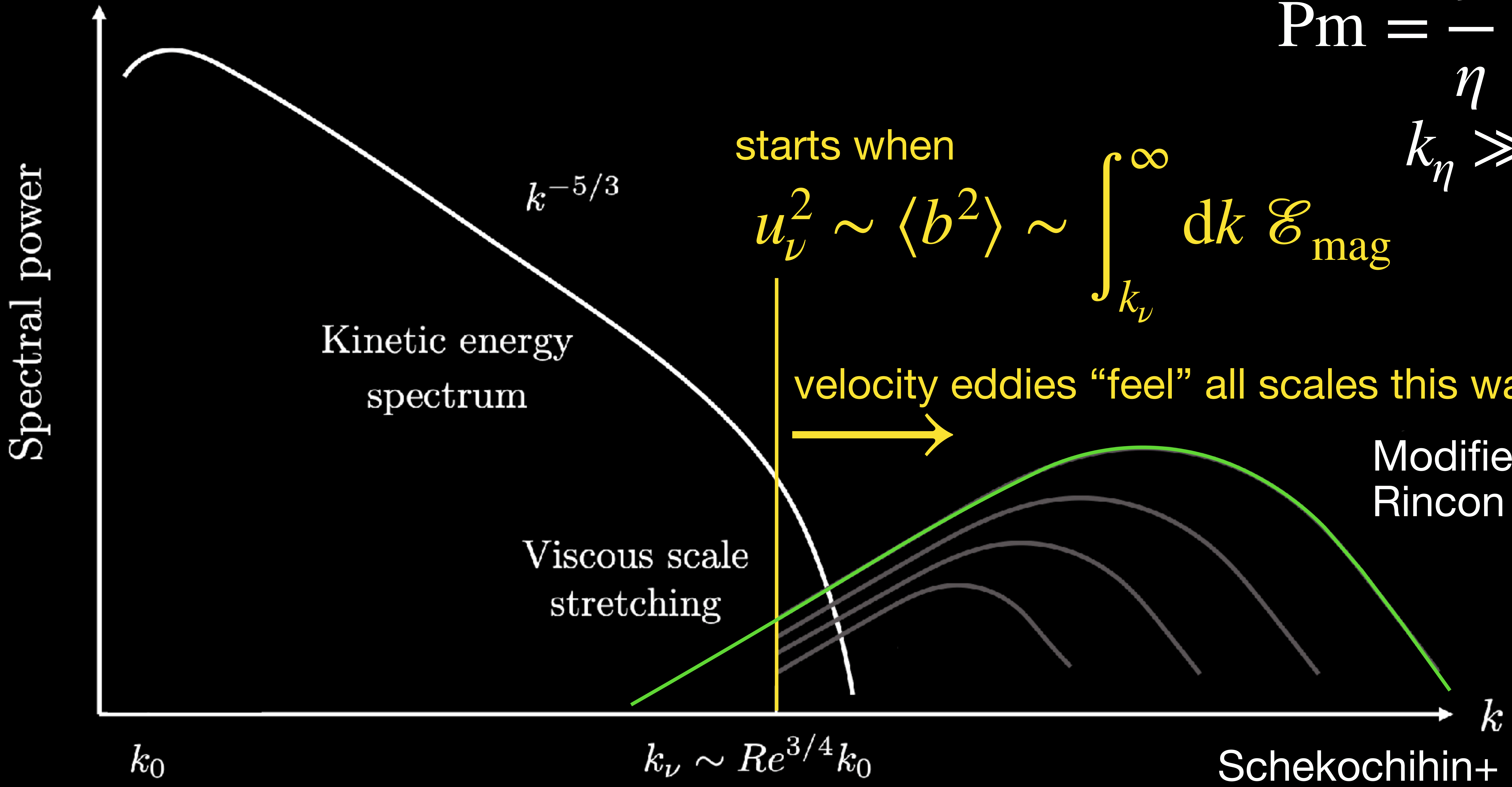
$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

$$k_\eta \gg k_\nu$$

starts when

$$u_\nu^2 \sim \langle b^2 \rangle \sim \int_{k_\nu}^{\infty} \mathrm{d}k\, \mathcal{E}_{\mathrm{mag}}$$

velocity eddies “feel” all scales this way



Modified from Rincon (2019)

Schekochihin+ (2002)

Small-scale dynamo

Nonlinear dynamo and backreaction

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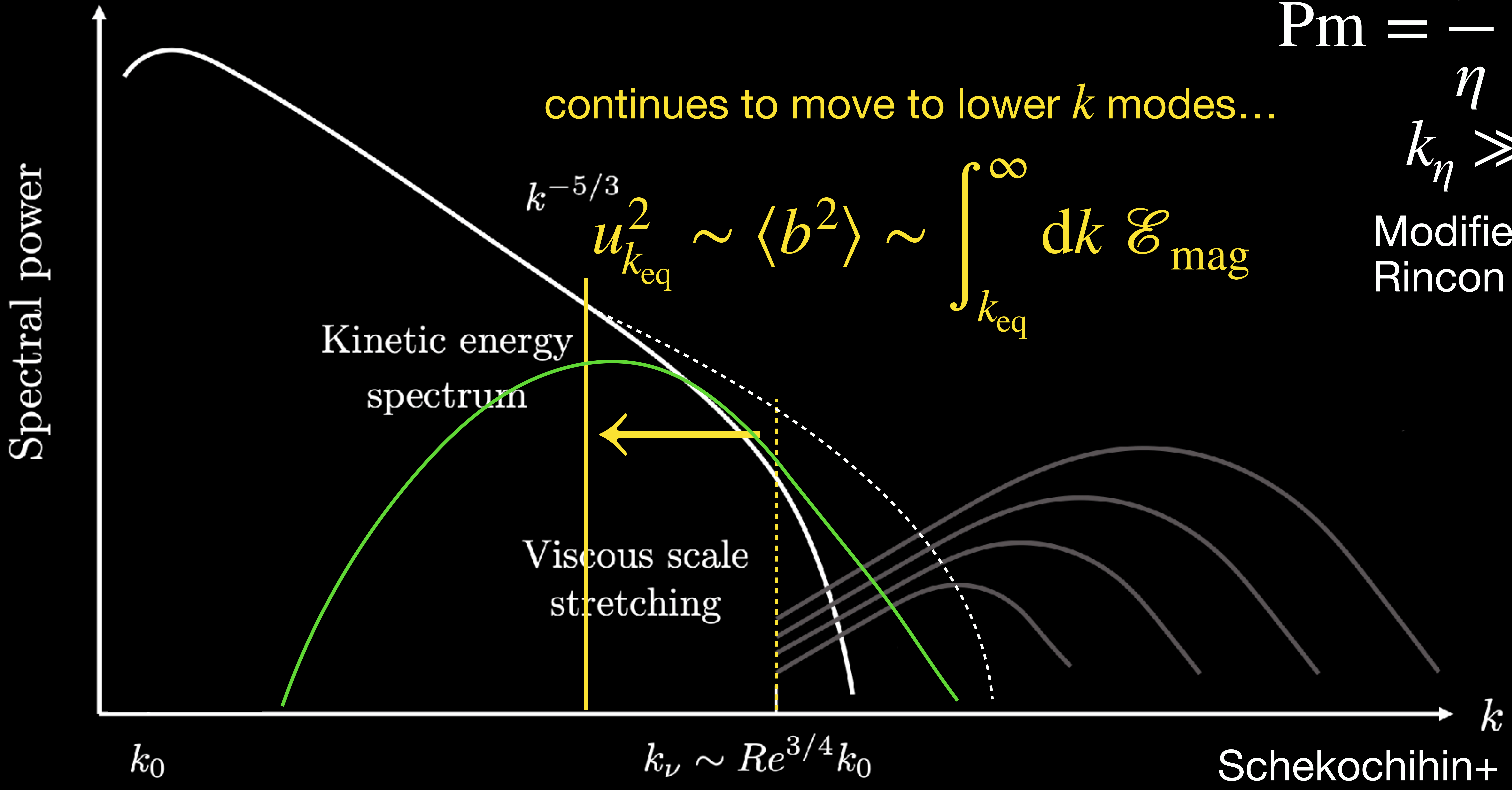
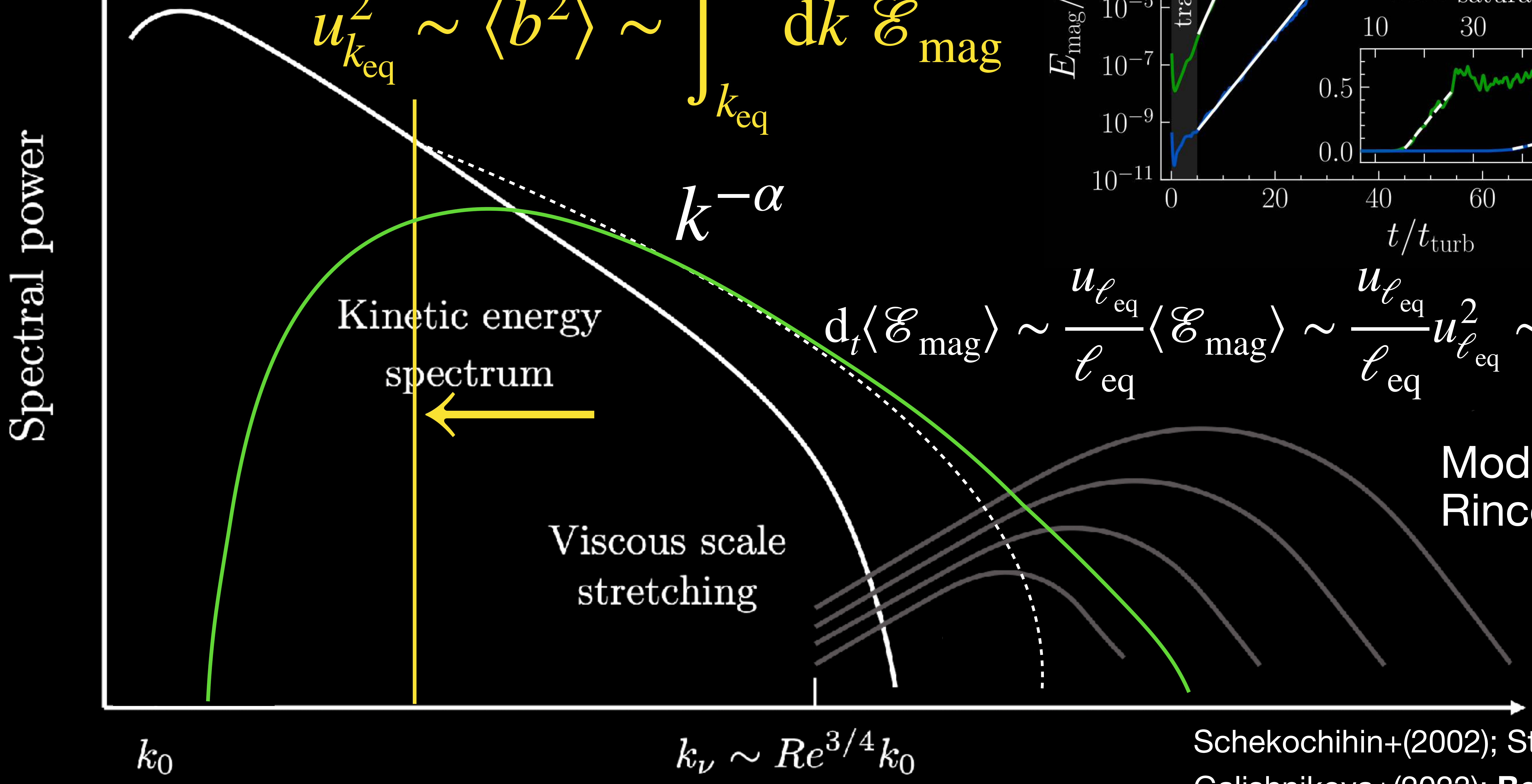


Schekochihin+ (2002)

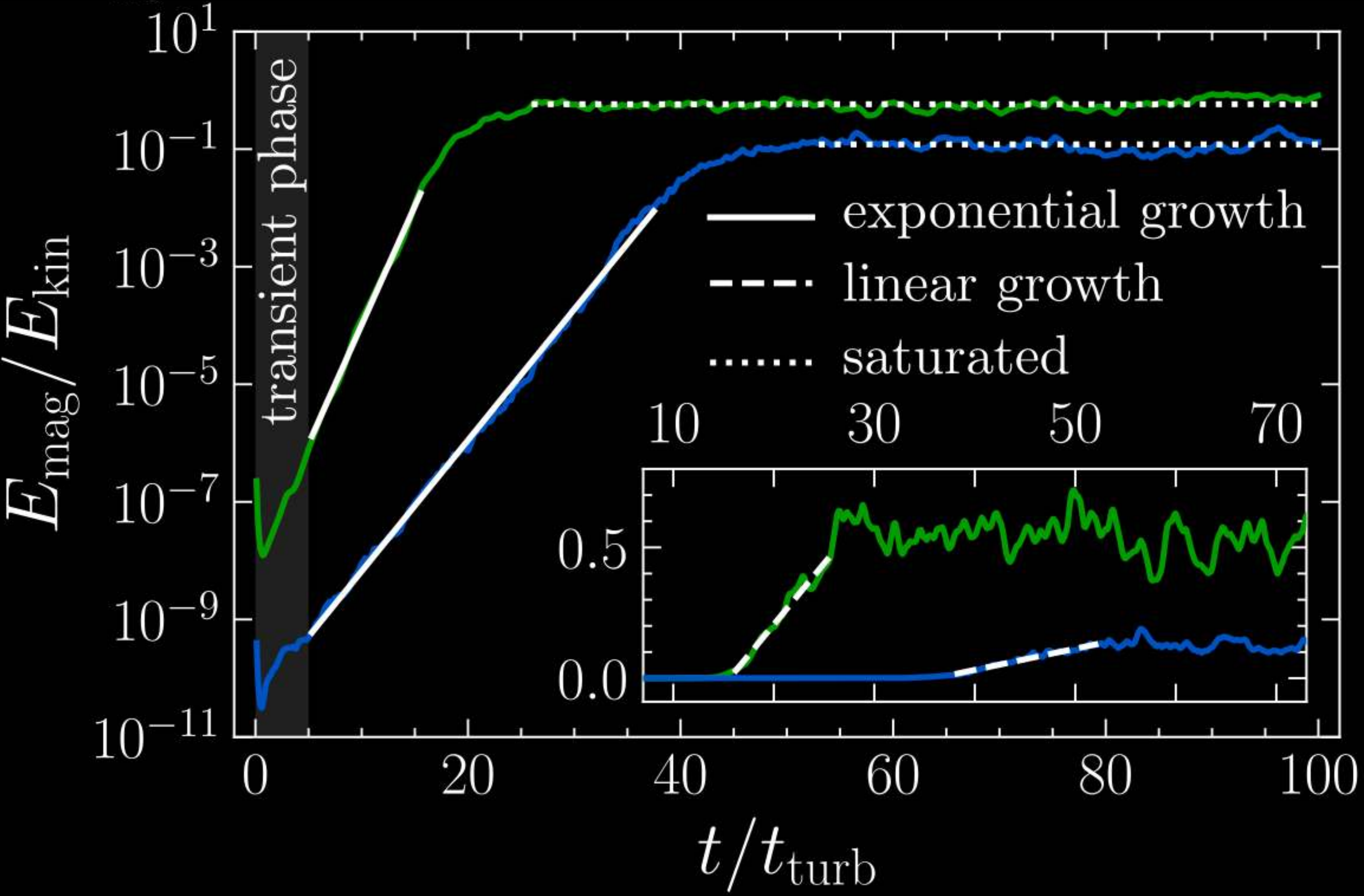
Small-scale dynamo

Nonlinear dynamo and backreaction

$$u_{k_{\rm eq}}^2 \sim \langle b^2 \rangle \sim \int_{k_{\rm eq}}^{\infty} \mathrm{d}k \, \mathscr{E}_{\rm mag}$$



$$\mathrm{d}_t \langle \mathscr{E}_{\rm mag} \rangle \sim \frac{u_{\ell_{\rm eq}}}{\ell_{\rm eq}} \langle \mathscr{E}_{\rm mag} \rangle \sim \frac{u_{\ell_{\rm eq}}}{\ell_{\rm eq}} u_{\ell_{\rm eq}}^2 \sim \varepsilon_{\rm eq} \sim \varepsilon,$$



Modified from Rincon (2019)

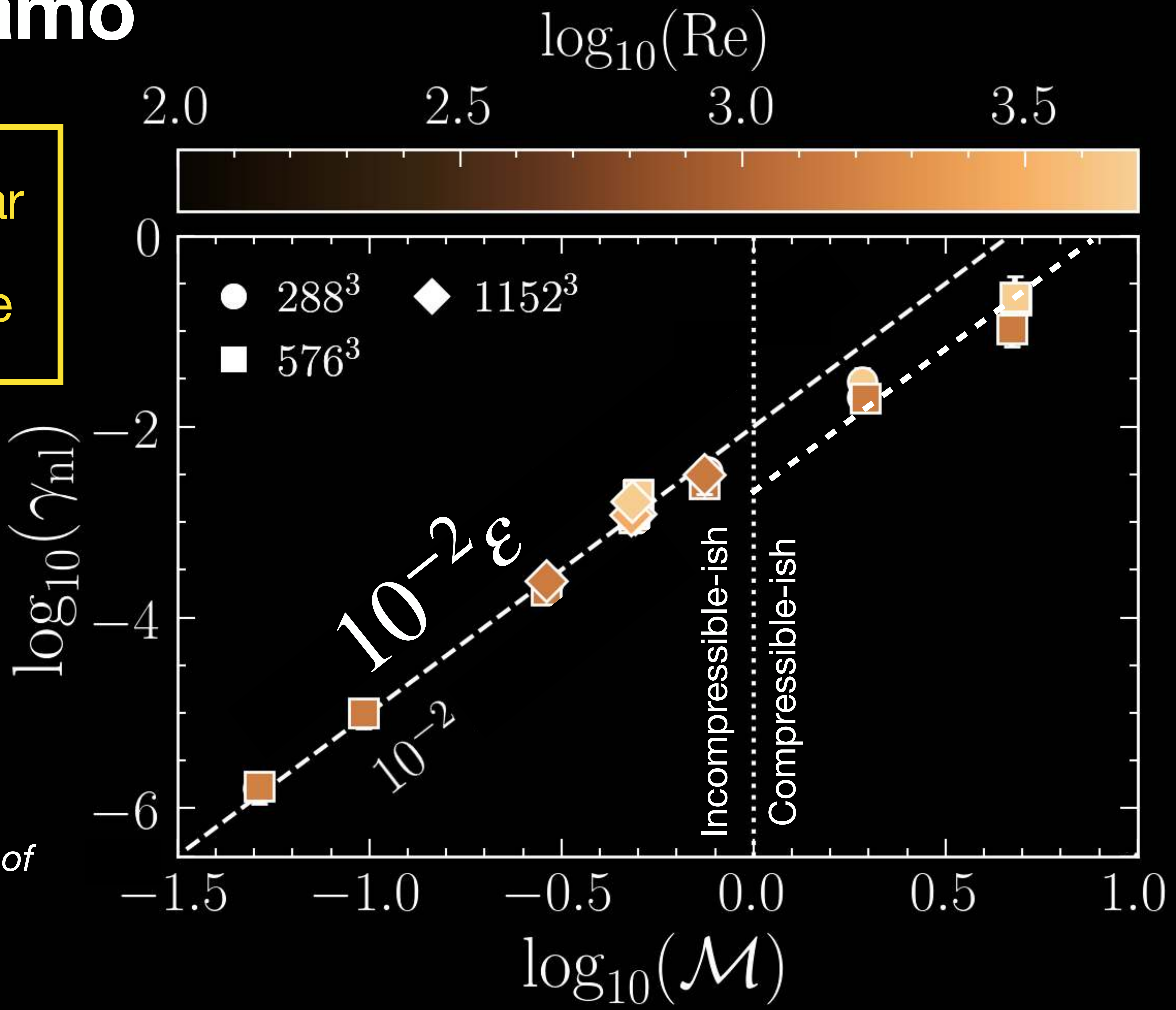
Schekochihin+(2002); St-Onge+(2020)
Galishnikova+(2023); **Beattie**+(2025)

Small-scale dynamo

Nonlinear dynamo

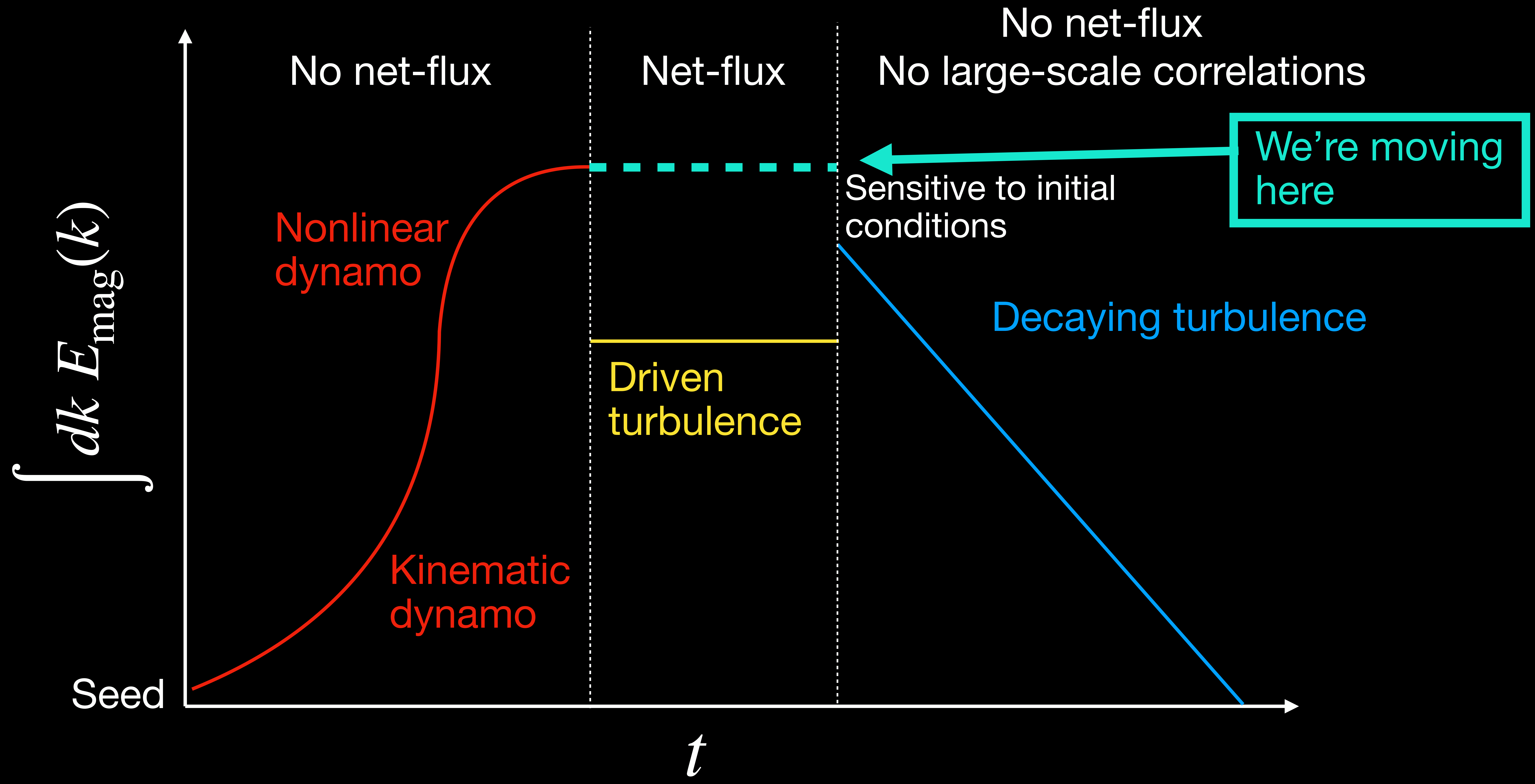
The engine of the nonlinear dynamo is the kinetic reservoir of the turbulence

$$\langle \mathcal{E}_{\mathrm{mag}} \rangle \sim \varepsilon t \, .$$



Kriel, **Beattie**+ (in prep.). *Growth rate of magnetic energy doing the nonlinear small-scale dynamo*

Driven no net-flux turbulence



$10{,}080^3$ magnetized supersonic turbulence simulation

Beattie, Federrath, Klessen, Cielo & Bhattacharjee

1. What is the scaling of the energy cascade in compressible MHD turbulence with no net flux?
2. How are the characteristic scales organized in the compressible supersonic turbulence?
3. What are the saturation physics of the compressible turbulent dynamo?

PI of a three total 190million core-hour projects on superMUC-NG

ILES of compressible MHD turbulence

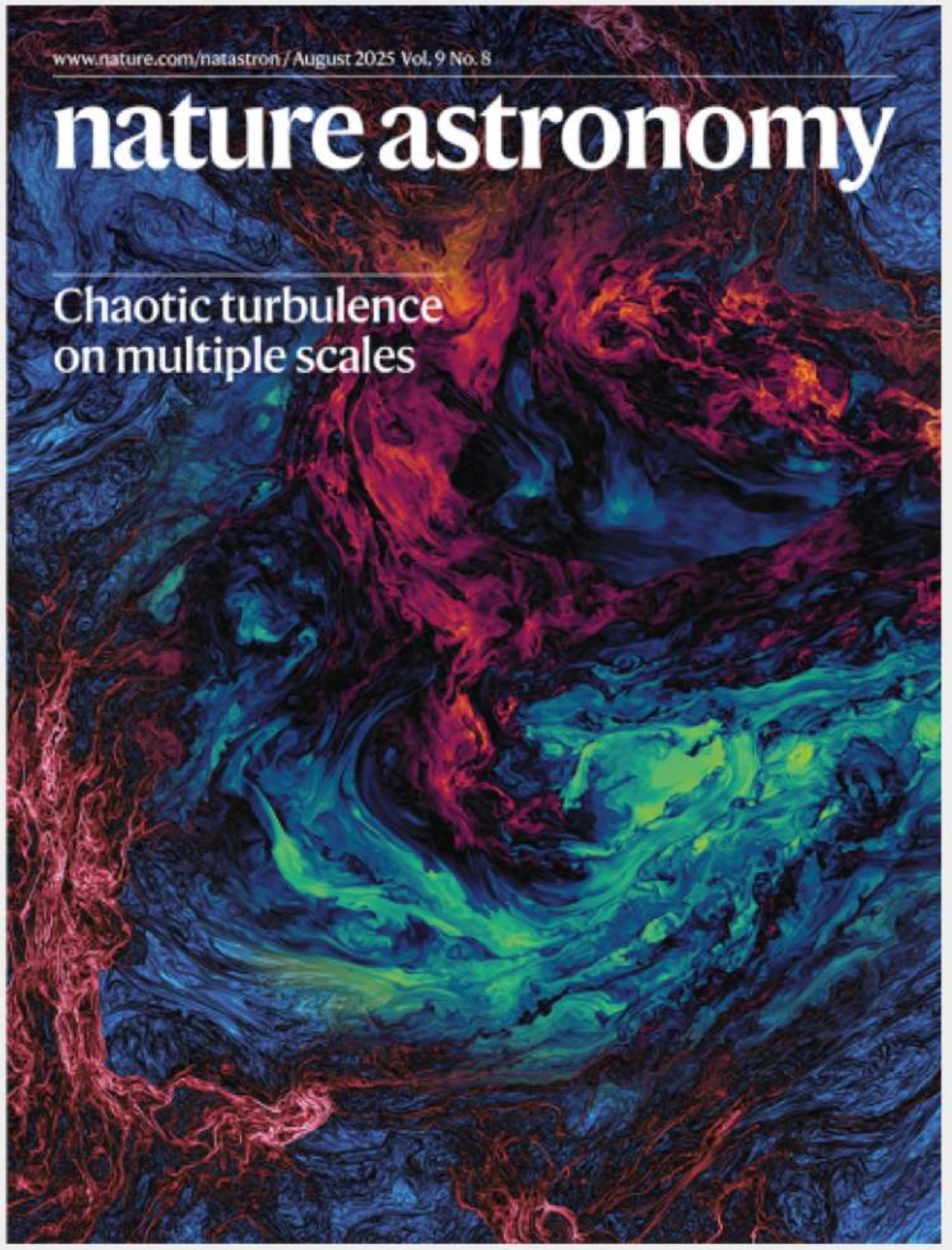
Turbulence: $\sigma_V / c_s \approx 4,\ \ell_0 = L/2$

Magnetic fields: $B = b_{\rm turb},\ \mathcal{M}_{\rm A} \approx 2$

Three experiments for convergence tests:

- **LOW-RES:** $2{,}520^3$ (0.3Mcore-h, 8,640cores)
- **MID-RES:** $5{,}040^3$ (4.0Mcore-h, 34,560cores)
- **HIGH-RES:** $10{,}080^3$ (80.0Mcore-h, 148,240cores)

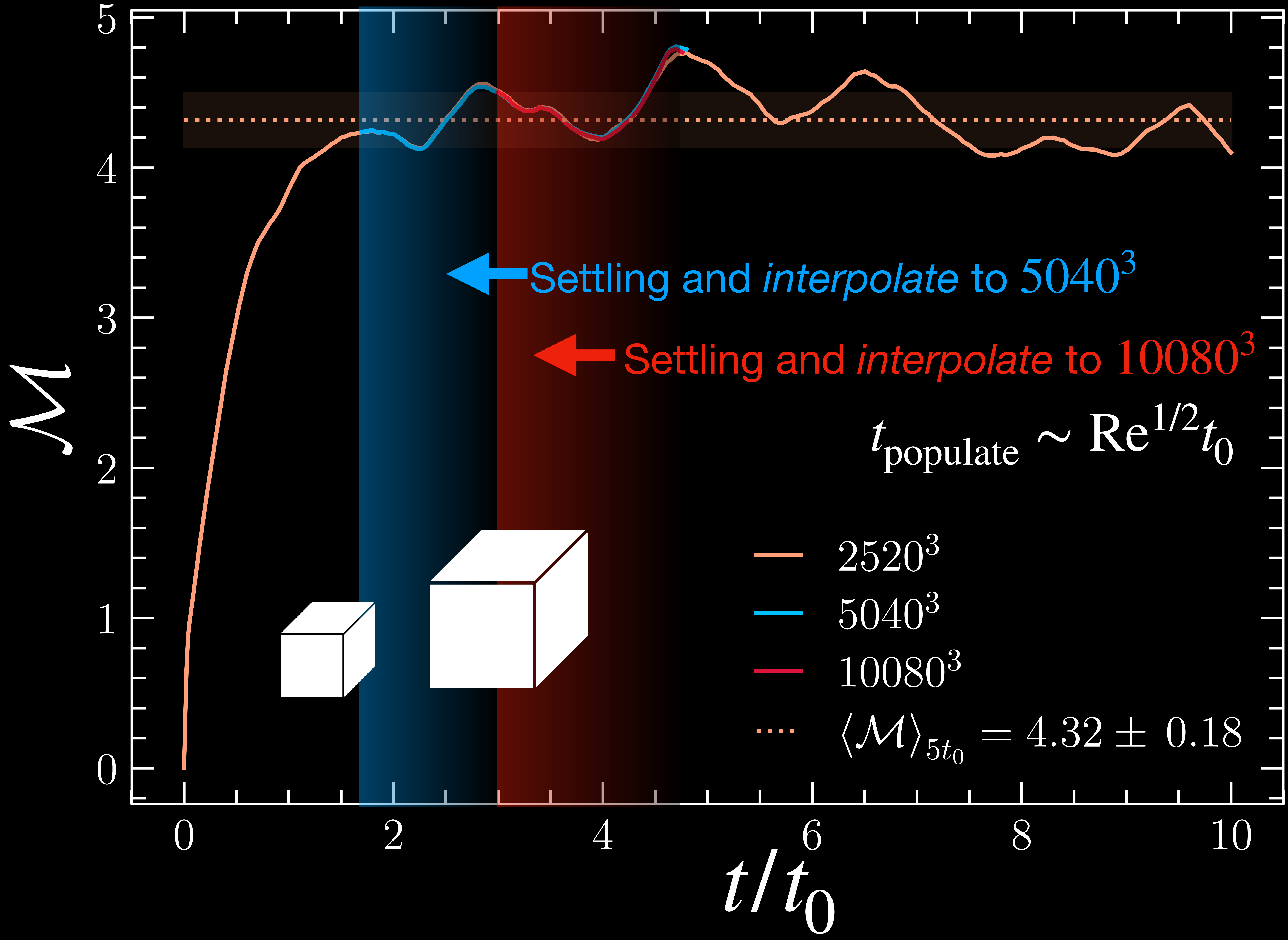
3.45PB in data products $\quad \mathrm{Rm} \sim \mathrm{Re} \gtrsim 10^6, \quad \mathrm{Pm} \sim 1-2$



Beattie+ 2025 (*Nature Astronomy*). The spectrum of magnetized turbulence in the interstellar medium

Volume integral Quantities

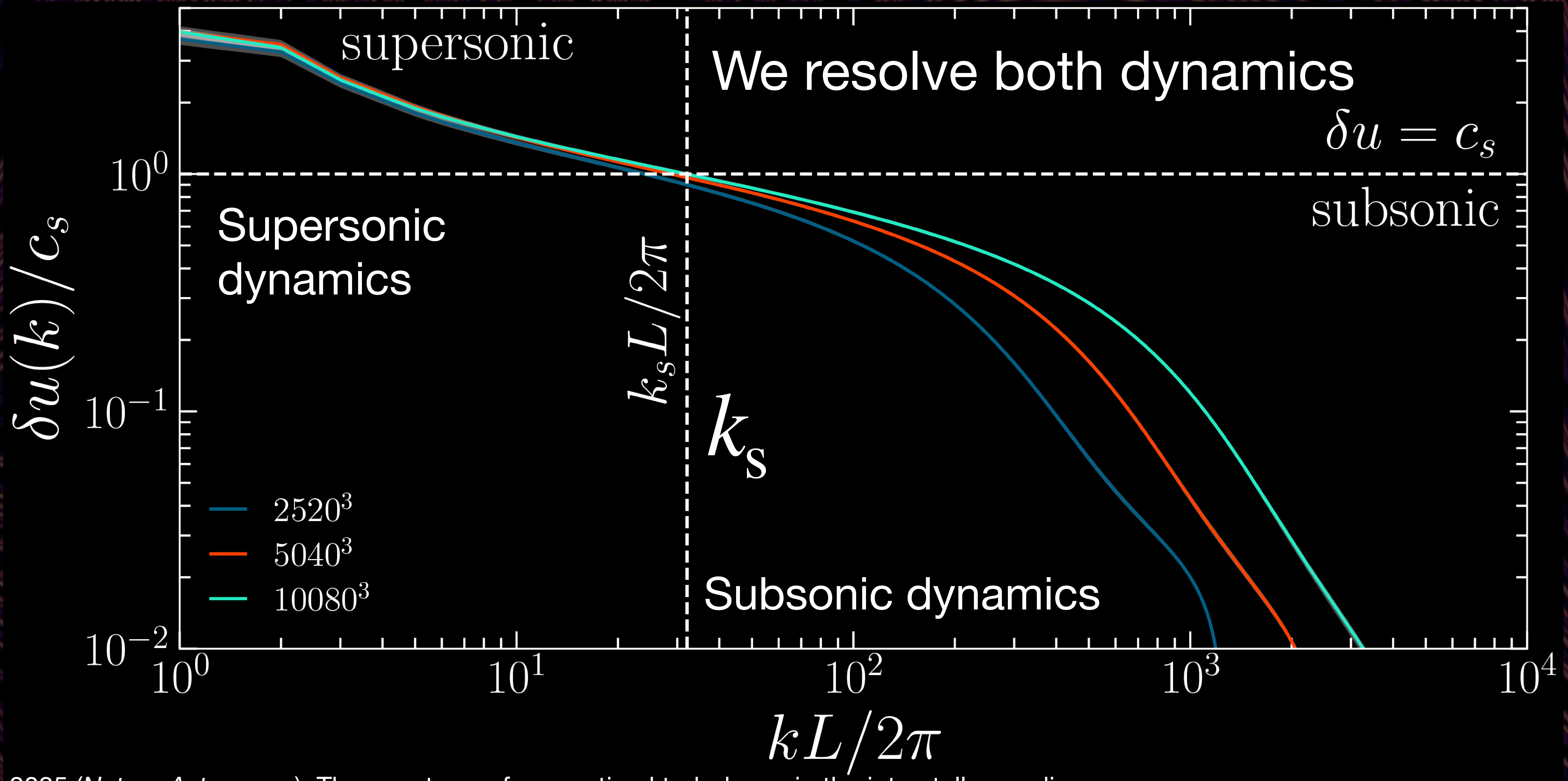
$$\mathscr{M} = \left\langle \frac{u^2}{c_s^2} \right\rangle_{\mathscr{V}}^{1/2}$$



Interpolation to generate ICs for successively higher resolution experiments

Beattie+ 2025 (*Nature Astronomy*). The spectrum of magnetized turbulence in the interstellar medium

Two important scales — the sonic scale



Beattie+ 2025 (*Nature Astronomy*). The spectrum of magnetized turbulence in the interstellar medium

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

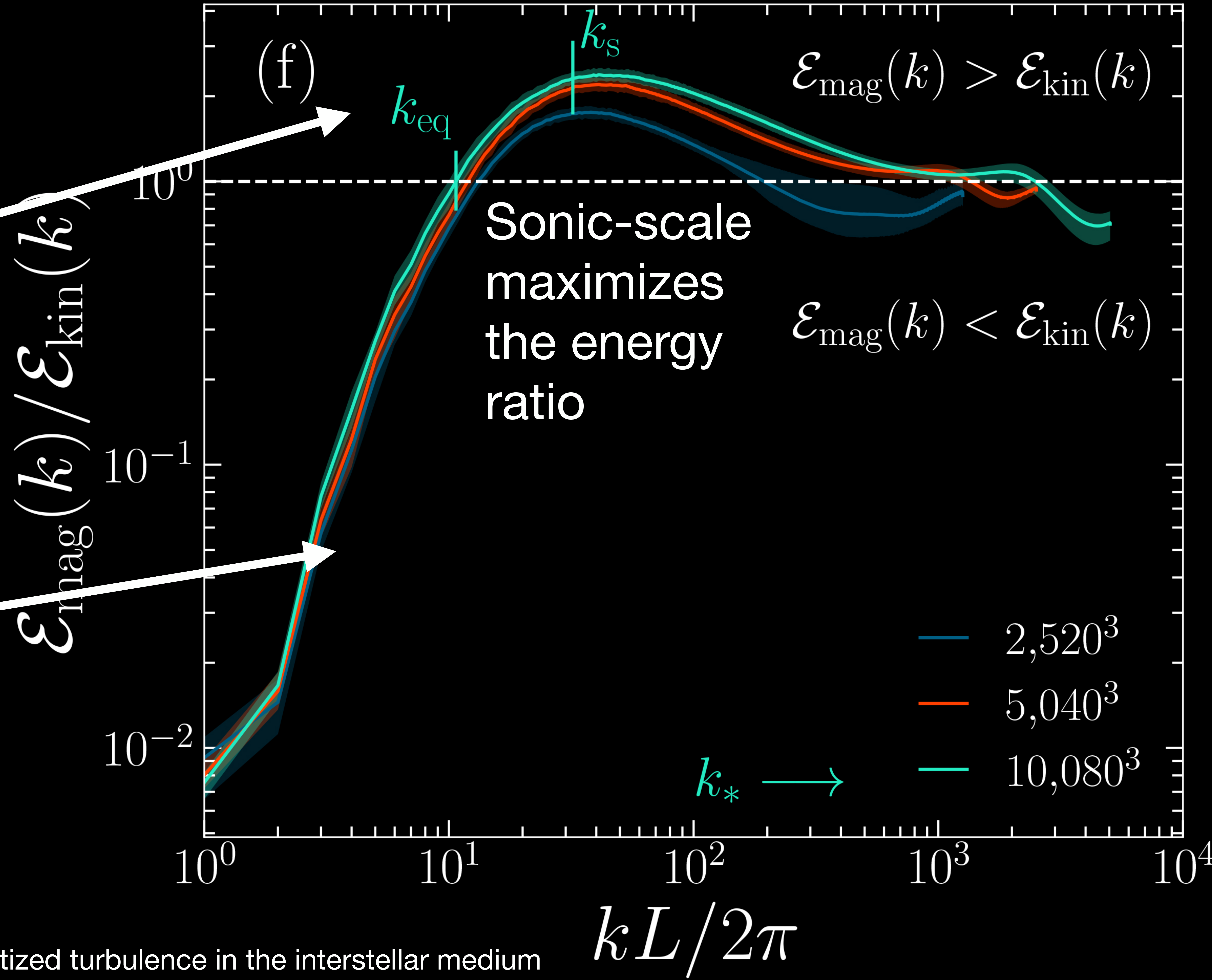
Two important scales — the Alfvén scale

sub-Alfvénic

k_{eq}

super-Alfvénic

Multiple regimes in a single domain!



Beattie+ 2025 (*Nature Astronomy*). The spectrum of magnetized turbulence in the interstellar medium

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Energy spectra: the disparate lives of u and b fluctuations

Two power laws

One power law, only emerging at very high Rm

Beattie+ 2025 (*Nature Astronomy*). The spectrum of magnetized turbulence in the interstellar medium

Energy spectra: the disparate lives of u and b fluctuations

Beattie+ 2025 (*Nature Astronomy*). The spectrum of magnetized turbulence in the interstellar medium

The subsonic spectrum (also ~ incompressible)

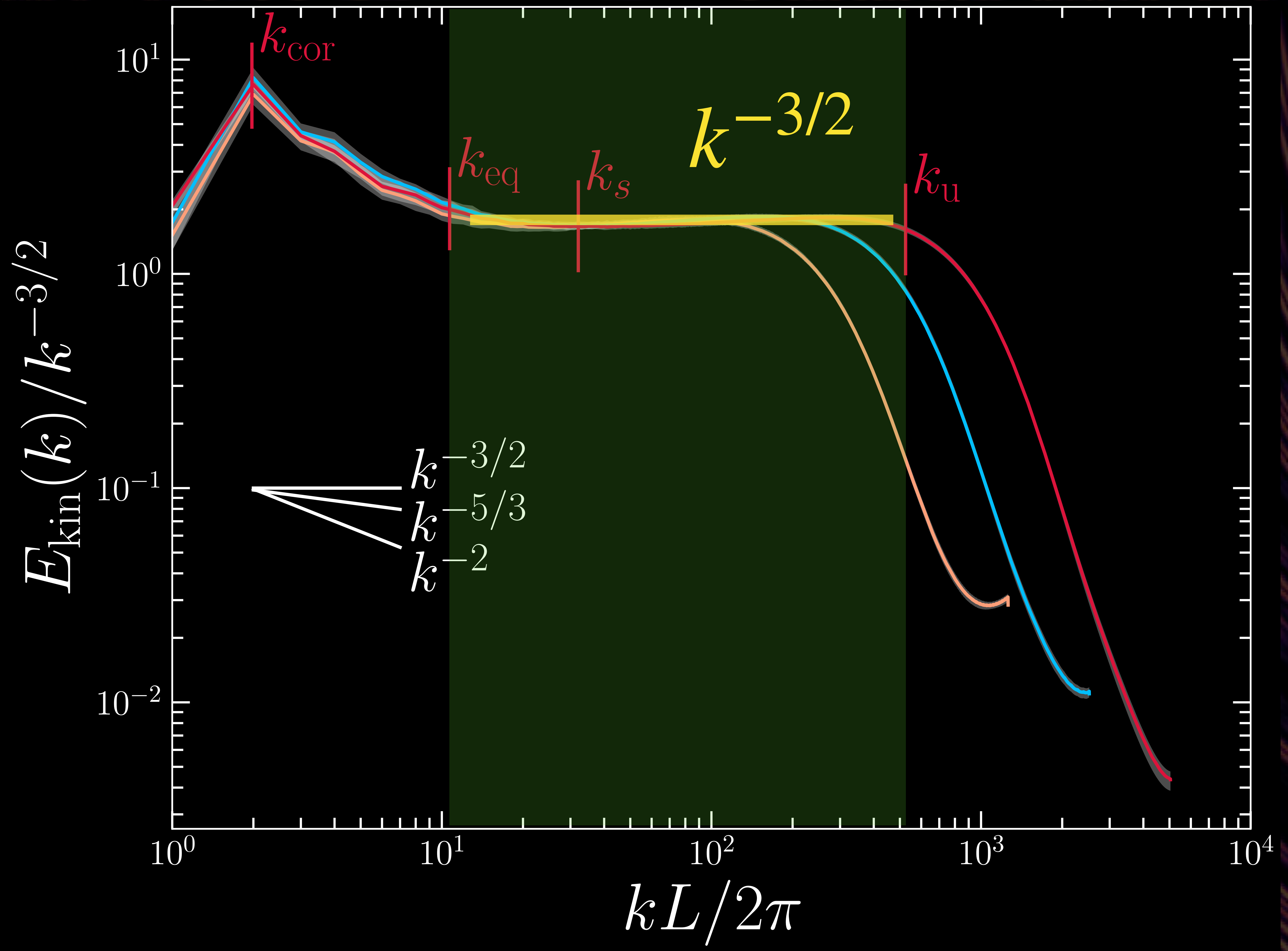
$$\mathcal{E}(k) \sim k^{-3/2},\ k > k_{\rm eq}$$

Unlikely fast-wave turbulence…
Galtier (2023)

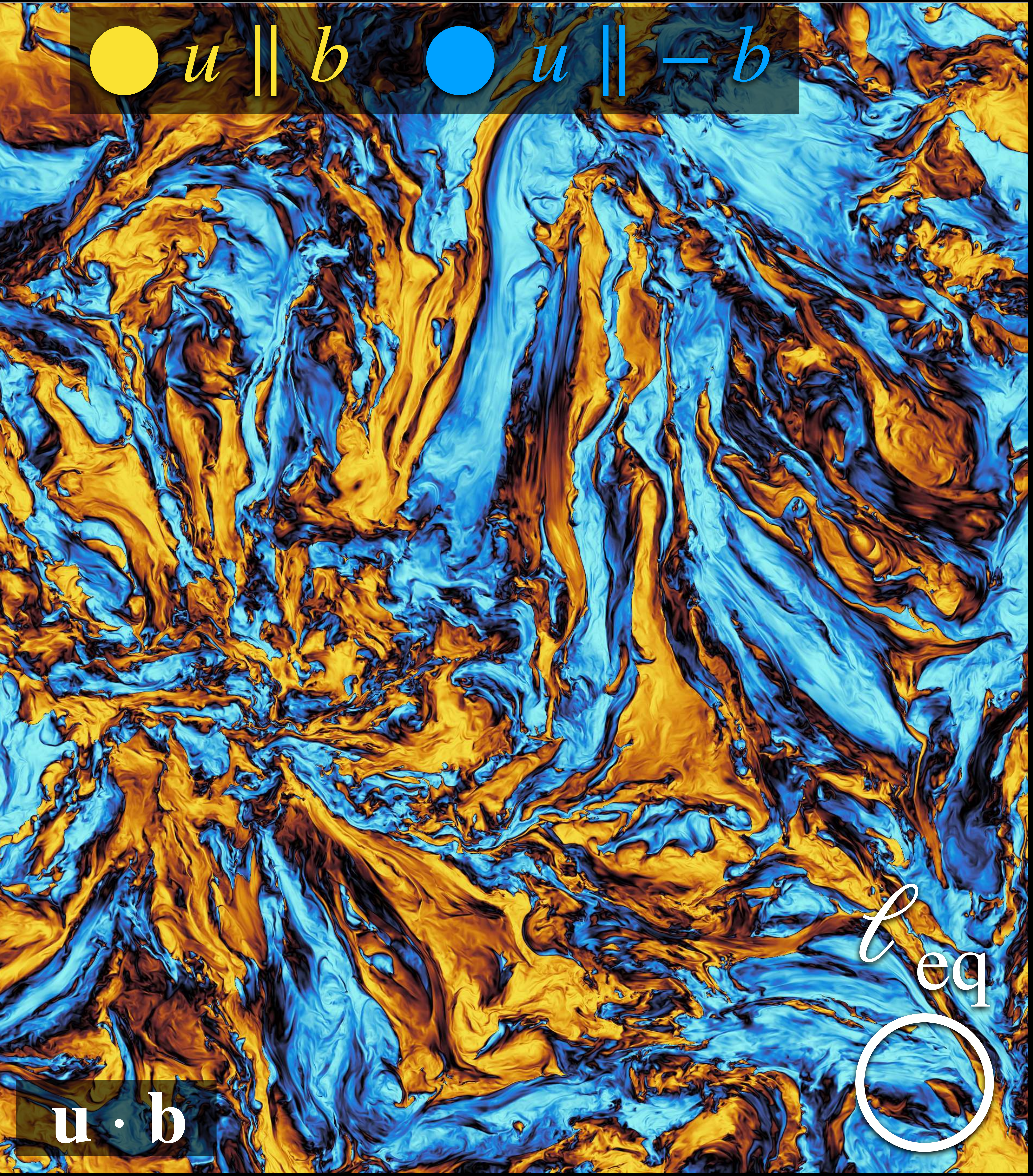
Scale-dependent alignment?

?

In general, we need:
A mechanism for depleting nonlinearities in the turbulence (turning $k^{-5/3} \rightarrow k^{-3/2}$)

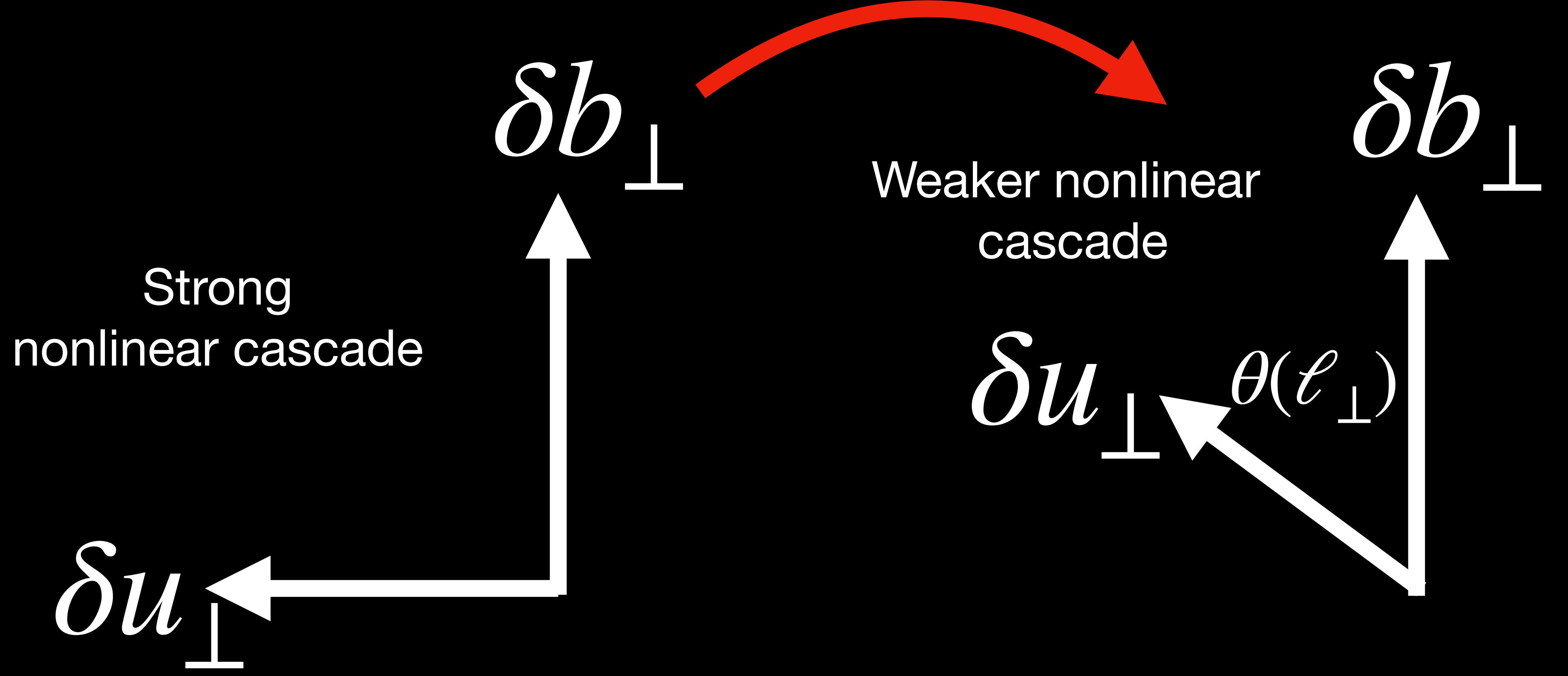


Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).



Dynamic alignment?

Shearing events between counter-propagating Alfvén waves / selective decay



$E_{\mathrm{kin}}(k_\perp) \propto k_\perp^{-5/3}$

Goldreich & Shridar (1995)

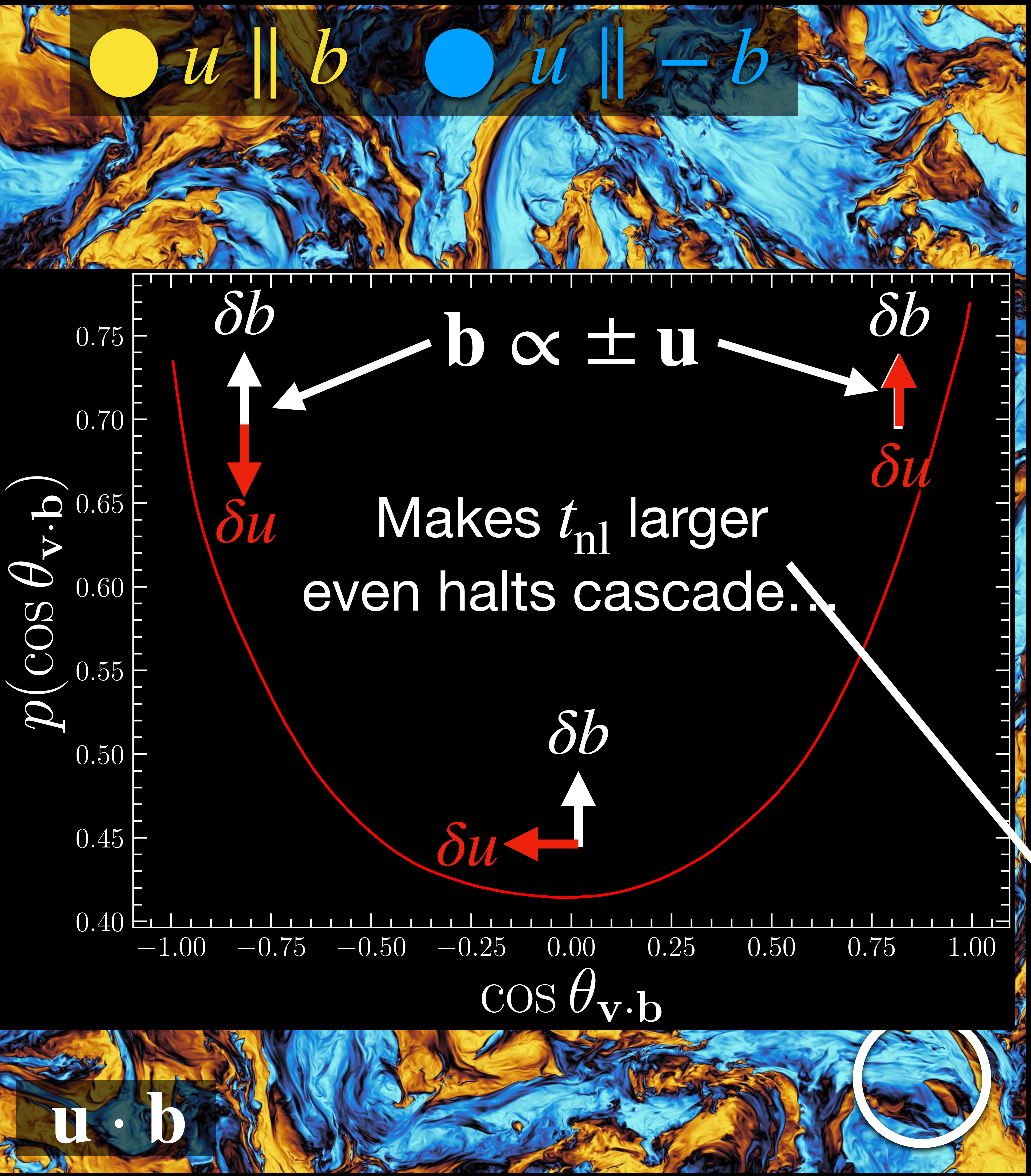
$E_{\mathrm{kin}}(k_\perp) \propto k_\perp^{-3/2}$

Boldyrev (2006)

Modifies the cascade timescale

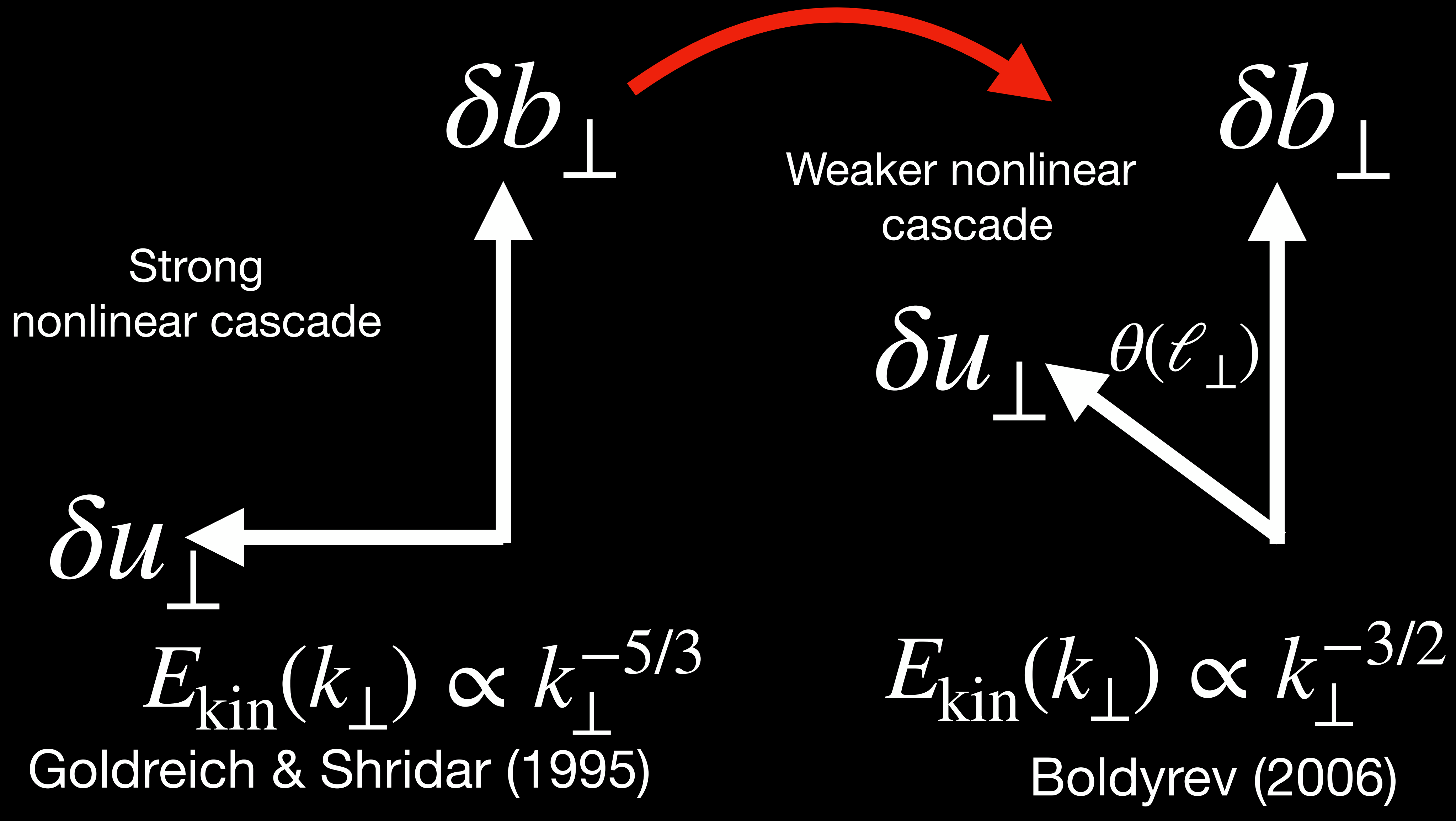
$$t_{\mathrm{nl}} \sim \frac{\ell_\perp}{z^{\mp}\sin\theta_{z^{\mp}}}, \quad z^{\mp} = (u \mp b)$$

Boldyrev (2006)



Dynamic alignment?

Shearing events between counter-propagating Alfvén waves / selective decay



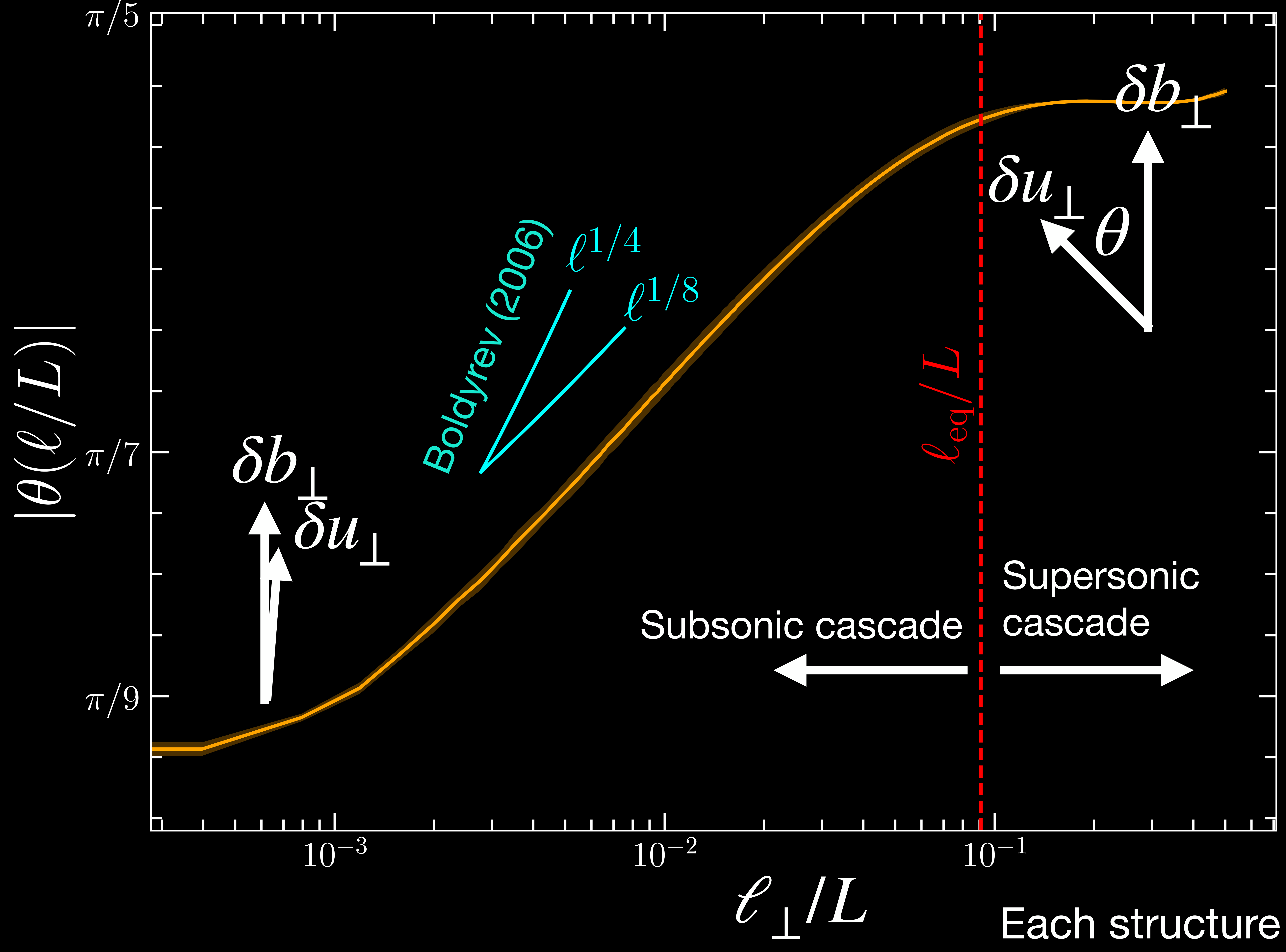
Modifies the cascade timescale

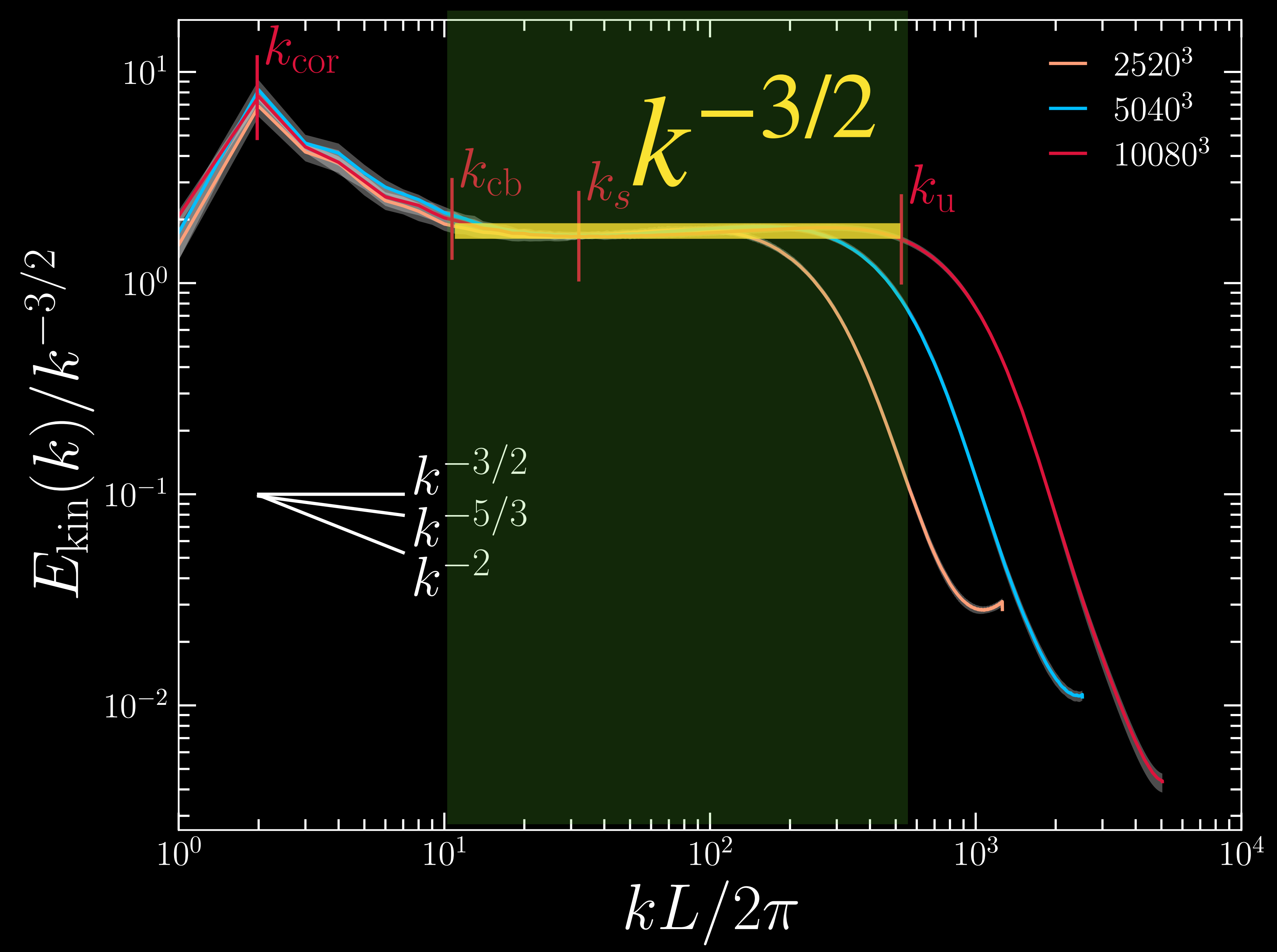
$$t_{\mathrm{nl}} \sim \frac{\ell_\perp}{z^{\mp}\sin\theta_{z^{\mp}}}, \quad z^{\mp} = (u \mp b)$$

Boldyrev (2006)

Steady state alignment

<u>Beattie</u> & Bhattacharjee *(subm. PRL.). Scale dependent alignment in compressible magnetohydrodynamic turbulence*





We find

$$\theta(\ell_\perp) \sim \ell_\perp^{1/8}$$

scale-dependent alignment, but not Boldyrev (2006).

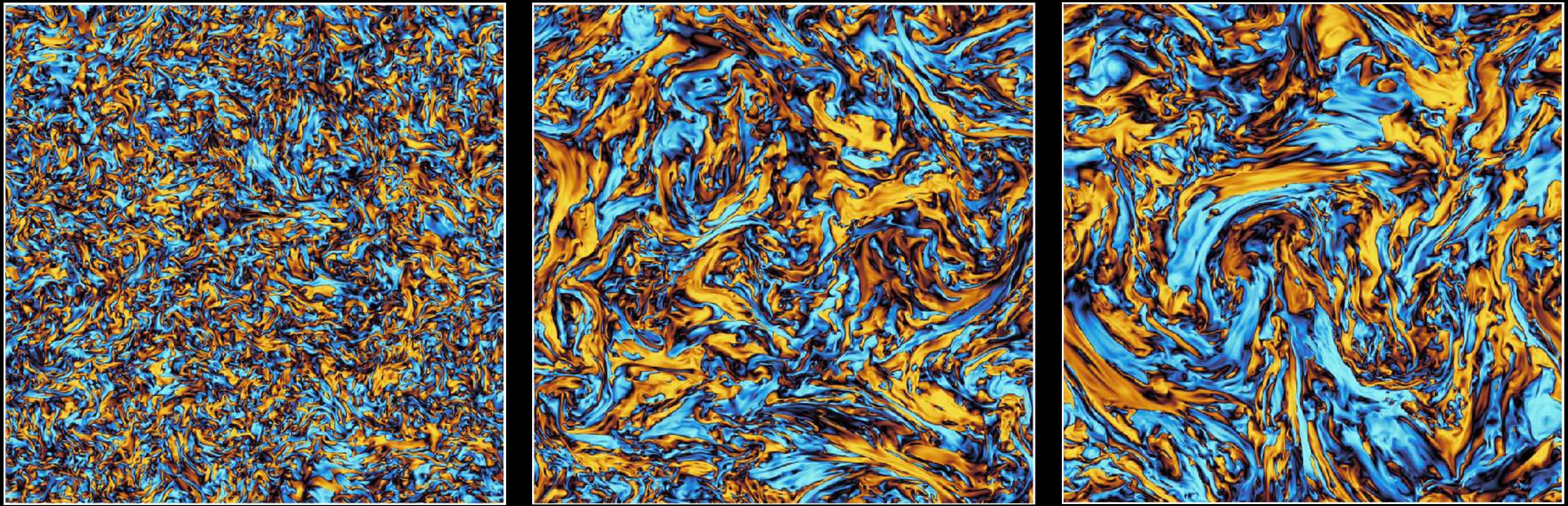
Each structure function costs 100,000 core hours!

Alignment through the nonlinear dynamo evolution

$t = t_{\text{kinematic}}$

$\mathbf{u} \parallel \pm \mathbf{b}$ $\mathbf{u} \parallel \pm \mathbf{b}$ $\mathbf{u} \parallel \pm \mathbf{b}$

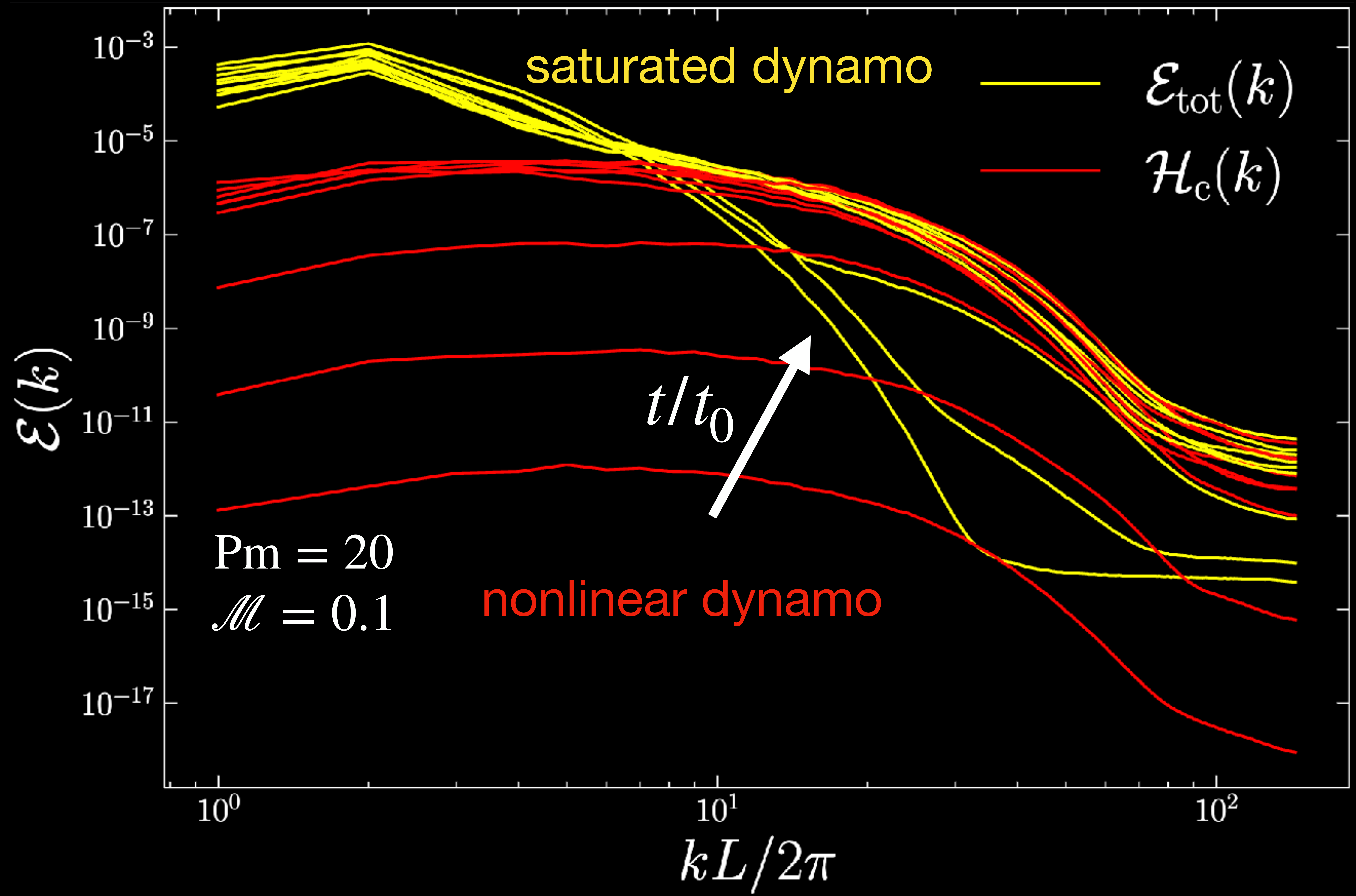
$t = t_{\text{saturate}}$



$\mathbf{u} \cdot \mathbf{b}$

$u \parallel b$ $u \parallel -b$

The dynamical saturation of the dynamo
Saturation through alignment

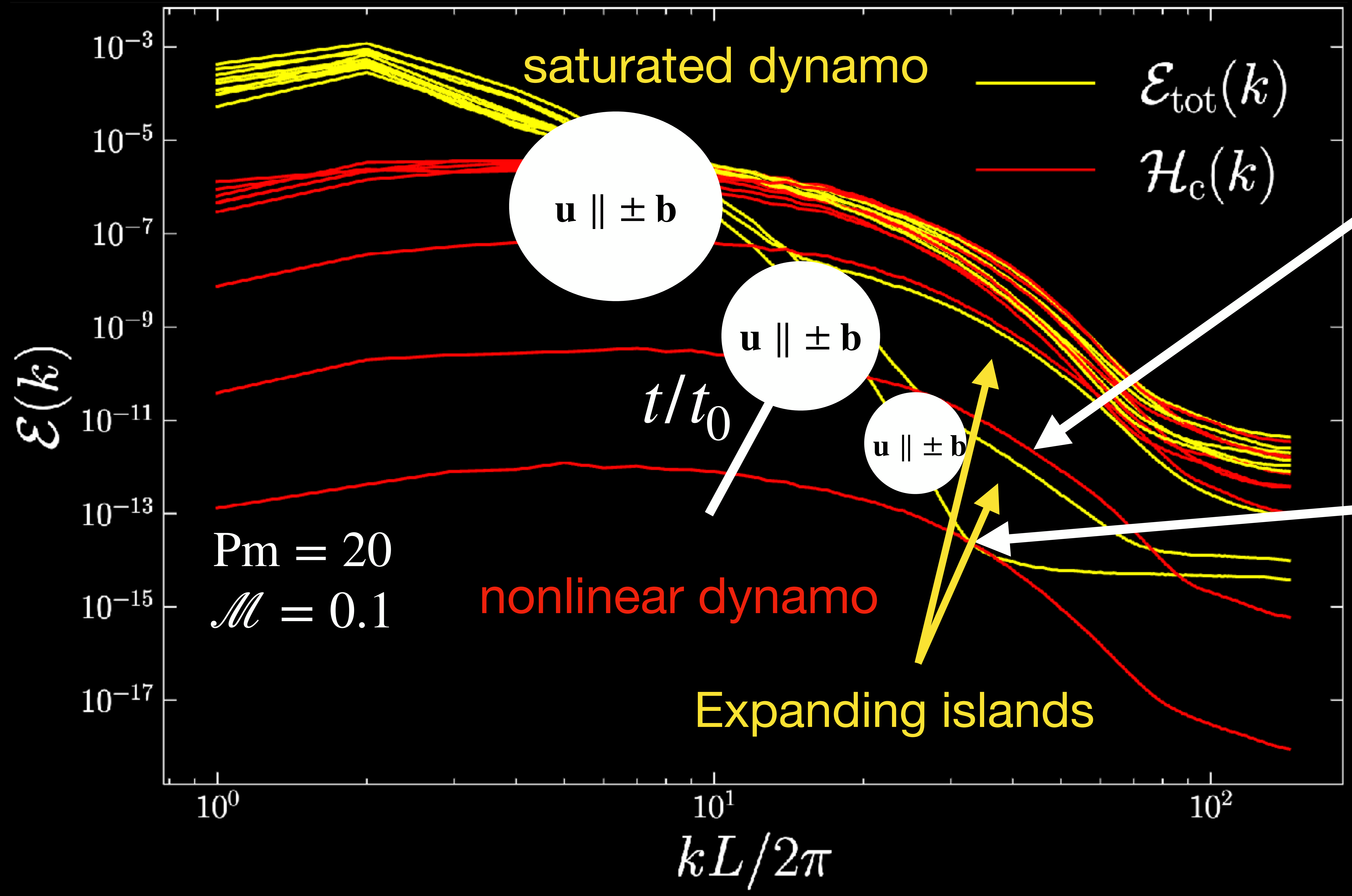


$$\mathcal{H}_c(k) = \mathbf{u}(k) \cdot \mathbf{b}^\dagger(k)$$

$\mathcal{H}_c$ grows $\Longrightarrow$ aligned scales develop where $\mathcal{H}_c > \mathcal{E}_{\rm tot} \Longrightarrow$ induction turns off

The dynamical saturation of the dynamo

Saturation through alignment



$$\mathscr{H}_c(k) = \mathbf{u}(k) \cdot \mathbf{b}^{\dagger}(k)$$

nonlinear dynamo
(backreaction turning on)

$\mathscr{H}_c$ grows $\Longrightarrow$ aligned scales develop where $\mathscr{H}_c > \mathscr{E}_{\mathrm{tot}} \Longrightarrow$ induction turns off

The dynamical saturation of the dynamo

Modified from Rincon (2019)

Saturation through alignment

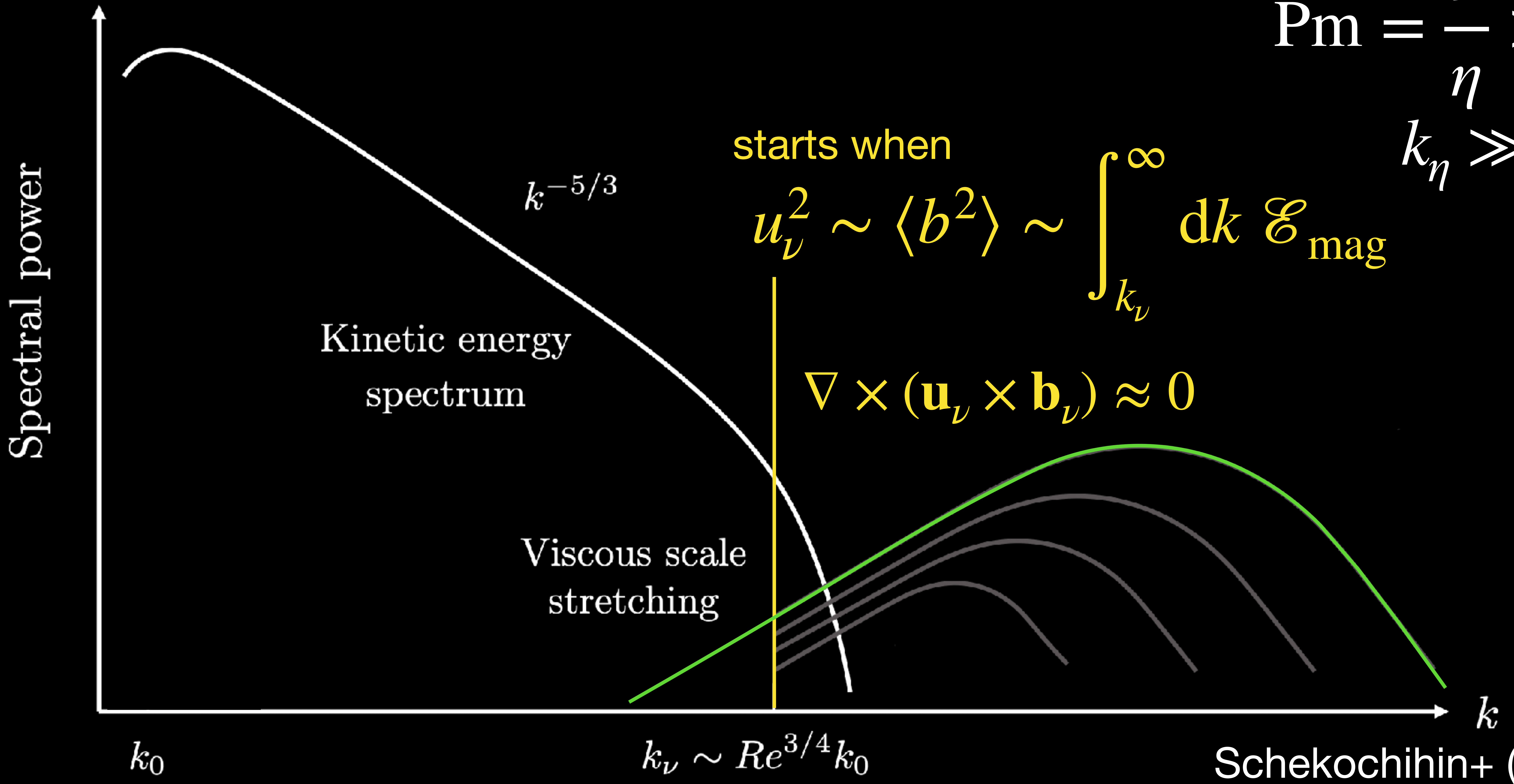
$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

$$k_\eta \gg k_\nu$$

starts when

$$u_\nu^2 \sim \langle b^2 \rangle \sim \int_{k_\nu}^{\infty} \mathrm{d}k\, \mathscr{E}_{\mathrm{mag}}$$

$$\nabla \times (\mathbf{u}_\nu \times \mathbf{b}_\nu) \approx 0$$



Schekochihin+ (2002)

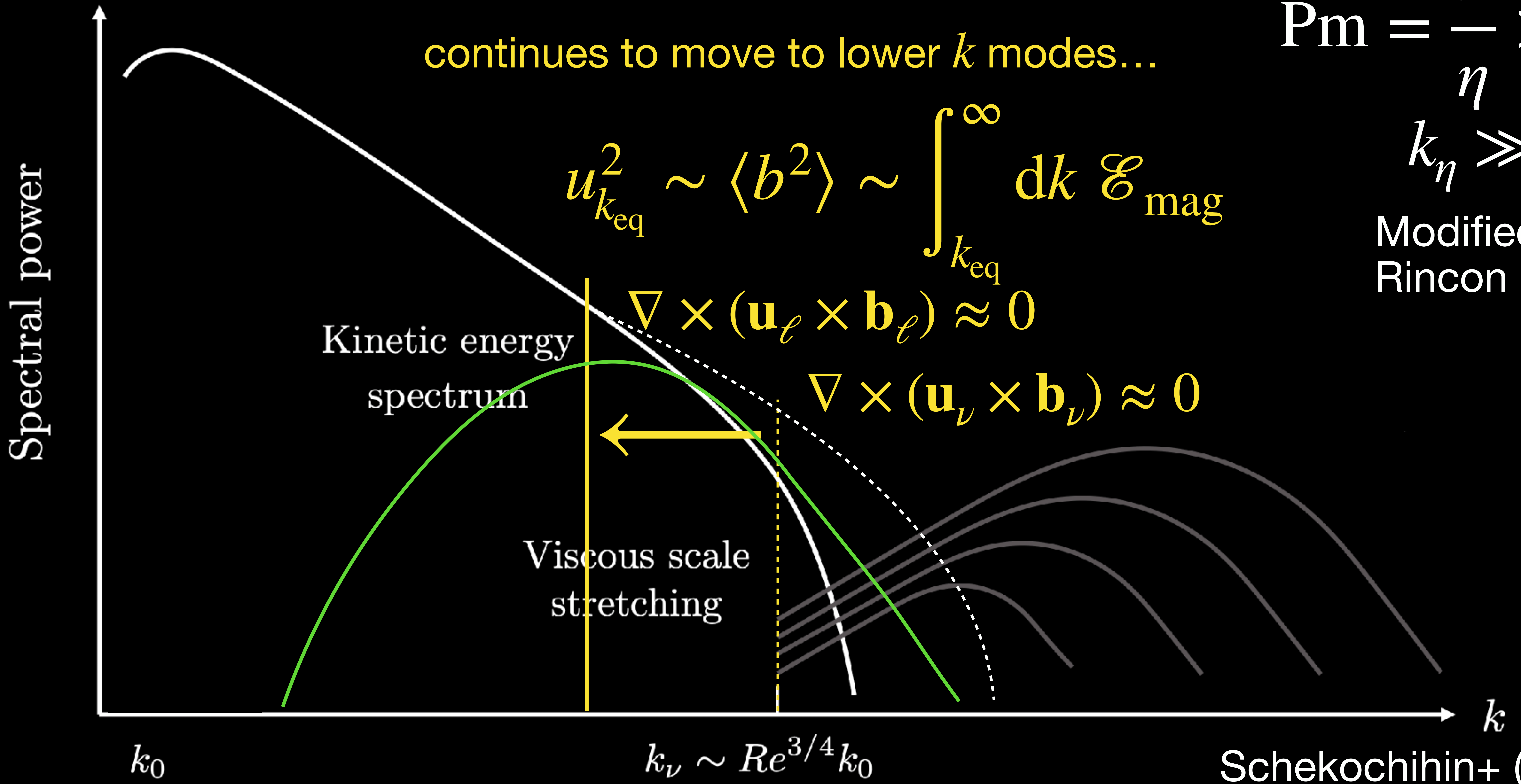
The dynamical saturation of the dynamo

Saturation through alignment

$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

$$k_\eta \gg k_\nu$$

Modified from Rincon (2019)



Schekochihin+ (2002)

The dynamical saturation of the dynamo

Saturation through alignment

$$u_{k_{\mathrm{eq}}}^2 \sim \langle b^2 \rangle \sim \int_{k_{\mathrm{eq}}}^{\infty} \mathrm{d}k \; \mathscr{E}_{\mathrm{mag}}$$

$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

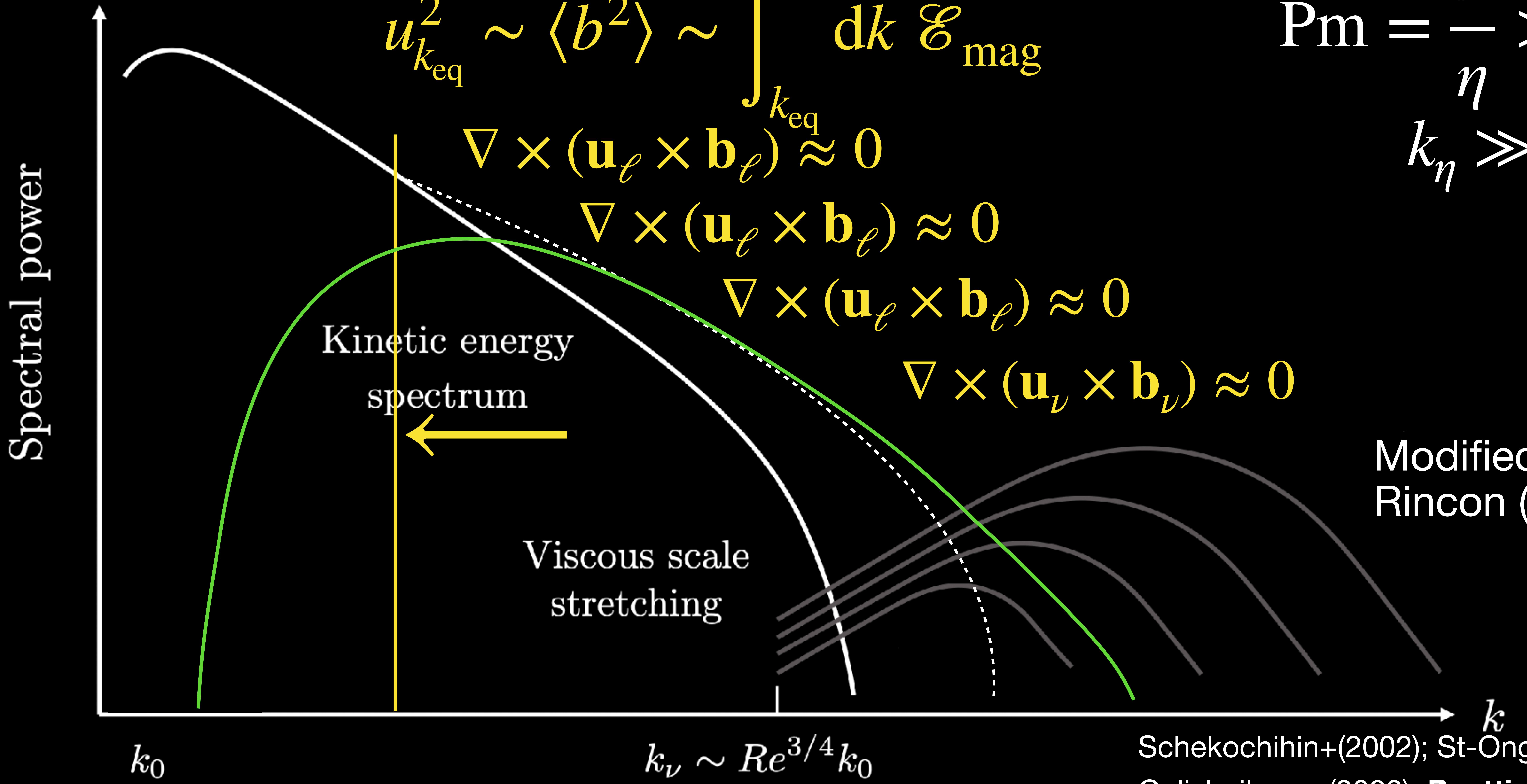
$$k_\eta \gg k_\nu$$

$$\nabla \times (\mathbf{u}_\ell \times \mathbf{b}_\ell) \approx 0$$

$$\nabla \times (\mathbf{u}_\ell \times \mathbf{b}_\ell) \approx 0$$

$$\nabla \times (\mathbf{u}_\ell \times \mathbf{b}_\ell) \approx 0$$

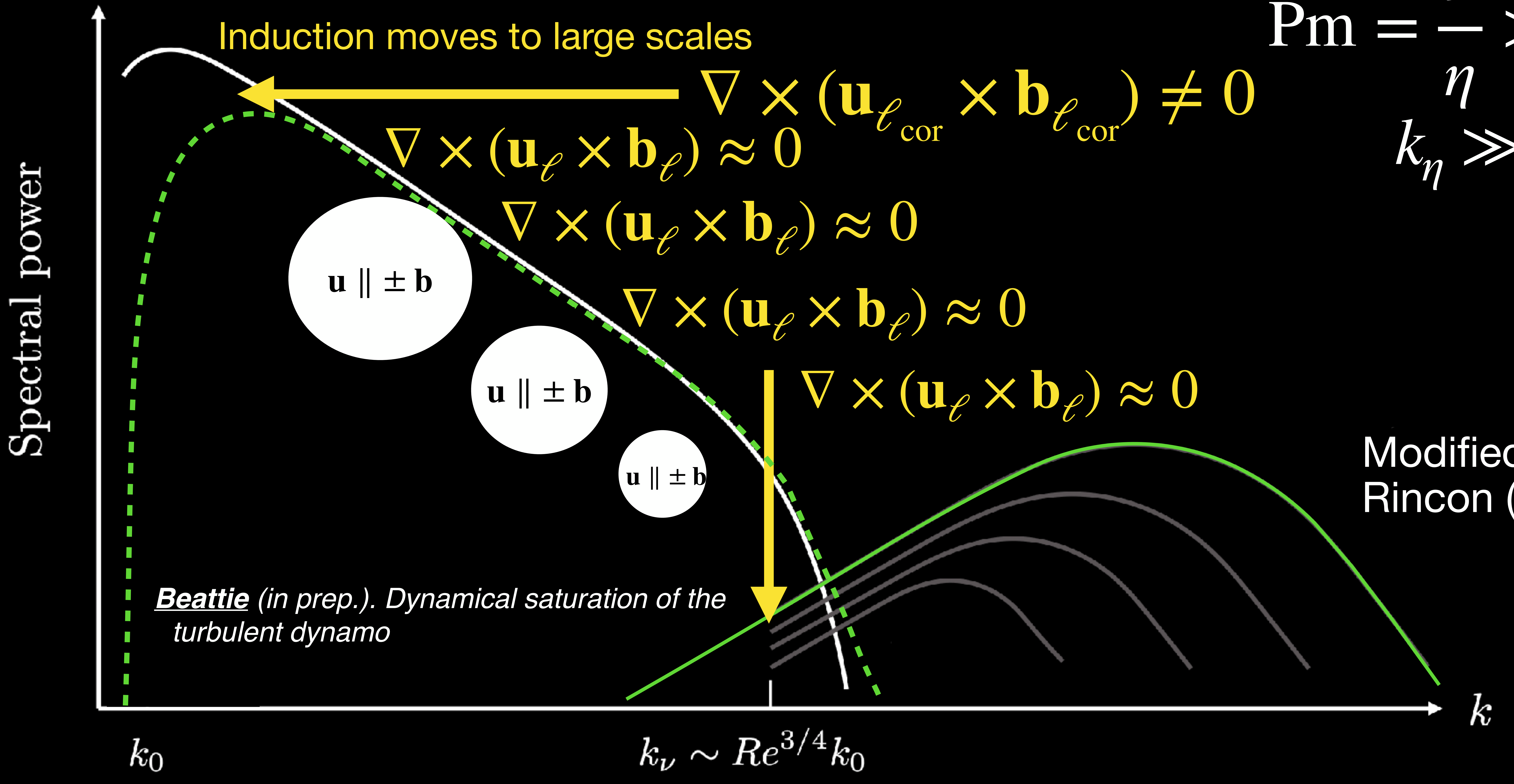
$$\nabla \times (\mathbf{u}_\nu \times \mathbf{b}_\nu) \approx 0$$



Modified from Rincon (2019)

Schekochihin+(2002); St-Onge+(2020)
Galishnikova+(2023); **Beattie**+(2024)

The dynamical saturation of the dynamo
Saturation through alignment



$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

$$k_\eta \gg k_\nu$$

Modified from Rincon (2019)

Energy spectra: the disparate lives of u and b fluctuations

Beattie+ 2025 (*Nature Astronomy*). The spectrum of magnetized turbulence in the interstellar medium

Conclusions for this early morning talk

1. Kinematic and nonlinear theory at the level of integral energy work really well, with subtle but expected changes between the subsonic and supersonic regimes.

2. The scale-dependent alignment, motivated for Alfvénic nonlinearity suppression, ends up playing an extremely important and generic role in saturating the turbulent dynamo independently of resistive processes.

    1. Potentially extremely universal, setting the saturation across alll of the plasmas I introduced at the start of the talk!

    2. Potentially can be probed in the laboratory — let's chat!

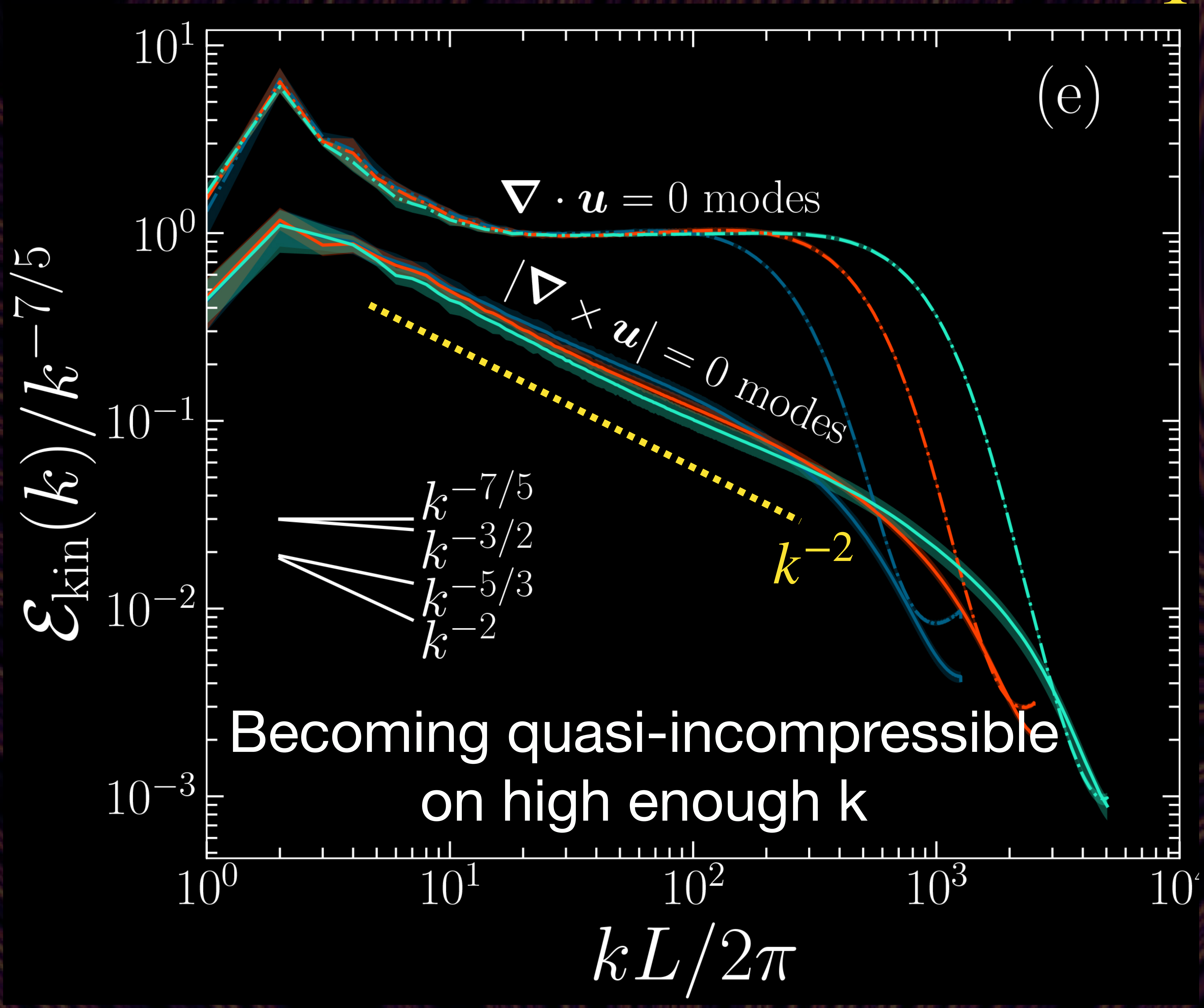
Thanks, questions?

james.beattie@princeton.
@astro_magnetism

Extra slides

The supersonic spectrum

$$\mathcal{E}(k) \sim k^{-2},\ k \leq k_{\mathrm{eq}}$$

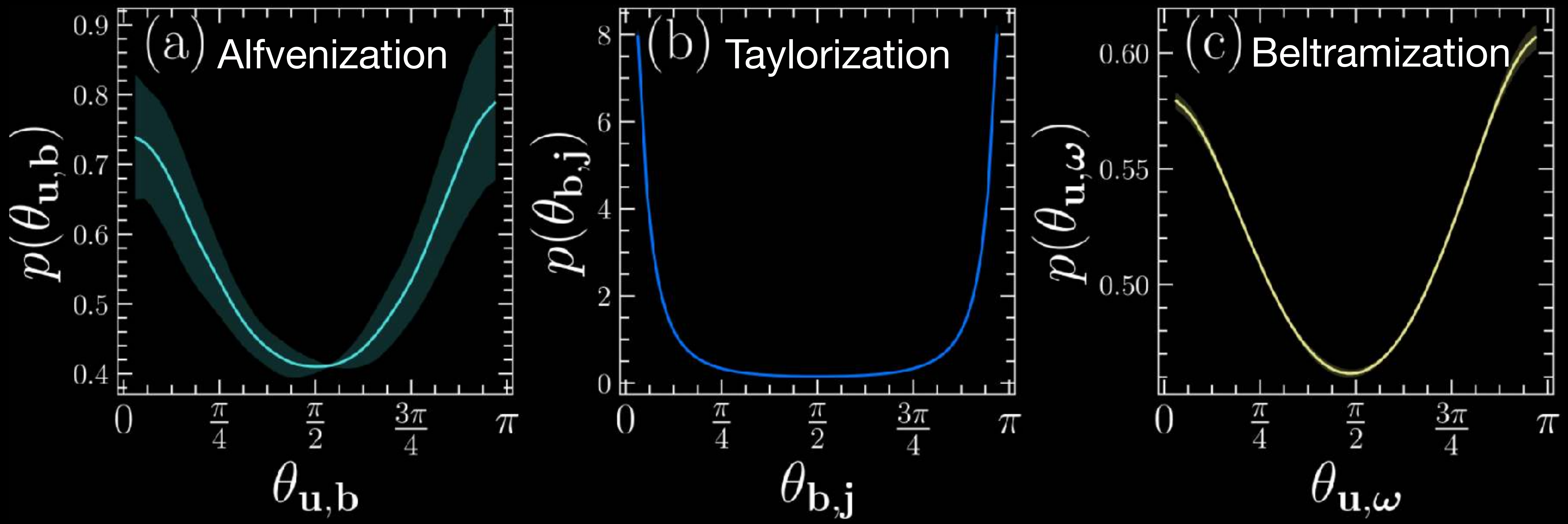


Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

But even more alignment than just u and b

Searching to weaken the nonlinearities

$$\boldsymbol{\omega} = \nabla \times \mathbf{u}$$

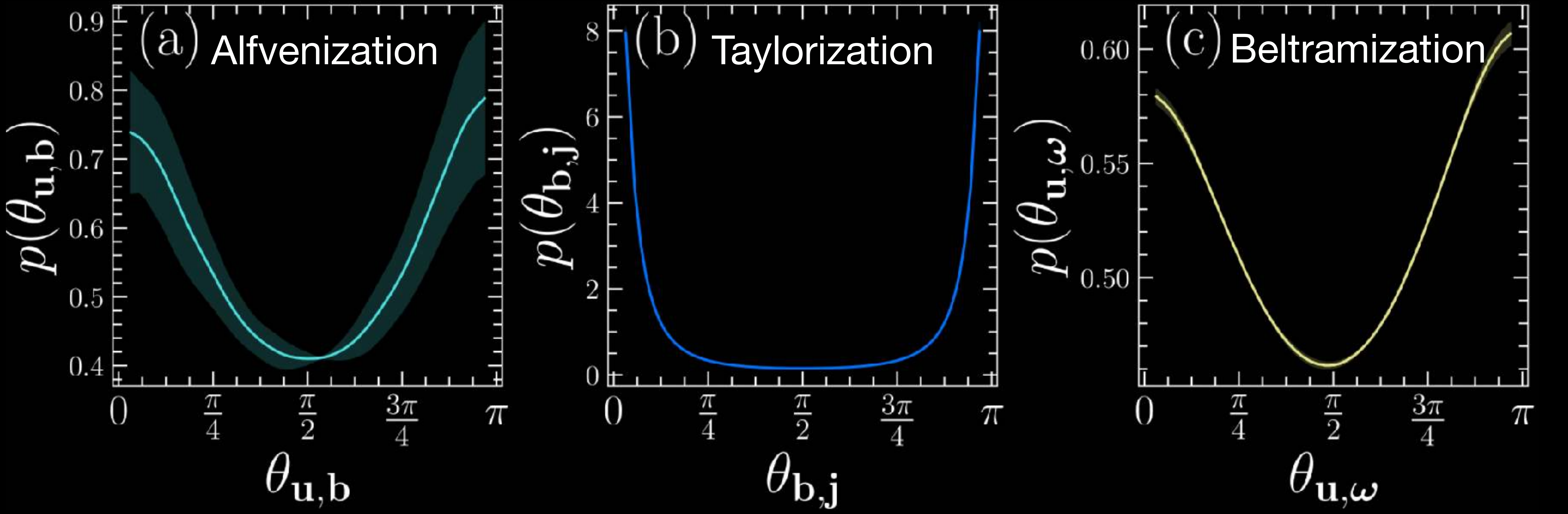


$$\mathbf{u} \propto \mathbf{b} \propto \mathbf{j} \propto \boldsymbol{\omega}$$

Beattie & Bhattacharjee *(submitted PRL.). Scale dependent alignment in compressible magnetohydrodynamic turbulence*

But even more alignment than just u and b

Searching to weaken the nonlinearities



$\nabla \times (\mathbf{u} \times \mathbf{b})$ — Induction

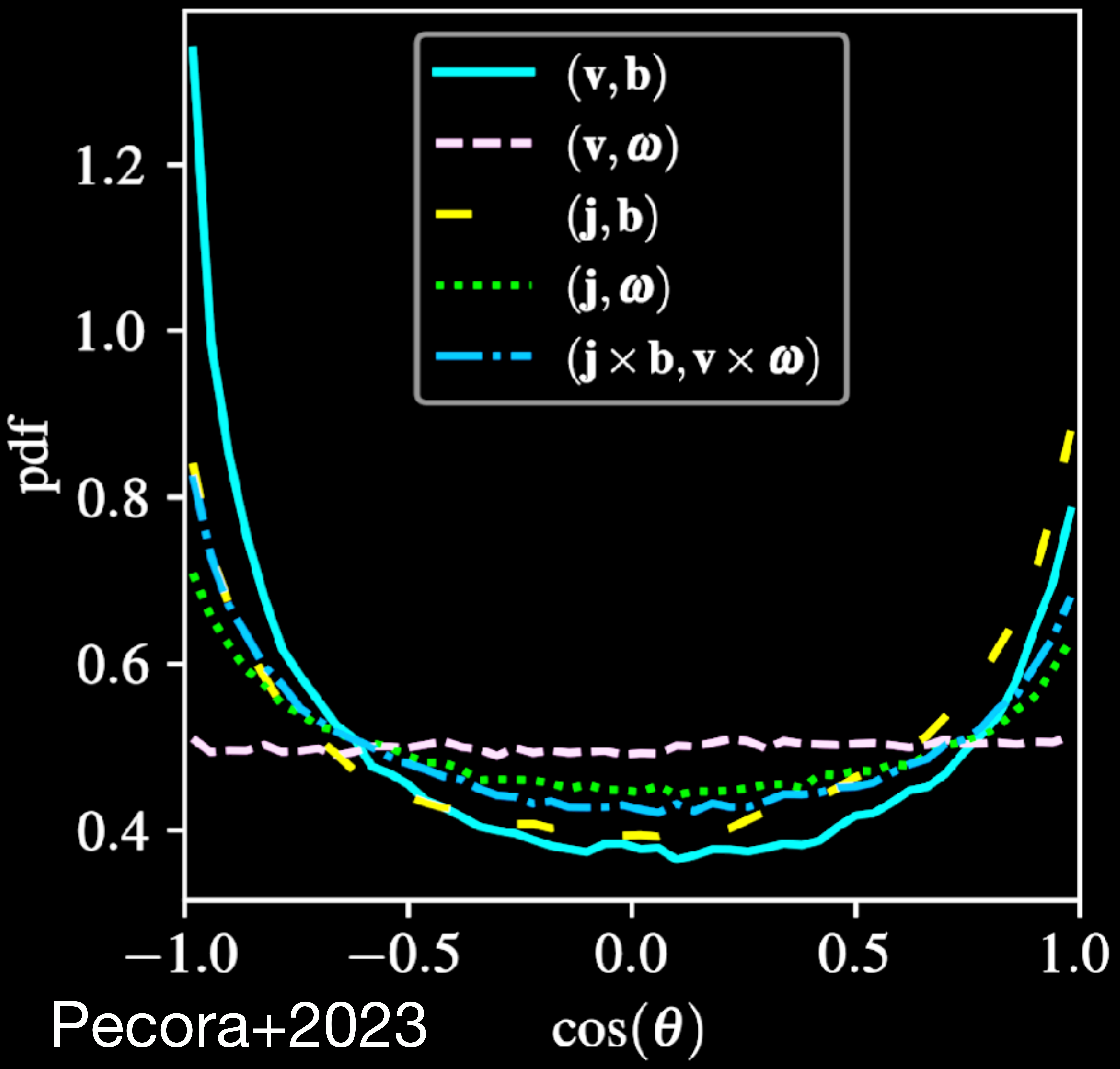
$\mathbf{j} \times \mathbf{b}$ — Lorentz force

$\nabla \cdot (\mathbf{u} \otimes \mathbf{u}) \sim \boldsymbol{\omega} \times \mathbf{u}$ — Reynolds nonlinearity

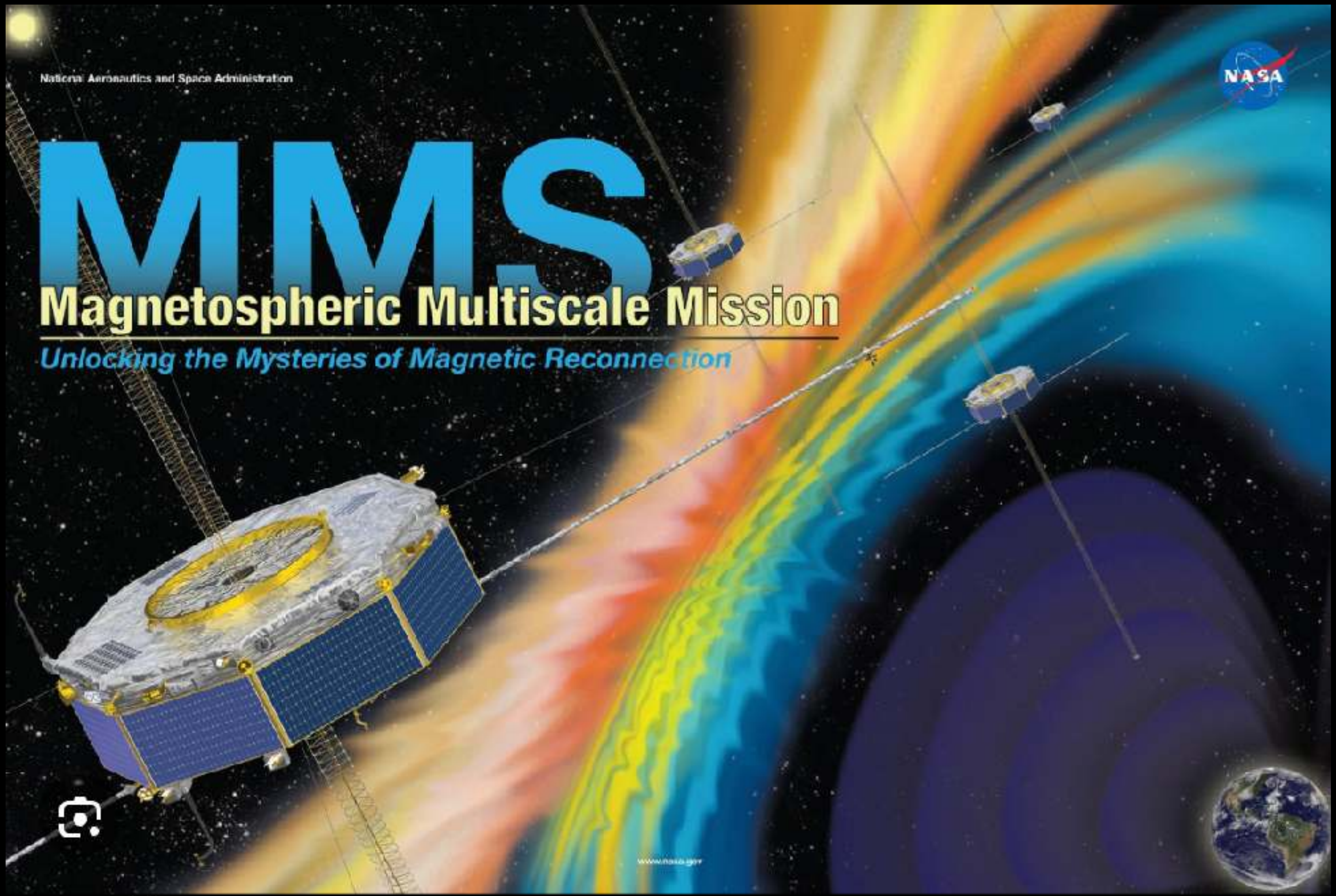
Beattie & Bhattacharjee *(submitted PRL.). Scale dependent alignment in compressible magnetohydrodynamic turbulence*

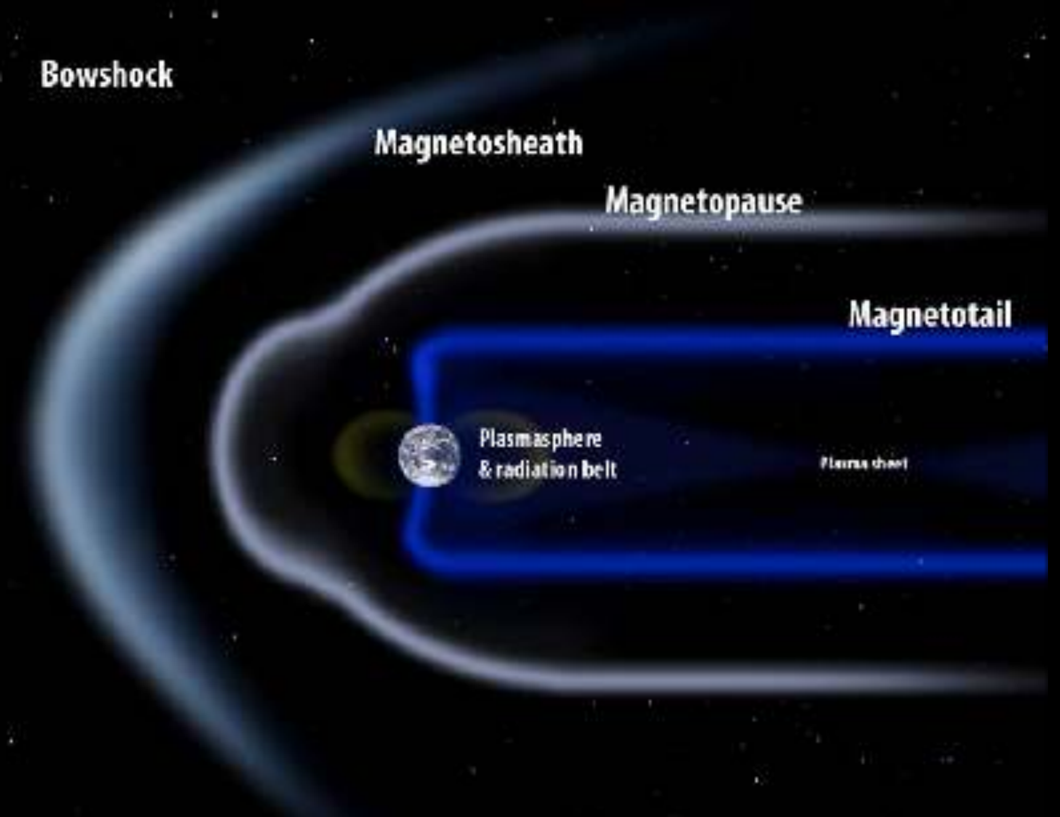
But even more alignment than just u and b

Searching to weaken the nonlinearities



Pecora+2023

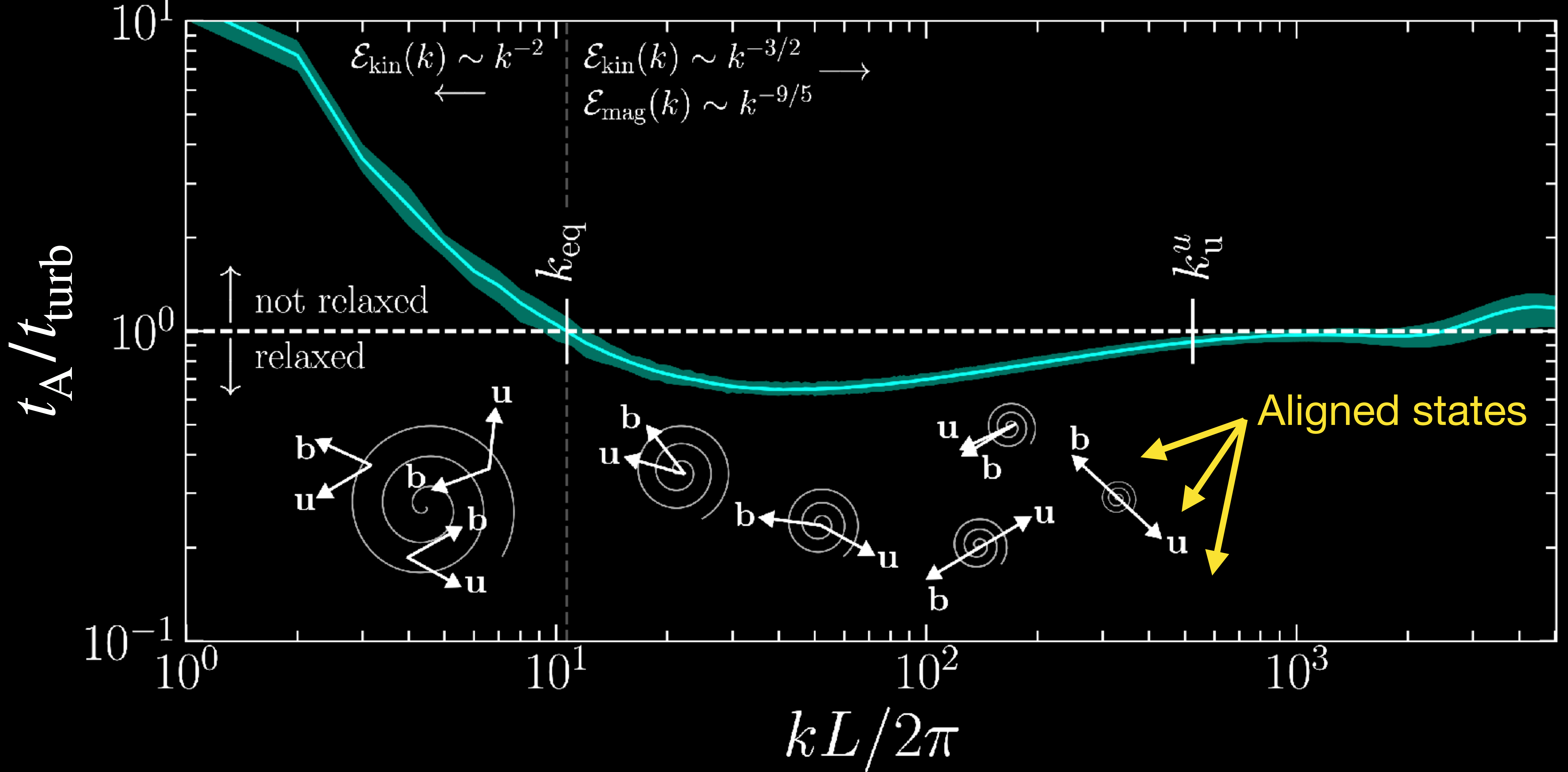




Highly-aligned states in magneotsheath turbulence!

The dynamical saturation of the dynamo

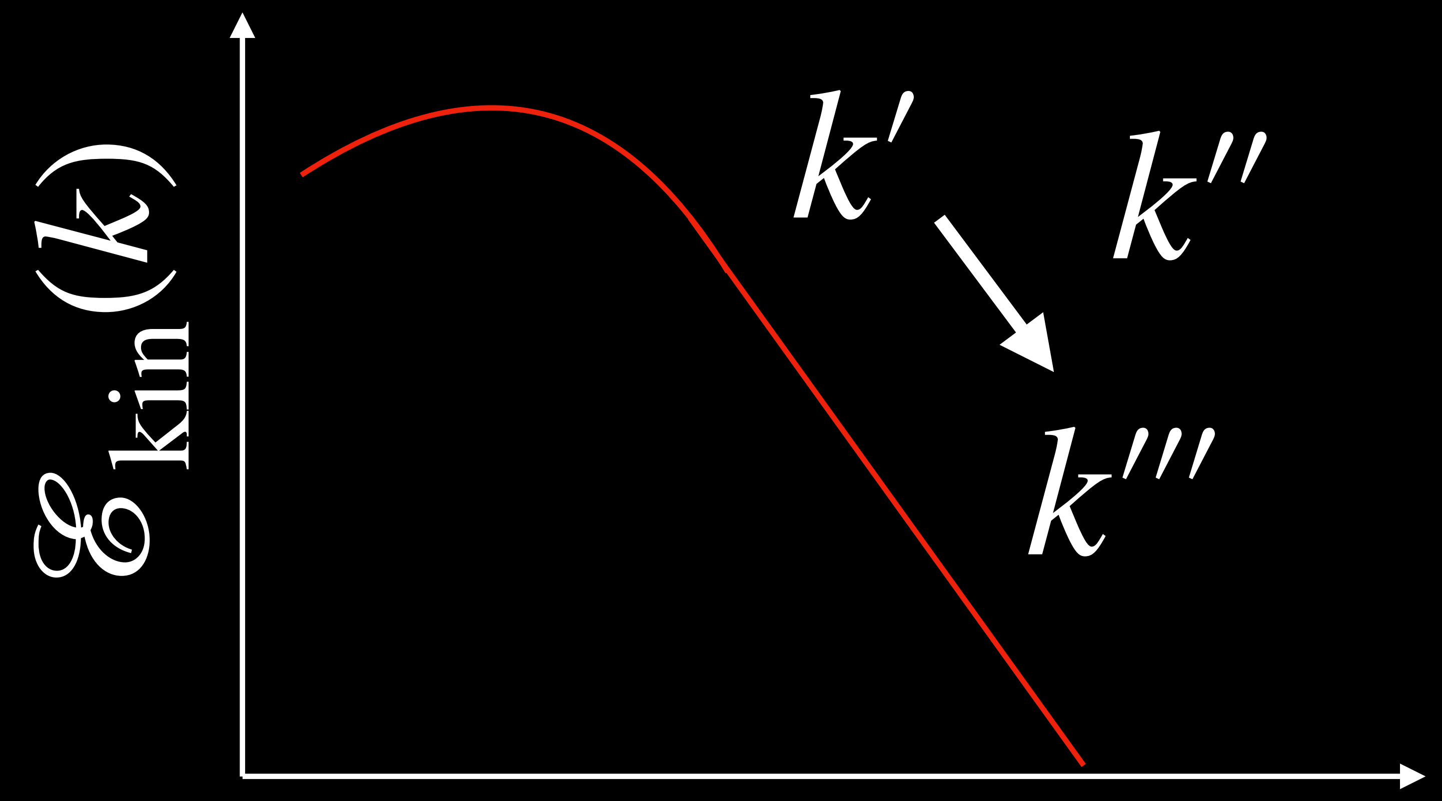
Saturation through alignment

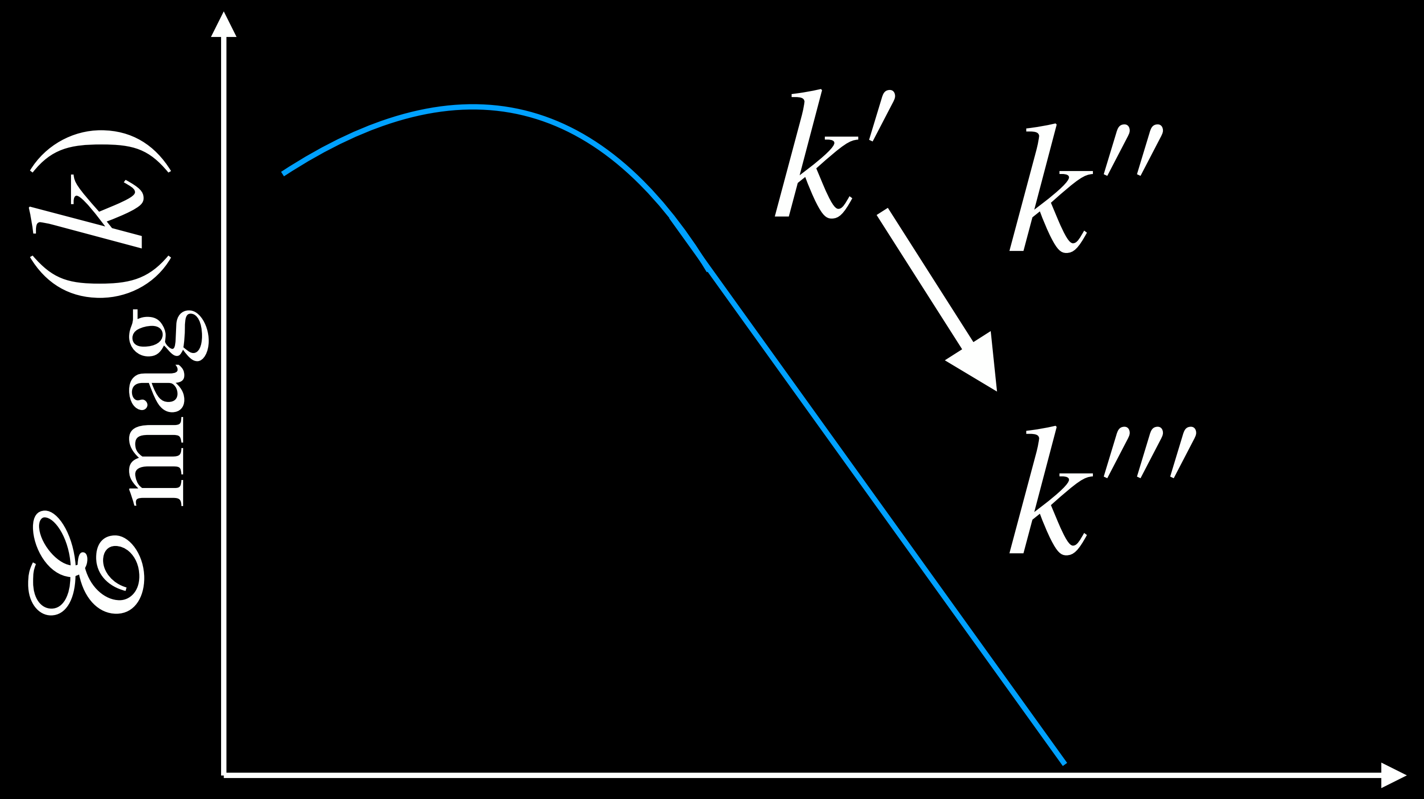


Energy spectra: the disparate lives of u and b fluctuations

Beattie+ 2025 (*Nature Astronomy*). The spectrum of magnetized turbulence in the interstellar medium

Energy spectra: the disparate lives of u and b fluctuations





$$\mathbf{w} = \sqrt{\rho}\mathbf{u}$$

$$T_{uu}(k', k''' \,|\, k'') = \int dV \; \overbrace{\mathbf{w}''' \otimes \mathbf{u}'' : \nabla \otimes \mathbf{w}' + \frac{1}{2}\mathbf{w}' \otimes \mathbf{w}''' : (\nabla \cdot \mathbf{u}'')}^{\text{kinetic cascade terms}}$$

$$T_{bb}(k', k''' \,|\, k'') = \int dV \; \overbrace{\mathbf{b}''' \otimes \mathbf{u}'' : \nabla \otimes \mathbf{b}' + \frac{1}{2}\mathbf{b}' \otimes \mathbf{b}''' : (\nabla \cdot \mathbf{u}'')}^{\text{magnetic cascade terms}}$$

C++ tool with FFTW backend

as defined in Grete + (2017)

Energy spectra: the disparate lives of u and b fluctuations

Beattie+ 2025 (*Nature Astronomy*). The spectrum of magnetized turbulence in the interstellar medium

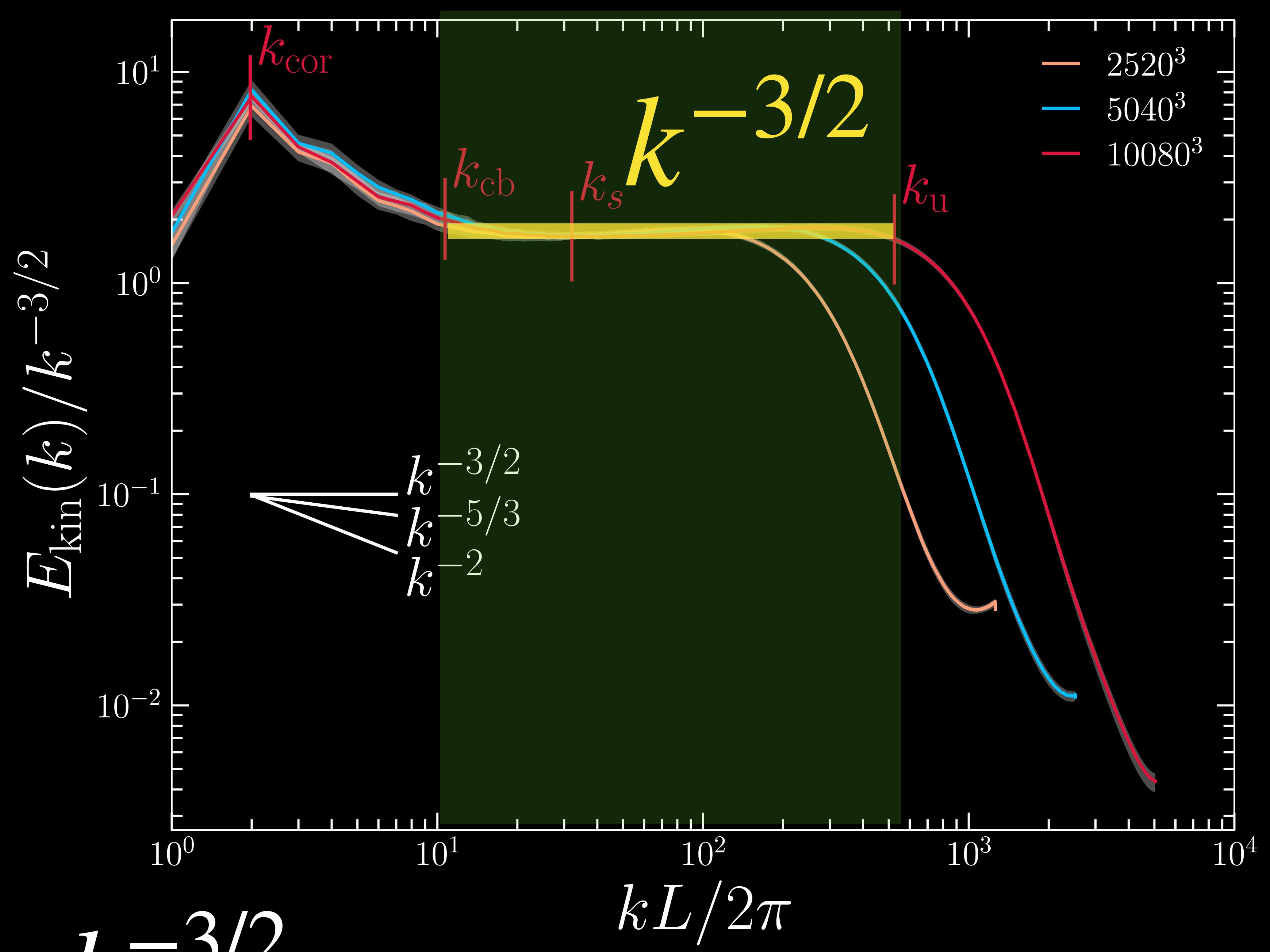
Putting it all together to understand the kinetic spectrum

Consider

$$E_{\rm kin} \propto \Pi_{uu}^{2/3}\theta_{u,b}^{-2/3}k^{-5/3}, \qquad \theta_{u,b}(\ell) \sim \ell^{1/8}$$

which means

$$E_{\rm kin} \propto \Pi_{uu}^{2/3}k^{-19/12} \propto k^{1.58},$$



inconsistent… not strong enough to suppress $k^{-5/3} \rightarrow k^{-3/2}$

We find

$$\theta(\ell_\perp) \sim \ell_\perp^{1/8}$$

scale-dependent alignment, but not Boldyrev (2006).

Crazy idea… what if….

$$\Pi_{uu}(\ell) \propto \ell^{\beta}$$

Very anti-Kolmogorov. Let's measure.

Putting it all together to understand the kinetic spectrum

Consider

$$E_{\mathrm{kin}} \propto \Pi_{uu}^{2/3} \theta_{u,b}^{-2/3} k^{-5/3},$$

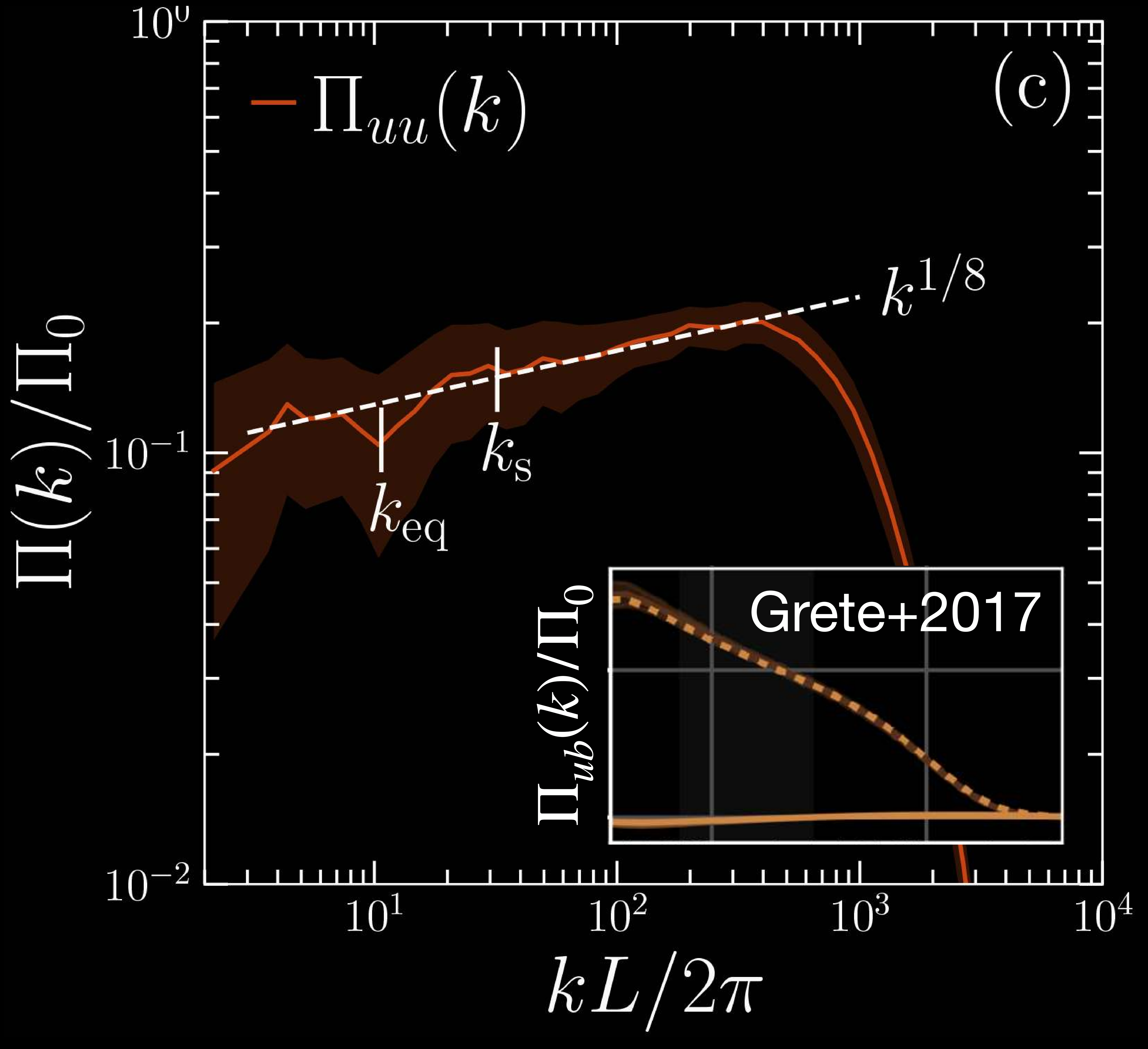
with

$$\theta_{u,b}(\ell) \sim \ell^{1/8}, \quad \Pi_{uu}(k) \propto k^{1/8}$$

implies

$$E_{\mathrm{kin}} \propto k^{-3/2},$$

The dynamo changes the nature of the cascade via scale-dependent ub fluxes.

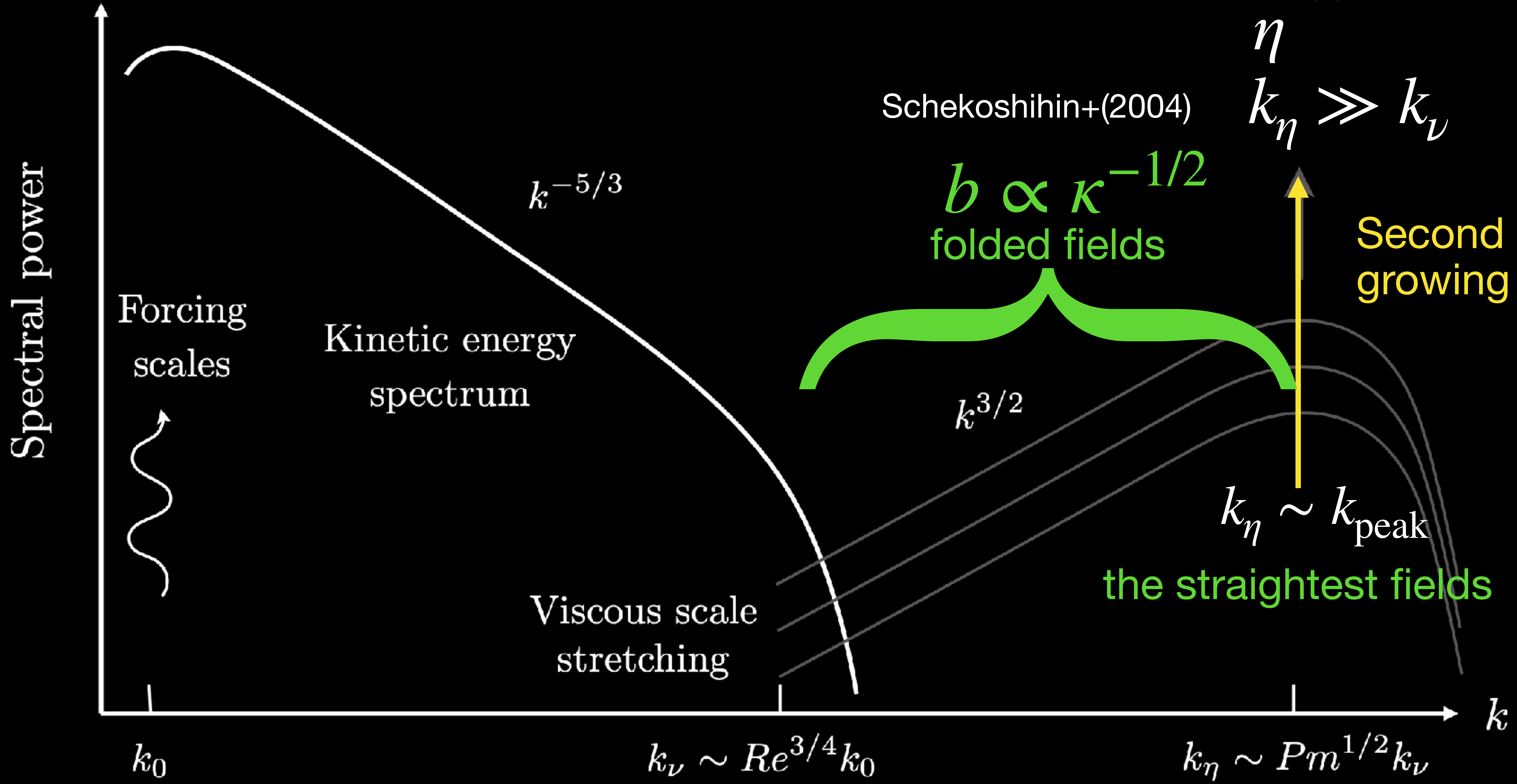


Small-scale dynamo
Folded and reversed fields

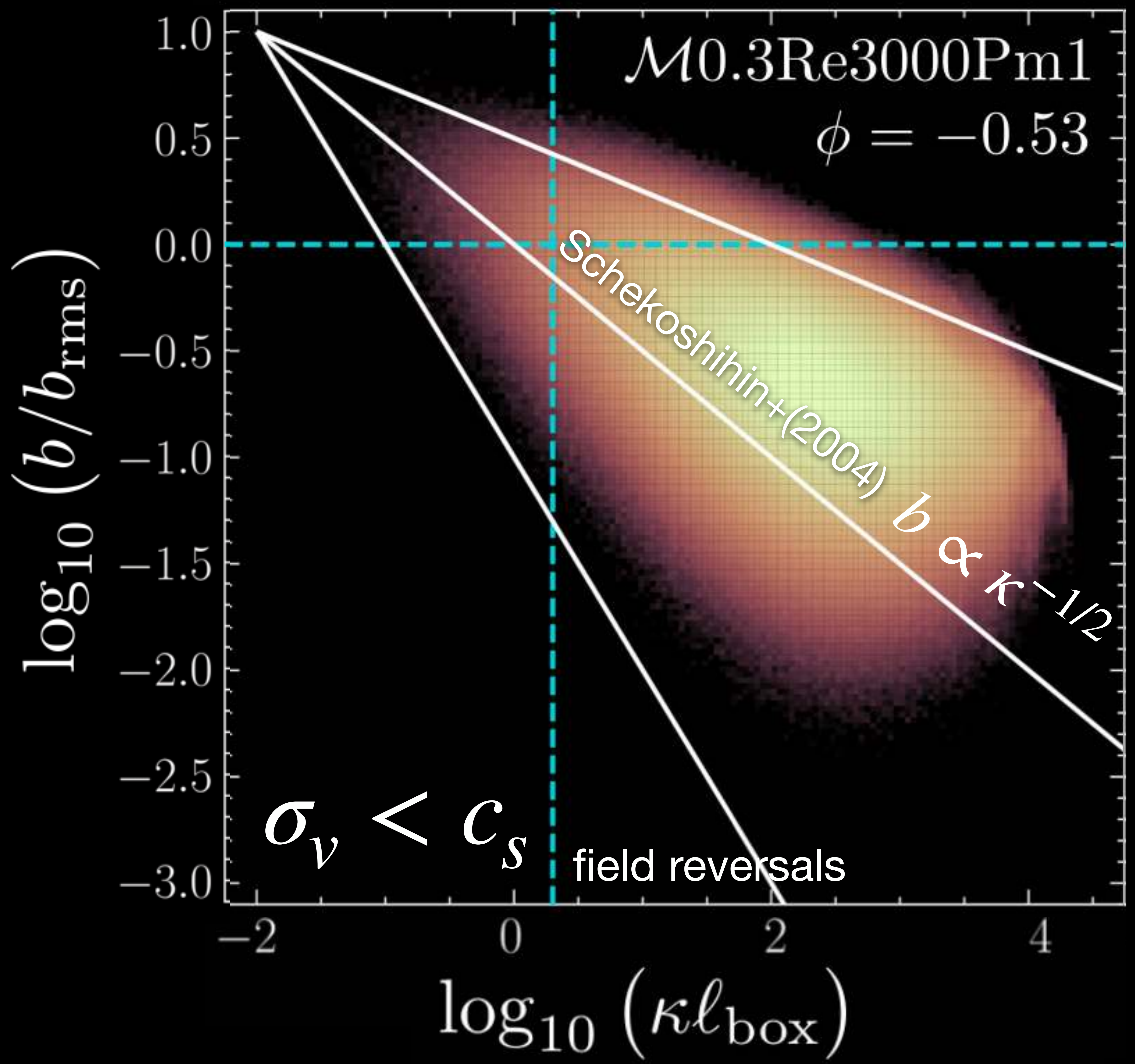
Modified from Rincon (2019)

$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

Schekoshihin+(2004) $k_\eta \gg k_\nu$

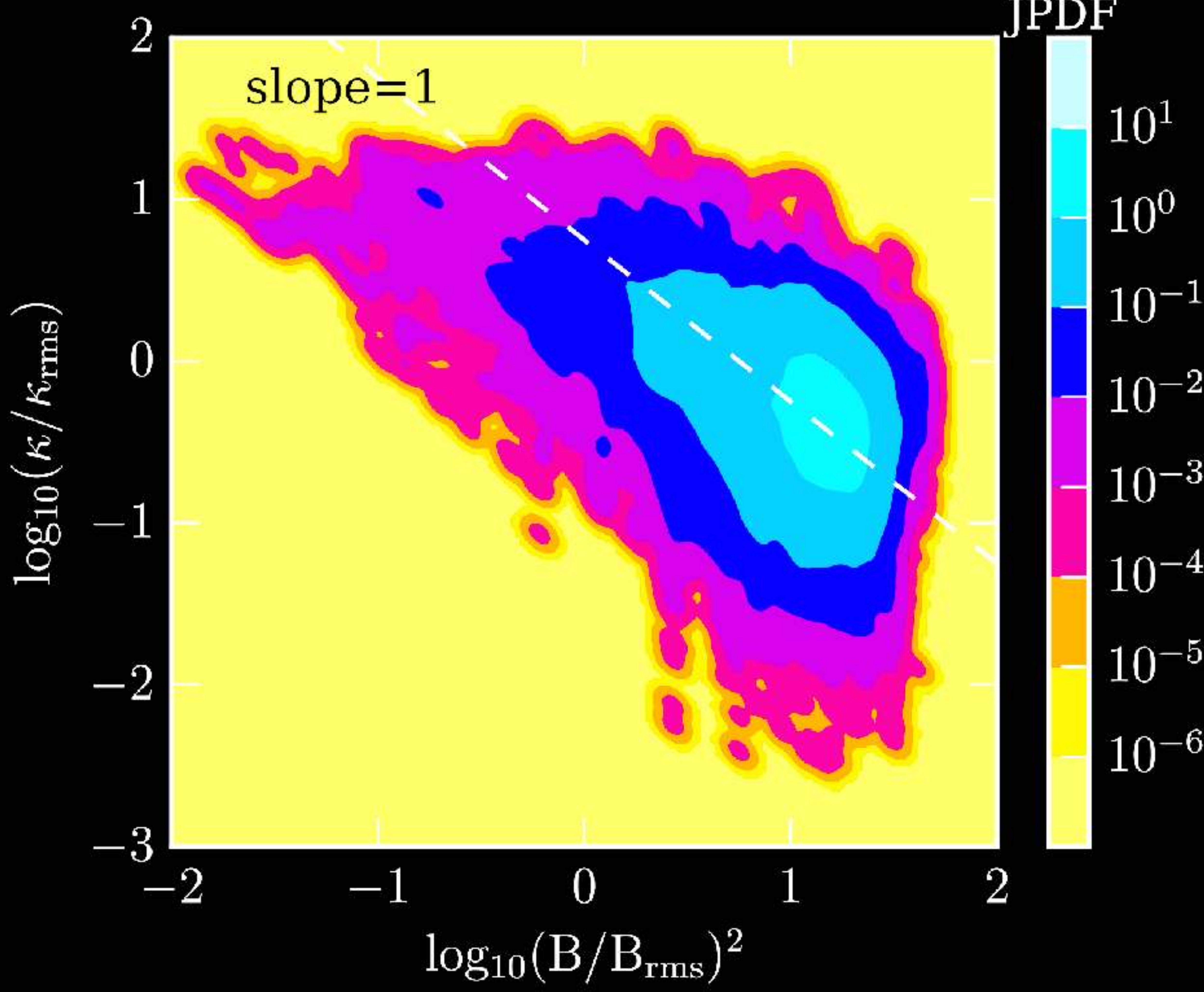


Small-scale dynamo
Folded and reversed fields



Kriel, **<u>Beattie</u>**+ (2025; MNRAS). *Fundamental scales II: the kinematic stage of the supersonic dynamo*

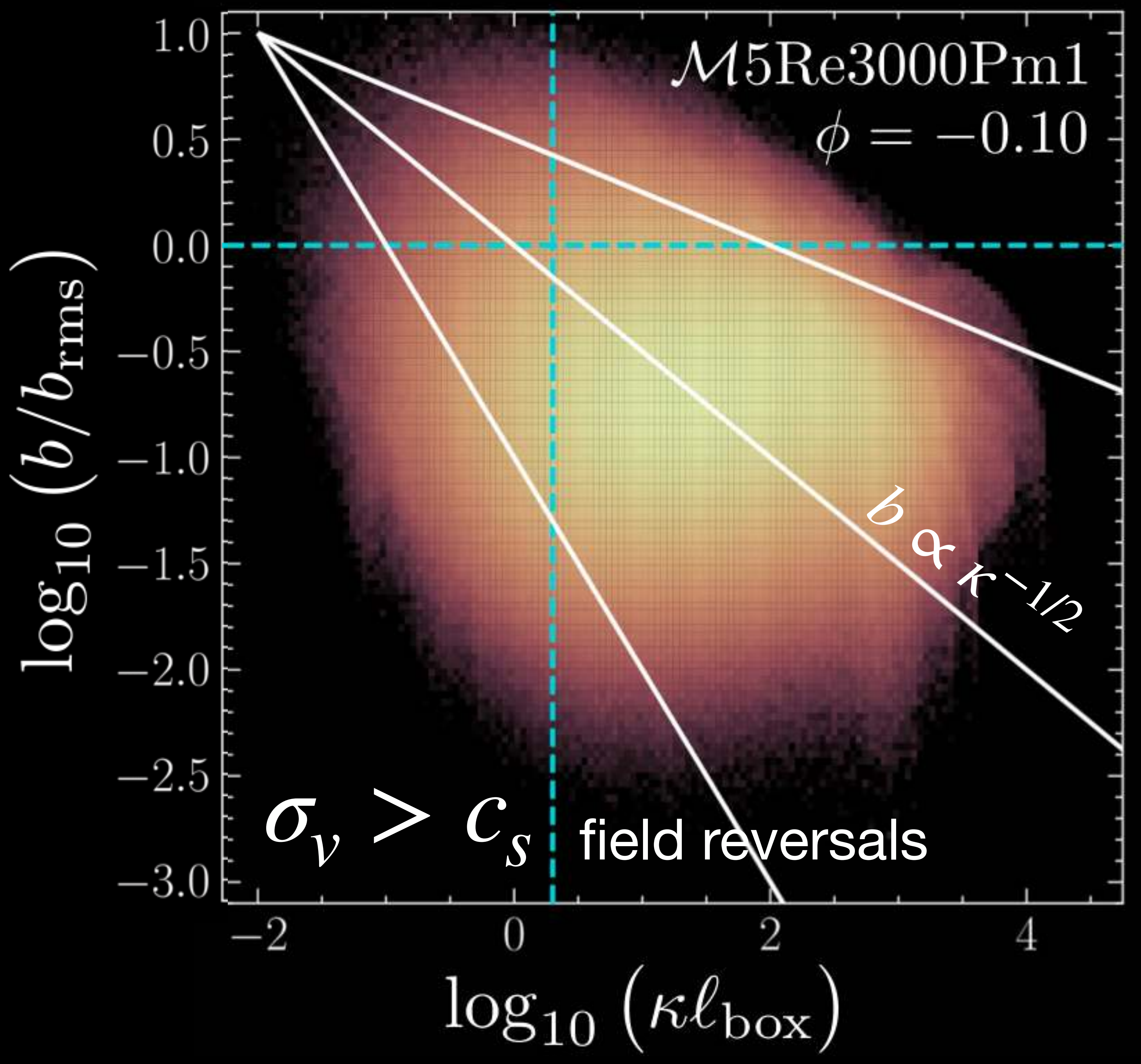
In-situ measurements of Earth's magnetosheath



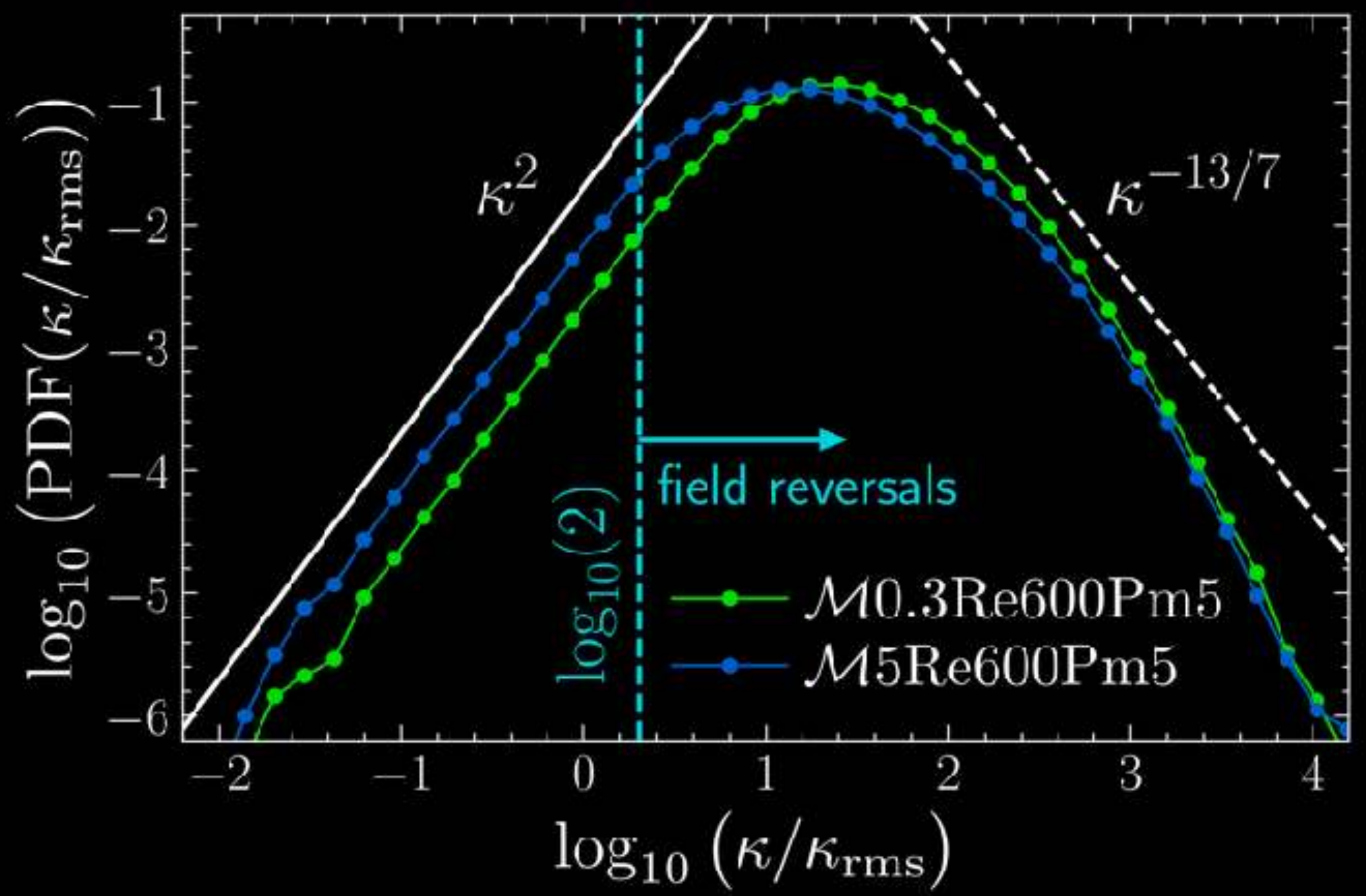
Bandyopadhyay+ (2024). *In situ measurement of curvature of magnetic field in turbulent space plasmas: a statistical study*

Turbulent dynamo

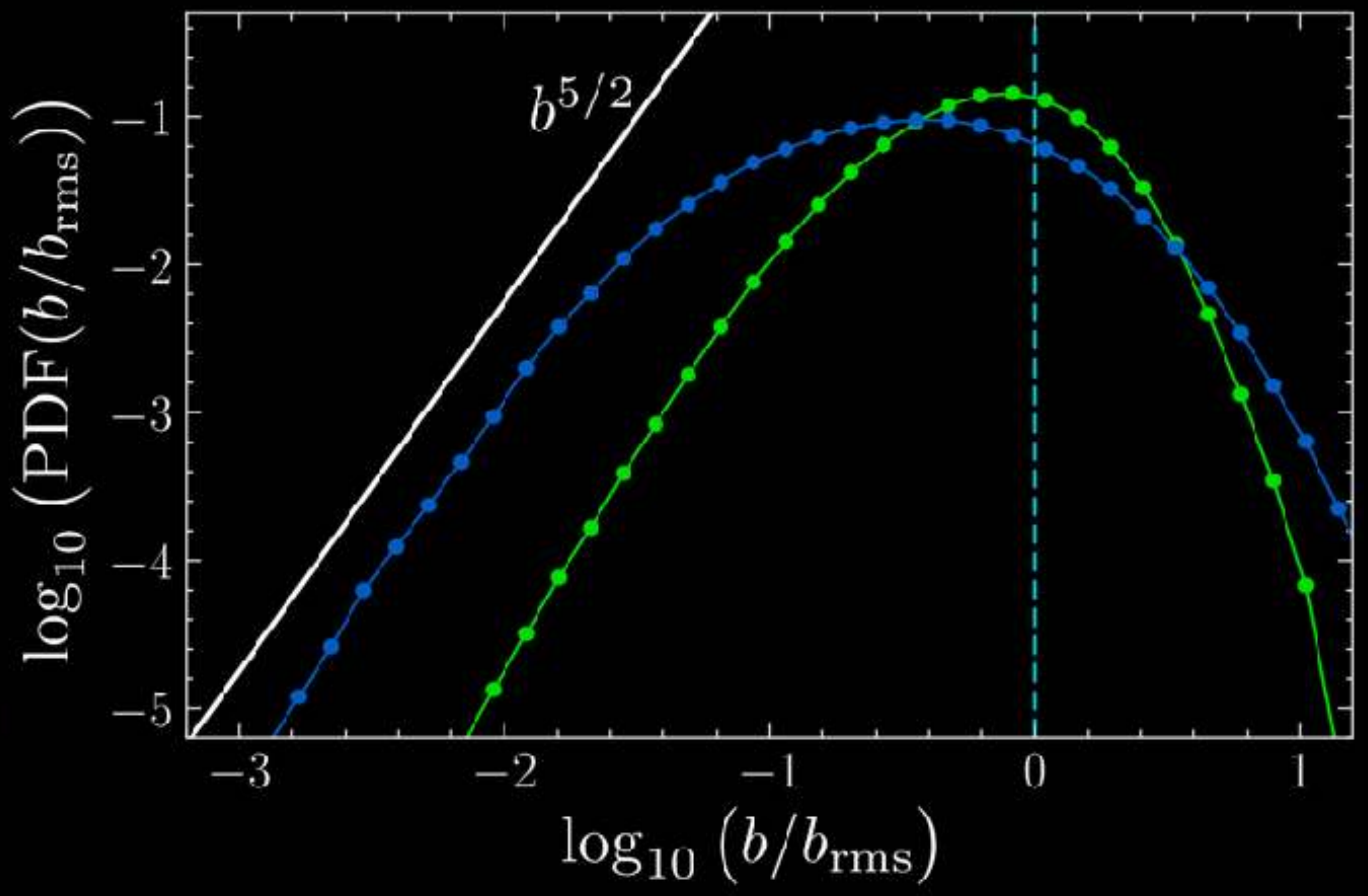
kinematic regime: folded fields



Folded field retained

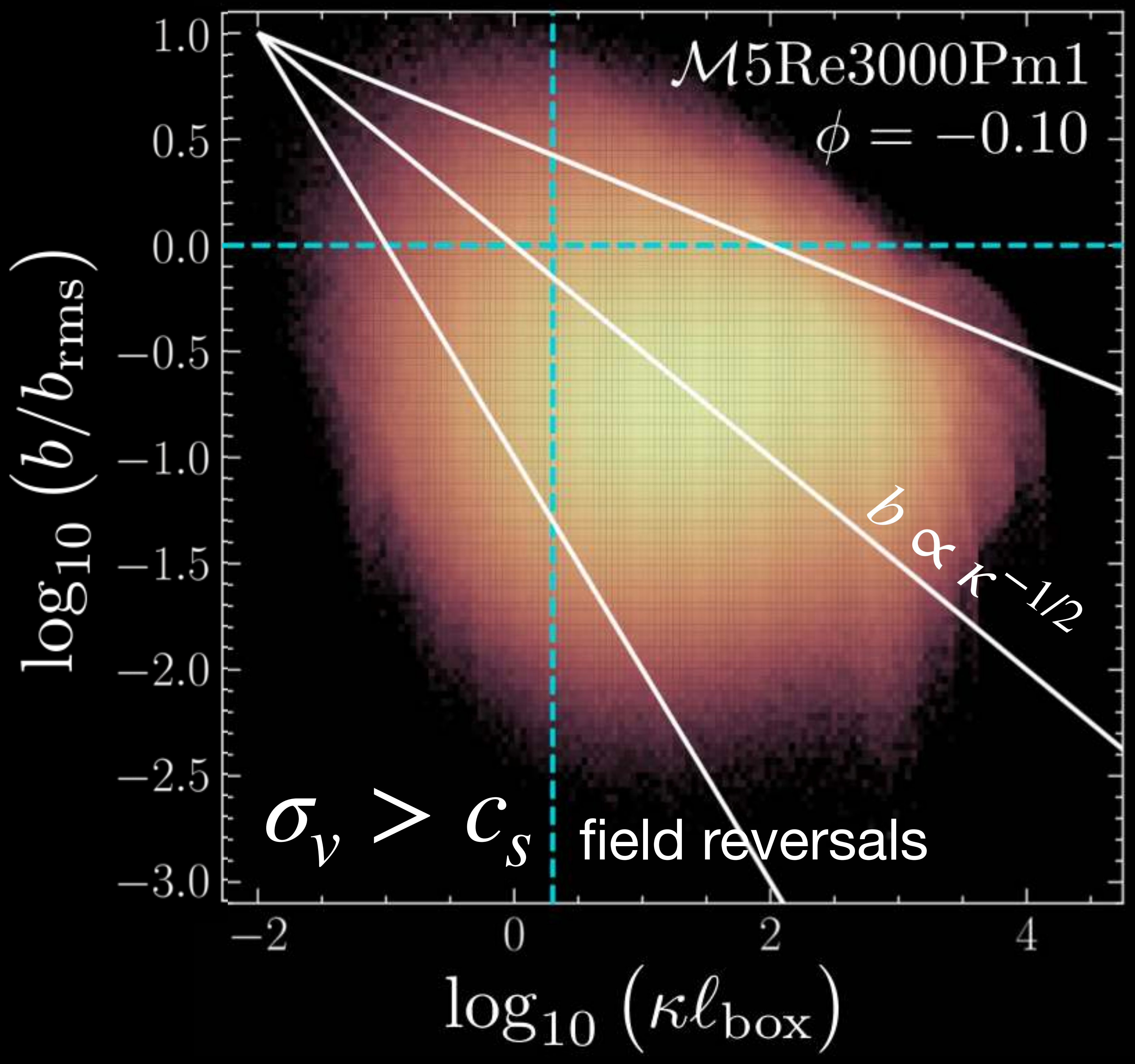


Amplitudes change (flux compressions)

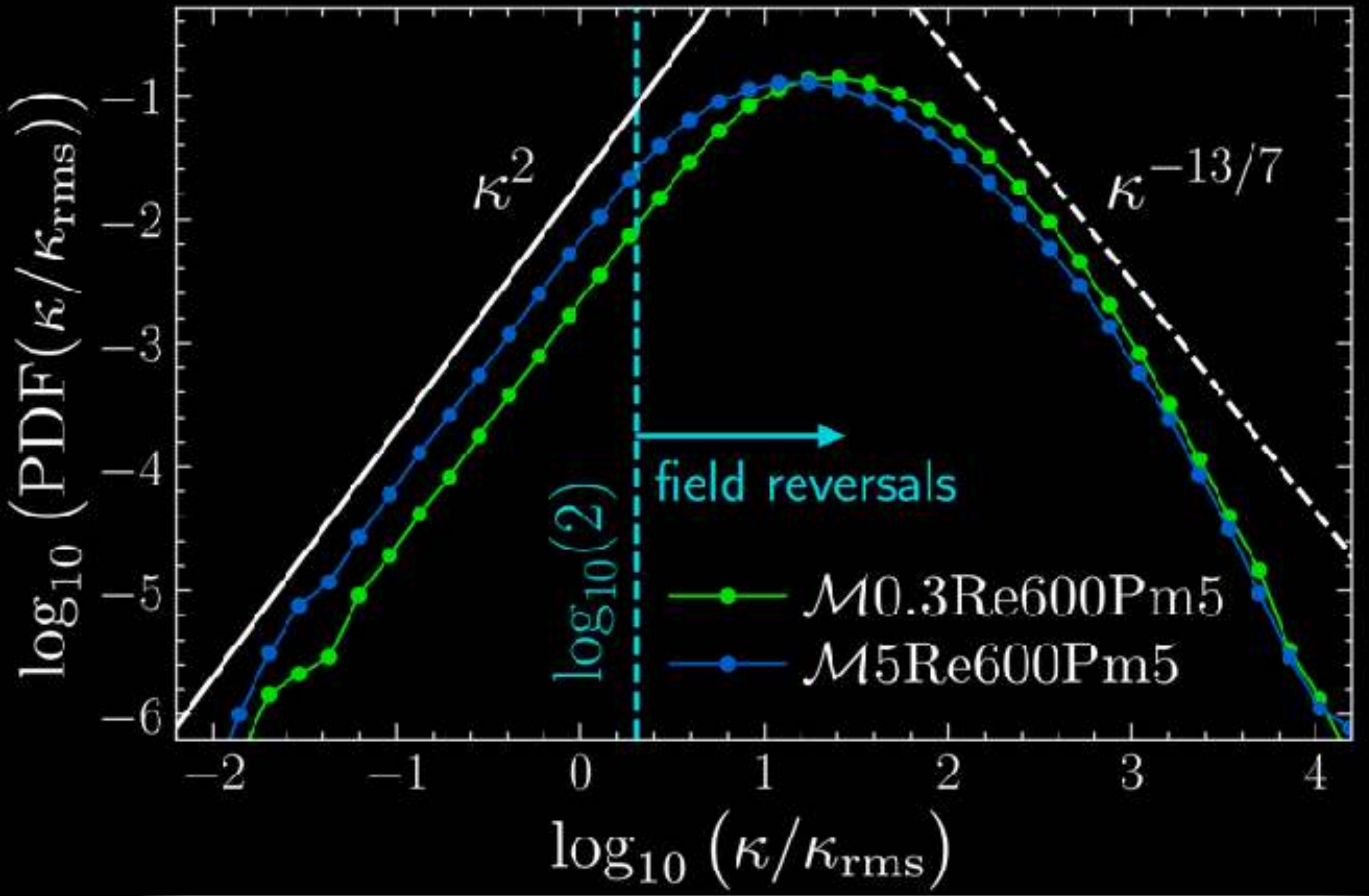


Kriel, **Beattie**+ (2025). *Fundamental scales II: the kinematic stage of the supersonic dynamo*

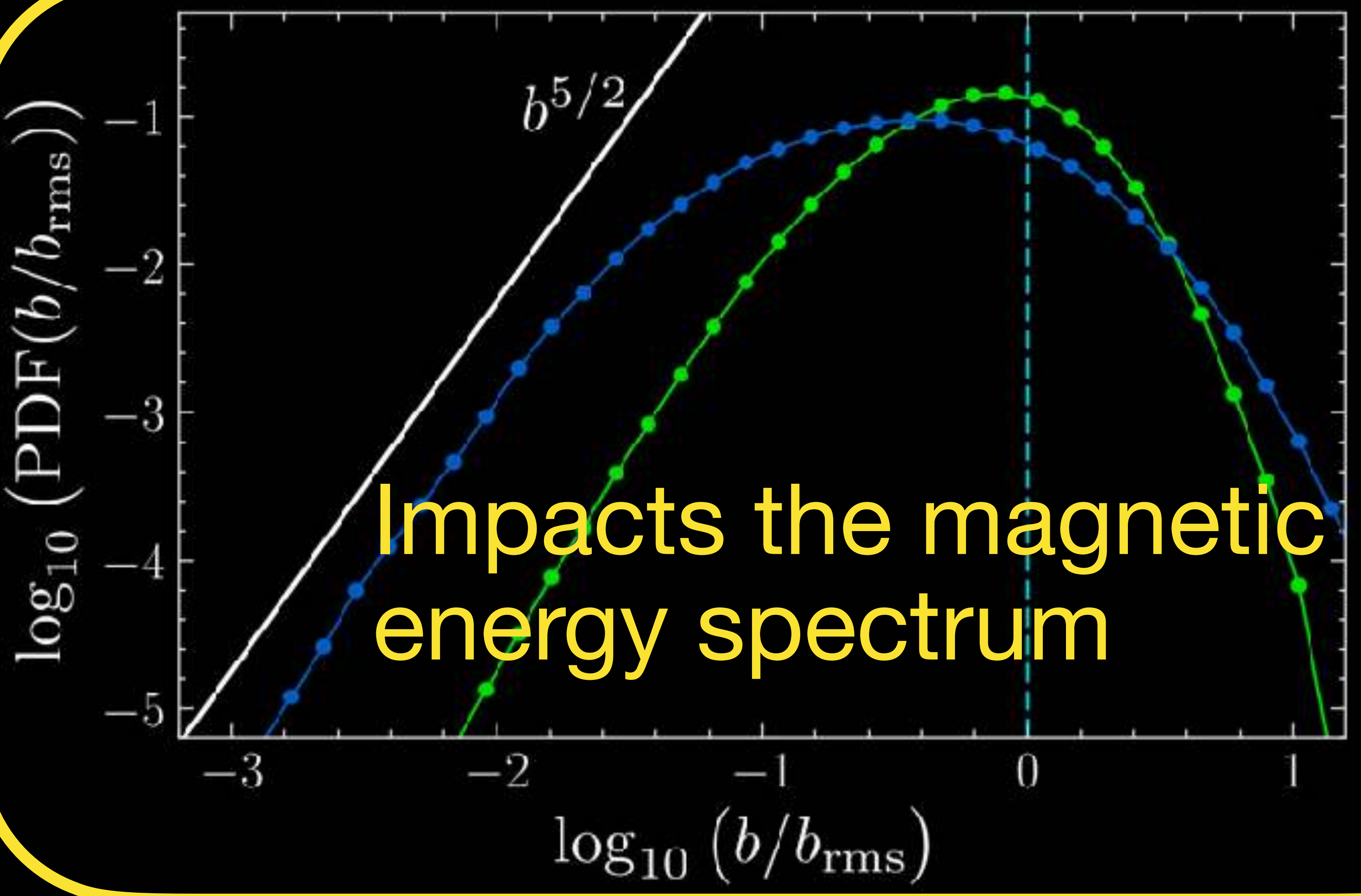
Turbulent dynamo
kinematic regime: folded fields



Folded field retained



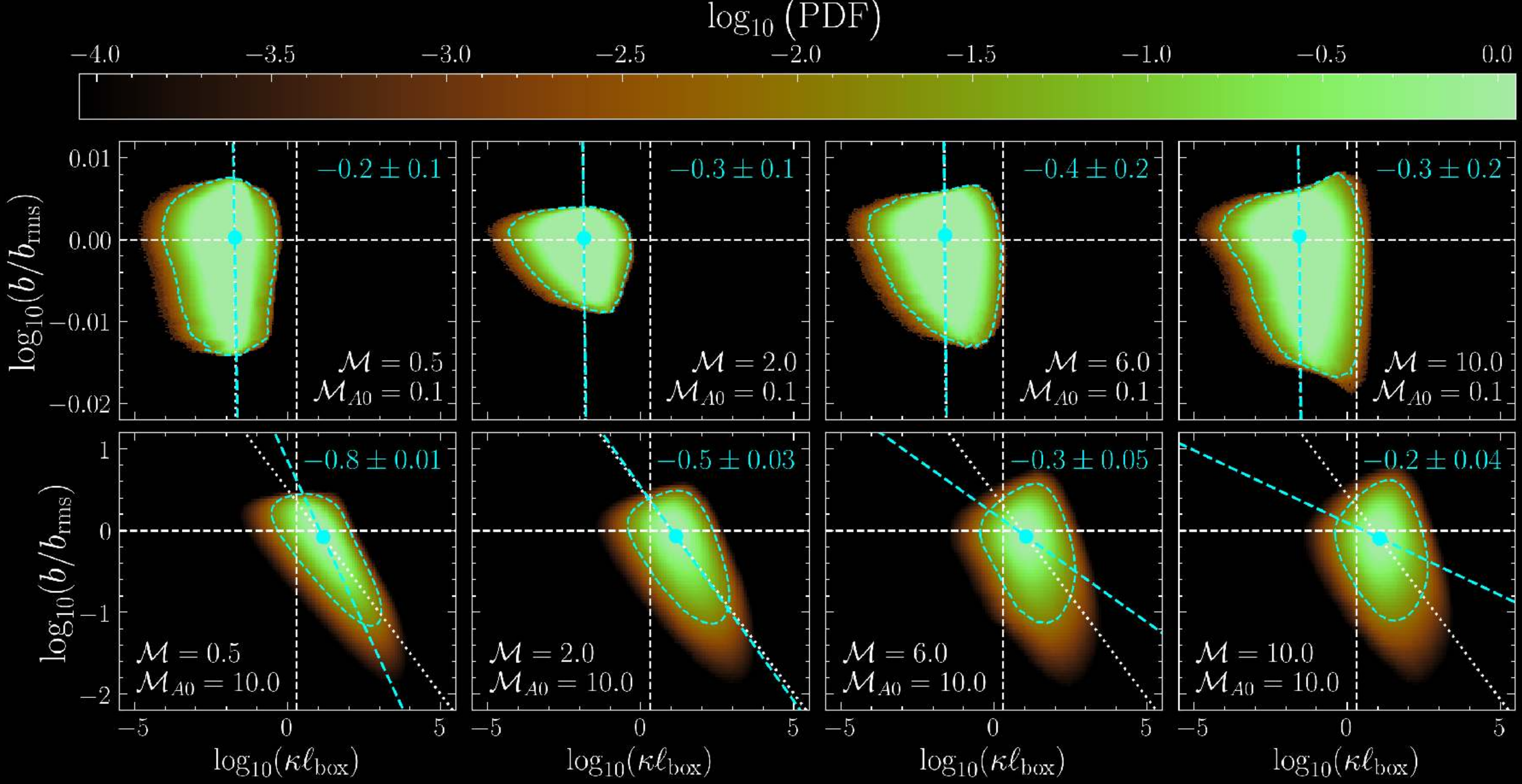
Amplitudes change (flux compressions)



Impacts the magnetic energy spectrum

Kriel, **Beattie**+ (2025). *Fundamental scales II: the kinematic stage of the supersonic dynamo*

The same correlations are not present in strong guide field simulations



Magnetic Relaxation?

Searching to weaken the nonlinearities

Relaxing into a saturation

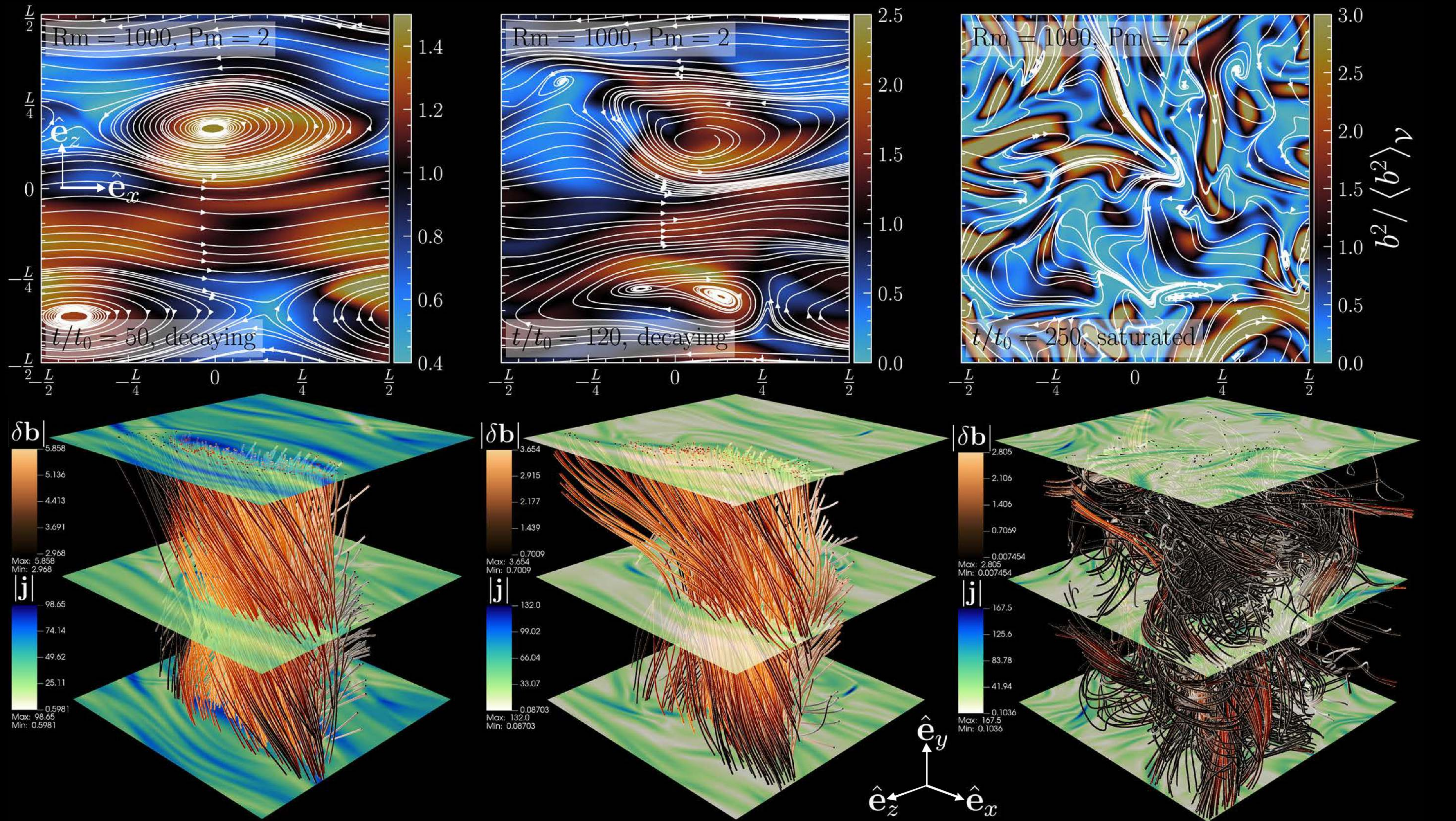
$$E_{\mathrm{mag}} \gg E_{\mathrm{kin}} \quad t = t_0 \longrightarrow t = t_{\mathrm{saturate}} \quad E_{\mathrm{mag}} \sim E_{\mathrm{kin}}$$

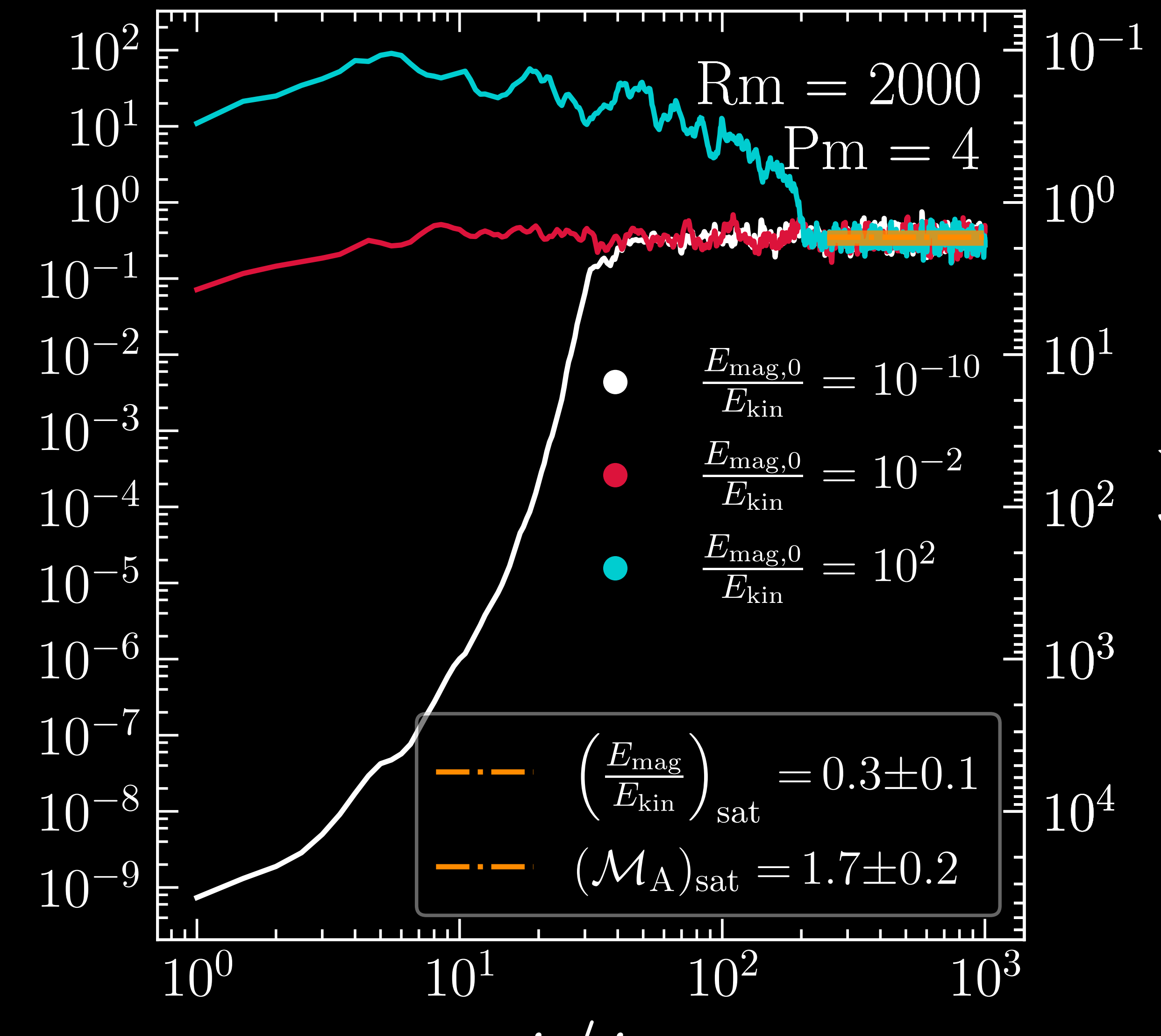
Almost perfect linear Taylor state

Saturation set by turbulent dynamo

$$\alpha_0 = \frac{\mathbf{j}\cdot\mathbf{b}}{b^2}$$

$$\mathbf{b} = \alpha_0 \mathbf{j}$$





$\langle E_{\rm mag}\rangle_{\mathcal{V}} / \langle E_{\rm kin}\rangle_{\mathcal{V}}$
$\mathcal{M}_{\rm A}$
t/t_0
Rm = 500
Pm = 1
$\left(\frac{E_{\rm mag}}{E_{\rm kin}}\right)_{\rm sat} = 0.09 \pm 0.03$
$(\mathcal{M}_{\rm A})_{\rm sat} = 3.4 \pm 0.6$
Rm = 1000
Pm = 2
$\left(\frac{E_{\rm mag}}{E_{\rm kin}}\right)_{\rm sat} = 0.3 \pm 0.1$
$(\mathcal{M}_{\rm A})_{\rm sat} = 2.0 \pm 0.2$
Rm = 2000
Pm = 4
$\frac{E_{\rm mag,0}}{E_{\rm kin}} = 10^{-10}$
$\frac{E_{\rm mag,0}}{E_{\rm kin}} = 10^{-2}$
$\frac{E_{\rm mag,0}}{E_{\rm kin}} = 10^{2}$
$\left(\frac{E_{\rm mag}}{E_{\rm kin}}\right)_{\rm sat} = 0.4 \pm 0.1$
$(\mathcal{M}_{\rm A})_{\rm sat} = 1.6 \pm 0.2$