No phenomenology is safe: surprises from extreme Reynolds number MHD turbulence simulations

Caltech Tea Talk

James Beattie

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In collaboration: Amitava Bhattacharjee (Princeton), Christoph Federrath (ANU), Ralf Klessen (UH), Salvatore Cielo (LRZ)

Objectives

1. Present a brief overview of MHD turbulence theory focussing on the two most popular phenomenologies: GS1995 and dynamical alignment (B2006)

2. Present the energy spectra (maybe time for just kinetic?) and alignment results from the world's largest supersonic MHD turbulence simulation.

3. *(Time-permitting; i.e., won't happen)* supernova-driven turbulence and new insights into dynamical turbulent dynamo saturation.

Beattie+(subm.). So Long Kolmogorov…
arXiv:2501.09855

Beattie & Bhattacharjee (subm.). Scale-dependent alignment and the depletion of nonlinearities in compressible magnetohydrodynamic turbulence

Beattie+*(2024). Magnetized compressible turbulence with a fluctuation dynamo and Reynolds numbers over a million* arXiv:2405.16626

Beattie+(2025; *Nature Astronomy, accepted*). The spectrum of magnetized turbulence in the interstellar medium arXiv:2504.07136

James Beattie. Magnetic reconnection in a turbulent fluid. Iterative LIC.

The ISM: A laboratory for interesting nonlinear physics

- **Multiphase trans-to-supersonic compressible plasma**
- $\mathrm{Re} = UL/\nu \gg 1 \implies$ highly-turbulent
- Turbulence driven by a zoo of sources
  - Gravitational instabilities, SN explosions, stellar feedback…
- **Pervaded by ultra-relativistic particles** with non-negligible energy densities (peaked number density in $\mathrm{GeV} - \mathrm{TeV}$ range)
- **Non-Gaussian gas-density structures** related to star formation (multi-scale filaments, ribbons, striations, sheets)
- **Dynamically important, maintained magnetic fields**
- $\mathrm{Rm} = UL/\eta \gg 1 \implies$ highly-turbulent magnetic fields
- ….

Krumholz & McKee (2005); Federrath & Klessen (2012); Tritsis+(2016); Xu & Lazarian (2016); Soler & Hennebelle (2017); Squire & Hopkins (2017); Mocz & Burkhart (2018), Burkhart & Mocz (2018); Heyer+(2012); Tritsis+(2018); Hu+(2019); Heyer+(2020); Krumholz+(2020); Beattie & Federrath (2020); Beattie+(2020); Körtgen & Soler (2020); Skalidis & Tassis (2020); Federrath+(2021); Bukhart (2021); Barreto-Mota+(2021); Hopkins+(2022); Beattie+(2022); Fielding+(2022); Sampson+ (2022); Kriel+(2022); Galishnikova+(2022); Beattie+(2023)

Reynolds number

quadratic nonlinearity

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u} + P\mathbb{I}) = 2\nabla \cdot (\nu \rho \mathbb{S})$$

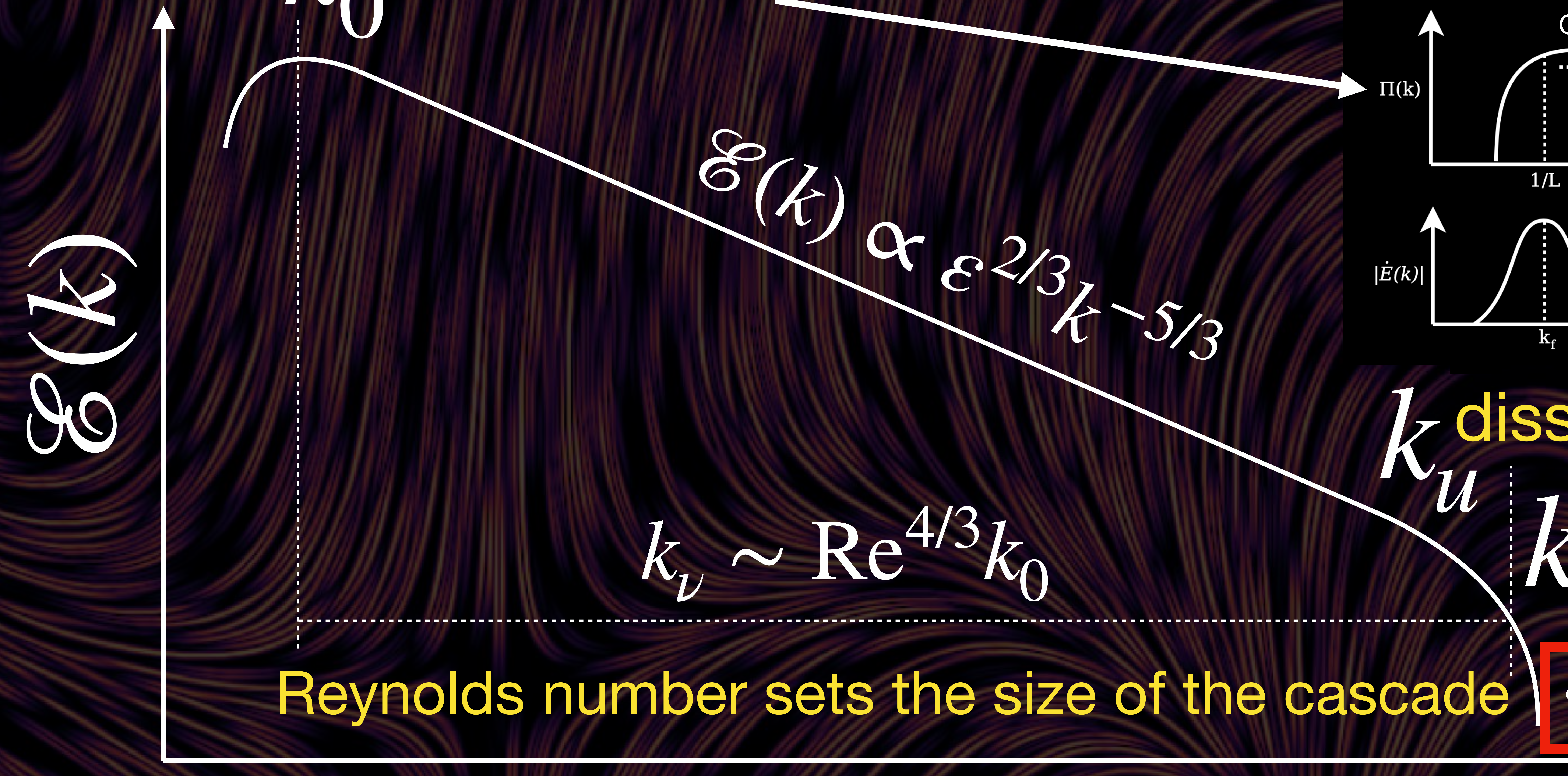
Creating nonlinear things in the fluid

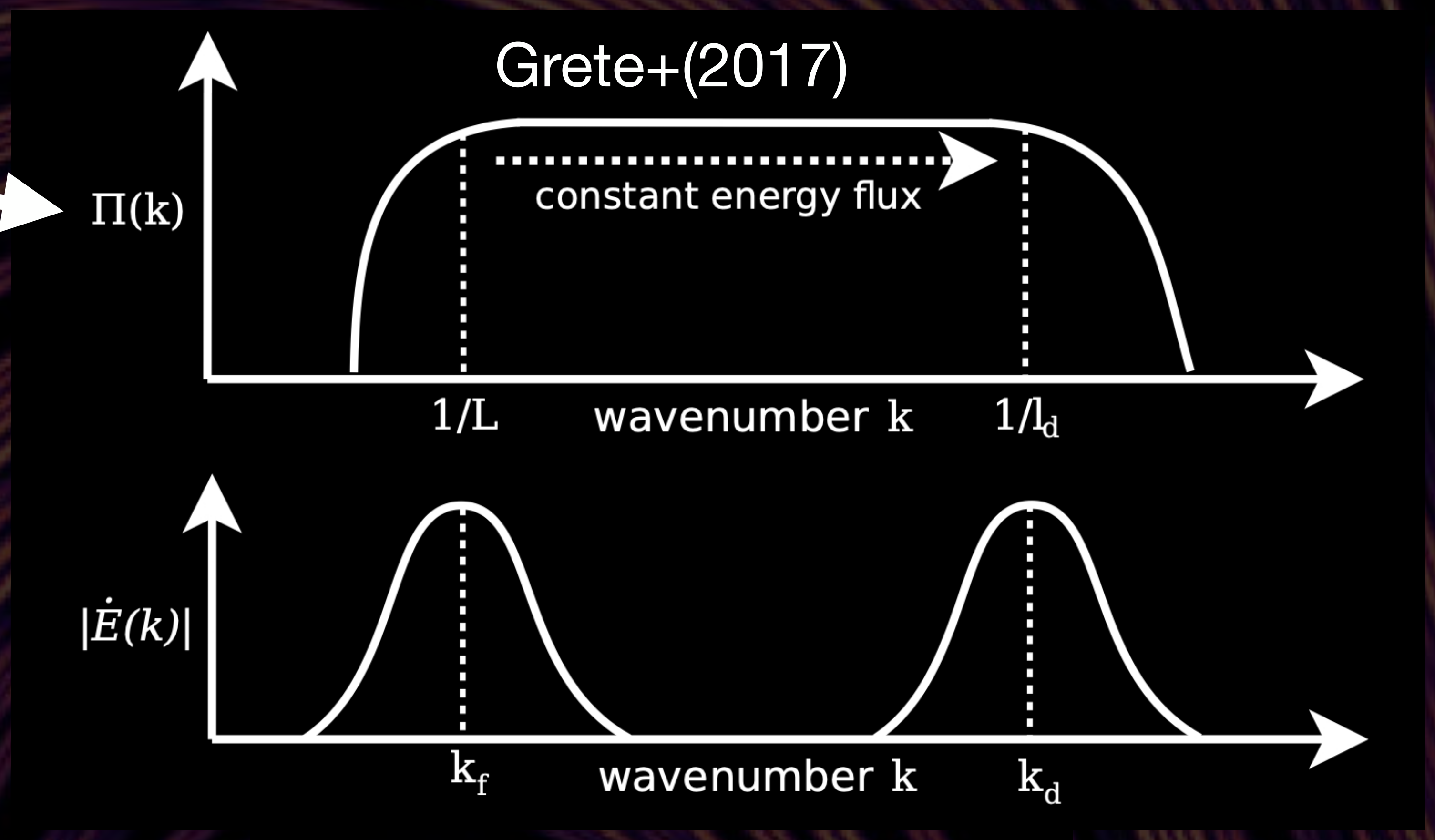
$$\mathrm{Re} = \frac{|\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})|}{|2\nu \nabla \cdot (\rho \mathbb{S})|} \sim \frac{UL}{\nu}$$

Smoothing out nonlinear things in the fluid

The Kolmogorov-type energy cascade

$$\varepsilon \sim u^3/\ell \implies u \sim (\varepsilon\ell)^{1/3} \implies u^2(k) \sim k^{-5/3}$$





dissipation scales

$$\boxed{\varepsilon_{\rm in} = \varepsilon_{\rm out} = Q_{\rm heating}}$$

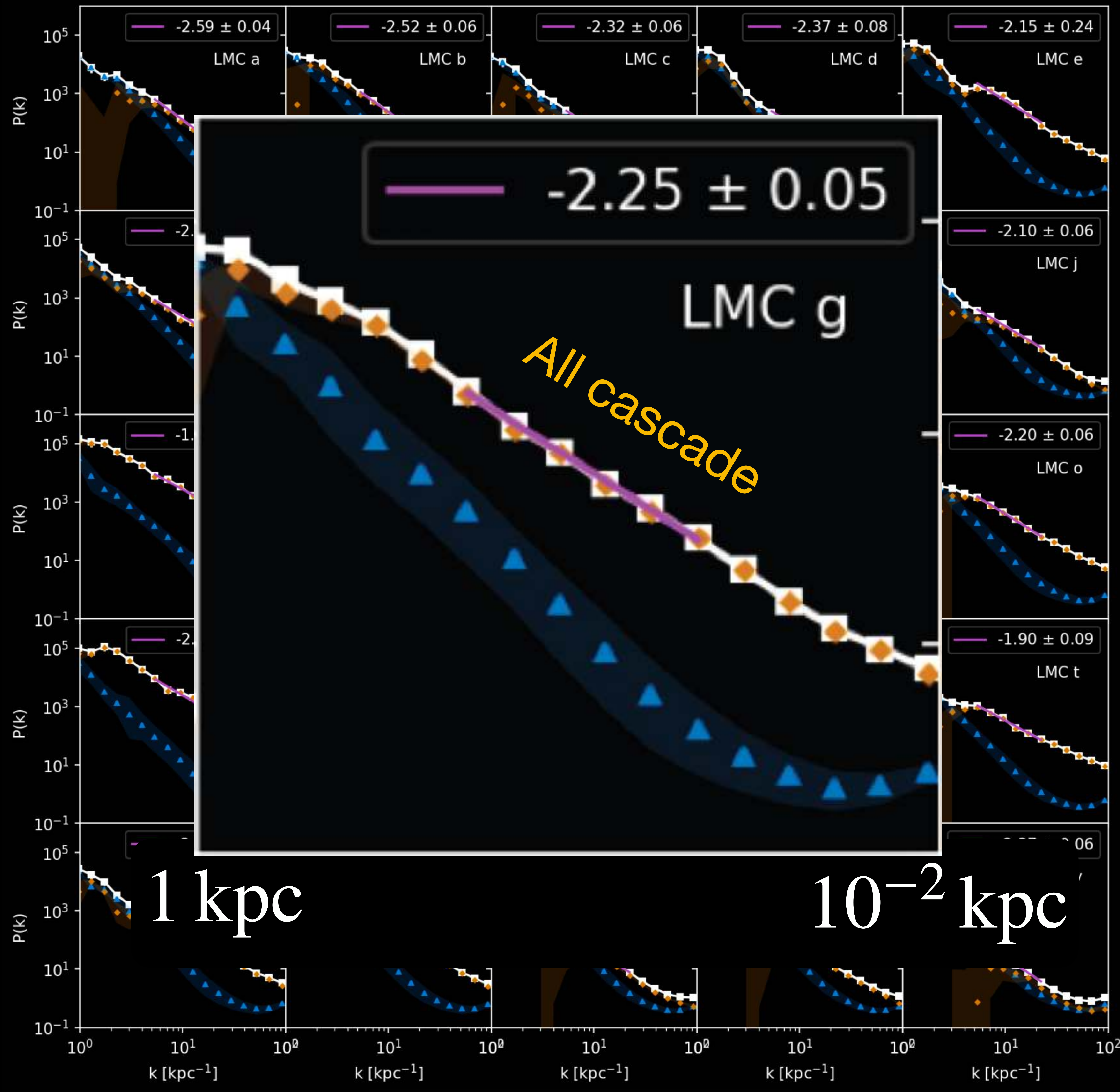
$\mathrm{Re} \gg 1$ $\quad k \sim \ell^{-1} \quad$ $\mathrm{Re} \sim 1$

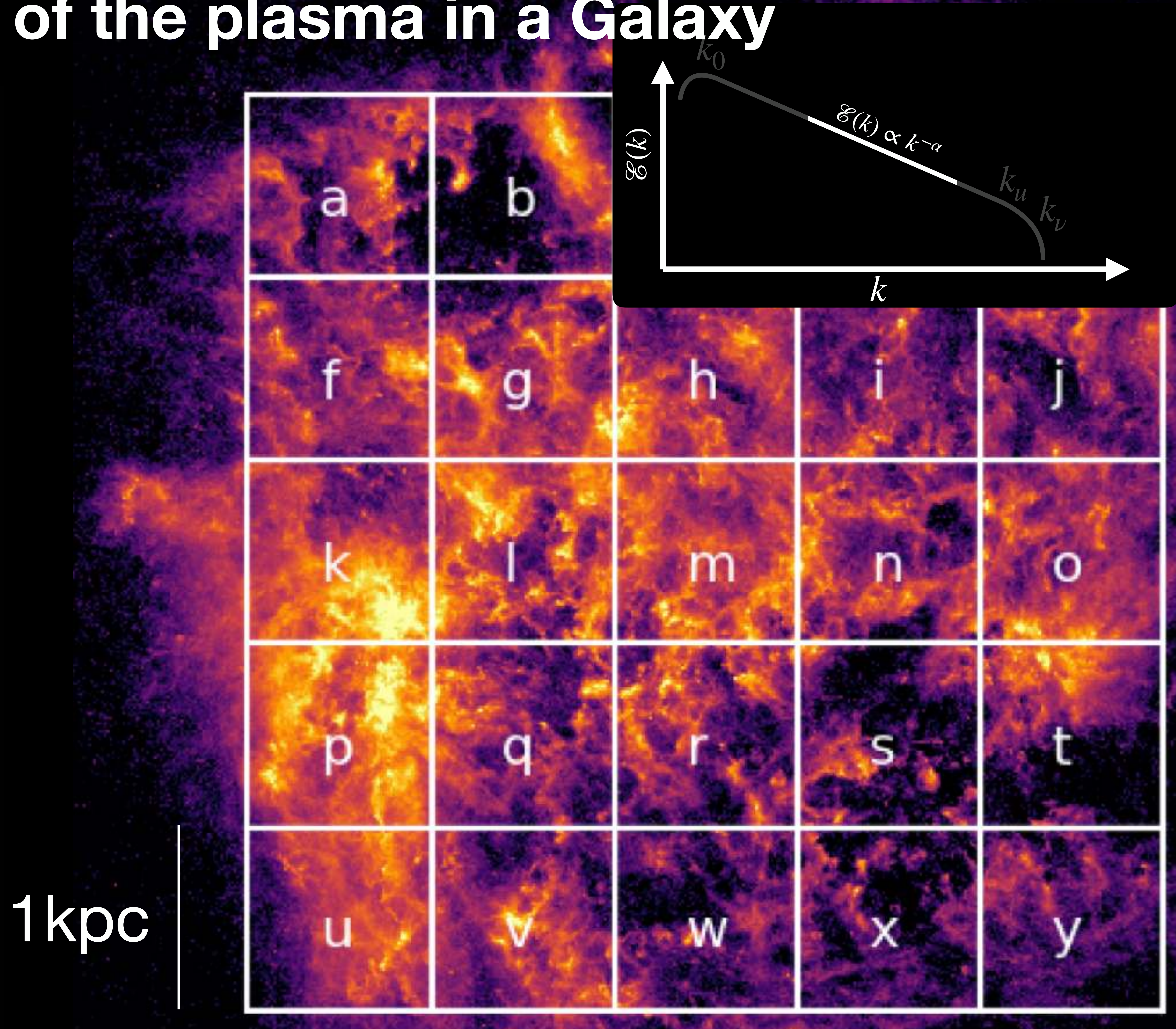
Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Turbulence

LMC: 500μm, *Herschel* (processing Gordon et al. 2014)

Galactic Cascade — the structure of the plasma in a Galaxy

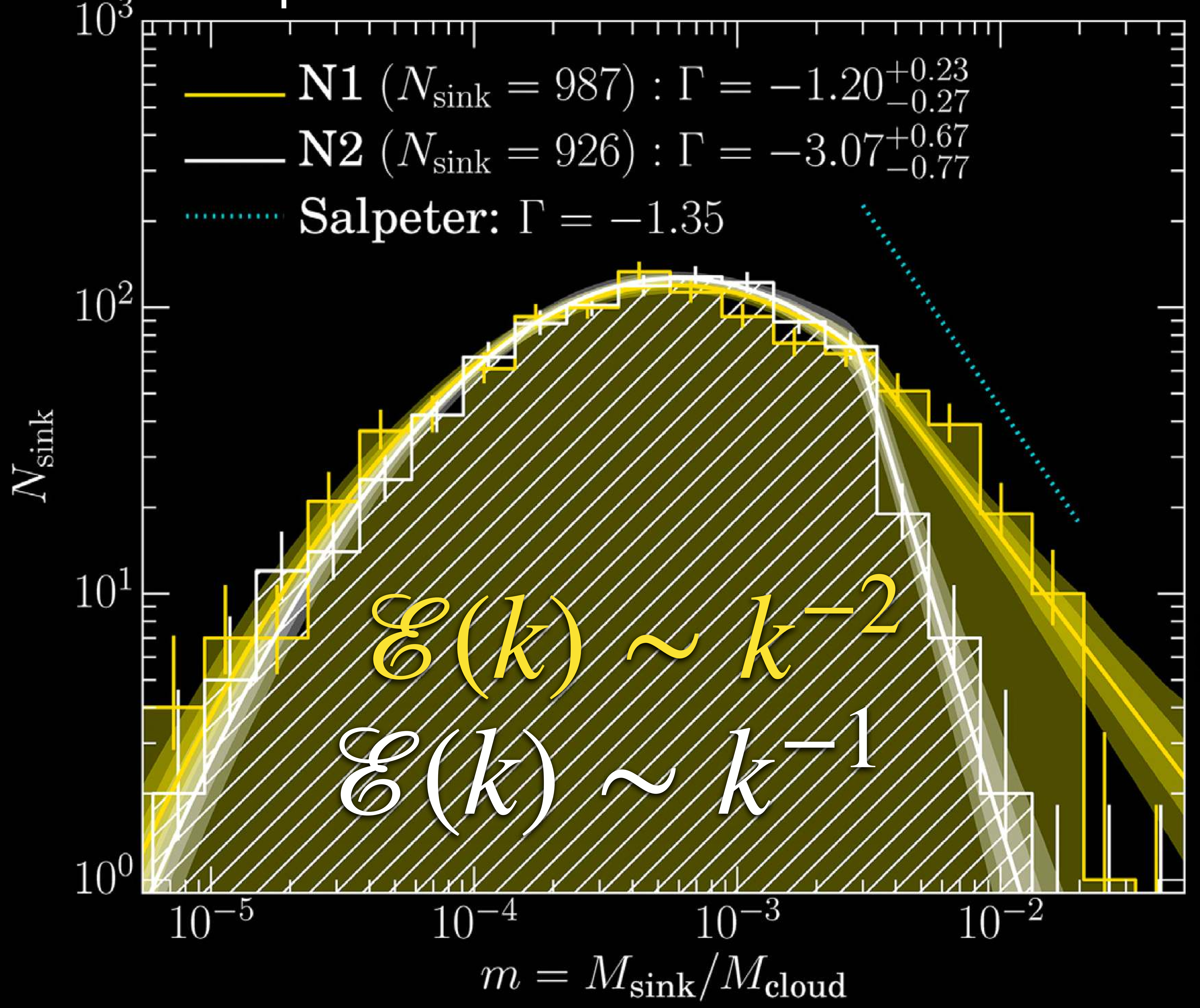




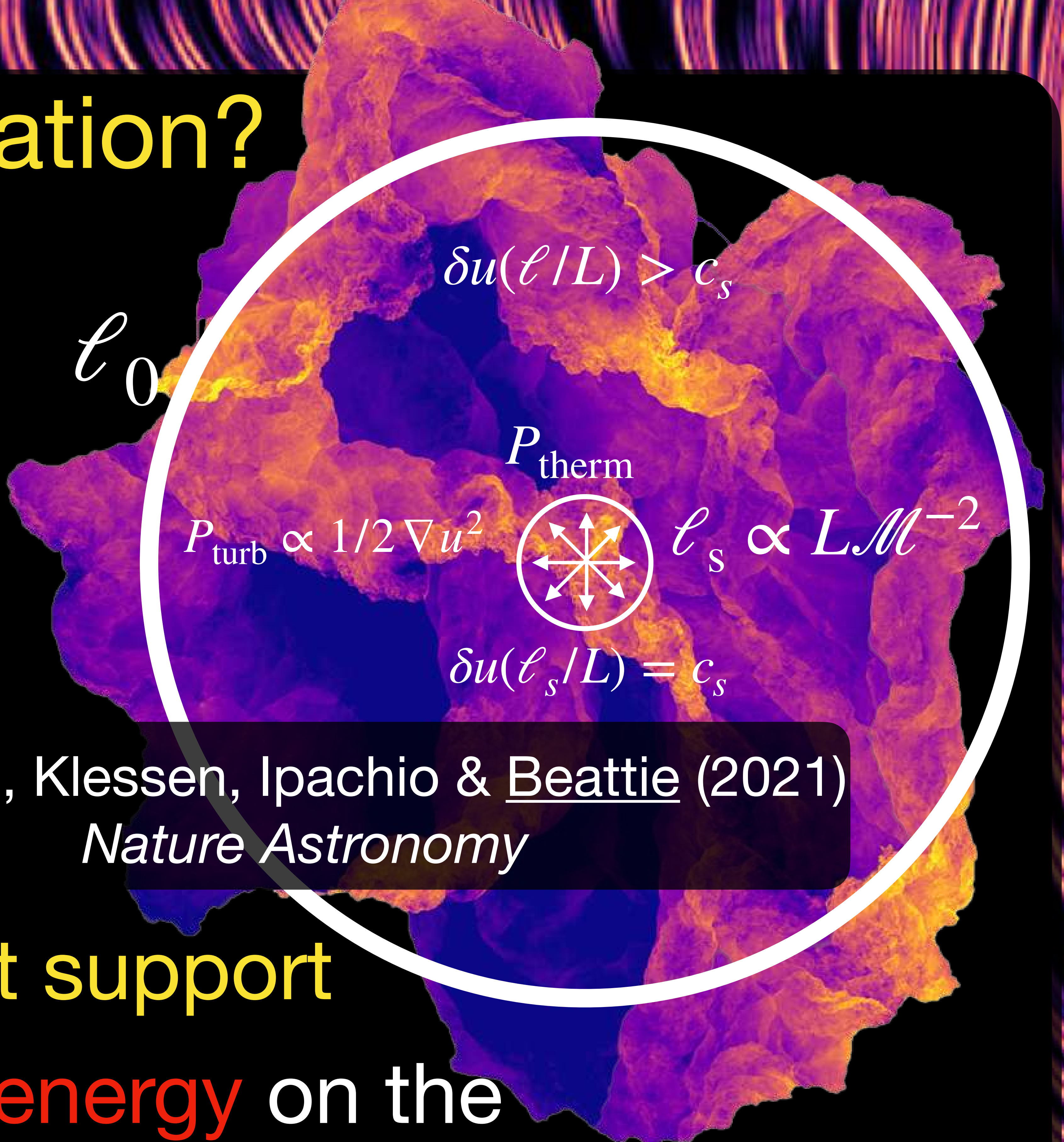
Colman, et al., 2022, MNRAS

Why should astrophysicists care — star formation?

Sink particle ∝ stellar IMF



Nam, Federrath & Krumholz (2021)



Federrath, Klessen, Ipachio & Beattie (2021)
Nature Astronomy

Turbulent support

More energy on the small-scales (more small-scale support) implies less high-mass stars formation

$$\mathbf{u}\cdot\nabla\otimes\mathbf{u} = \frac{1}{2}\nabla u^2 - \mathbf{u}\times\nabla\times\mathbf{u}$$

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Why should astrophysicists care — cosmic ray transport?

Testing Physical Models for Cosmic Ray Transport Coefficients on Galactic Scales: Self-Confinement and Extrinsic Turbulence at ∼GeV Energies

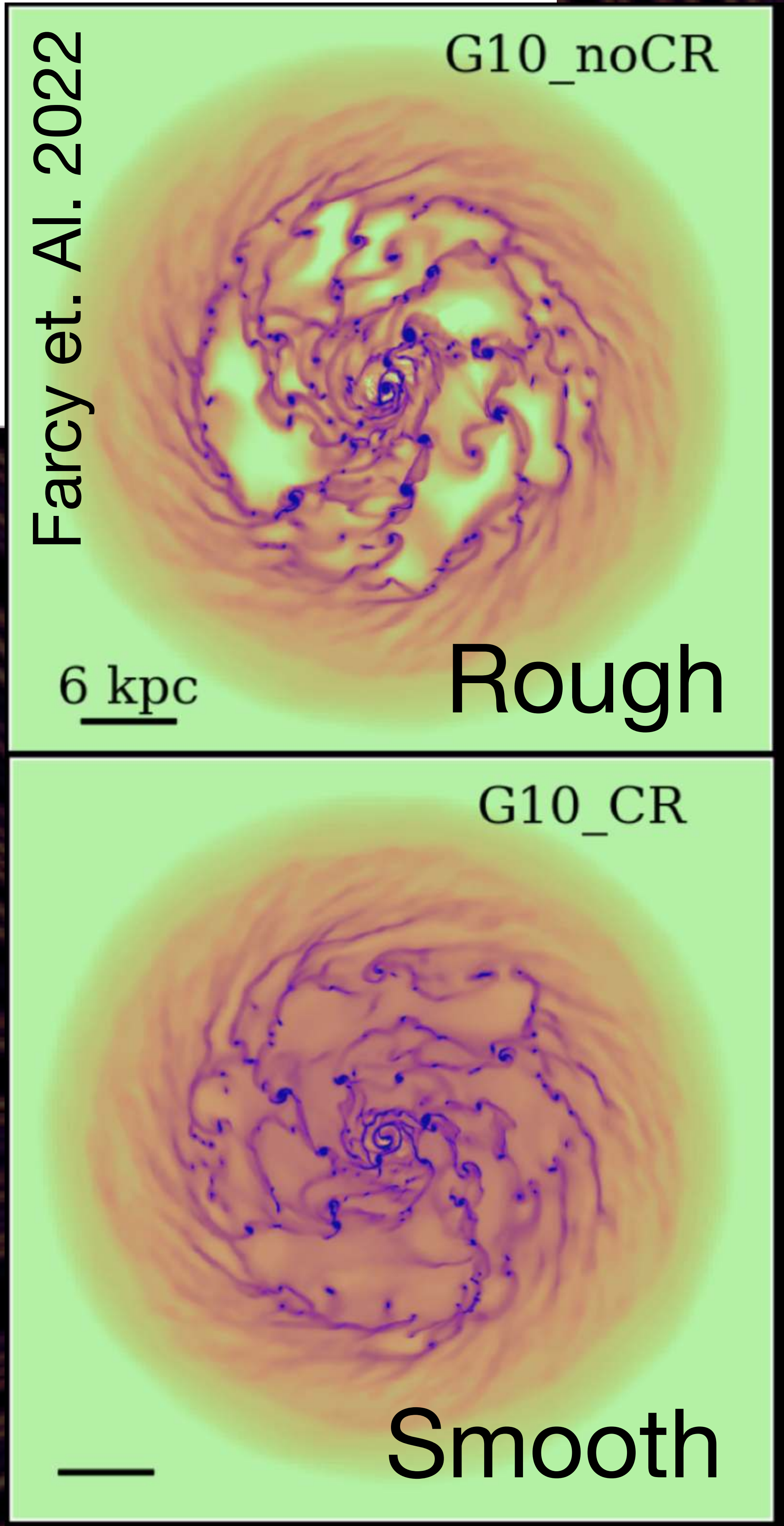
Philip F. Hopkins[1], Jonathan Squire[2], T. K. Chan[3,4], Eliot Quataert[5], Suoqing Ji[1], Dušan Kereš[2], Claude-André Faucher-Giguère[6]

Sampson, Beattie et al. (2023, MNRAS). *Turbulent diffusion of streaming cosmic rays in compressible, partially ionised plasma*

Provides extra source of scattering on large-scales

Determines the properties of turbulence

CRs add extra pressure support + smooth plasma gradients



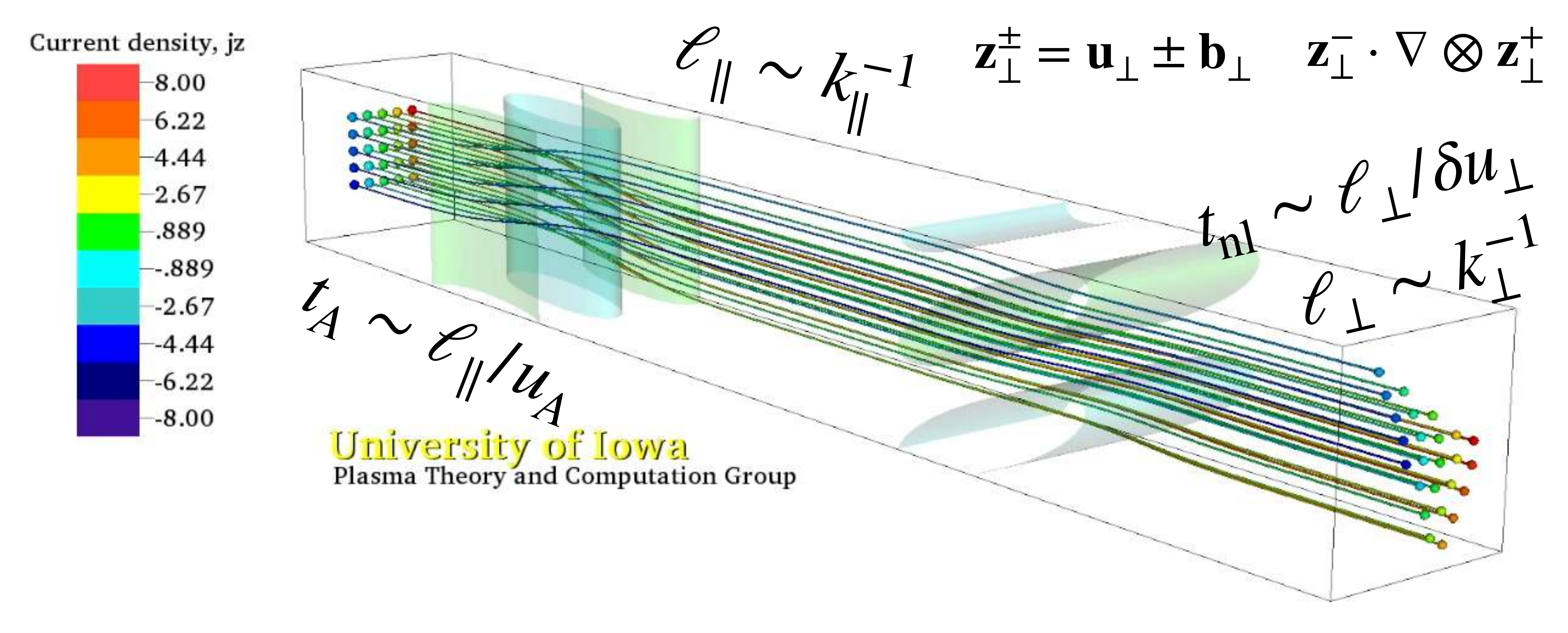
Beattie et al. (2023, Frontiers). *Ion Alfvén velocity fluctuations and implications for the diffusion of streaming cosmic rays*

The energy cascade — fundamental for CR transport

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

The incompressible MHD energy cascade

Interacting Alfvén waves (visualization from Greg G. Howes)



$\ell_{\parallel} \sim k_{\parallel}^{-1}$ $\quad \mathbf{z}_{\perp}^{\pm} = \mathbf{u}_{\perp} \pm \mathbf{b}_{\perp} \quad \mathbf{z}_{\perp}^{-} \cdot \nabla \otimes \mathbf{z}_{\perp}^{+}$

$t_{\mathrm{nl}} \sim \ell_{\perp} / \delta u_{\perp}$

$\ell_{\perp} \sim k_{\perp}^{-1}$

$t_A \sim \ell_{\parallel} / u_A$

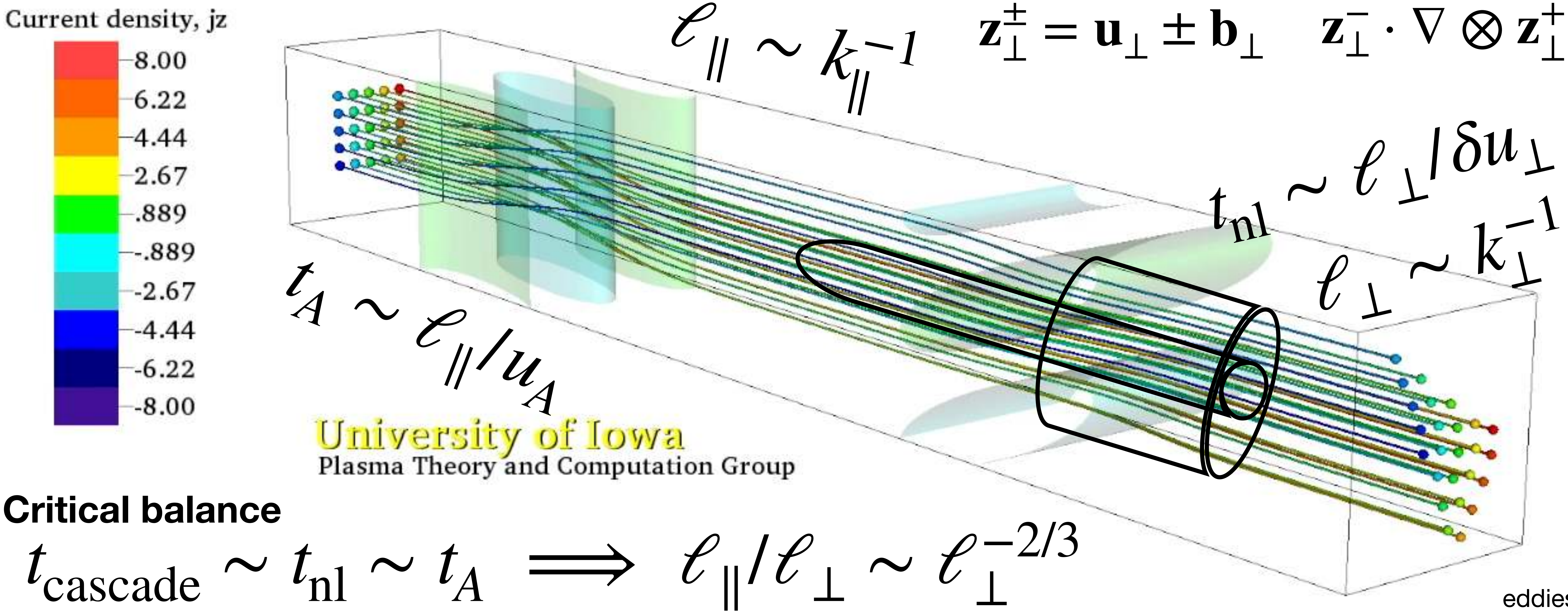
MHD (can be) globally and (is) locally anisotropic!

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

The incompressible MHD energy cascade

Goldreich & Shridar (1995)

Interacting Alfvén waves (visualization from Greg G. Howes)



$$\ell_\parallel \sim k_\parallel^{-1} \qquad \mathbf{z}_\perp^\pm = \mathbf{u}_\perp \pm \mathbf{b}_\perp \qquad \mathbf{z}_\perp^- \cdot \nabla \otimes \mathbf{z}_\perp^+$$

$$t_{\mathrm{nl}} \sim \ell_\perp / \delta u_\perp$$

$$\ell_\perp \sim k_\perp^{-1}$$

$$t_A \sim \ell_\parallel / u_A$$

Critical balance

$$t_{\mathrm{cascade}} \sim t_{\mathrm{nl}} \sim t_A \implies \ell_\parallel / \ell_\perp \sim \ell_\perp^{-2/3}$$

eddies

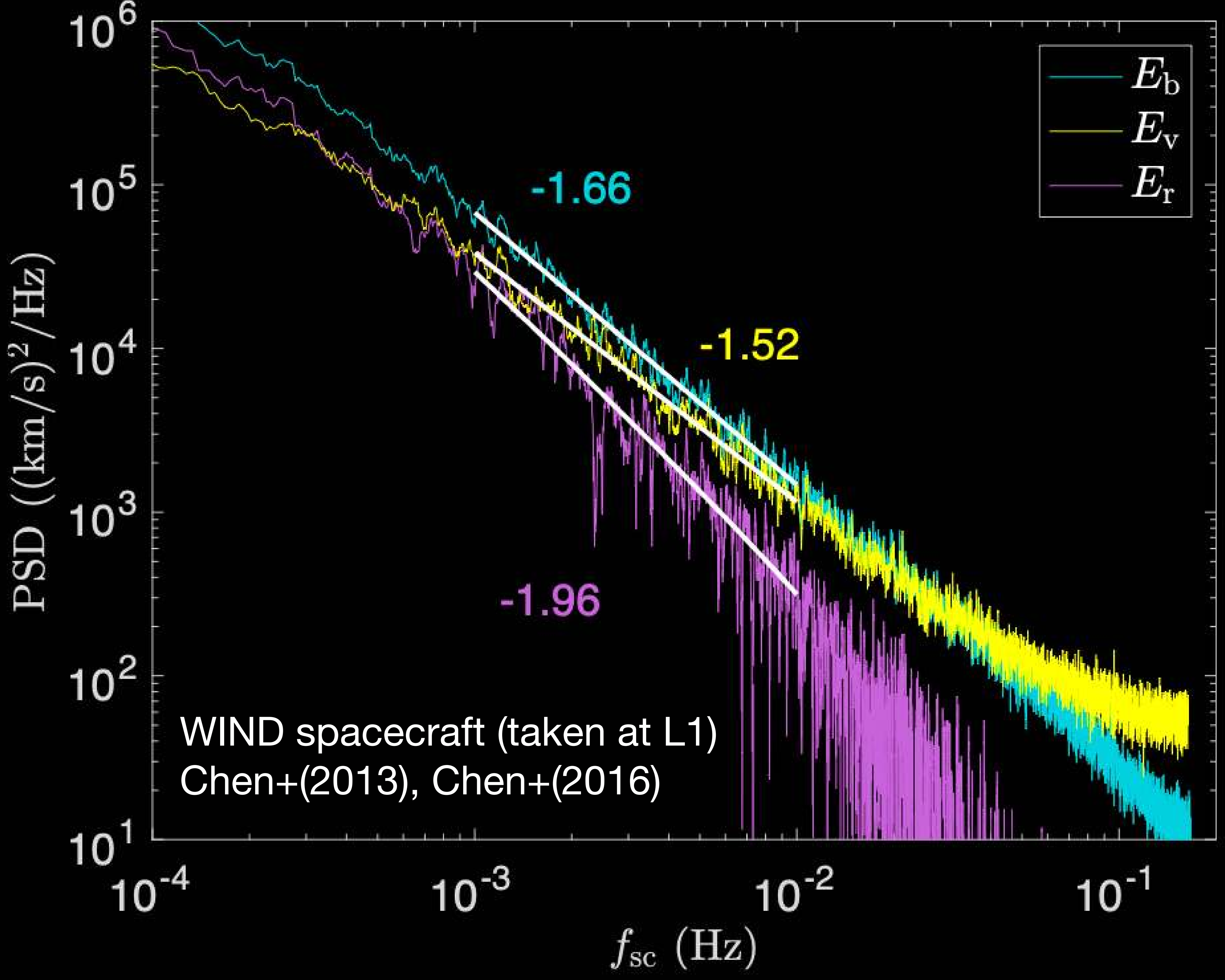
$$\delta u_\perp^2 / t_{\mathrm{nl}} \sim \varepsilon \iff \delta u_\perp \sim (\varepsilon \ell_\perp)^{1/3} \iff u^2(k_\perp) \sim \varepsilon^{2/3} k_\perp^{-5/3}$$

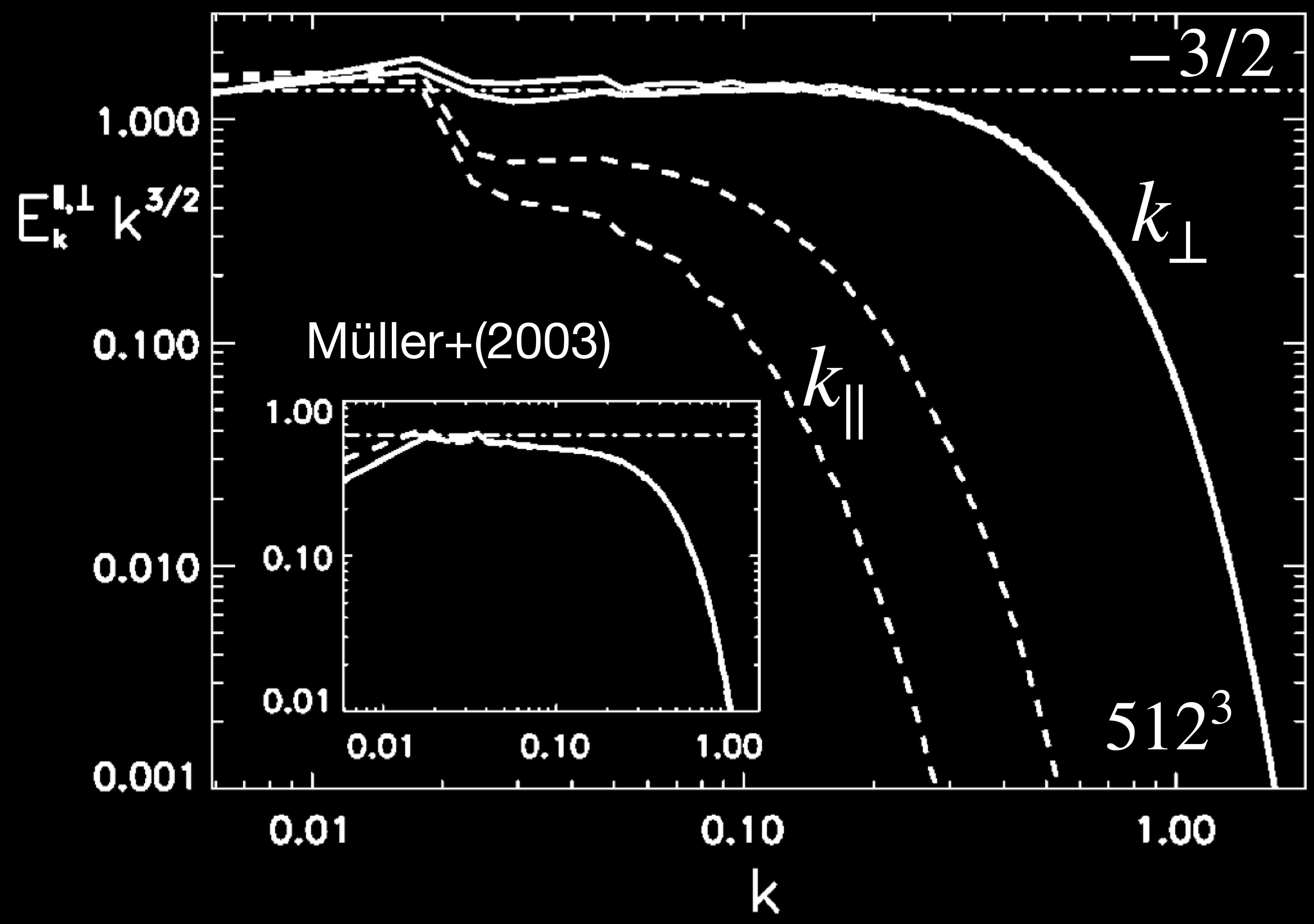
Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

The incompressible MHD energy cascade

Goldreich & Shridar (1995)??

High-resolution simulations and solar wind observations





Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

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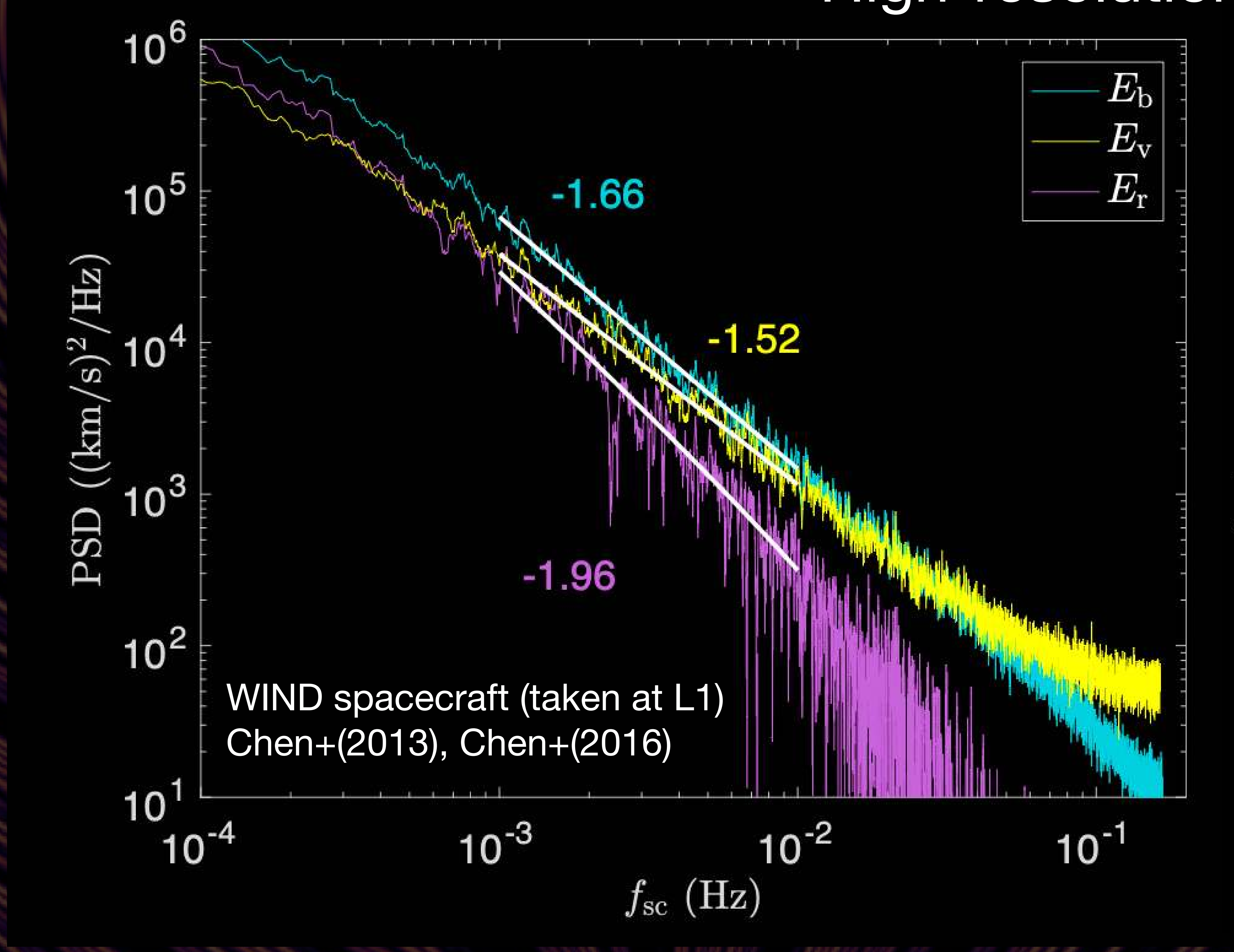
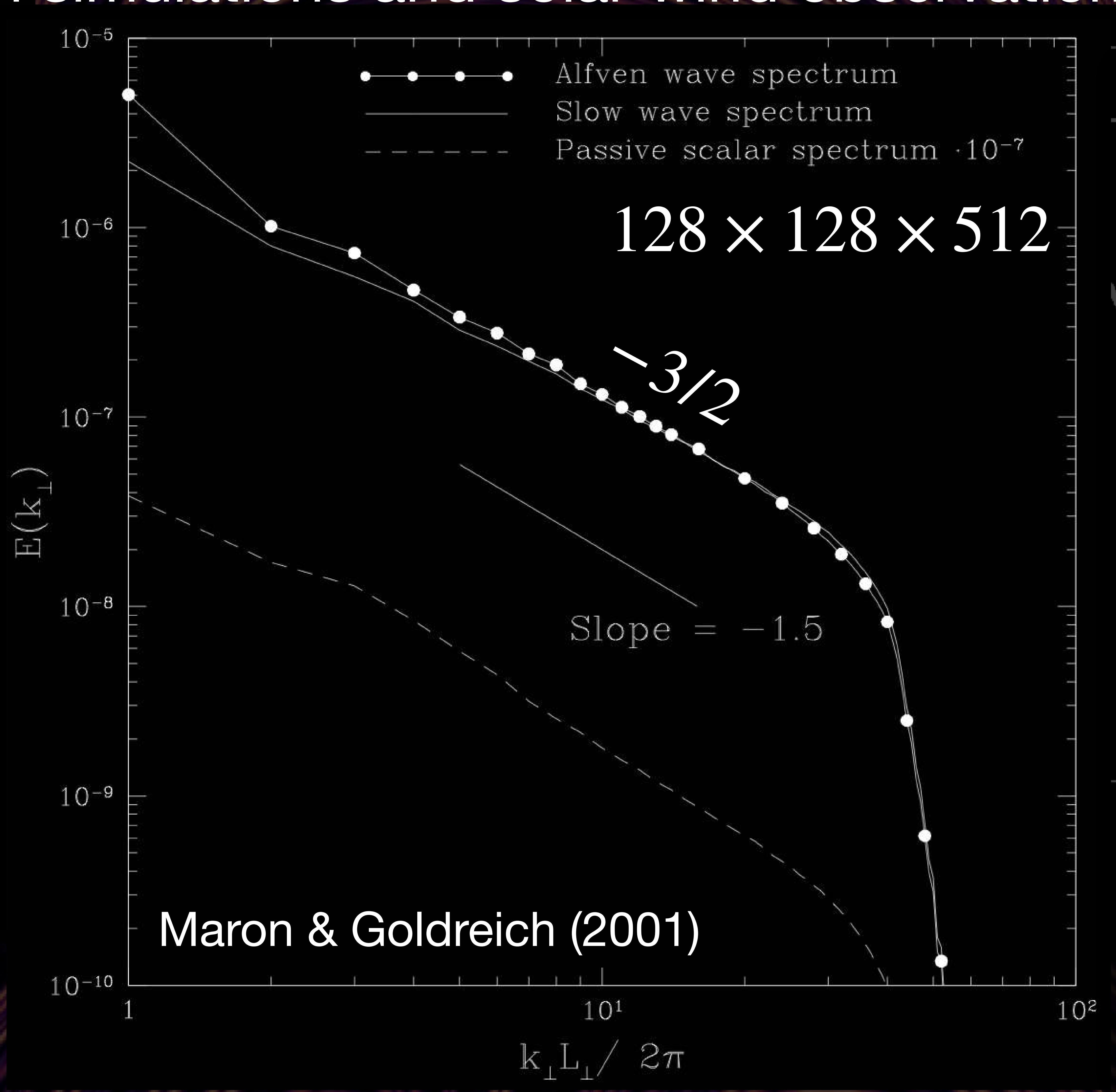
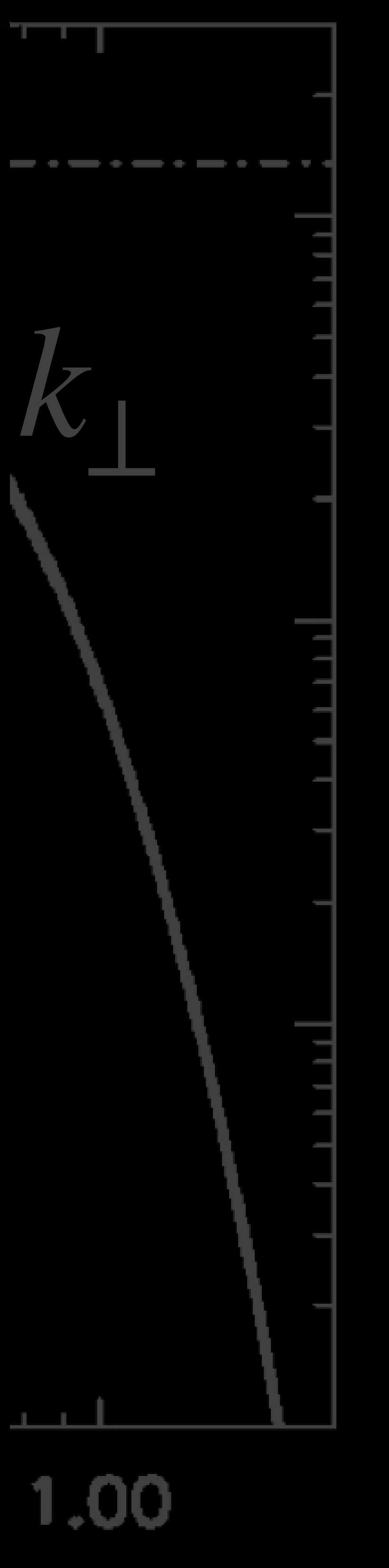
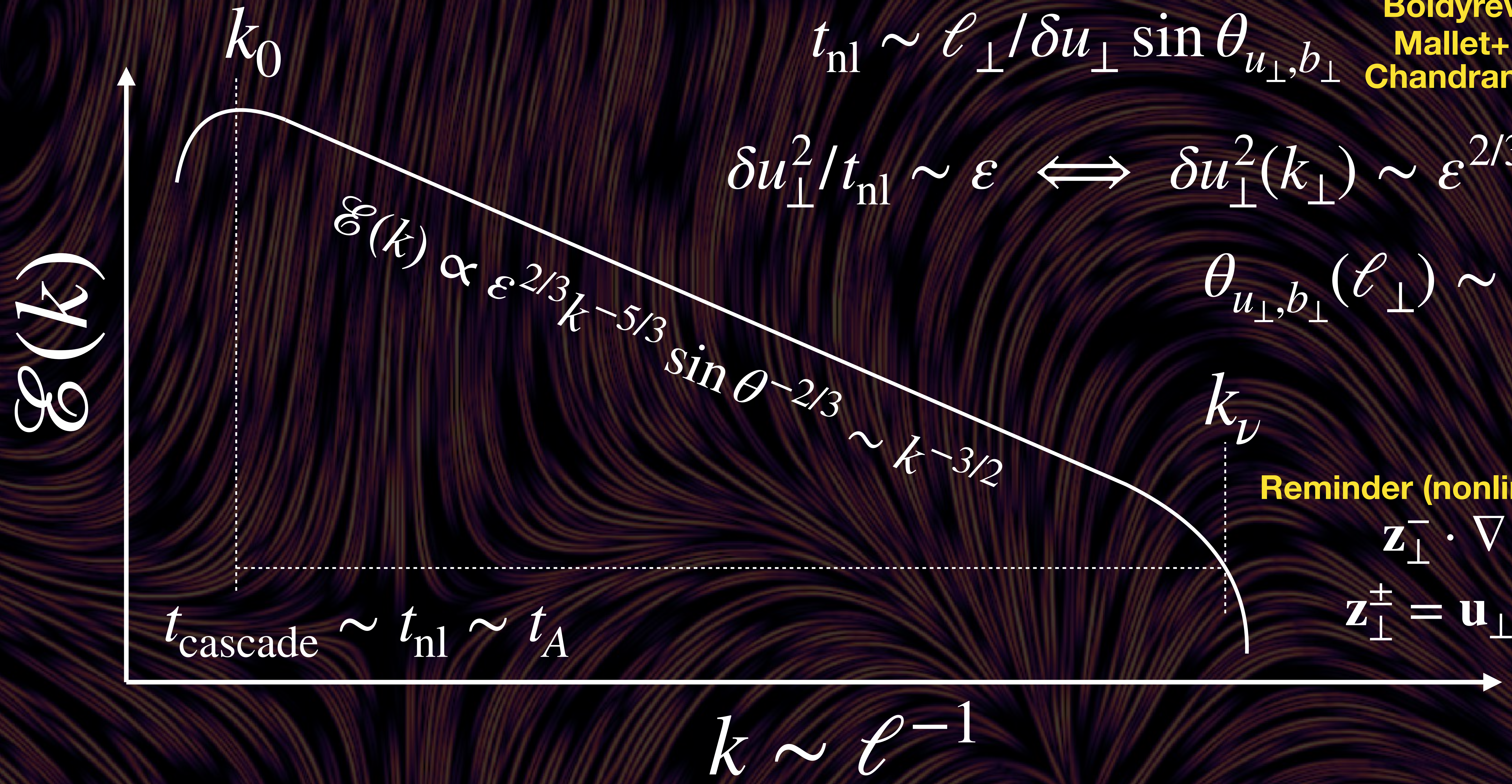






Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

How to deplete the nonlinearities? Dynamic Alignment!



$$t_{\rm nl} \sim \ell_\perp / \delta u_\perp \sin\theta_{u_\perp, b_\perp}$$

Boldyrev (2006)
Mallet+ (2015)
Chandran+ (2015)

$$\delta u_\perp^2 / t_{\rm nl} \sim \varepsilon \iff \delta u_\perp^2(k_\perp) \sim \varepsilon^{2/3} k_\perp^{-3/2}$$

$$\theta_{u_\perp, b_\perp}(\ell_\perp) \sim \ell_\perp^{1/4}$$

Reminder (nonlinearity):

$$\mathbf{z}_\perp^- \cdot \nabla \otimes \mathbf{z}_\perp^+$$

$$\mathbf{z}_\perp^\pm = \mathbf{u}_\perp \pm \mathbf{b}_\perp$$

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

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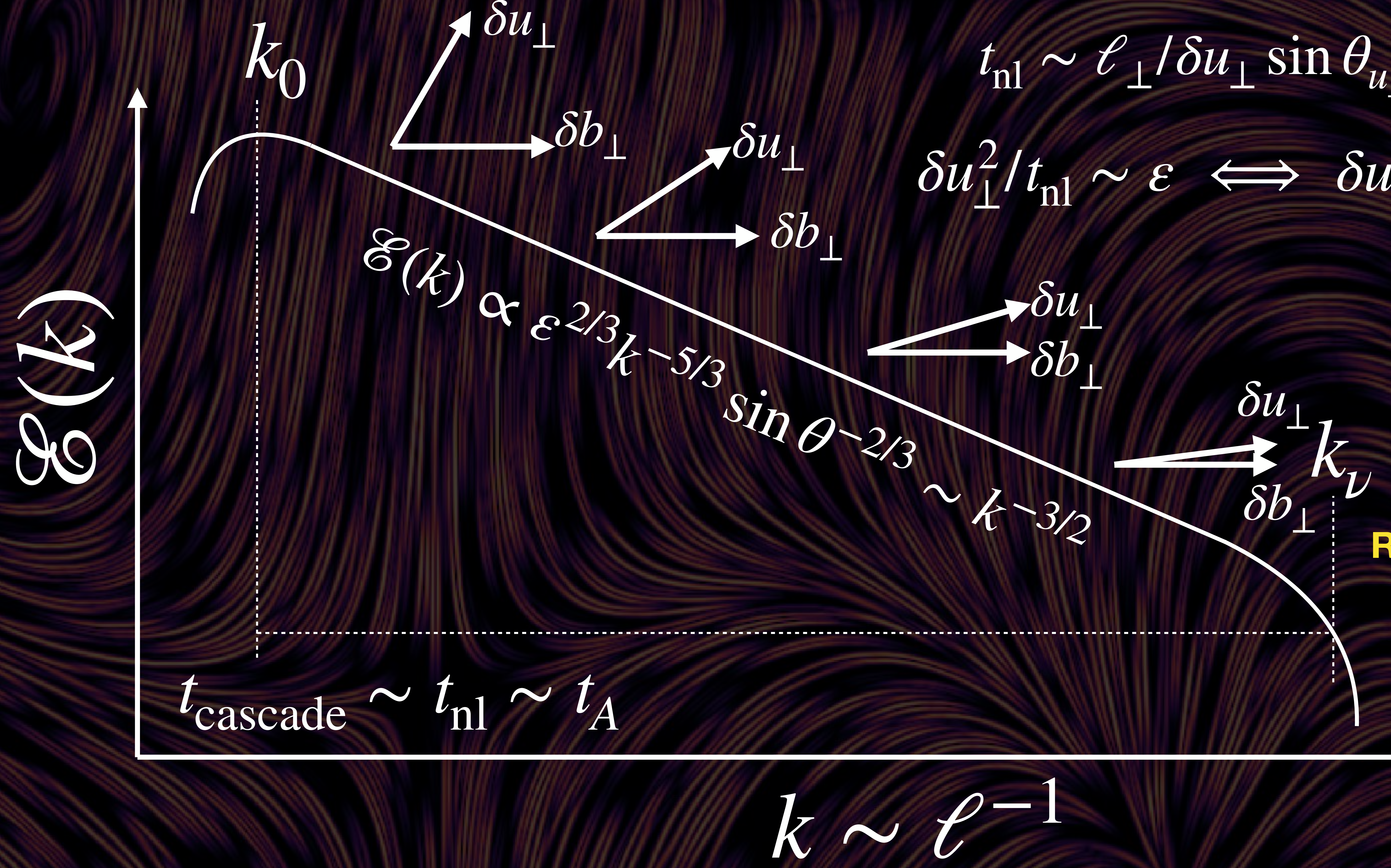

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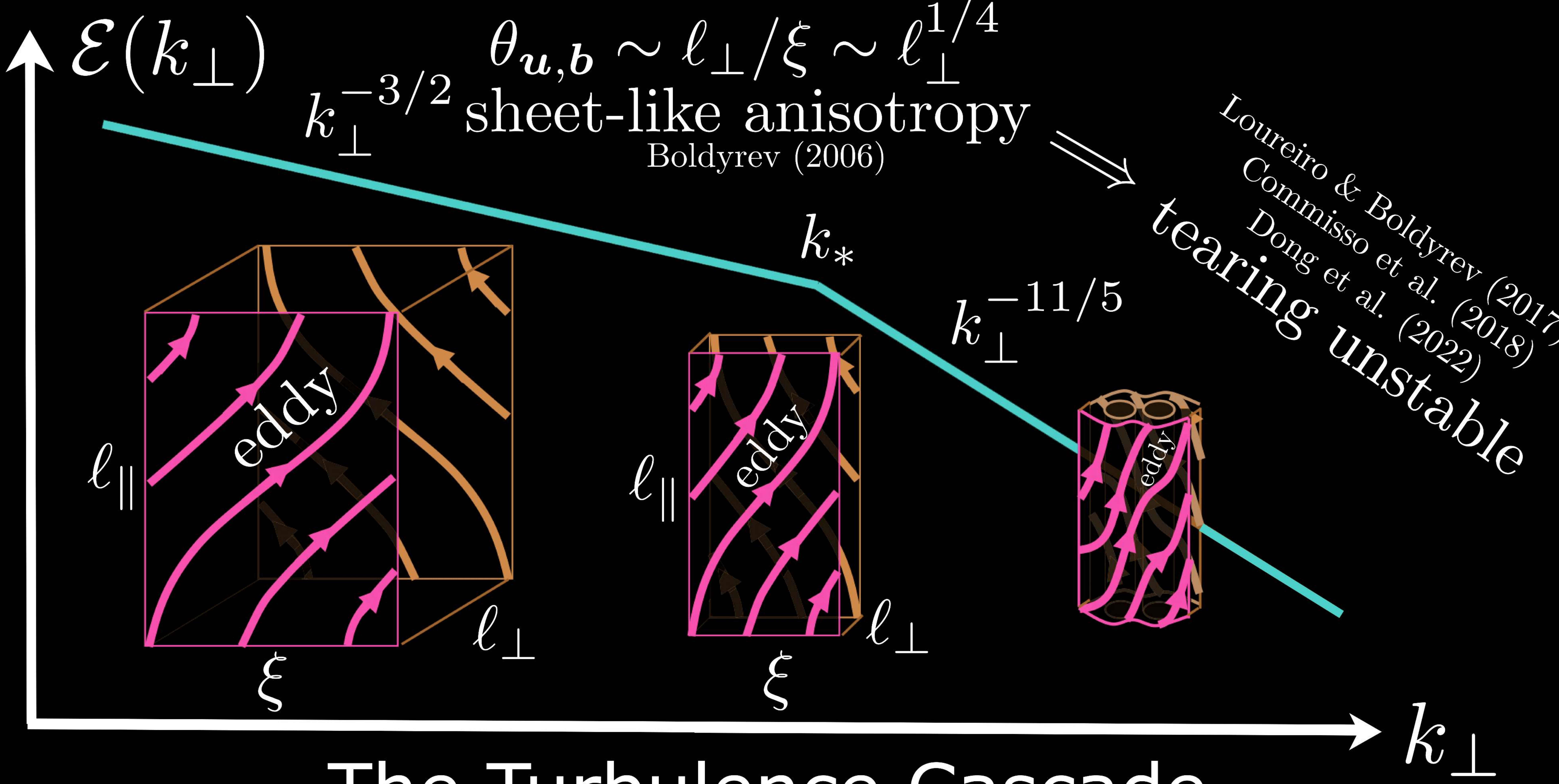
Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).



The Turbulence Cascade

Modified from Loureiro & Boldyrev (2017)

$10{,}080^3$ magnetized compressible turbulence

Beattie, Federrath, Klessen, Cielo & Bhattacharjee

1. What is the scaling of the energy cascade in highly-compressible, ISM-style MHD turbulence?
2. **How are the characteristic scales organized in the compressible interstellar medium turbulence?**
3. **What are the saturation physics of the turbulent dynamo?**

PI of a 100million core-hour project on superMUC-NG

ILES of compressible MHD turbulence

Turbulence: $\sigma_V / c_s \approx 4,\ \ell_0 = L/2$

Magnetic fields: $B = b_{\mathrm{turb}},\ \mathcal{M}_A \approx 2$

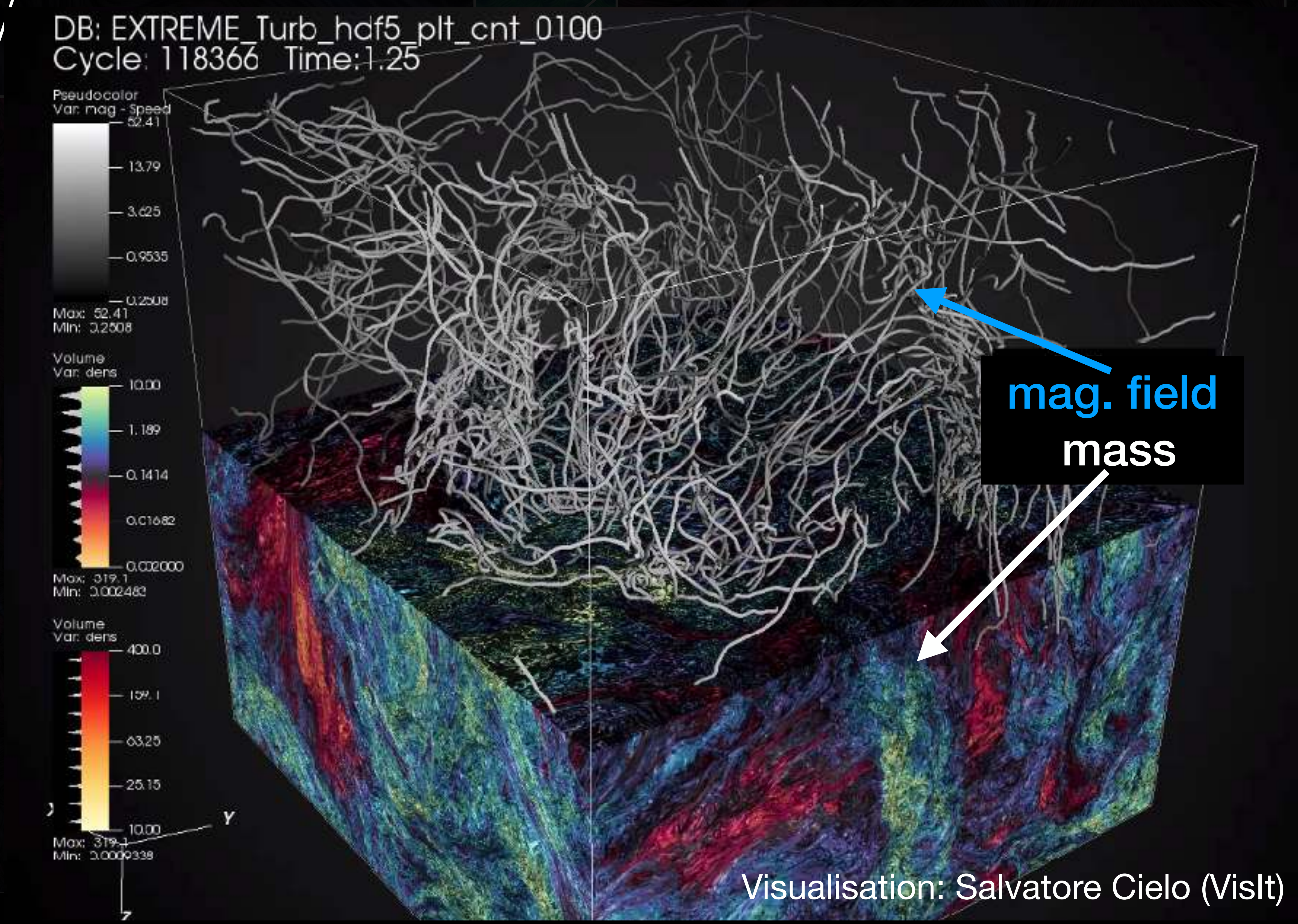
Three experiments for convergence tests:

- **LOW-RES:** $2{,}520^3$ **(0.3Mcore-h, 8,640cores)**
- **MID-RES:** $5{,}040^3$ **(4.0Mcore-h, 34,560cores)**
- **HIGH-RES:** $10{,}080^3$ **(80.0Mcore-h, 148,240cores)**
  - **Resolving 10pc down to ~200au everywhere on the grid**
  - **Factor of 4 higher linear grid resolution than IllustrisTNG**

3.45PB **in data products**

Broad Code details:

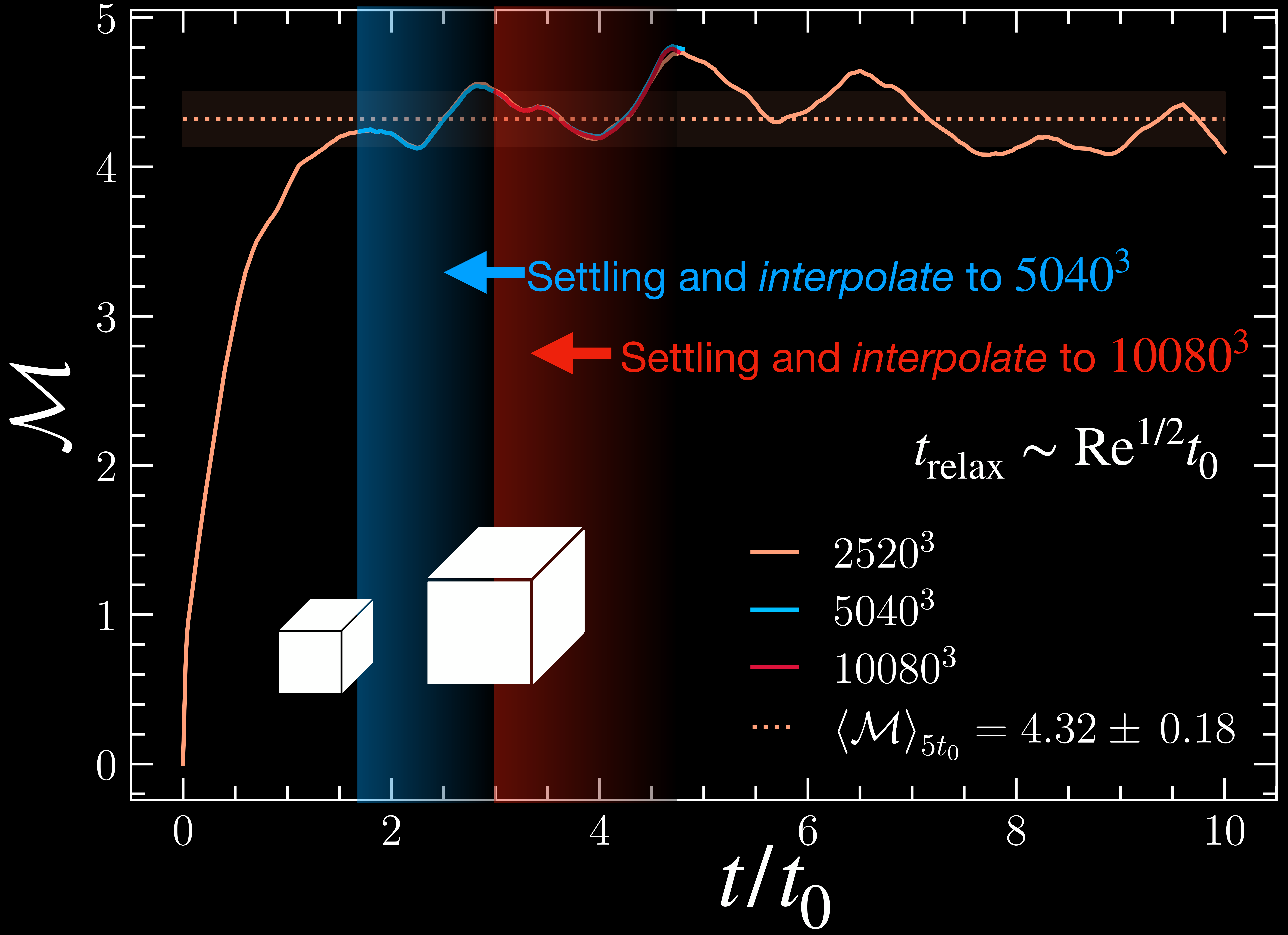
- Highly-modified version of finite volume code *FLASH*, second-order in space approximate Riemann (PPM) solver with framework outlined in Bouchut+2010, tested in *FLASH* in Waagen+2011.
- Ideal (ILES) compressible non-helical, isothermal MHD turbulence with finite correlation time (OU process; Federrath2022).



Volume integral Quantities

$$\mathscr{M} = \left\langle \frac{u^2}{c_s^2} \right\rangle_{\mathscr{V}}^{1/2}$$

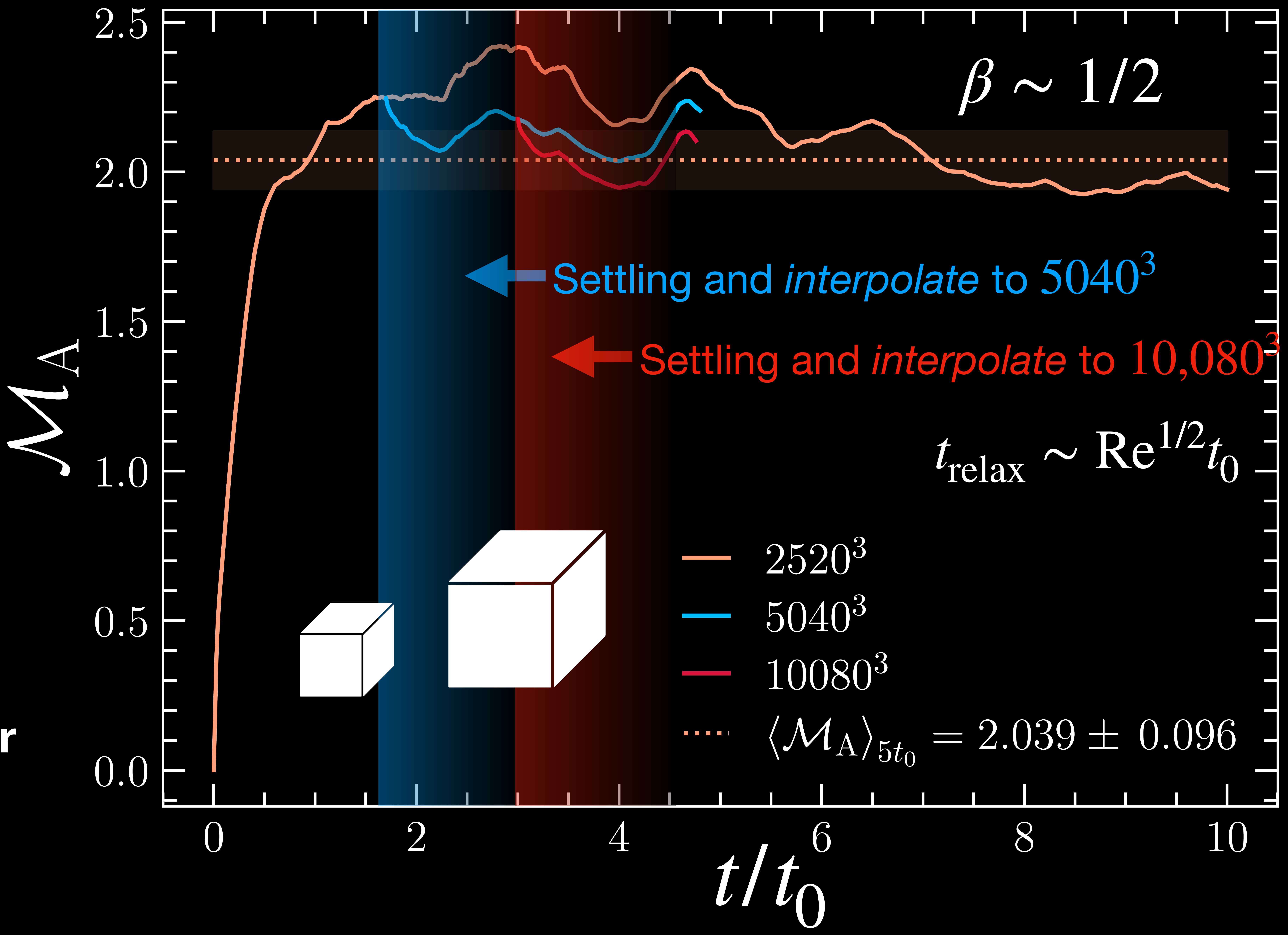
Interpolation to generate ICs for successively higher resolution experiments



Volume integral Quantities

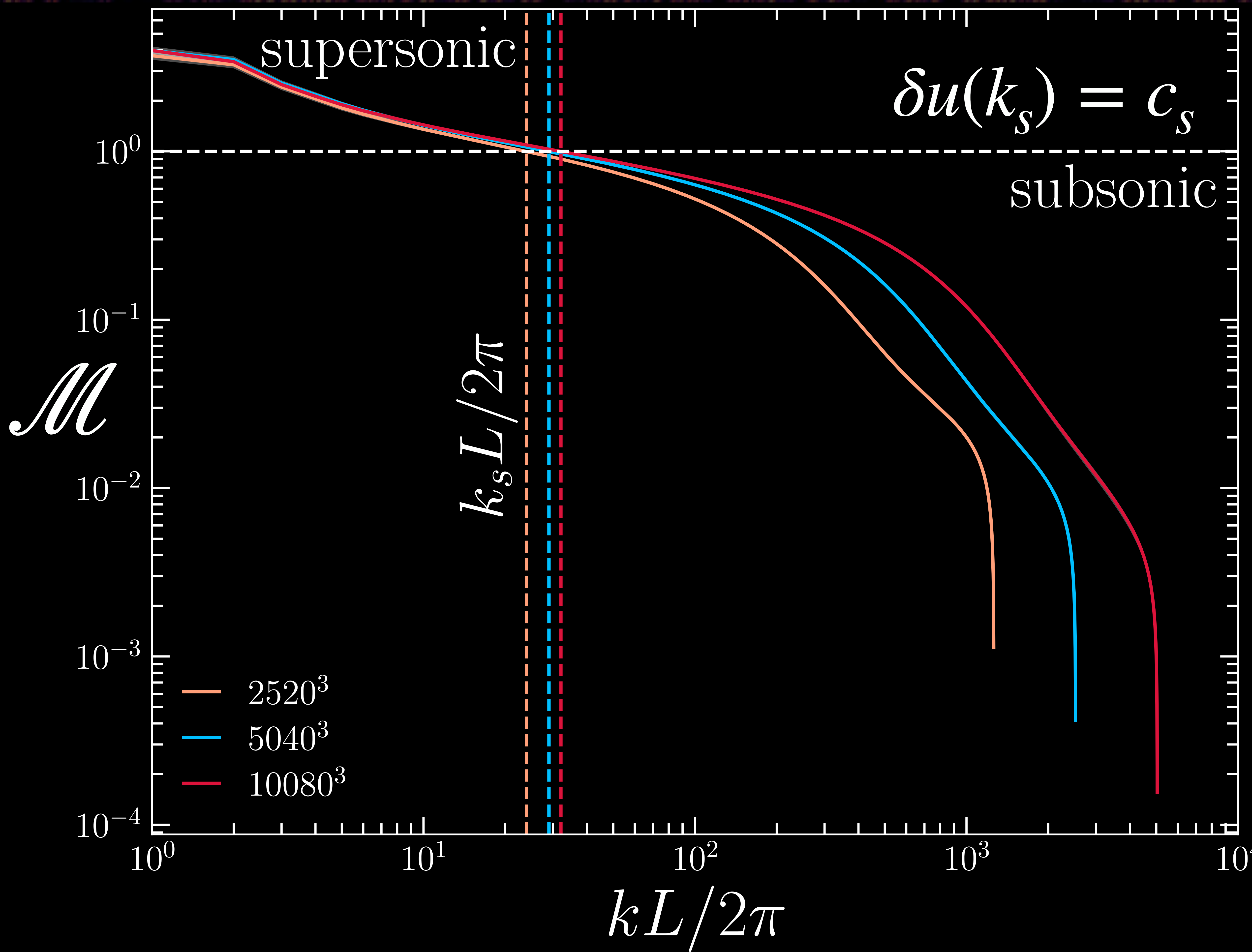
$$\mathscr{M}_A = \sqrt{\frac{E_{\mathrm{kin}}}{E_{\mathrm{mag}}}}$$

Interpolation to generate ICs for successively higher resolution experiments




$\ln(\rho/\rho_0)$

Energy spectrum: two important scales



Sonic scale

Turbulent motions are supersonic

$$\delta u(k) > c_s$$

Transition scale

$$\delta u(k_s) = c_s$$

Turbulent motions are subsonic

$$\delta u(k_s) < c_s$$

Conclusion: no significant change compared to pure hydro.

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Energy spectrum: two important scales

Kinetic energy dominates, $\mathscr{E}_{\mathrm{mag}} < \mathscr{E}_{\mathrm{kin}}$

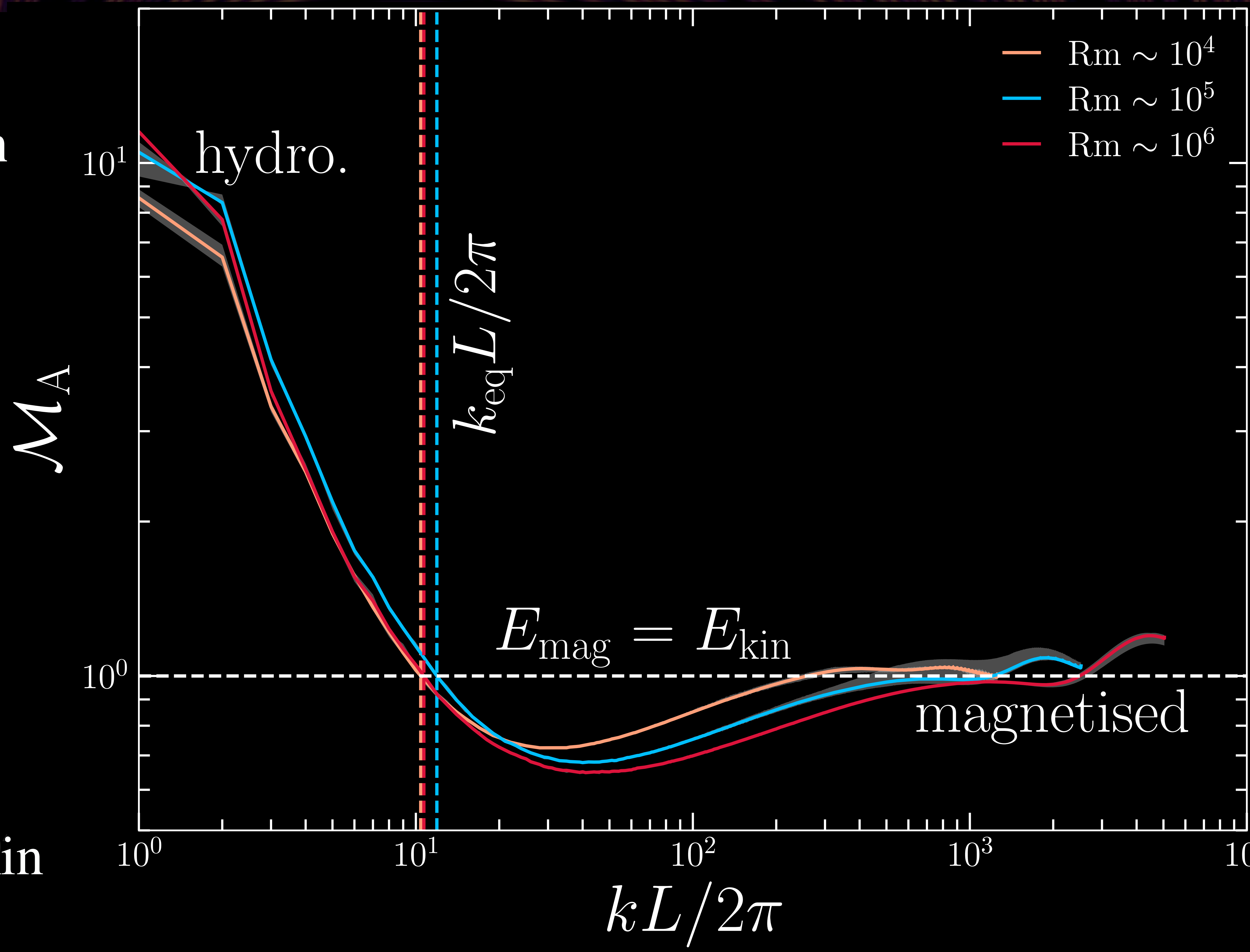
$$\mathscr{M}_A > 1$$

Transition into magnetically dominated turbulence $\mathscr{E}_{\mathrm{mag}} = \mathscr{E}_{\mathrm{kin}}$

$$\mathscr{M}_A = 1$$

Magnetic energy dominates, $\mathscr{E}_{\mathrm{mag}} > \mathscr{E}_{\mathrm{kin}}$

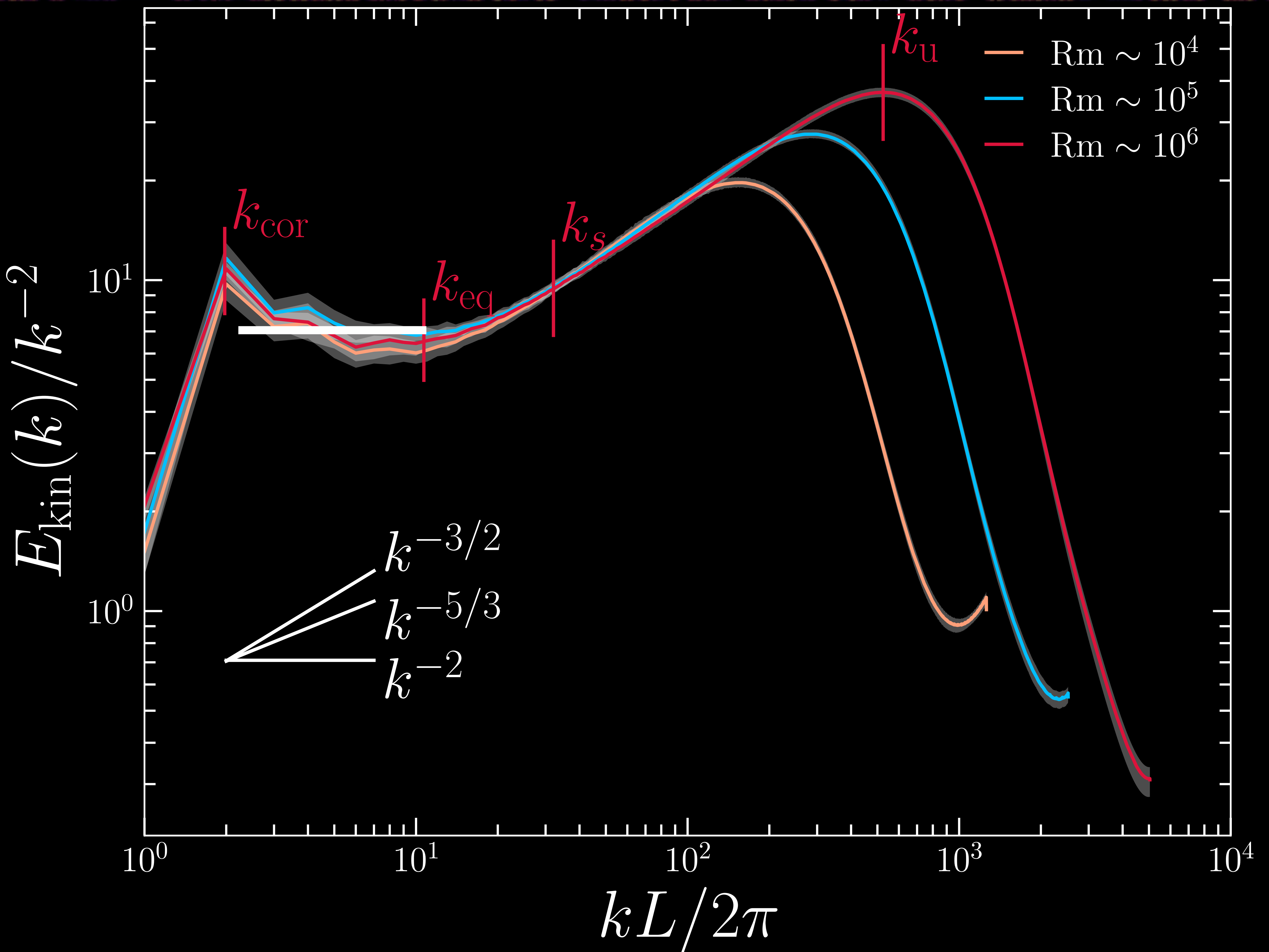
$$\mathscr{M}_A < 1$$

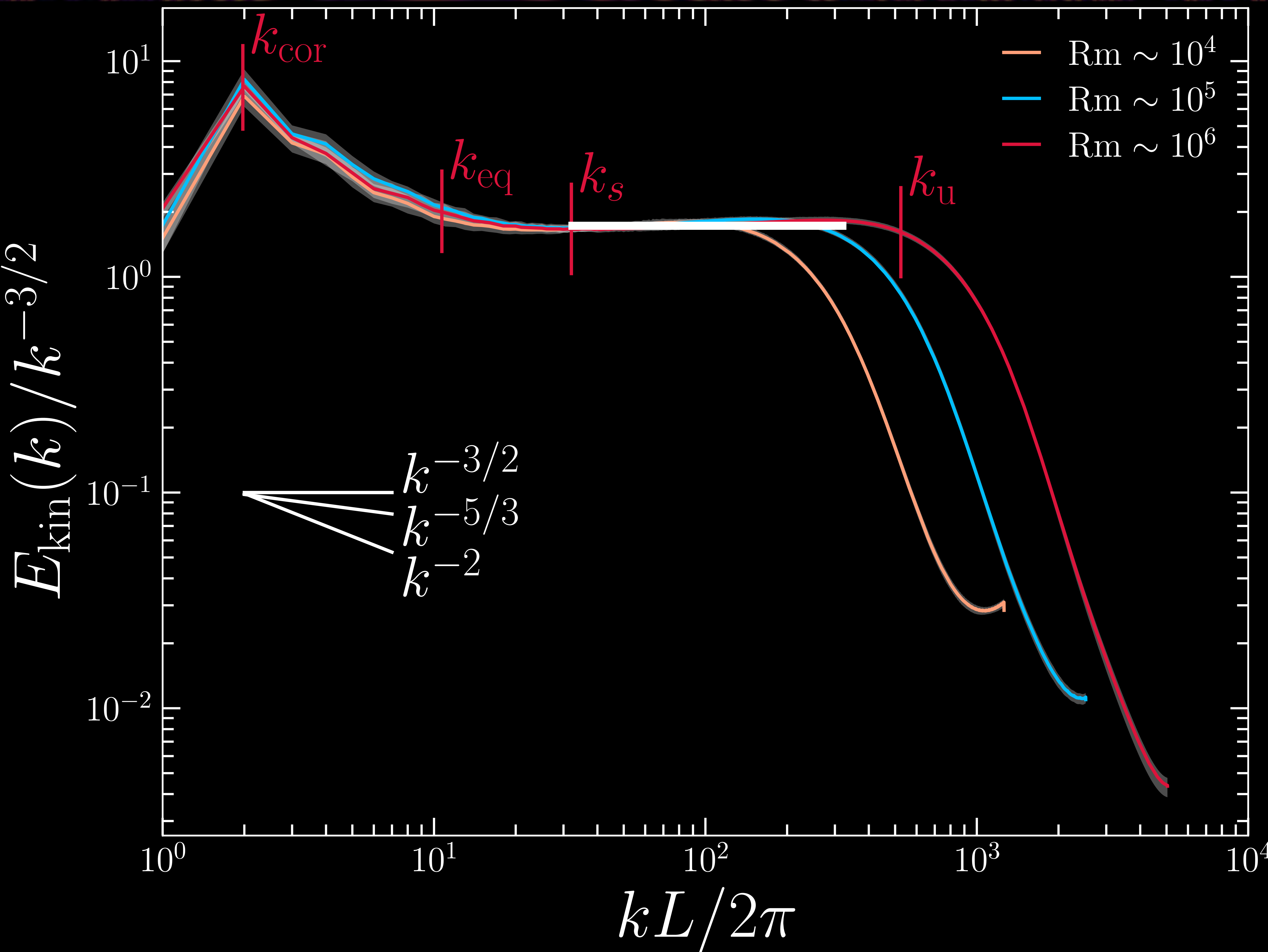


Alfvénic (equipartition) scale

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

The kinetic energy spectrum

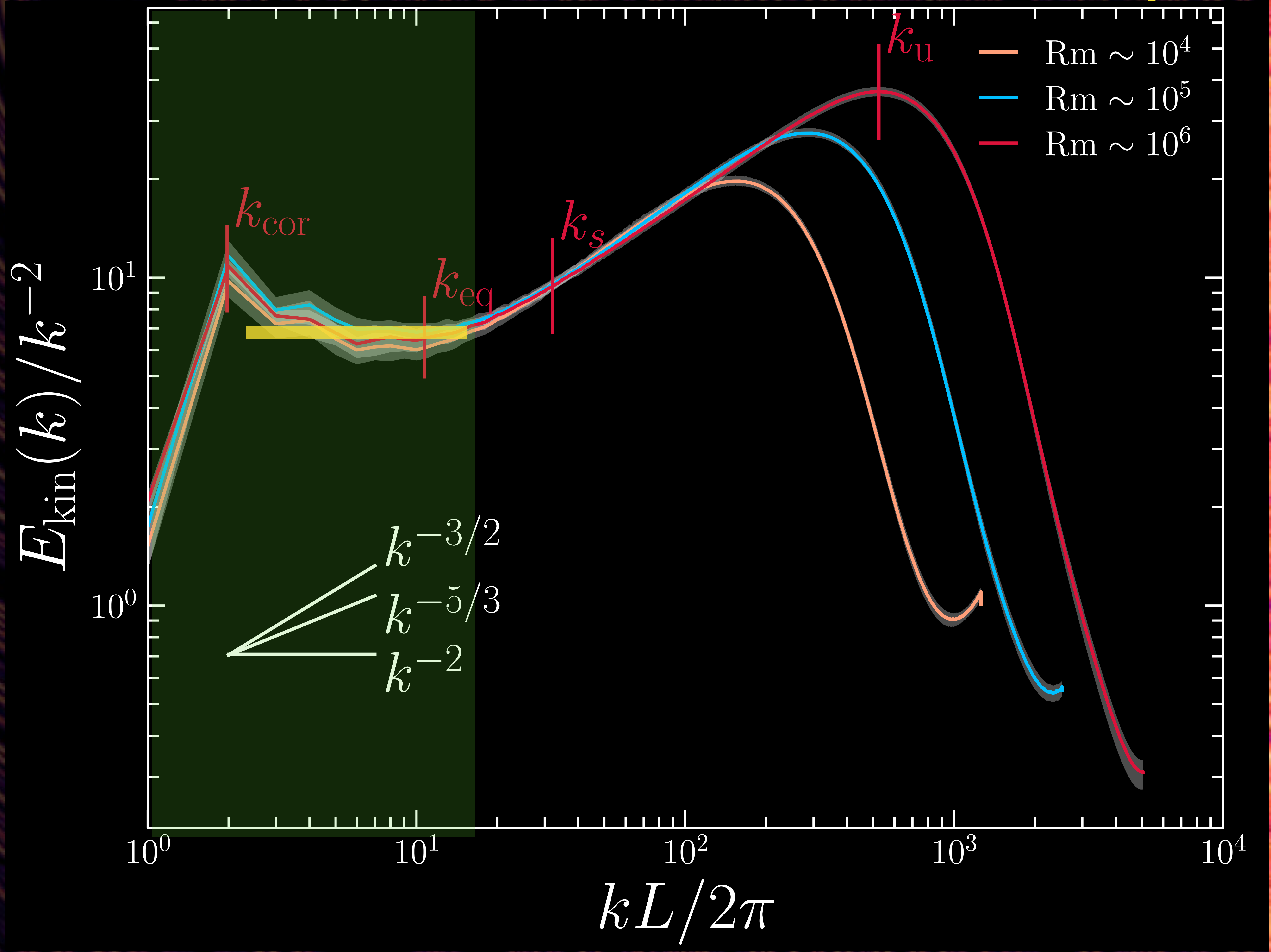




Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

The supersonic component

$$\mathcal{E}(k) \sim k^{-2},\ k \leq k_{\mathrm{eq}}$$



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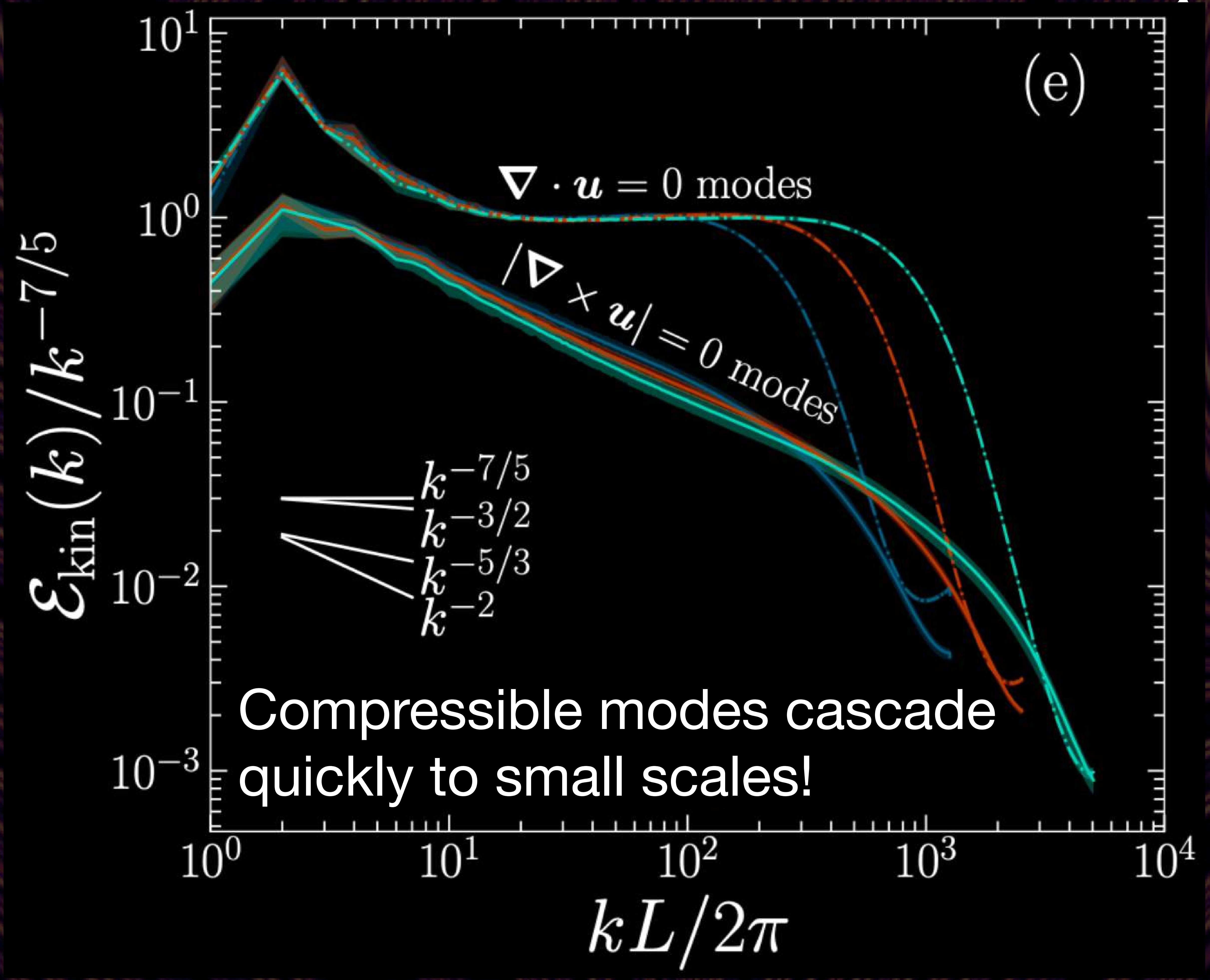


Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

The subsonic component

$$\mathcal{E}(k) \sim k^{-3/2},\ k \gtrsim k_s$$

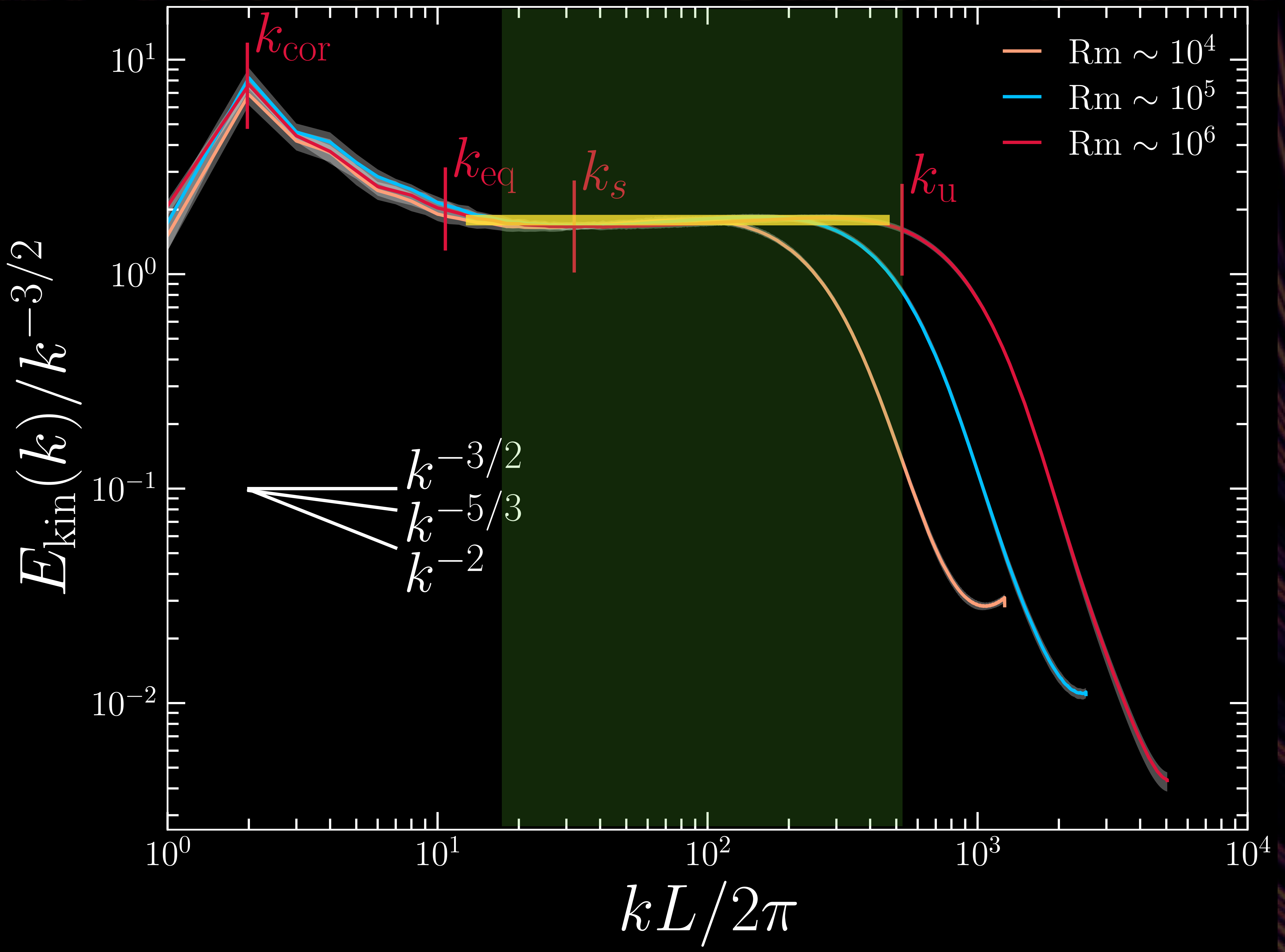
Quasi-incompressible on these scales since steep k^{-2} $|\nabla \times \mathbf{u}| = 0$ mode spectrum.

$$|u_{\mathrm{comp}}| \ll |u_{\mathrm{sol}}|$$

And Alfvénic to sub-Alfvénic

$$\delta u \sim \delta b$$

Perfect conditions for **Alfvén mode theories**. So, dynamical alignment?

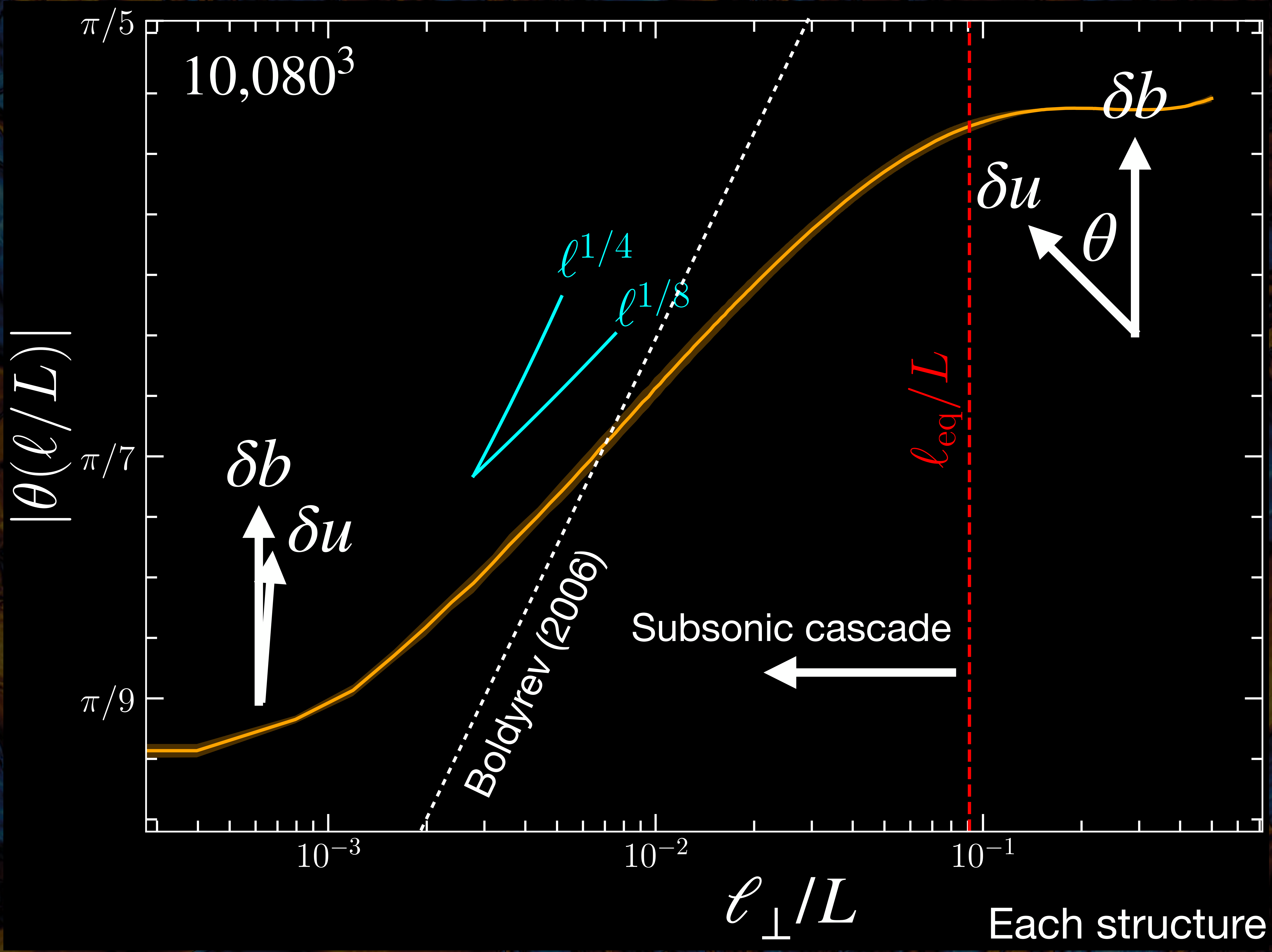


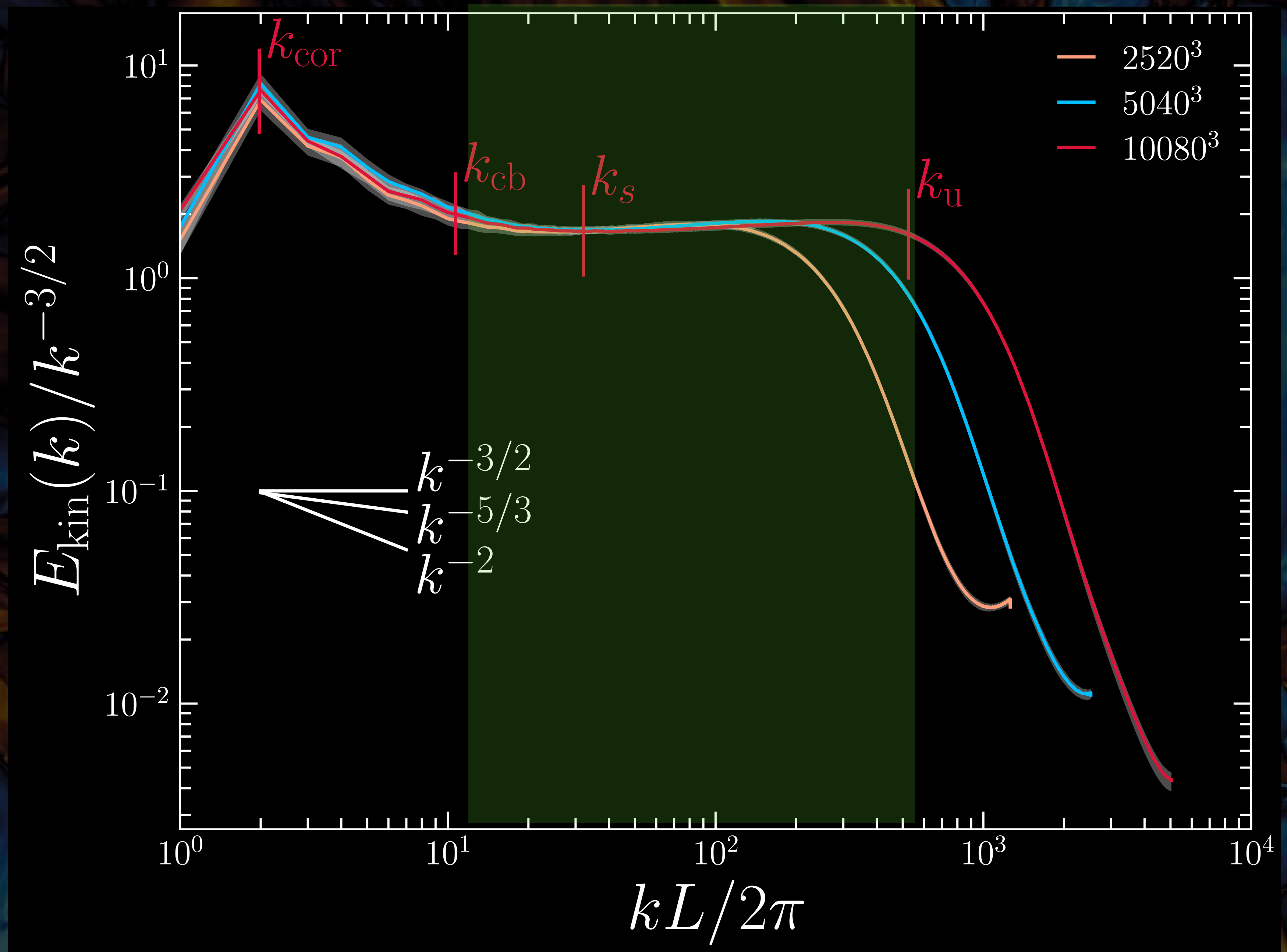
Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

$u \parallel b$
$u \parallel -b$
λ_{eq}
$\mathbf{u} \cdot \mathbf{b}$
(a)
GS1995
$p(\theta_{\mathbf{u},\mathbf{b}})$
$\theta_{\mathbf{u},\mathbf{b}}$
0.9
0.8
0.7
0.6
0.5
0.4
0
$\frac{\pi}{4}$
$\frac{\pi}{2}$
$\frac{3\pi}{4}$
π

Scale dependent alignment

Beattie & Bhattacharjee *(subm.). Scale-dependent alignment and the depletion of nonlinearities in compressible*





We find

$$\theta(\ell_\perp) \sim \ell_\perp^{1/8}$$

scale-dependent alignment, but not Boldyrev (2006).

Each structure function costs 100,000 core hours!

How to deplete the nonlinearities?

Inconsistent with a $k^{-3/2}$ spectrum!

$$\mathcal{E}(k) \propto \varepsilon^{2/3} k^{-5/3} \theta^{-2/3} \sim \boxed{k^{-19/12},} \qquad \theta \sim \ell^{1/8}$$

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

How to deplete the nonlinearities?

Inconsistent with a $k^{-3/2}$ spectrum!

$19/12 \approx 1.58$

$$\mathscr{E}(k) \propto \varepsilon^{2/3} k^{-5/3} \theta^{-2/3} \sim k^{-19/12}, \qquad \theta \sim \ell^{1/8}$$

Crazy idea… what if alignment changes energy flux…

$$\frac{\delta u_\perp^2}{t_{\mathrm{nl}}} \sim \varepsilon \sim \ell^\beta \iff \delta u_\perp(k_\perp) \sim (k^{-\beta})^{2/3} k^{-5/3} (k^{-1/8})^{-2/3}$$

$$\boxed{\varepsilon \sim \Pi_{uu}(k) \sim k^{1/8} \iff \beta = -1/8}$$

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

How to deplete the nonlinearities?

Indepedendently measure kinetic energy flux for all cascade terms following Grete+(2017)

What we want:

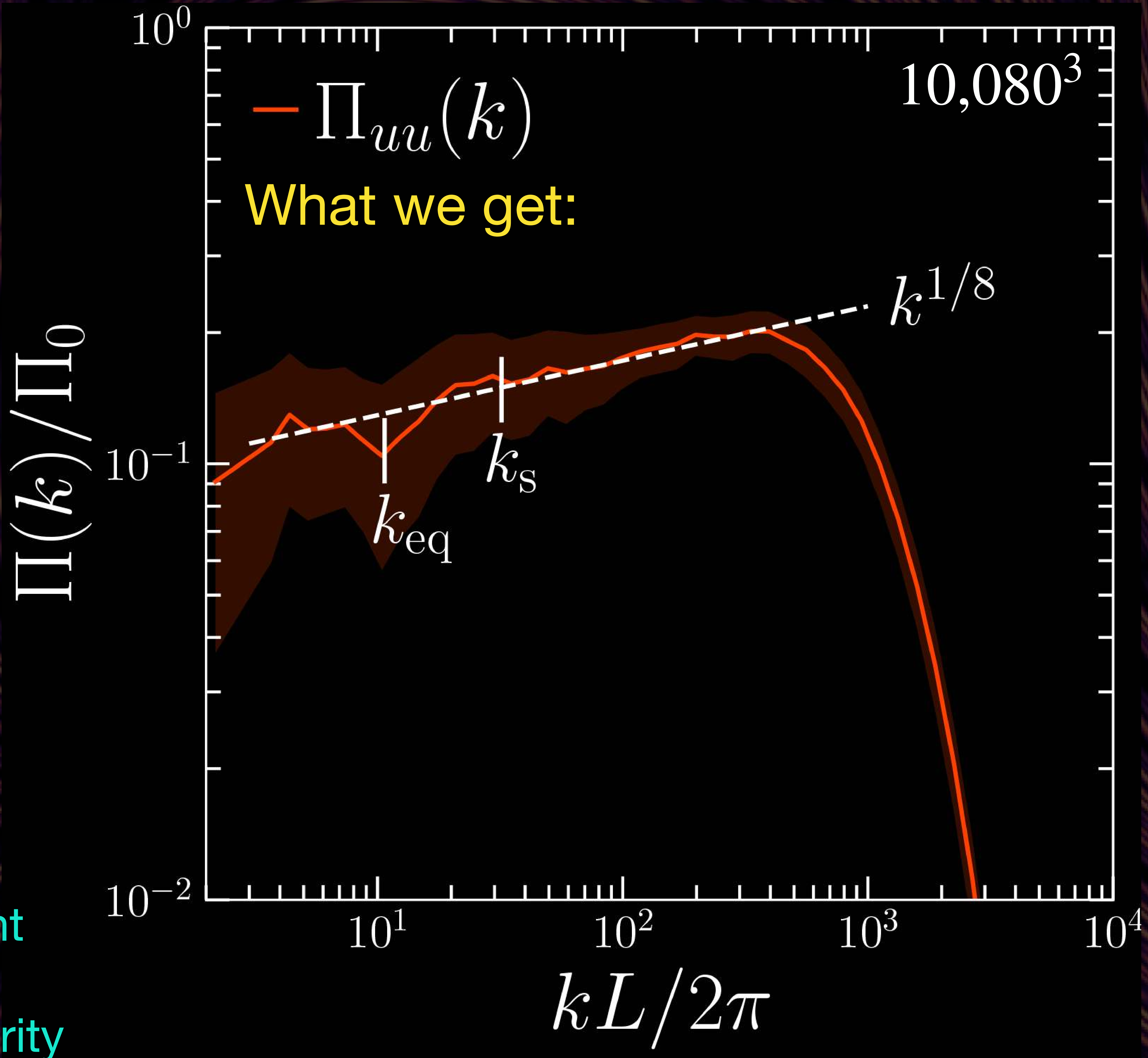
$$\boxed{\varepsilon \sim \Pi_{uu}(k) \sim k^{1/8}}$$

Scale-dependent depletion of energy reservoir

$$\mathcal{E}(k) \propto \boxed{\varepsilon^{2/3}} k^{-5/3} \boxed{\theta^{-2/3}}$$

$$\propto k^{-3/2}$$

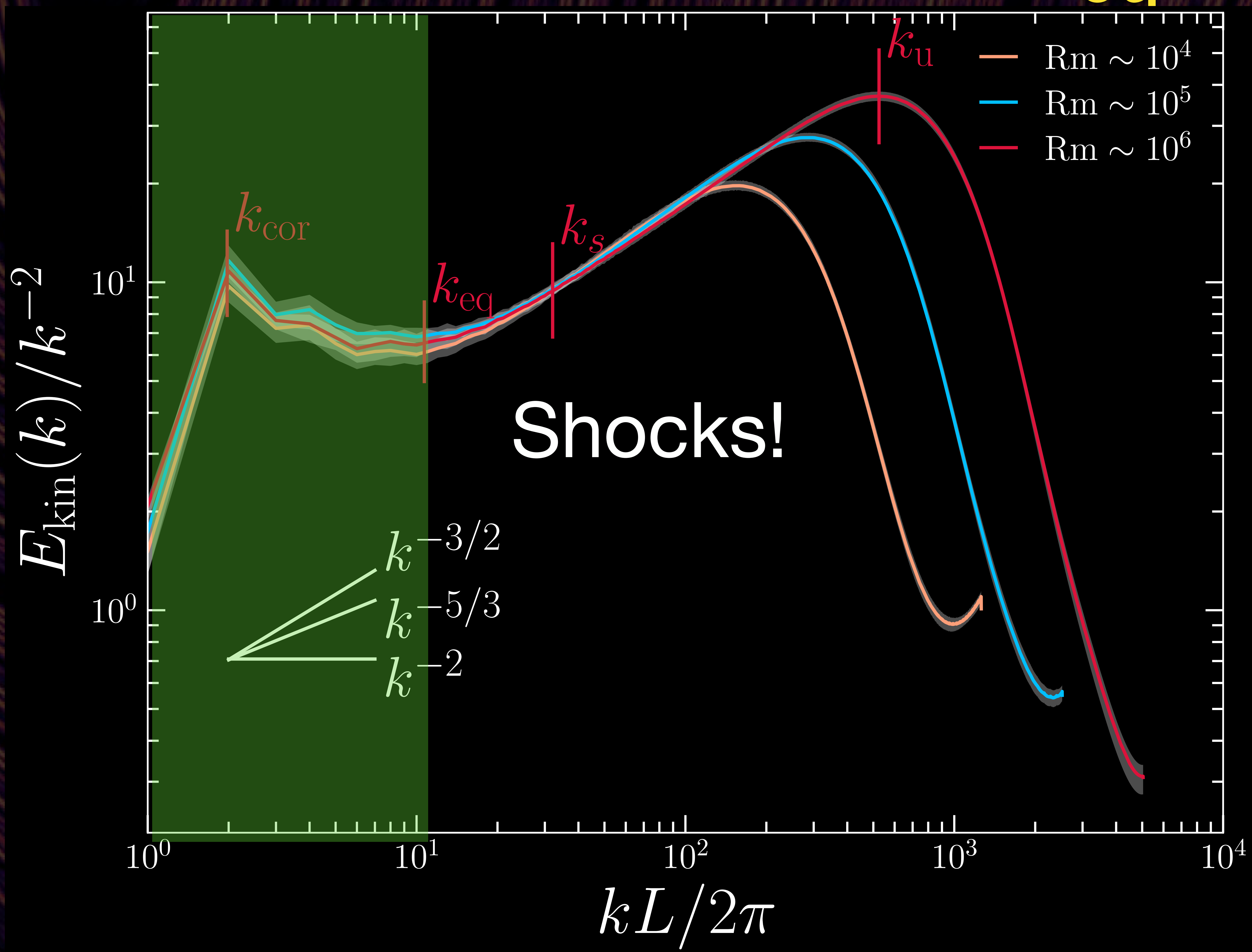
Scale-dependent depletion of turbulent nonlinearity

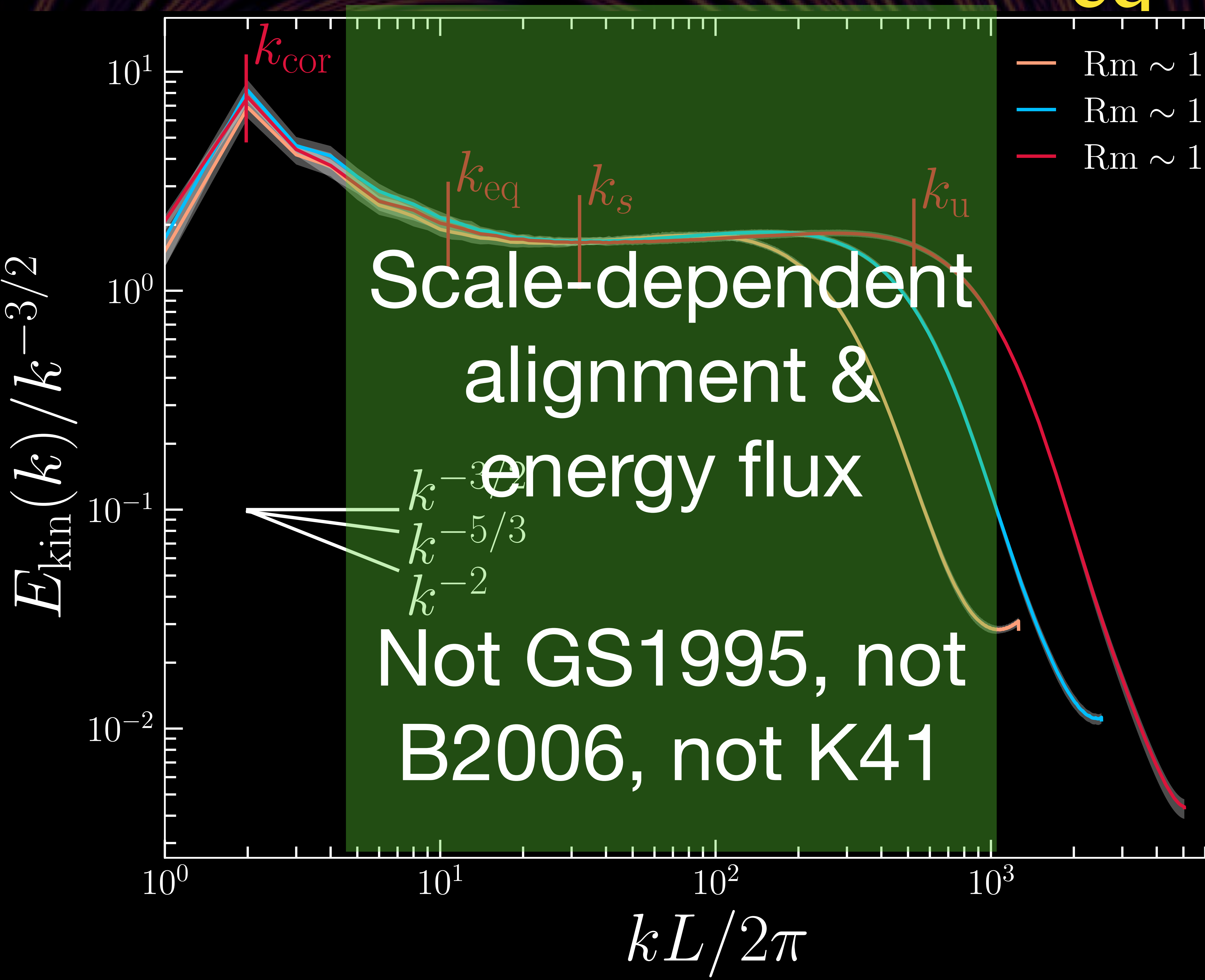


Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Compressible turbulent kinetic energy spectrum at $\mathrm{Re} \sim 10^6$

$\mathscr{E}(k) \sim k^{-2},\ k \leq k_{\mathrm{eq}}$ $\mathscr{E}(k) \sim k^{-3/2},\ k > k_{\mathrm{eq}}$





Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Thanks, questions?

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Lots more interesting aspects that I could not cover

1. Why is the flux scale-dependent?
2. Is the Burgers' spectrum actually a cascade?
3. What does this mean for transition to reconnection and why $\theta \sim \ell^{1/8}$?
4. What happens when you have a strong large-scale magnetic field?
5. You didn't show the magnetic spectrum?
6. What is the asymptotic state of turbulence at $\mathrm{Re} \sim \mathrm{Rm} \to \infty$

Carina Nebula, JWST

Thanks, questions?

james.beattie@princeton.
@astro_magnetism

Lots more interesting aspects that I could not cover

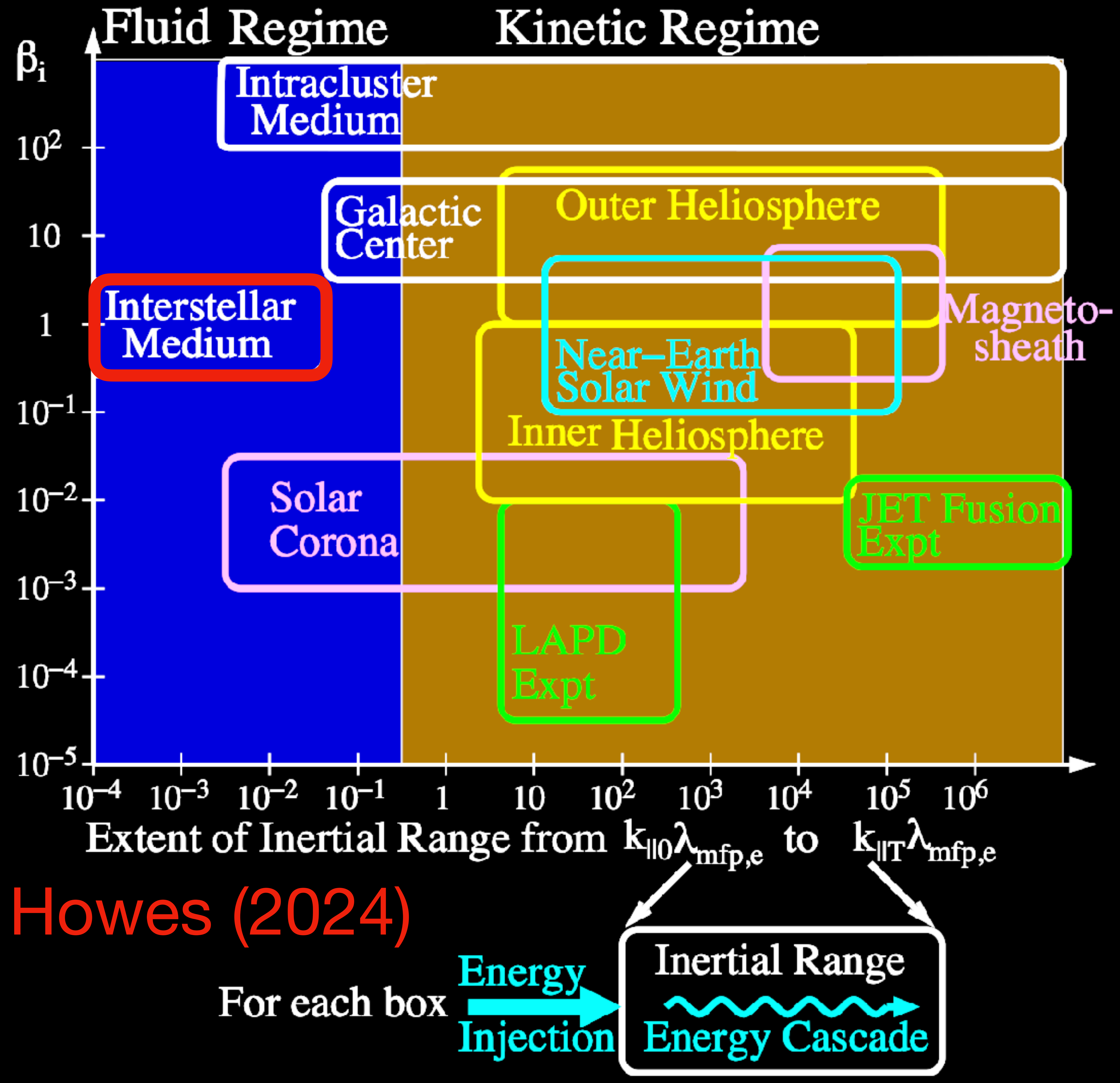
1. Why is the flux scale-dependent? Both the dynamo term and nonlocal compressible fluxes contribute. More work to do. Related to dynamo saturation.
2. Is the Burgers' spectrum actually a cascade? It's a strongly nonlocal one… I calculated the flux transfers.
3. What does this mean for transition to reconnection and why $\theta \sim \ell^{1/8}$? Elias Most and I are thinking about $\theta \sim \ell^{\alpha}$
4. What happens when you have a strong large-scale magnetic field? The next big sims are running as we speak
5. You didn't show the magnetic spectrum? Next time. Invite me to give a longer talk.
6. What is the asymptotic state of turbulence at $\mathrm{Re} \sim \mathrm{Rm} \to \infty$
Depletion of nonlinearities do not beat viscous truncation, but there are some interesting behaviors with other nonlinearities in the MHD equations that have not been explored before.

Carina Nebula, JWST

The ISM: A laboratory for ir

- **Multiphase trans-to-sup**
- $\mathrm{Re} = UL/\nu \gg 1 \implies$ hi
- Turbulence driven by a zo
  - Gravitational instabilities
- **Pervaded by ultra-relativ** energy densities (peaked
- **Non-Gaussian gas-dens** (multi-scale filaments, ribb
- **Dynamically important, r**
- $\mathrm{Rm} = UL/\eta \gg 1 \implies$ h
- ....

Krumholz & McKee (2005); Federrath & Klessen (2012); Tri
Mocz & Burkhart (2018), Burkhart & Mocz (2018); Heyer+(
(2020); Beattie+(2020); Körtgen & Soler (2020); Sk
Hopkins+(2022); Beattie+(2022); Fielding+(20



Howes (2024)