Magnetic dynamos
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Contents

1 Magnetic dynamos

Magnetic dynamos are ubiquitous across many scales in the Universe. They describe the transfer of energy from fluid motions into the magnetic field. One of the main distinctions between dynamo mechanisms is whether they preferentially amplify modes at short wavelengths, near or below the viscous cutoff, or coherent modes at wavelengths comparable to the outer scale of the system. In this lecture, I will move through both types of dynamos and then connect them through the mean electromotive force. At a formal level, both problems are closure problems: in mean-field theory one seeks to express correlations of the form $\langle uB \rangle$ in terms of the large-scale magnetic flux, whereas in the small-scale theory one seeks to express transfer terms of the form $\langle uBB \rangle$ in terms of the magnetic-energy density.

These lecture notes take inspiration from Rincon (2019) and Brandenburg and Ntormousi (2023) and Alex Schekochihin's notes on kinetic and fluid plasmas. I should apologize in advance that the citation list is not intended to be exhaustive; rather, I mostly highlight a few papers that I and my collaborators have found useful, or have written ourselves, and even then not extensively, in order to keep the discussion focused.

1.1 Goals for this lecture

- Outline the difference between large-scale and small-scale dynamo theory.
- Define the theoretical frameworks for each theory, such that you can apply them to your problems.

2 The induction equation

The central equation of dynamo theory is the induction equation,

$$\partial_t \boldsymbol{B} = \boldsymbol{\nabla} \times (\boldsymbol{u} \times \boldsymbol{B}) + \frac{1}{\mathrm{Rm}} \nabla^2 \boldsymbol{B}, \qquad \boldsymbol{\nabla} \cdot \boldsymbol{B} = 0, \tag{1}$$

where

$$\mathrm{Rm} = \frac{UL}{\eta} \tag{2}$$

is the magnetic Reynolds number, with U and L characteristic velocity and length scales and η the microscopic magnetic diffusivity. Using the full vector identity

$$\boldsymbol{\nabla} \times (\boldsymbol{u} \times \boldsymbol{B}) = (\boldsymbol{B} \cdot \boldsymbol{\nabla})\boldsymbol{u} - (\boldsymbol{u} \cdot \boldsymbol{\nabla})\boldsymbol{B} + \boldsymbol{u}(\boldsymbol{\nabla} \cdot \boldsymbol{B}) - \boldsymbol{B}(\boldsymbol{\nabla} \cdot \boldsymbol{u}), \tag{3}$$

and then imposing $\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0$, the induction equation may be rewritten as

$$(\partial_t + \boldsymbol{u} \cdot \boldsymbol{\nabla}) \boldsymbol{B} = (\boldsymbol{B} \cdot \boldsymbol{\nabla})\boldsymbol{u} - \boldsymbol{B}(\boldsymbol{\nabla} \cdot \boldsymbol{u}) + \frac{1}{\mathrm{Rm}} \nabla^2 \boldsymbol{B}. \tag{4}$$

Dotting Equation 4 with $\boldsymbol{B}$ gives the exact magnetic-energy identity

$$\frac{1}{2} (\partial_t + \boldsymbol{u} \cdot \boldsymbol{\nabla}) B^2 = \overbrace{(\hat{\boldsymbol{B}} \otimes \hat{\boldsymbol{B}} : \boldsymbol{\nabla} \otimes \boldsymbol{u})}^{\gamma} B^2 - (\boldsymbol{\nabla} \cdot \boldsymbol{u}) B^2 + \frac{1}{\mathrm{Rm}} \boldsymbol{B} \cdot \nabla^2 \boldsymbol{B}, \tag{5}$$

where

$$\gamma \equiv \hat{\boldsymbol{B}} \otimes \hat{\boldsymbol{B}} : \boldsymbol{\nabla} \otimes \boldsymbol{u} \tag{6}$$

is the local stretching rate along the magnetic-field direction. In the incompressible limit, $\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0$, the compressive term drops out and only stretching and resistive diffusion remain:

$$\frac{1}{2} (\partial_t + \boldsymbol{u} \cdot \boldsymbol{\nabla}) B^2 = \gamma B^2 + \frac{1}{\mathrm{Rm}} \boldsymbol{B} \cdot \nabla^2 \boldsymbol{B}. \tag{7}$$

A dynamo corresponds to the existence of magnetic modes that grow or are maintained in time.

The remainder of this section is heuristic and is included only to motivate the scale-by-scale picture used later. To understand how growth can arise, it is useful to compare inductive stretching and resistive diffusion at wavenumber k. In Fourier space, resistive diffusion acts locally on each mode as

$$\gamma_\eta(k) = -\eta k^2, \tag{8}$$

while the inductive term couples different scales through the velocity field. If one assumes that the transfer is predominantly local in scale, so that magnetic structures of size $\ell \sim k^{-1}$ are stretched mainly by eddies of comparable size, then the characteristic strain rate is estimated by

$$\gamma_{\mathrm{str}}(k) \sim k u_k, \tag{9}$$

where u_k is the characteristic velocity increment associated with scales near k^{-1}. The net growth rate of magnetic fluctuations at scale k is therefore, schematically,

$$\gamma(k) \sim k u_k - \eta k^2. \tag{10}$$

Magnetic amplification at scale k requires

$$\gamma(k) > 0 \iff k u_k > \eta k^2 \iff \mathrm{Rm}_k \equiv \frac{u_k}{\eta k} > 1. \tag{11}$$

This defines a scale-dependent magnetic Reynolds number Rm_k, and implies the existence of a resistive cutoff scale k_η such that

$$\gamma(k_\eta) = 0. \tag{12}$$

Induction dominates for $k < k_\eta$, while diffusion dominates for $k > k_\eta$. This estimate should not be read as a rigorous dynamo criterion: the actual onset problem is an eigenvalue problem for the induction operator, and growing modes cannot in general be inferred from a pointwise condition on $\gamma(k)$ alone. It does, however, capture the basic idea that there must be a band of scales for which inductive stretching beats resistive diffusion. Equivalently, one expects a critical outer-scale magnetic Reynolds number Rm_c above which the induction operator admits exponentially growing modes.

In the mean-field sections below I continue to use the nondimensional diffusion coefficient $1/\mathrm{Rm}$. In the small-scale Kazantsev section, however, I revert to the conventional dimensional notation η for magnetic diffusivity and introduce the hydrodynamic Reynolds number $\mathrm{Re} = UL/\nu$, because those are the standard variables in that literature.

3 Small-scale dynamo (kinematic only)

3.1 Magnetic energy spectrum evolution

In component form, this may be written as

$$\partial_t B_i = \partial_j (u_i B_j - u_j B_i) + \eta \nabla^2 B_i. \tag{13}$$

Introducing the Fourier transform,

$$B_i(\boldsymbol{k}, t) = \int \mathrm{d}^3 x\, e^{-i\boldsymbol{k}\cdot\boldsymbol{x}} B_i(\boldsymbol{x}, t), \tag{14}$$

the induction equation becomes

$$\partial_t B_i(\boldsymbol{k}) = i k_j \int \frac{\mathrm{d}^3 k'\, \mathrm{d}^3 k''}{(2\pi)^3} \delta(\boldsymbol{k} - \boldsymbol{k}' - \boldsymbol{k}'') \Big[\hat{u}_i(\boldsymbol{k}') \hat{B}_j(\boldsymbol{k}'') - \hat{u}_j(\boldsymbol{k}') \hat{B}_i(\boldsymbol{k}'') \Big] - \eta k^2 \hat{B}_i(\boldsymbol{k}). \tag{15}$$

The magnetic energy associated with mode $\boldsymbol{k}$ is

$$E_B(\boldsymbol{k}, t) \equiv \frac{1}{2}|\hat{\boldsymbol{B}}(\boldsymbol{k}, t)|^2. \tag{16}$$

Its evolution is therefore

$$\partial_t E_B(\boldsymbol{k}, t) = \mathrm{Re}\left[\hat{B}_i^\dagger(\boldsymbol{k})\partial_t \hat{B}_i(\boldsymbol{k})\right] \tag{17}$$

$$= \mathrm{Re}\left\{ik_j \hat{B}_i^\dagger(\boldsymbol{k}) \int \frac{\mathrm{d}^3k'\ \mathrm{d}^3k''}{(2\pi)^3}\delta(\boldsymbol{k} - \boldsymbol{k}' - \boldsymbol{k}'')\left[\hat{u}_i(\boldsymbol{k}')\hat{B}_j(\boldsymbol{k}'') - \hat{u}_j(\boldsymbol{k}')\hat{B}_i(\boldsymbol{k}'')\right]\right\} - 2\eta k^2 E_B(\boldsymbol{k}, t). \tag{18}$$

To pass from the mode-by-mode budget to an isotropic shell budget, define the one-dimensional magnetic energy spectrum by integrating over the spherical shell $|\boldsymbol{k}| = k$:

$$M(k, t) = \frac{1}{2}\int k^2\, \mathrm{d}\Omega_k\, |\hat{\boldsymbol{B}}(\boldsymbol{k}, t)|^2, \tag{19}$$

so that

$$M(k, t) = \int k^2\, \mathrm{d}\Omega_k\, E_B(\boldsymbol{k}, t). \tag{20}$$

Differentiating at fixed k gives

$$\partial_t M(k, t) = \int k^2\, \mathrm{d}\Omega_k\, \partial_t E_B(\boldsymbol{k}, t), \tag{21}$$

and substituting the mode-energy equation yields the exact shell-integrated budget

$$\partial_t M(k, t) = \mathcal{T}_B(k, t) - 2\eta k^2 M(k, t). \tag{22}$$

It is useful to isolate the triadic integrand

$$T_B(\boldsymbol{k}; \boldsymbol{k}', \boldsymbol{k}'', t) \equiv \mathrm{Re}\left\{ik_j \hat{B}_i^\dagger(\boldsymbol{k})\delta(\boldsymbol{k} - \boldsymbol{k}' - \boldsymbol{k}'')\left[\hat{u}_i(\boldsymbol{k}')\hat{B}_j(\boldsymbol{k}'') - \hat{u}_j(\boldsymbol{k}')\hat{B}_i(\boldsymbol{k}'')\right]\right\}, \tag{23}$$

where

$$\mathcal{T}_B(k, t) = \int k^2\, \mathrm{d}\Omega_k \int \frac{\mathrm{d}^3k'\ \mathrm{d}^3k''}{(2\pi)^3} T_B(\boldsymbol{k}; \boldsymbol{k}', \boldsymbol{k}'', t). \tag{24}$$

The delta function enforces the triad condition $\boldsymbol{k} = \boldsymbol{k}' + \boldsymbol{k}''$, so $\mathcal{T}_B(k, t)$ is the total induction contribution into the shell of radius k after summing over all velocity–magnetic–magnetic triads that end on that shell. The resistive term is local in k, while the inductive term is nonlocal in Fourier space and depends on mixed third-order correlations. In the kinematic problem this term describes both redistribution among magnetic wavenumbers and net amplification by the prescribed velocity field, so it is not closed as written. A statistical closure is therefore required to obtain a tractable evolution equation for $M(k, t)$.

3.2 The Kazantsev (1968) $k^{3/2}$ model

3.2.1 Closure of the third-order correlators

The exact spectral evolution equation

$$\partial_t M(k, t) = \mathcal{T}_B(k, t) - 2\eta k^2 M(k, t) \tag{25}$$

is not closed, since the inductive transfer term $\mathcal{T}_B$ depends on mixed third-order moments of the form $\langle uBB\rangle$. Thus, the evolution of the second-order magnetic correlator depends on higher-order correlations, and a closure is required.

The Kazantsev–Kraichnan model provides such a closure by replacing the velocity field with a prescribed random process that is Gaussian, homogeneous, isotropic, non-helical, and δ-correlated in time. Its two-point covariance is

$$\langle u_i(\boldsymbol{x},t)u_j(\boldsymbol{x}',t')\rangle = \kappa_{ij}(\boldsymbol{r})\,\delta(t-t'), \qquad \boldsymbol{r} \equiv \boldsymbol{x}' - \boldsymbol{x}, \tag{26}$$

with the isotropic tensor decomposition

$$\kappa_{ij}(\boldsymbol{r}) = \kappa_N(r)\left(\delta_{ij} - \frac{r_i r_j}{r^2}\right) + \kappa_L(r)\frac{r_i r_j}{r^2}. \tag{27}$$

For an incompressible flow, $\partial_i u_i = 0$, so the transverse and longitudinal correlators are not independent:

$$\kappa_N(r) = \kappa_L(r) + \frac{r}{2}\kappa_L'(r). \tag{28}$$

In the kinematic regime, the velocity is statistically independent of the magnetic field. The δ-correlation in time implies that the velocity has no memory, so the magnetic field experiences a Markovian sequence of random stretching events.

The most natural object to evolve is the equal-time magnetic two-point correlator,

$$H_{ij}(\boldsymbol{r},t) \equiv \langle B_i(\boldsymbol{x},t)B_j(\boldsymbol{x}',t)\rangle\,. \tag{29}$$

Isotropy implies the analogous decomposition

$$H_{ij}(\boldsymbol{r},t) = H_N(r,t)\left(\delta_{ij} - \frac{r_i r_j}{r^2}\right) + H_L(r,t)\frac{r_i r_j}{r^2}, \tag{30}$$

and $\boldsymbol{\nabla}\cdot\boldsymbol{B} = 0$ further gives

$$H_N(r,t) = H_L(r,t) + \frac{r}{2}H_L'(r,t). \tag{31}$$

Thus the whole isotropic problem reduces to the single scalar function $H_L(r,t)$.

Under these assumptions, Gaussian integration by parts (the Furutsu–Novikov formula; "Gaussian integration by parts"; Martins Afonso et al. 2019) gives

$$\langle u_i(\boldsymbol{x},t)\,B_j(\boldsymbol{x},t)B_k(\boldsymbol{x}',t)\rangle = \int \mathrm{d}t'\ \mathrm{d}^3x'\ \langle u_i(\boldsymbol{x},t)u_m(\boldsymbol{x}',t')\rangle \left\langle \frac{\delta\big(B_j(\boldsymbol{x},t)B_k(\boldsymbol{x}',t)\big)}{\delta u_m(\boldsymbol{x}',t')}\right\rangle. \tag{32}$$

The point of this identity is that, for a Gaussian velocity field, every occurrence of u_i multiplying a functional of the velocity can be traded for a two-point covariance and a functional derivative. Here that functional is B_jB_k. The $\delta(t-t')$ covariance collapses the time integral to the instantaneous response at $t' = t$, so the only remaining task is to understand how B_jB_k changes under a perturbation of the velocity field.

From the induction equation, the velocity enters through $\partial_\ell(u_iB_\ell - u_\ell B_i)$, so varying with respect to u_m brings down one spatial derivative acting on the magnetic two-point correlator. Schematically,

$$\frac{\delta\big(B_jB_k\big)}{\delta u_m} \ \sim\ \partial_\ell\big(B_jB_k\big). \tag{33}$$

This is the key closure step: the third-order object $\langle uBB\rangle$ is converted into derivatives of the second-order object $\langle BB\rangle$. Substituting this into the Furutsu–Novikov relation yields

$$\langle u_iB_jB_k\rangle \ \sim\ \int \mathrm{d}^3x'\,\kappa_{im}(\boldsymbol{x}-\boldsymbol{x}')\,\partial_\ell\,\langle B_j(\boldsymbol{x})B_k(\boldsymbol{x}')\rangle\,. \tag{34}$$

In the smooth-flow limit, the velocity correlator may be expanded for small separations $|\boldsymbol{r}| \ll 1$,

$$\kappa_{im}(\boldsymbol{r}) = \kappa_{im}(0) + \frac{1}{2}r_\ell r_n\,\partial_\ell\partial_n\kappa_{im}(0) + \cdots\,. \tag{35}$$

The leading nontrivial contribution is quadratic in $\boldsymbol{r}$, and, after integrating by parts, introduces a second spatial derivative acting on the correlator,

$$\langle u_i B_j B_k \rangle \;\sim\; \partial_\ell \partial_n \langle B_j B_k \rangle \,. \tag{36}$$

In an isotropic flow, the tensor $\partial_\ell \partial_n$ reduces to isotropic combinations of $\delta_{\ell n}$ and $\hat{r}_\ell \hat{r}_n$, so that upon contraction it yields an effective second-order scalar operator. Thus, the third-order correlator may be expressed in terms of spatial derivatives of the second-order correlator, schematically,

$$\langle uBB \rangle \;\sim\; \kappa\, \boldsymbol{\nabla}\boldsymbol{\nabla} \langle BB \rangle \,, \tag{37}$$

which closes the hierarchy at second order. Carrying this calculation through and projecting onto the longitudinal scalar correlator gives the closed Kazantsev equation

$$\partial_t H_L = \kappa(r) H_L'' + \left(\frac{4\kappa(r)}{r} + \kappa'(r) \right) H_L' + \left(\kappa''(r) + \frac{4\kappa'(r)}{r} \right) H_L, \tag{38}$$

where

$$\kappa(r) = 2\eta + \kappa_L(0) - \kappa_L(r) \tag{39}$$

is twice the effective turbulent-plus-microscopic diffusivity. Equation 38 is the mathematically precise closed form of the isotropic, non-helical Kazantsev model.

The isotropic magnetic energy spectrum $M(k,t)$ is then obtained from H_{ij} by Fourier transform and angular integration. To get a simple spectral equation, one now specializes to the smooth-flow, large-Pm Batchelor regime, in which the magnetic field lives at sub-viscous scales and the velocity is differentiable across the magnetic structures. In that limit the velocity covariance can be expanded at small r:

$$\begin{aligned} \kappa_L(r) &= \kappa_0 - \frac{\kappa_2}{4} r^2 + \cdots \,, \\ \kappa_N(r) &= \kappa_0 - \frac{\kappa_2}{2} r^2 + \cdots \,. \end{aligned} \tag{40}$$

Equivalently,

$$\kappa_{ij}(\boldsymbol{r}) = \kappa_0 \delta_{ij} - \frac{\kappa_2 r^2}{2} \left(\delta_{ij} - \frac{1}{2} \frac{r_i r_j}{r^2} \right) + \cdots \,. \tag{41}$$

In this regime, carrying the isotropic reduction through to Fourier space leaves a scalar differential operator acting on $M(k,t)$. The advantage of working with second-order correlators is that the Kazantsev closure then yields a closed expression for the triadic transfer term,

$$\mathcal{T}_B(k,t) = \frac{\gamma}{5} \left(k^2 \frac{\partial^2}{\partial k^2} - 2k \frac{\partial}{\partial k} + 6 \right) M(k,t), \tag{42}$$

so that the spectral evolution equation becomes

$$\partial_t M = \frac{\gamma}{5} \left(k^2 \frac{\partial^2 M}{\partial k^2} - 2k \frac{\partial M}{\partial k} + 6M \right) - 2\eta k^2 M. \tag{43}$$

This spectral form is not the general Kazantsev equation; it is the Batchelor-limit spectral equivalent of Equation 38. The operator in Equation 43 is the spectral counterpart of diffusion plus drift in $\ln k$: the second derivative spreads the spectrum in logarithmic wavenumber, while the first-derivative and zeroth-order pieces encode systematic drift and net amplification. The velocity statistics enter only through

$$\gamma = \frac{5}{4} \kappa_2 = \frac{5}{2} |\kappa_L''(0)|, \tag{44}$$

which is of order the viscous-scale strain rate. In isotropic turbulence,

$$\gamma \sim \frac{\delta u_\nu}{\ell_\nu}, \qquad \gamma t_0 \sim \mathrm{Re}^{1/2}, \tag{45}$$

where $t_0 \sim L/U$ is the outer-scale turnover time and Kolmogorov scaling has been used in the last estimate. Equation 43 may therefore be interpreted as a Fokker–Planck equation in $\ln k$, in which stochastic stretching drives diffusion and drift in wavenumber space, while resistivity provides an absorbing boundary at large k.

3.2.2 Diffusion-free regime

For $k \ll k_\eta$, resistive damping is negligible and the evolution is governed by stochastic stretching alone. In the Batchelor regime this means the relevant interval is the sub-viscous but still diffusion-free range $k_\nu \ll k \ll k_\eta$. The easiest way to see the structure of the solution is to change variables to $y = \ln k$ and factor out the stationary power-law prefactor by writing $M = e^{3y/2}\Psi(y,t)$. Equation 43 then becomes an ordinary diffusion equation for Ψ in y, together with a uniform growth term $3\gamma/4$. In this regime, the Fokker–Planck equation admits the exact Green's function solution

$$M(k,t) = e^{3\gamma t/4} \int_0^\infty \frac{\mathrm{d}k_0}{k_0}\, M_0(k_0) \left(\frac{k}{k_0}\right)^{3/2} \sqrt{\frac{5}{4\pi\gamma t}} \exp\left[-\frac{5\ln^2(k/k_0)}{4\gamma t}\right]. \tag{46}$$

This solution describes a lognormal diffusion process in $\ln k$. The Gaussian factor is the Green's function of diffusion in $y = \ln k$, while the prefactor $(k/k_0)^{3/2}$ comes from the integrating-factor transformation that removes the drift term. In particular, three distinct rates are encoded in the kernel. First, each fixed mode grows exponentially at rate

$$\gamma_{\mathrm{mode}} = \frac{3\gamma}{4}. \tag{47}$$

Second, the Gaussian kernel broadens in logarithmic wavenumber with variance

$$\sigma^2_{\ln k} = \frac{2\gamma t}{5}, \tag{48}$$

so that the spectral width grows as

$$\sigma_{\ln k} = \sqrt{\frac{2\gamma t}{5}}. \tag{49}$$

Third, the peak of the Green's-function kernel drifts to larger wavenumber according to

$$\ln\left(\frac{k_{\mathrm{peak}}}{k_0}\right) = \frac{3\gamma t}{5}, \qquad k_{\mathrm{peak}}(t) \sim k_0 e^{3\gamma t/5}. \tag{50}$$

Thus, in the diffusion-free regime, the spectrum grows, broadens, and drifts toward smaller spatial scales. Because the width in $\ln k$ also grows in time, the total magnetic energy integrated over k grows faster than a single mode, namely $\int M(k,t)\, \mathrm{d}k \propto e^{2\gamma t}$.

3.2.3 Resistively truncated regime (what most people measure as a kinematic dynamo)

At the end of the diffusion-free regime, the spectrum reaches the resistive scale k_η, beyond which magnetic diffusion can no longer be neglected. Once the resistive term is restored, the Kazantsev equation no longer describes unbounded diffusion toward large k. Instead, one looks for separable solutions of the form $M(k,t) = e^{\lambda\gamma t}\phi(k)$, which turns the PDE into an eigenvalue problem for the spatial profile $\phi(k)$. The resistive cutoff supplies the large-k boundary condition that selects a discrete growing eigenmode. In this regime, the long-time asymptotic form of the spectrum is

$$M(k,t) \propto \left(\frac{k}{k_\eta}\right)^{3/2} K_0\left(\frac{k}{k_\eta}\right) e^{\lambda\gamma t}, \tag{51}$$

where K_0 is a modified Bessel function of the second kind and

$$k_\eta = \sqrt{\frac{\gamma}{10\eta}} \sim \mathrm{Pm}^{1/2} k_\nu, \tag{52}$$

and λ is a dimensionless growth-rate prefactor. The spectrum therefore retains its $k^{3/2}$ behavior on scales $k_\nu \ll k \ll k_\eta$, but is now resistively truncated: it turns over near the resistive scale and decays exponentially for $k \gtrsim k_\eta$, because $K_0(x) \sim e^{-x}/\sqrt{x}$ for large x. In contrast to the diffusion-free regime, where the spectrum continually broadens in time, the presence of the resistive cutoff selects a fixed spectral shape corresponding to an eigenmode of the dynamo operator. The time dependence is then carried almost entirely by the overall factor $e^{\lambda\gamma t}$, while the shape $\phi(k)$ remains approximately stationary. As a result, the number of dynamically active modes is approximately fixed, and the total magnetic energy grows at essentially the same rate as each mode, namely $\sim e^{3\gamma t/4}$. By construction, this model describes random amplification of magnetic energy at small scales, but it does not produce coherent magnetic structure on scales larger than the forcing scale. This limitation motivates the development of large-scale dynamo theory.

3.3 Comments

A useful point of contact with simulations is provided by the series of papers by Neco Kriel and collaborators, (Kriel et al., 2022, 2025a,b, including aspects of the nonlinear regime in the most recent work). The scaling $M(k) \propto k^{3/2}$ is found in many numerical studies. A less frequently emphasized consequence is that the predicted spectrum is highly sensitive to the viscous and resistive scales of the plasma model. In most global, and even many local, simulations those dissipation scales are numerical rather than physical, so the peak of the magnetic spectrum can be substantially nonphysical even when the inertial-range prediction is qualitatively correct. That same sensitivity is also why the Kazantsev model is attractive as a sub-grid guide for unresolved magnetic structure.

Most of my deep understanding, and indeed, most of the insights, come from reading in-detail Schekochihin et al. (2001); Schekochihin et al. (2002, 2004) and Rincon (2019), which took roughly my entire Ph.D to properly digest.

4 Large-scale dynamo

4.1 Mean-field induction equation and the EMF

The Kazantsev model explains how random stretching amplifies magnetic energy, but it does not by itself generate coherent magnetic structure on scales larger than the forcing scale. To describe large-scale organization, we decompose the fields into mean and fluctuating parts,

$$\boldsymbol{B} = \overline{\boldsymbol{B}} + \delta\boldsymbol{b}, \qquad \boldsymbol{u} = \overline{\boldsymbol{U}} + \delta\boldsymbol{u}, \qquad \overline{\delta\boldsymbol{b}} = 0, \qquad \overline{\delta\boldsymbol{u}} = 0. \tag{53}$$

We also assume that the averaging operator satisfies the Reynolds rules and commutes with ∂_t and $\boldsymbol{\nabla}$.

Averaging Equation 1 gives the mean-field induction equation

$$\partial_t \overline{\boldsymbol{B}} = \boldsymbol{\nabla} \times \left(\overline{\boldsymbol{U}} \times \overline{\boldsymbol{B}} + \overline{\boldsymbol{\mathcal{E}}}\right) + \frac{1}{\mathrm{Rm}} \nabla^2 \overline{\boldsymbol{B}}, \tag{54}$$

where the electromotive force (EMF) is

$$\overline{\boldsymbol{\mathcal{E}}} \equiv \overline{\delta\boldsymbol{u} \times \delta\boldsymbol{b}}. \tag{55}$$

The EMF is the closure problem of mean-field electrodynamics: it couples the small-scale fluctuations to the mean magnetic field.

Using Equation 3 and $\boldsymbol{\nabla} \cdot \overline{\boldsymbol{B}} = 0$, Equation 54 can first be written as

$$\partial_t \overline{\boldsymbol{B}} = (\overline{\boldsymbol{B}} \cdot \boldsymbol{\nabla})\overline{\boldsymbol{U}} - (\overline{\boldsymbol{U}} \cdot \boldsymbol{\nabla})\overline{\boldsymbol{B}} - \overline{\boldsymbol{B}}(\boldsymbol{\nabla} \cdot \overline{\boldsymbol{U}}) + \boldsymbol{\nabla} \times \overline{\boldsymbol{\mathcal{E}}} + \frac{1}{\mathrm{Rm}} \nabla^2 \overline{\boldsymbol{B}}, \tag{56}$$

and then rearranged into material-derivative form,

$$\left(\partial_t + \overline{\boldsymbol{U}} \cdot \boldsymbol{\nabla}\right) \overline{\boldsymbol{B}} = (\overline{\boldsymbol{B}} \cdot \boldsymbol{\nabla})\overline{\boldsymbol{U}} - \overline{\boldsymbol{B}}(\boldsymbol{\nabla} \cdot \overline{\boldsymbol{U}}) + \boldsymbol{\nabla} \times \overline{\boldsymbol{\mathcal{E}}} + \frac{1}{\mathrm{Rm}} \nabla^2 \overline{\boldsymbol{B}}. \tag{57}$$

Dotting this with $\overline{\boldsymbol{B}}$ gives the local large-scale magnetic-energy budget,

$$\frac{1}{2}\left(\partial_t + \overline{\boldsymbol{U}}\cdot\boldsymbol{\nabla}\right)|\overline{\boldsymbol{B}}|^2 = \overline{B}_i\overline{B}_j\partial_j\overline{U}_i - |\overline{\boldsymbol{B}}|^2(\boldsymbol{\nabla}\cdot\overline{\boldsymbol{U}}) + \overline{\boldsymbol{B}}\cdot(\boldsymbol{\nabla}\times\overline{\boldsymbol{\mathcal{E}}}) + \frac{1}{\mathrm{Rm}}\overline{\boldsymbol{B}}\cdot\nabla^2\overline{\boldsymbol{B}}. \tag{58}$$

For periodic boundaries, integration by parts yields

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\langle|\overline{\boldsymbol{B}}|^2\rangle = \left\langle \overline{B}_i\overline{B}_j\partial_j\overline{U}_i - \frac{1}{2}|\overline{\boldsymbol{B}}|^2(\boldsymbol{\nabla}\cdot\overline{\boldsymbol{U}})\right\rangle + \langle\overline{\boldsymbol{\mathcal{E}}}\cdot(\boldsymbol{\nabla}\times\overline{\boldsymbol{B}})\rangle - \frac{1}{\mathrm{Rm}}\langle|\boldsymbol{\nabla}\times\overline{\boldsymbol{B}}|^2\rangle, \tag{59}$$

and therefore

$$\langle\overline{\boldsymbol{B}}\cdot(\boldsymbol{\nabla}\times\overline{\boldsymbol{\mathcal{E}}})\rangle = \langle\overline{\boldsymbol{\mathcal{E}}}\cdot(\boldsymbol{\nabla}\times\overline{\boldsymbol{B}})\rangle = \mu_0\langle\overline{\boldsymbol{\mathcal{E}}}\cdot\overline{\boldsymbol{J}}\rangle, \qquad \overline{\boldsymbol{J}} \equiv \frac{1}{\mu_0}\boldsymbol{\nabla}\times\overline{\boldsymbol{B}}. \tag{60}$$

The sign is positive in a periodic domain because $\boldsymbol{\nabla}\cdot(\overline{\boldsymbol{\mathcal{E}}}\times\overline{\boldsymbol{B}}) = \overline{\boldsymbol{B}}\cdot(\boldsymbol{\nabla}\times\overline{\boldsymbol{\mathcal{E}}}) - \overline{\boldsymbol{\mathcal{E}}}\cdot(\boldsymbol{\nabla}\times\overline{\boldsymbol{B}})$.

4.2 Exact fluctuation equation and the FOSA closure

To determine the EMF, subtract Equation 54 from the full induction equation. After inserting Equation 53, the unaveraged induction equation becomes

$$\partial_t(\overline{\boldsymbol{B}} + \delta\boldsymbol{b}) = \boldsymbol{\nabla}\times\left(\overline{\boldsymbol{U}}\times\overline{\boldsymbol{B}} + \overline{\boldsymbol{U}}\times\delta\boldsymbol{b} + \delta\boldsymbol{u}\times\overline{\boldsymbol{B}} + \delta\boldsymbol{u}\times\delta\boldsymbol{b}\right) + \frac{1}{\mathrm{Rm}}\nabla^2(\overline{\boldsymbol{B}} + \delta\boldsymbol{b}). \tag{61}$$

The mean-field equation removes the $\overline{\boldsymbol{U}}\times\overline{\boldsymbol{B}}$ contribution and replaces $\delta\boldsymbol{u}\times\delta\boldsymbol{b}$ by its average. Subtracting the mean equation therefore gives the exact evolution equation for the fluctuating magnetic field,

$$\partial_t\delta\boldsymbol{b} = \boldsymbol{\nabla}\times\left(\overline{\boldsymbol{U}}\times\delta\boldsymbol{b} + \delta\boldsymbol{u}\times\overline{\boldsymbol{B}} + \delta\boldsymbol{u}\times\delta\boldsymbol{b} - \overline{\boldsymbol{\mathcal{E}}}\right) + \frac{1}{\mathrm{Rm}}\nabla^2\delta\boldsymbol{b}. \tag{62}$$

Kinematic evolution alone is not enough to make this equation linear: "kinematic" only means that the velocity field is prescribed and receives no Lorentz-force backreaction. The term $\boldsymbol{\nabla}\times(\delta\boldsymbol{u}\times\delta\boldsymbol{b} - \overline{\boldsymbol{\mathcal{E}}})$ still couples magnetic fluctuations to themselves and is therefore nonlinear in the fluctuating magnetic field.

Suppose now that the mean flow is spatially uniform, $\overline{\boldsymbol{U}} = \text{const}$. In the comoving Galilean frame, the constant advection term can be removed. If we then adopt a first-order smoothing approximation (FOSA), also called a quasilinear closure, we make one additional assumption beyond kinematicity: we neglect the nonlinear term $\boldsymbol{\nabla}\times(\delta\boldsymbol{u}\times\delta\boldsymbol{b} - \overline{\boldsymbol{\mathcal{E}}})$ relative to the source term $\boldsymbol{\nabla}\times(\delta\boldsymbol{u}\times\overline{\boldsymbol{B}})$. The fluctuation equation reduces to

$$\partial_t\delta\boldsymbol{b} - \frac{1}{\mathrm{Rm}}\nabla^2\delta\boldsymbol{b} = \boldsymbol{\nabla}\times(\delta\boldsymbol{u}\times\overline{\boldsymbol{B}}). \tag{63}$$

This is the first point at which the linear-response assumption actually enters the derivation.

Let $\boldsymbol{r} = \boldsymbol{x} - \boldsymbol{x}'$ and $\tau = t - t' > 0$. The causal Green's function for the linear operator in Equation 63 gives

$$\delta b_k(\boldsymbol{x}, t) = \int_0^\infty \mathrm{d}\tau \int \mathrm{d}^3r\, G_{km}(\boldsymbol{r}, \tau)\Big[\partial'_s\big(\delta u_m\overline{B}_s\big) - \partial'_r\big(\delta u_r\overline{B}_m\big)\Big]_{(\boldsymbol{x}', t')}, \tag{64}$$

where $\boldsymbol{x}' = \boldsymbol{x} - \boldsymbol{r}$ and $t' = t - \tau$. Substituting Equation 64 into Equation 55 shows that the EMF is a causal linear functional of the mean field:

$$\overline{\boldsymbol{\mathcal{E}}}_i(\boldsymbol{x}, t) = \int_0^\infty \mathrm{d}\tau \int \mathrm{d}^3r\, \Big[K_{ij}(\boldsymbol{r}, \tau)\,\overline{B}_j(\boldsymbol{x} - \boldsymbol{r}, t - \tau) + L_{ij\ell}(\boldsymbol{r}, \tau)\,\partial_\ell\overline{B}_j(\boldsymbol{x} - \boldsymbol{r}, t - \tau)\Big]. \tag{65}$$

The kernels are built from the Green's function and second-order velocity correlators, schematically $K \sim \overline{\delta\boldsymbol{u}\, G\,(\boldsymbol{\nabla}\delta\boldsymbol{u})}$.

4.3 Scale expansion and isotropic reduction

If the turbulence is statistically homogeneous, then the kernels depend only on the separations $(\boldsymbol{r}, \tau)$. If their support is concentrated on a correlation length ℓ_c and correlation time τ_c, while the mean field varies on much larger scales L_M and T_M, with $\ell_c/L_\mathrm{M} \ll 1$ and $\tau_c/T_\mathrm{M} \ll 1$, then $\overline{\boldsymbol{B}}(\boldsymbol{x}-\boldsymbol{r}, t-\tau)$ can be Taylor expanded about $(\boldsymbol{x}, t)$:

$$\overline{B}_j(\boldsymbol{x}-\boldsymbol{r}, t-\tau) = \overline{B}_j(\boldsymbol{x}, t) - r_\ell \partial_\ell \overline{B}_j(\boldsymbol{x}, t) - \tau \partial_t \overline{B}_j(\boldsymbol{x}, t) + \frac{1}{2} r_\ell r_m \partial_\ell \partial_m \overline{B}_j(\boldsymbol{x}, t) + \cdots . \tag{66}$$

Substituting Equation 66 into Equation 65 yields the local gradient expansion

$$\overline{\boldsymbol{\mathcal{E}}}_i = a_{ij}\,\overline{B}_j + b_{ij\ell}\,\partial_\ell \overline{B}_j + c_{ij}\,\partial_t \overline{B}_j + d_{ij\ell m}\,\partial_\ell \partial_m \overline{B}_j + \cdots , \tag{67}$$

where

$$\begin{aligned}
a_{ij} &= \int_0^\infty \mathrm{d}\tau \int \mathrm{d}^3 r\; K_{ij}(\boldsymbol{r}, \tau), \\
b_{ij\ell} &= \int_0^\infty \mathrm{d}\tau \int \mathrm{d}^3 r\; L_{ij\ell}(\boldsymbol{r}, \tau) - \int_0^\infty \mathrm{d}\tau \int \mathrm{d}^3 r\; r_\ell K_{ij}(\boldsymbol{r}, \tau), \\
c_{ij} &= -\int_0^\infty \mathrm{d}\tau \int \mathrm{d}^3 r\; \tau K_{ij}(\boldsymbol{r}, \tau), \\
d_{ij\ell m} &= \frac{1}{2} \int_0^\infty \mathrm{d}\tau \int \mathrm{d}^3 r\; r_\ell r_m K_{ij}(\boldsymbol{r}, \tau).
\end{aligned} \tag{68}$$

Under statistical homogeneity and isotropy, the tensors reduce to their isotropic forms:

$$a_{ij} = \alpha\,\delta_{ij}, \qquad b_{ij\ell} = \beta\,\epsilon_{ij\ell}, \qquad c_{ij} = \gamma_t\,\delta_{ij}. \tag{69}$$

The leading-order closure therefore becomes

$$\overline{\boldsymbol{\mathcal{E}}} = \alpha\,\overline{\boldsymbol{B}} - \beta\,\boldsymbol{\nabla} \times \overline{\boldsymbol{B}} + \gamma_t\,\partial_t \overline{\boldsymbol{B}} + \cdots , \tag{70}$$

where α is a pseudoscalar and β acts as a turbulent magnetic diffusivity. In this isotropic limit, Equation 70 is the simplest mean-field closure.

4.4 Derivation of α and β under FOSA

To derive explicit expressions for the scalar coefficients, assume incompressibility, $\boldsymbol{\nabla} \cdot \delta\boldsymbol{u} = 0$, and a short correlation time τ_c so that the diffusive evolution of $\delta\boldsymbol{b}$ can be neglected over one correlation interval. Then Equation 63 gives

$$\delta\boldsymbol{b} \simeq \tau_c \left[(\overline{\boldsymbol{B}} \cdot \boldsymbol{\nabla})\delta\boldsymbol{u} - (\delta\boldsymbol{u} \cdot \boldsymbol{\nabla})\overline{\boldsymbol{B}}\right]. \tag{71}$$

Substituting Equation 71 into Equation 55 yields

$$\overline{\boldsymbol{\mathcal{E}}}_i = \tau_c \epsilon_{ijk} \left[\overline{\delta u_j \partial_\ell \delta u_k}\,\overline{B}_\ell - \overline{\delta u_j \delta u_\ell}\,\partial_\ell \overline{B}_k\right]. \tag{72}$$

Comparing Equation 72 with Equation 67 identifies

$$\begin{aligned}
a_{i\ell} &= \tau_c\,\epsilon_{ijk}\,\overline{\delta u_j \partial_\ell \delta u_k}, \\
b_{ik\ell} &= -\tau_c\,\epsilon_{ijk}\,\overline{\delta u_j \delta u_\ell}.
\end{aligned} \tag{73}$$

Isotropy then implies

$$\begin{aligned}
\overline{\delta u_j \partial_\ell \delta u_k} &= \frac{1}{6}\overline{\delta\boldsymbol{u} \cdot (\boldsymbol{\nabla} \times \delta\boldsymbol{u})}\,\epsilon_{j\ell k}, \\
\overline{\delta u_j \delta u_\ell} &= \frac{1}{3}\overline{\delta\boldsymbol{u}^2}\,\delta_{j\ell},
\end{aligned} \tag{74}$$

and the required contractions can be shown explicitly. For the α term,

$$a_{i\ell} = \tau_c \epsilon_{ijk} \left(\frac{1}{6} \overline{\delta \boldsymbol{u} \cdot (\boldsymbol{\nabla} \times \delta \boldsymbol{u})}\, \epsilon_{j\ell k} \right) = -\frac{\tau_c}{3} \overline{\delta \boldsymbol{u} \cdot (\boldsymbol{\nabla} \times \delta \boldsymbol{u})}\, \delta_{i\ell}, \tag{75}$$

because $\epsilon_{ijk}\epsilon_{j\ell k} = -2\delta_{i\ell}$. Likewise, for the β term,

$$b_{ik\ell} = -\tau_c \epsilon_{ijk} \left(\frac{1}{3} \overline{\delta \boldsymbol{u}^2}\, \delta_{j\ell} \right) = \frac{\tau_c}{3} \overline{\delta \boldsymbol{u}^2}\, \epsilon_{ik\ell}. \tag{76}$$

Thus the familiar FOSA estimates are

$$\begin{aligned} \alpha &= -\frac{\tau_c}{3} \overline{\delta \boldsymbol{u} \cdot (\boldsymbol{\nabla} \times \delta \boldsymbol{u})}, \\ \beta &= \frac{\tau_c}{3} \overline{\delta \boldsymbol{u}^2}. \end{aligned} \tag{77}$$

With spatially uniform α and β, the mean-field induction equation becomes

$$\partial_t \overline{\boldsymbol{B}} = \boldsymbol{\nabla} \times (\overline{\boldsymbol{U}} \times \overline{\boldsymbol{B}}) + \alpha\, \boldsymbol{\nabla} \times \overline{\boldsymbol{B}} + \left(\frac{1}{\mathrm{Rm}} + \beta \right) \nabla^2 \overline{\boldsymbol{B}}. \tag{78}$$

The α term can drive ordered large-scale field growth, while β enhances the effective magnetic diffusion. It is convenient to define the total large-scale diffusivity

$$\eta_{\mathrm{T}} \equiv \frac{1}{\mathrm{Rm}} + \beta, \tag{79}$$

so that the model can be compared directly with the standard dimensional notation in which $\eta_{\mathrm{T}} = \eta + \beta$.

4.5 Why inhomogeneous mean flows are harder

The constant-flow derivation above relies on translational invariance. If the mean flow varies in space, then even the linearized fluctuation equation contains spatially varying coefficients:

$$\partial_t \delta \boldsymbol{b} = \boldsymbol{\nabla} \times \left(\overline{\boldsymbol{U}}(\boldsymbol{x}) \times \delta \boldsymbol{b} + \delta \boldsymbol{u} \times \overline{\boldsymbol{B}} \right) + \frac{1}{\mathrm{Rm}} \nabla^2 \delta \boldsymbol{b} + \boldsymbol{\nabla} \times \left(\delta \boldsymbol{u} \times \delta \boldsymbol{b} - \overline{\boldsymbol{\mathcal{E}}} \right). \tag{80}$$

Even after a FOSA-style linearization, the corresponding Green's function depends on the two positions separately, $G = G(\boldsymbol{x}, t; \boldsymbol{x}', t')$, rather than only on the separations $(\boldsymbol{r}, \tau)$. Once translational invariance is lost, Equation 66 is no longer automatically justified, so additional approximations are needed. This is the point at which the simple homogeneous closure ceases to be reliable.

4.6 Inferring transport coefficients from data

More generally, one may write

$$\overline{\mathcal{E}}_i = \alpha_{ij}\, \overline{B}_j - \beta_{ij}\, \overline{J}_j, \qquad \overline{J}_j \equiv (\nabla \times \overline{\boldsymbol{B}})_j. \tag{81}$$

This expresses the EMF as a linear function of the mean field and its gradients. Multiplying Equation 81 by $\overline{B}_m$ and $\overline{J}_m$ and averaging gives

$$\begin{aligned} \left\langle \overline{\mathcal{E}}_i \overline{B}_m \right\rangle &= \alpha_{ij} \left\langle \overline{B}_j \overline{B}_m \right\rangle - \beta_{ij} \left\langle \overline{J}_j \overline{B}_m \right\rangle, \\ \left\langle \overline{\mathcal{E}}_i \overline{J}_m \right\rangle &= \alpha_{ij} \left\langle \overline{B}_j \overline{J}_m \right\rangle - \beta_{ij} \left\langle \overline{J}_j \overline{J}_m \right\rangle. \end{aligned} \tag{82}$$

For each i, this is a linear system for the tensor components α_{ij} and β_{ij}. In practice, the coefficients are obtained with least-squares fits to simulation data, making the closure problem a regression problem once suitable mean fields have been defined.

5 Some simple large-scale dynamos that may or may not exist

5.1 The α^2 dynamo

For vanishing mean flow, $\overline{\boldsymbol{U}} = 0$, and spatially uniform transport coefficients, the mean-field induction equation reduces to

$$\frac{\partial \overline{\boldsymbol{B}}}{\partial t} = \alpha \, \boldsymbol{\nabla} \times \overline{\boldsymbol{B}} + \eta_{\mathrm{T}} \nabla^2 \overline{\boldsymbol{B}}. \tag{83}$$

Seeking plane-wave solutions of the form

$$\overline{\boldsymbol{B}}(\boldsymbol{x}, t) = \mathrm{Re}\left[\hat{\boldsymbol{B}}\, e^{\lambda t + i\boldsymbol{k}\cdot\boldsymbol{x}}\right], \qquad \boldsymbol{k} \cdot \hat{\boldsymbol{B}} = 0, \tag{84}$$

one obtains

$$\lambda \hat{\boldsymbol{B}} = \alpha \, i\boldsymbol{k} \times \hat{\boldsymbol{B}} - \eta_{\mathrm{T}} k^2 \hat{\boldsymbol{B}}. \tag{85}$$

Choosing $\boldsymbol{k} = k\hat{\boldsymbol{z}}$ and $\hat{\boldsymbol{B}} = (\hat{B}_x, \hat{B}_y, 0)$ gives a 2×2 system for $(\hat{B}_x, \hat{B}_y)$. Equivalently, the helical combinations $\hat{B}_{\pm} = \hat{B}_x \pm i\hat{B}_y$ diagonalize the operator, giving the eigenvalues

$$\lambda_{\pm} = \pm|\alpha|k - \eta_{\mathrm{T}} k^2. \tag{86}$$

Thus, the α^2 dynamo grows whenever

$$|\alpha|k > \eta_{\mathrm{T}} k^2. \tag{87}$$

The fastest-growing mode occurs at

$$k_{\max} = \frac{|\alpha|}{2\eta_{\mathrm{T}}}, \tag{88}$$

with growth rate

$$\lambda_{\max} = \frac{\alpha^2}{4\eta_{\mathrm{T}}}. \tag{89}$$

5.2 The $\alpha\Omega$ dynamo

For a shearing mean flow,

$$\overline{\boldsymbol{U}} = Sx\,\hat{\boldsymbol{y}}, \tag{90}$$

the mean-field equation contains an additional stretching term due to differential rotation. Assuming the mean field depends only on z and writing

$$\overline{\boldsymbol{B}} = B_x(z,t)\,\hat{\boldsymbol{x}} + B_y(z,t)\,\hat{\boldsymbol{y}}, \tag{91}$$

the mean-field equations become

$$\partial_t B_x = -\alpha\, \partial_z B_y + \eta_{\mathrm{T}} \partial_z^2 B_x, \tag{92}$$

$$\partial_t B_y = S B_x + \alpha\, \partial_z B_x + \eta_{\mathrm{T}} \partial_z^2 B_y. \tag{93}$$

Looking for solutions proportional to $e^{\lambda t + ikz}$ yields

$$\lambda B_x = -i\alpha k\, B_y - \eta_{\mathrm{T}} k^2 B_x, \tag{94}$$

$$\lambda B_y = S B_x + i\alpha k\, B_x - \eta_{\mathrm{T}} k^2 B_y. \tag{95}$$

Eliminating (B_x, B_y) gives the dispersion relation

$$(\lambda + \eta_{\mathrm{T}} k^2)^2 = \alpha^2 k^2 - i\alpha S k. \tag{96}$$

In the strong-shear limit, $|S| \gg |\alpha k|$, the $\alpha^2 k^2$ term is subdominant, so the growth rate scales as

$$\lambda \sim \sqrt{|\alpha S k|} - \eta_{\mathrm{T}} k^2, \tag{97}$$

up to order-unity factors set by the phase of the square root. This shows that shear and kinetic helicity can cooperate to produce a more efficient large-scale dynamo than the α^2 mechanism alone.

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